

Recent Developments in Dynamical Supersymmetry Breaking

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Some formal aspects of supersymmetry breaking are reviewed. The classic “requirements” for supersymmetry breaking include chiral matter, a dynamical superpotential, and a classical superpotential which completely lifts the moduli space. These “requirements” may be evaded in theories with large matter representations. The mechanisms of supersymmetry breaking by confinement and quantum deformation of the moduli space are explained, with emphasis on the importance of identifying the relevant degrees of freedom in the ground state. Supersymmetry breaking and the behavior of the Witten index in non-chiral theories are discussed. The quantum removal of directions which are classically unlifted is also illustrated. Examples of product gauge group theories that admit dual descriptions of the non-supersymmetric ground state are also presented.

1 Introduction

If supersymmetry is a symmetry of nature, it is clearly broken in the ground state. Understanding the mechanisms by which supersymmetry may be spontaneously broken is crucial in deciphering what types of supersymmetric theories may describe nature. Recently there has been considerable progress in more formal aspects of supersymmetry breaking. Most of these have followed from the recent revolution in our understanding of strong non-perturbative dynamics in $N = 1$ supersymmetric gauge theories.¹ In this paper I review recent developments in understanding the formal requirements for, and mechanisms of, supersymmetry breaking.²

In the next section the motivation for studying dynamical supersymmetry breaking by non-perturbative gauge dynamics is reviewed. The importance of identifying the relevant degrees of freedom in the ground state is emphasized. The classic “requirements” for dynamical supersymmetry breaking are reviewed, and the general means by which these may be evaded are discussed. An example of breaking by the mechanism of a dynamical superpotential over a classical moduli space is also given. In §3 the mechanisms of supersymmetry breaking by confinement and quantum deformation of the moduli space are reviewed. Supersymmetry is broken in these cases even though the exact non-perturbative superpotential vanishes in the absence of a tree level super-

potential. Non-chiral theories which break supersymmetry are discussed in §4. The manner in which the Witten index can vanish in such theories is explained. Finally, in §5 product gauge group theories are given which admit two dual descriptions of the non-supersymmetric ground state. As a function of the parameters of the theory, these dual descriptions do not have overlapping regions of applicability.

2 Dynamical Supersymmetry Breaking

The Witten index for a supersymmetric theory, $\text{Tr}(-1)^F$, counts the number of supersymmetric ground states.³ This index is not modified at any order in perturbation theory. So supersymmetry is either broken classically at tree level, or by non-perturbative dynamics. In nature, if supersymmetry has any relevance to the hierarchy problem, the supersymmetry breaking scale is certainly well below the Planck scale. This would not be the case with tree level breaking in the absence of very small parameters. However, in theories which exhibit dimensional transmutation (such as asymptotically free non-Abelian gauge theories) the dynamical scale can naturally be hierarchically smaller than any fundamental scale.

Since the scale for supersymmetry breaking is well below the Planck scale, one can hope that the breaking admits an effective field theory description in the rigid supersymmetric limit. This is not necessarily the case if some fields have Planck scale expectation values, in which case supergravity effects can become important. In addition, in the global limit we give up any hope of understanding the smallness of the cosmological constant. However, even in the local context, it is non-perturbative field theory dynamics which is believed to break supersymmetry. For all these reasons it is useful to study supersymmetry breaking in global non-Abelian gauge theories.

2.1 Identifying the Relevant Degrees of Freedom

Since the Hamiltonian is related to the supercharge, $H = \frac{1}{2}\{Q, Q^\dagger\}$ for translationally invariant states, supersymmetry breaking implies non-zero vacuum energy in the rigid limit, $Q^\alpha|0\rangle \neq 0 \Rightarrow H|0\rangle \neq 0$. Stable supersymmetry breaking therefore requires a ground state with non-zero energy. The potential is an incoherent sum of D - and F -terms

$$V = \frac{1}{2}g^2 D^a D^a + F_\phi K^{\phi\phi^*} F_\phi^* \quad (1)$$

where $D^a = K_\phi T^a \phi$ and $F_\phi = \partial_\phi W$. Note that the D - and F -terms can not interfere. In order to check for a supersymmetric ground state with vanishing

energy, it is therefore sufficient to consider the F -terms on the subspace of the full field space for which $D^a = 0$. The question of supersymmetry breaking may then be reduced to whether or not all the auxiliary equations of motion can be simultaneously satisfied with $F = 0$ on the D -flat moduli space.

