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HEAVY QUARK PHYSICS*

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ABSTRACT

Various aspects of the physics of heavy quarks and of CP violation are reviewed.

1. INTRODUCTION

The field of high energy or elementary particle physics has undergone a spectacular advance over the course of the last few decades, culminating in the identification of the basic constituents of matter as quarks and leptons and the description of their interactions in terms of strong and electroweak gauge theories. This is often dubbed the standard model.

The course of the 1970s and '80s is replete with experiments of increasing accuracy and higher energy checking the predictions based on the standard model. This includes the discovery in the '80s of the W and Z bosons at their expected masses. Together with the photon, they form the basic quanta of the electroweak theory.

While the standard model has emerged without the necessity of modification after being subjected to tighter and tighter scrutiny, it is incomplete in two important ways. First, some of the particles required in the model are yet to be discovered. While six leptons are known, only five of the six corresponding quarks

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have been found. The sixth quark, the top quark, is actively being pursued at this moment. Even more fundamental, as it is tied to the mechanism of generating masses in the standard model, is the Higgs boson.

But aside from what is missing inside the standard model, there is a second, deeper incompleteness, in that we are not satisfied with it as a final theory of matter and forces. Many parameters, such as the masses of quarks and leptons, are simply put in by hand. The relation between quarks and leptons, which seem to come in three families in which pairs of leptons are matched to pairs of quarks, is not explained. Even the overall scale of masses of the W and the Z is not understood and seems to be "fine tuned" to produce the observed situation. Could not the interactions be further unified? Some of these questions are addressed in the context of theories that would encompass the standard model, but there are many possible paths to follow, none of them entirely satisfactory or compelling.

This defines much of what is happening in high energy physics today and envisage in the 1990s. One needs to complete the standard model by finding the top quark and the Higgs boson. At the same time, it is essential to look for the physics that lies beyond it (including possibly in the Higgs sector), or which at least requires significant extensions. In looking for such new physics, there are two principal avenues:

(1) The high-energy route involves the direct observation of new quarks, new leptons, heavy Higgs bosons ... Of necessity, this involves accelerators which are at the high-energy frontier. That frontier, in the past few years, has begun to yield quark-quark (from proton-antiproton colliders) collisions at total energies of order 100 GeV. The near future should see lepton-lepton (from electron-positron colliders) collisions in this range and quark-quark collisions probing physics up to several hundred GeV. Experiments at the SSC will be allow us to explore physics at the 1000 GeV scale and above. This is the natural continuation of the field of high energy physics to higher-mass scales and it is essential that this path be continued.

(2) The "low-energy" route also can involve the direct observation of new particles such as additional light neutrinos. The confirmation of nonzero neutrino mass and mixing would indicate physics beyond the standard model as well. However, much of the work at "low energy" aims to be sensitive to new physics through the indirect effects of virtual, heavy particles. These, through precision measurements, give us a window on the high-energy world which others attack directly. While deemed a "low-energy" or low-mass-scale route, in many cases it is implemented at high-energy experimental facilities, using them to produce intense fluxes of low-mass particles whose properties with respect to electroweak interactions are then studied.

In this latter mode, we search for physics beyond the standard model through:

(a) Processes forbidden in the standard model, such as would be induced by lepton-flavor changing neutral currents.

(b) Indications that CP-violating phenomena have an origin other than from the nontrivial phase in the quark flavor mixing matrix of the standard model.

(c) Deviations from expected rates, especially for rare processes which are sensitive to heavy virtual particles (from a fourth-generation, supersymmetry, left-right electroweak gauge symmetry, *etc.*) This is especially true of CP-violating processes, which in some cases are especially sensitive to the top quark and possible other high-mass particles.

As we pin down and measure the parameters associated with each of the particles in the standard model, we use these numbers, together with our improved calculational skills, to obtain updated predictions. Then we can return to the former perspective of looking for physics beyond the standard model by comparing these predictions with all previous data and by pointing to further experiments which are yet more sensitive to new physics.

2. THE KOBAYASHI–MASKAWA MATRIX

In the standard model with $SU(2)\times U(1)$ as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates, and the matrix connecting them has become known as the Kobayashi–Maskawa[1] (K–M) matrix since an explicit parametrization in the six-quark case was first given by them in 1973. It generalizes the four-quark case, where the matrix is parametrized by a single angle, the Cabibbo angle.[2]

By convention, the three-charge 2/3 quarks (u, c, and t) are unmixed, and all the mixing is expressed in terms of a 3×3 unitary matrix V operating on the charge $-1/3$ quarks (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad . \tag{1}$$

There are several parametrizations of the K–M matrix. In the 1988 edition of the *Review of Particle Properties,* a "standard" form is advocated:[3]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \quad . \tag{2}$$

This was first proposed by Chau and Keung.[4] The choice of rotation angles follows earlier work of Maiani,[5] and the placement of the phase follows that of Wolfenstein.[6] The