The moduli space of D -flat directions is special for a number of reasons. First, the gauge symmetry generally exhibits a definite pattern of breaking, $G \rightarrow H$, on the moduli space. Second, the microscopic fields of the theory break up into heavy fields which are eaten by the super-Higgs mechanism, plus light fields which parameterize $D^a = 0$. These parameters are related to the ring of gauge invariant chiral operators, $\{X_i\}$. It has recently been shown that the classical moduli space of D -flat directions is in one to one correspondence with the classical chiral ring.⁴ This considerably simplifies the analysis of gauge theories since the set of gauge invariant operators plus relations is typically much easier to find than an explicit parameterization of the D -flat directions.

It is important to note that with broken supersymmetry the fields need not sit precisely on a D -flat direction in the ground state. However, physically for small Yukawa couplings, $\lambda \ll g$, the excitations taking the system away from $D^a = 0$ are typically very heavy and can be integrated out, leaving the light moduli as effective degrees of freedom.

In a theory with a supersymmetric ground state, much of the power in analyzing strong dynamics comes from the holomorphy and $U(1)_R$ symmetry of the superpotential. These constraints, plus global symmetries and limits are often enough to uniquely fix the full non-perturbative superpotential. However, in theories in which supersymmetry is broken, a quantitative description of the ground state and excited spectrum also requires knowledge of the Kähler potential. Even though the Kähler potential is not protected by any non-renormalization theorems, its leading behavior can be determined in two important limits. The first is for large expectation values along the D -flat directions. In this limit the gauge group is highly Higgsed, and weak at the scale of the expectation values. The relevant Kähler potential is then just the classical canonical one for the microscopic fields, ϕ , projected onto the D -flat directions

$$K = \phi^\dagger \phi|_{D^a=0} \quad \phi \gg \Lambda \quad (2)$$

The Kähler potential (2) receives small, calculable, quantum corrections in this limit. This is the standard limit in which all theories of global supersymmetry breaking have been analyzed in the past.

For small expectation values, $\phi \ll \Lambda$, the gauge group is strongly coupled, and the Kähler potential in general receives large uncalculable corrections. However, the dominant contribution to the Kähler is sometimes calculable at strong coupling. This occurs at points of enhanced global symmetry. Clas-

sically the gauge group is typically enhanced at such points, and the classical Kähler potential for the moduli becomes singular. Quantum mechanically however, non-perturbative degrees of freedom, φ , (including confined and magnetic chiral multiplets and magnetic gauge multiplets) often become massless at enhanced symmetry points in order to saturate global anomalies. In terms of these degrees of freedom the Kähler potential is smooth. If the quantum theory at the enhanced symmetry point is infrared free, then in terms of the non-perturbative degrees of freedom

$$K(\varphi = 0) = \varphi^\dagger \varphi \quad (3)$$

This non-perturbative information about the Kähler potential is crucial in analyzing non-supersymmetric ground states at strong coupling. For the mechanisms of supersymmetry breaking discussed in §3 which do not rely on a dynamically generated superpotential, in a certain sense the information that supersymmetry is in fact broken is contained in the Kähler potential. At strong coupling it is therefore very important to identify the relevant non-perturbative degrees of freedom.

2.2 Classic Requirements for Supersymmetry Breaking

As discussed in §2.1 supersymmetry is broken if all the auxiliary equations of motion can not be simultaneously satisfied on the moduli space. While this requirement may seem rather innocuous, in the past it was believed that building theories with stable dynamical supersymmetry breaking was rather difficult. This difficulty followed from a number of conditions which were thought to be requirements. The classic “requirements” for supersymmetry breaking are

- Chiral matter
- A non-perturbative superpotential generated over the classical moduli space
- A tree level superpotential which completely lifts the classical moduli space
- $U(1)_R$ symmetry

The reasons for these “requirements” are reviewed below.

The Witten index, $\text{Tr}(-1)^F$, counts the number of supersymmetric ground states, and necessarily vanishes if supersymmetry is broken. Non-zero energy states do not contribute to the index.³ If supersymmetry were broken in a

theory which contained vector matter, it would then appear that completely integrating out this sector of the theory can not change the index, or the conclusion that supersymmetry is broken. Supersymmetry breaking by gauge dynamics therefore seems to require chiral representations. The loop hole in this argument and behavior of the Witten index in non-chiral theories is discussed in §4.

Supersymmetry breaking requires a non-zero potential. Non-perturbative gauge dynamics (specifically gaugino condensation or a single instanton) are known to generate a non-zero potential over classical moduli spaces. The existence of such non-perturbative dynamics therefore seems a reasonable requirement in order to break supersymmetry. This puts a strict constraint on the matter content however. To see this, it is useful to write the full non-perturbative superpotential as $W_{NP} = f(\Lambda, X_i)$ where f is a holomorphic function and X_i are the (classical) gauge invariant chiral operators. It is convenient to assign vanishing R -charge to all chiral superfields, $R[X_i] = 0$. In this case the R -charge of the gauginos and matter fermions are $R[\lambda] = 1$ and $R[\psi] = -1$. This R -charge assignment is in general anomalous, $\partial_\mu j_R^\mu = (C/16\pi^2) F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$, where $C \equiv C_\lambda - \sum_\psi C_\psi$ with $\text{Tr}(T^a T^b) = C\delta^{ab}$. Now the dynamical scale is defined as the pole of the holomorphic (one-loop) gauge β -function

$$\left(\frac{\Lambda}{\mu}\right)^b = e^{-8\pi^2/g^2(\mu)+i\theta} \quad (4)$$

where b is the β -function coefficient $b = 3C_\lambda - 2\sum C_\psi$. Because of the anomaly, the θ term transforms under a $U(1)_R$ transformation as $\theta \rightarrow \theta + 2C\alpha$. The dynamical scale (4) then inherits an R -charge from the anomaly, $R[\Lambda^b] = 2C$. In the limit $\Lambda \rightarrow 0$, the non-perturbative superpotential should vanish or reproduce the classical moduli space. For a dynamical superpotential generated by gaugino condensation or an instanton this implies Λ must appear raised to a positive power in W_{NP} . Since $R[W] = 2$ and $R[\Lambda^b] = 2C$, this implies lifting of the moduli space requires $\sum_\psi C_\psi < C_\lambda$ for an asymptotically free theory. Large matter representations therefore do not generate a dynamical superpotential, and were believed not to break supersymmetry. However, as discussed in §3, such theories can in fact break supersymmetry by mechanisms other than a dynamically generated superpotential.

The magnitude of a dynamical superpotential in the ground state depends on the expectation values on the moduli space. Subgroups of the gauge group which are Higgsed on the moduli space become weaker as the expectation values increase. If these subgroups generate a dynamical superpotential, then the dynamical scale and potential approach zero as the expectation values go to infinity. If such directions on the classical moduli space are unlifted by

a tree level superpotential the theory exhibits run away behavior to infinite expectation values with zero coupling and unbroken supersymmetry. Lifting all classical flat directions with a tree level superpotential therefore seems to be a reasonable requirement for supersymmetry breaking. However, it is possible that along certain directions in moduli space some fields gain mass from a tree level superpotential. The subgroups under which these fields transform become more confining and more strongly coupled along these directions if $b_L - b_H > 0$, where b_L and b_H are the β -function coefficients in the high energy theory and effective low energy theory with the heavy matter integrated out. In this case the dynamical potential can grow as the expectation values increase along these directions. The quantum removal of directions which are classically unlifted in theories which break supersymmetry is discussed in §4.

Supersymmetry breaking requires non-vanishing auxiliary expectation values on the moduli space, as discussed in the previous section. For n fields, the vanishing of the auxiliary equations of motion, $F_\phi = \partial_\phi W = 0$, amount to n equations in n unknowns. Generically this system has a solution, and supersymmetry is unbroken. However, if the theory possesses a $U(1)_R$ symmetry *and* a field ϕ_i carrying non-zero R -charge has an expectation value, it is possible to redefine the superpotential by⁵

$$W = \phi_i^{2/R_i} W(\phi_j / \phi_i^{R_j/R_i}) \quad (5)$$

The auxiliary equations of motion then become n equations in only $n - 1$ unknowns. Generically this system does not have a solution, and supersymmetry is broken. Note that a non- R -symmetry would reduce both the number of equations and unknowns.

While a spontaneously broken $U(1)_R$ symmetry generically implies broken supersymmetry, this is not a necessary condition.⁵ However, almost every known example of dynamical supersymmetry breaking has a $U(1)_R$ symmetry in some limit.

It is now apparent that the classic “requirements” which are sufficient, are certainly not necessary for supersymmetry breaking. The realization that supersymmetry may be broken without chiral matter, a dynamically generated superpotential, or a classical potential which completely lifts the moduli space has followed the recent improved understanding of strongly coupled supersymmetric theories. The mechanisms by which supersymmetry may be broken without these are detailed in §3 and §4.

2.3 The $SU(3) \times SU(2)$ Model

The simplest, and best studied, model which illustrates supersymmetry breaking by the classic mechanism of a dynamically generated superpotential is the $SU(3) \times SU(2)$ model of Affleck, Dine, and Seiberg.⁶ This theory satisfies all the classic requirements for supersymmetry breaking outlined in the previous section.

The matter content of the model is

$$\begin{array}{ccc}
 & & SU(3) \times SU(2) \\
 P & (\square, \square) & \\
 L & (1, \square) & \\
 \overline{Q}_i & (\overline{\square}, 1) & i = 1, 2
 \end{array} \tag{6}$$

This is just the one generation supersymmetric standard model without hypercharge, the positron, or Higgs bosons. Classically, there is a moduli space parameterized by three invariants: $Z = P^2 \overline{Q}_1 \overline{Q}_2$, $X_i = PL \overline{Q}_i$. The gauge group is completely broken at generic points on the moduli space. At tree level there is a single renormalizable coupling which can be added to the superpotential, $W_{tree} = \lambda X_1$. This superpotential leaves invariant non-anomalous accidental $U(1)_R$ and $U(1)$ flavor symmetries, and completely lifts the classical moduli space. Classically, there is a supersymmetric ground state at the origin, with the gauge symmetries unbroken.

In the quantum theory the exact non-perturbative superpotential over the classical moduli space is fixed by holomorphy and symmetries to be

$$W = \frac{\Lambda_3^7}{Z} + \lambda X_1 \tag{7}$$

where Λ_3 is the $SU(3)$ dynamical scale. The dynamical term gives a potential which grows at small expectation values, while the tree level terms gives a potential which grows at large expectation values. Since the model has a $U(1)_R$ symmetry, the ground state has non-zero vacuum energy and supersymmetry is broken. Parametrically, for $\lambda \ll 1$, the field expectation values and vacuum energy scale as $\phi \sim \Lambda_3 / \lambda^{1/7}$ and $V \sim |\lambda^2 (\Lambda_3 / \lambda^{1/7})^4| = |\lambda^{10/7} \Lambda_3^4|$.⁶

It is important to note that $U(1)_R$ symmetry plays a very important role in this model. It is very easy to construct models with both tree level and non-perturbative terms which stabilize the ground state. However, without a $U(1)_R$ symmetry there is generally an interference between various contributions to F -terms, leading to $F = 0$ at a finite expectation value with supersymmetry unbroken.

Notice that the exact superpotential (7) is independent of Λ_2 . While it follows from holomorphy and symmetries that the classical moduli space is unlifted by $SU(2)$ dynamics, it seems rather odd that in the limit $\Lambda_2 \gg \Lambda_3$, the $SU(2)$ dynamics would have no effect on the non-supersymmetric ground state. This was the original puzzle that led to the observation of supersymmetry breaking by the quantum deformation of the moduli space, as discussed in §3.2.

3 Supersymmetry Breaking Without a Dynamical Superpotential

Perhaps the most important of the classic requirements for supersymmetry breaking is a dynamical superpotential generated by gaugino condensation or an instanton over the classical moduli space. In the absence of a tree level superpotential these are the only non-perturbative effects which can lift the moduli space and generate a potential. However, many other types of non-perturbative dynamics are now understood. In the presence of a tree level superpotential these can in principle give rise to additional mechanisms for supersymmetry breaking. In this section the mechanisms of supersymmetry breaking by confinement and by the quantum deformation of the moduli space are reviewed.

3.1 Confinement

Asymptotically free gauge theories are necessarily strongly coupled at the origin of moduli space, at least in terms of the microscopic degrees of freedom. However, as discussed in §2.1, non-perturbative degrees of freedom often become massless at the origin of moduli space in order to saturate global anomalies, and represent the infrared degrees of freedom of the theory. If these non-perturbative degrees of freedom do not transform under any (magnetic) gauge symmetry, the infrared theory is a free Wess-Zumino model with confined chiral multiplets, and perhaps with superpotential couplings. A tree level superpotential in the microscopic theory in general leads to a modification of the effective superpotential in the confined theory. Even if the microscopic tree level superpotential has no linear terms, it is possible that the effective superpotential is linear in a confined field, thereby inducing a non-zero auxiliary component. Supersymmetry may then be broken by the O’Raifeartaigh mechanism in the confined theory. In the classical theory there is a supersymmetric ground state at the origin, while in the confined quantum theory the superpotential is not stationary at the origin and supersymmetry is broken. The mechanism for supersymmetry breaking is therefore confinement.

The simplest example of supersymmetry breaking by confinement is for the theory⁷

$$SU(2) \quad (8)$$

$$Q \quad \square \square \square$$

This theory has a single flat direction parameterized by $X = Q^4$. At the origin of moduli space, $X = 0$, the gauge invariant composite X has the correct global quantum numbers to saturate the global anomalies. It is therefore consistent to postulate that at the origin X is in fact a canonically normalized confined degree of freedom so that $K(X = 0) = X^\dagger X$.⁷ The lowest order term that can be added to the tree level superpotential is X itself, $W_{tree} = \gamma X$. In the classical theory this completely lifts the moduli space, leaving a supersymmetric vacuum at the origin. However, in the quantum theory, with X confined, the potential does not vanish at the origin, $V = \gamma^2 \Lambda_2^6$, and supersymmetry is broken.

Almost any theory which breaks supersymmetry by a dynamically generated superpotential can be deformed to one in which the relevant description is tree level O’Raifeartaigh breaking in terms of confined degrees of freedom. This is accomplished by integrating in enough light vector matter so that all the gauge groups confine.⁹

3.2 Quantum Deformation of Moduli Space

The patterns of spontaneous breaking for global or gauge symmetries on a quantum moduli space may differ from those on the classical moduli space. In this case the theory is said to have a quantum deformed moduli space. For example, at the origin of a classical moduli space, all fields have zero expectation value, and the global symmetries are unbroken. However, in the quantum theory some of the global symmetries can remain broken everywhere on the moduli space.⁸ Points which are part of the classical moduli space can therefore be removed by the quantum deformation. If tree level interactions give vanishing potential and auxiliary components only at points on the classical moduli space which are removed by quantum deformation, supersymmetry is broken in the quantum theory.⁹ The mechanism for supersymmetry breaking is therefore quantum deformation of the moduli space.

The mechanism of supersymmetry breaking by quantum deformation of the moduli space is actually contained within the $SU(3) \times SU(2)$ model discussed in §2.3. To see this consider the limit $\Lambda_2 \gg \Lambda_3$. In this limit the $SU(3)$ is weakly gauged at the scale Λ_2 . Treating the $SU(3)$ as a weakly gauged global symmetry, the $SU(2)$ then has $N_f = 2$ flavors (four fields) in

the fundamental representation, namely P and L . This theory has a quantum deformed moduli space.⁸ In terms of $SU(2)$ singlet moduli, it is described by $\hat{q} = PL/\Lambda_2 \in \mathbf{3}$ of $SU(3)$ and $\hat{\bar{q}} = P^2/\Lambda_2 \in \bar{\mathbf{3}}$ of $SU(3)$, subject to the constraint $\hat{q}\hat{\bar{q}} = \Lambda_2^2$. Since $\hat{q}$ and $\hat{\bar{q}}$ transform under $SU(3)$, on the quantum moduli space the $SU(3)$ is generically completely broken. The maximal unbroken subgroup occurs at the point $\bar{U} = \bar{D} = 0$ and $\hat{q} = \hat{\bar{q}} = \Lambda_2$ for which there is an unbroken $SU(2)' \subset SU(3)$. As mentioned in §2.3, the classical potential vanishes only at the point for which $SU(3)$ is unbroken. This point is removed from the quantum moduli space. In this limit supersymmetry is therefore broken by the quantum deformation of the $SU(2)$ moduli space.^{9,10} The tree level superpotential at the maximal symmetry point is $W = \lambda\Lambda_2^2 S_d$, where S_d is the $SU(2)'$ singlet component of $\bar{D}$. With this, quantum deformation of the $SU(2)$ moduli space induces an auxiliary expectation value with vacuum energy $V \sim |\lambda^2 \Lambda_2^4|$.

In the limit $\Lambda_2 \gg \Lambda_3$ it is the non-perturbative $SU(2)$ dynamics which breaks supersymmetry, even though these dynamics do not lift the classical moduli space (c.f. Eq. (7)). This is a clear example that the exact superpotential over the classical moduli does not always contain all the relevant information.

4 Supersymmetry Breaking with Vector Matter

As discussed in §2.2, it appears that supersymmetry breaking by gauge dynamics requires chiral representations. It is however actually possible to break supersymmetry with vector matter. The supersymmetric vacua which should exist in such a theory in some sense reside at the boundary of field space where some fields have infinite value (as shown explicitly in the example below). An unregulated calculation of $\text{Tr}(-1)^F$ is therefore not well defined. But a regulated calculation, in which only finite field values are weighted, can be suitably defined, and may vanish. If in fact there are no supersymmetric vacua for finite field values, then supersymmetry is broken.

The simplest example of supersymmetry breaking with vector matter is for the theory

$$\begin{array}{ccc}
 SU(2) & & \\
 Q_i & \square & i = 1, \dots, 4 \\
 S^{[ij]} & 1 & i, j = 1, \dots, 4
 \end{array} \tag{9}$$

Classically there is a moduli space parameterized by S^{ij} and $M_{ij} = Q_i Q_j$ subject to $\text{Pf}M = 0$. At tree level the superpotential $W_{tree} = \lambda S^{ij} M_{ij}$ completely lifts the M_{ij} but leaves S^{ij} undetermined. In the quantum theory, for

$\lambda = 0$, the M_{ij} moduli space is deformed, and the classical constraint is modified to $\text{Pf}M = \Lambda_2^4$.⁸ Holomorphy and symmetries may be used to show that the quantum constraint is not modified for $\lambda \neq 0$.⁹ The S^{ij} auxiliary equations of motions, $\lambda M_{ij} = 0$, are then incompatible with the quantum constraint $\text{Pf}M = \Lambda_2^4$. The classical moduli space is completely lifted for $\lambda \neq 0$, and supersymmetry is broken. The tree level superpotential on the quantum moduli space is⁹

$$W = \lambda M_5 S_5 \pm 2\lambda \Lambda_2^2 S_0 \quad (10)$$

where $S^{ij} = \{S_5, S_0\}$ with $S_5, M_5 \in \mathbf{5}$ of $SP(2)_F \subset SU(4)_F$ global flavor symmetry, and $S_0 \in \mathbf{1}$ of $SP(2)_F$. The first term pairs the $\widehat{M}_5 = M_5/\Lambda_2$ and S_5 moduli into a massive Dirac state, while the second term induces an auxiliary expectation value and vacuum energy $V \sim |\lambda^2 \Lambda_2^4|$. This theory demonstrates both supersymmetry breaking with vector matter,^{9,10,11} and by the quantum deformation of the moduli space.^{9,10}

This theory has a pseudo-flat direction corresponding to the S_0 component of S^{ij} along which $V \sim |\lambda^2 \Lambda_2^4|$. This direction would be precisely flat if the Kähler potential for S were precisely canonical, but is lifted by quantum corrections. The index $\text{Tr}(-1)^F$ can change since vacua can move continuously in or out from infinity along this direction under small deformations of the theory. To see this consider the effective theory along the pseudo-flat direction S_0 with $W = \pm 2\lambda \Lambda_2^2 S_0 + \epsilon S_0^2$. For $\epsilon = 0$, the vacuum energy along the entire S_0 direction is $V \sim |\lambda^2 \Lambda_2^4|$. However, for $\epsilon \neq 0$ there are two supersymmetric ground states at $S_0 = \pm \lambda \Lambda_2^2/\epsilon$. For $\epsilon \rightarrow 0$ these ground states are sent to ∞ along the pseudo-flat direction. The theory with $\epsilon = 0$ (enforced by discrete or continuous symmetries) breaks supersymmetry, while that with $\epsilon \neq 0$ does not. In this way the properly defined, regulated, $\text{Tr}(-1)^F$ is discontinuous at $\epsilon = 0$. The existence of a pseudo-flat direction along which the index can change is generic to non-chiral models of supersymmetry breaking.

This theory also exhibits quantum removal of classical flat directions.^{9,10} The S^{ij} moduli are not lifted by the tree level superpotential, and are only lifted by quantum effects. The existence of these classically unlifted directions contradicts one of the classic “requirements” for supersymmetry breaking discussed in §2.2. In contrast to flat directions along which a gauge group is Higgsed and becomes weaker, here the matter fields become more massive and the theory becomes more strongly coupled, leading to a vacuum energy which does not vanish even infinitely far along the pseudo-flat direction. It is also possible to construct theories with classically unlifted directions along which one subgroup is Higgsed and becomes weaker, while another subgroup becomes more confining and stronger, with the result that the total potential actually grows at large expectation values.⁹

5 Dual Descriptions of Supersymmetry Breaking

Identifying the relevant low energy degrees of freedom in models of supersymmetry breaking is important in giving a proper description of the ground state. It is now understood that certain asymptotically free gauge theories flow in the infrared to magnetic gauge theories with different gauge symmetry and matter representations.¹² In a theory with multiple scales, if the magnetic scale is well above the supersymmetry breaking scale, the relevant degrees in the non-supersymmetric ground state are the infrared magnetic ones, rather than the ultraviolet electric ones. By interchanging the magnetic and supersymmetry breaking scales, the magnetic description of supersymmetry breaking can often be continuously connected to an electric description. In this way seemingly disparate models of supersymmetry breaking can be related by duality.

Examples of dual descriptions of supersymmetry breaking are for the theories¹³

$$\begin{array}{cc}
 A & (\square, 1) \\
 \overline{P} & (\square, \square) \\
 L & (1, \square) \\
 \overline{U} & (\square, 1)
 \end{array} \tag{11}$$

with $N \geq 11$ and odd. These theories are just the Affleck, Dine, Seiberg, $SU(N)$ theories with an $\square$ and $N - 4$ $\overline{\square}$ ⁶ with the maximal $SP(\frac{1}{2}(N - 5))$ flavor symmetry acting on the $\overline{\square}$ promoted to a gauge symmetry, and additional matter to cancel anomalies. The classical moduli space is parameterized by V^k and $QV^{k-1}L$, $k = 1, \dots, \frac{1}{2}(N - 5)$ where $V_{\alpha\beta} = A\overline{P}_\alpha\overline{P}_\beta$, and $Q_\alpha = A\overline{P}_\alpha\overline{U}$, and α, β are $SP(M)$ indices. On the moduli space the gauge group is generically broken to $SU(5) \subset SU(N)$, with $\square$ and $\overline{\square}$ of $SU(5)$ remaining. At tree level there is a single renormalizable coupling which can be added to the superpotential $W_{tree} = \lambda V$. This superpotential leaves invariant a non-anomalous $U(1)_R$ symmetry, and completely lifts the classical moduli space. Classically there is a supersymmetric ground state at the origin.

Quantum mechanically, the non-perturbative $SU(N)$ dynamics lift the classical supersymmetric ground state at the origin and supersymmetry is broken. The low energy description of supersymmetry breaking in the ground state depends on the relative importance of the $SU(N)$ and $SP(\frac{1}{2}(N - 5))$ non-perturbative dynamics. If the $SP(M)$ is weakly coupled in the ground state, it may be treated classically. In this case the unbroken $SU(5)$ with $\square$ and $\overline{\square}$ generates a potential and breaks supersymmetry.⁶ The position of the ground state is then determined by a balance between this dynamically gener-

ated potential and the tree level potential. This is the electric description of the theory in terms of the underlying ultraviolet degrees of freedom.

If the $SP(\frac{1}{2}(N-5))$ is strongly coupled in the ground state, its non-perturbative dynamics can not be ignored. For $\Lambda_{SP} \gg \Lambda_{SU}$, $SU(N)$ is weakly gauged at the scale Λ_{SP} , and may be treated as a weakly gauged flavor symmetry. The $SP(\frac{1}{2}(N-5))$ therefore has $\frac{1}{2}(N+1)$ flavors ($N+1$ $\square$) and for $N \geq 11$ flows in the infrared towards a weakly coupled theory in a free magnetic phase. The weakly coupled magnetic description has gauge group $SU(N) \times \widetilde{SU}(2)$ with “mesons” $\widehat{A} = \overline{A}/\Lambda_{SP} = \overline{P}^2/\Lambda_{SP} \in \overline{\square}$ of $SU(N)$ and $\widehat{\overline{D}} = \overline{D}/\Lambda_{SP} = \overline{P}L/\Lambda_{SP} \in \overline{\square}$ of $SU(N)$, and dual “magnetic” quarks $\tilde{P} \in (\square, \square)$ of $SU(N) \times \widetilde{SU}(2)$ and $\tilde{L} \in \square$ of $\widetilde{SU}(2)$. For expectation values much less than Λ_{SP} these fields, along with the electric fields A and $\overline{U}$, make up the canonically normalized degrees of freedom. The matter content of this free magnetic phase is just an $SU(N) \times SU(2)$ generalization of the $SU(3) \times SU(2)$ model discussed in §2.3, with an additional flavor of $\square$ and $\overline{\square}$ of $SU(N)$. This is the magnetic description of the theory.

In the absence of the electric tree level superpotential, the moduli space of the free magnetic theory is parameterized by $Z = \tilde{P}^2 \overline{U} \overline{D}$, $X_1 = \tilde{P} \tilde{L} \overline{D}$, $X_2 = \tilde{P} \tilde{L} \overline{U}$, $\overline{V} = \overline{A} \tilde{P} \tilde{P}$, and $V = \overline{A} A$, subject to the dual tree level superpotential

$$W_{\widetilde{tree}} = \frac{1}{\Lambda_{SP}} (\overline{V} + X_1) \quad (12)$$

This superpotential, along with the non-perturbative $\widetilde{SU}(2)$ dynamics ensures that the moduli space of the free magnetic theory coincides with the classical moduli space of the electric theory. With the electric tree level superpotential, the full tree level superpotential in the magnetic theory is $W_{tree} = W_{\widetilde{tree}} + \lambda V$. It follows from symmetries, holomorphy, and limits that there are no additional contributions to the magnetic tree level superpotential. The final term is a Dirac mass $m = \lambda \Lambda_{SP}$ for the pair A and $\widehat{A}$. For $\lambda \Lambda_{SP} \gg \Lambda_{SU}$ the Dirac pair is much heavier than the dynamical scale in the free magnetic theory and may be integrated out. Below the scale $\lambda \Lambda_{SU}$, the effective magnetic theory is then just the $SU(N) \times SU(2)$ theory. In this limit the $SU(N) \times SP(\frac{1}{2}(N-5))$ theory with an antisymmetric representation is dual to the $SU(N) \times SU(2)$ theory with only fundamental representations.

It is important to note that in this example as a function of the parameters of the microscopic theory the two descriptions of the supersymmetry breaking ground state do not have overlapping regions of applicability. In many cases duality in supersymmetry breaking models with product gauge groups can be used as a generator to give other models of supersymmetry breaking.¹³

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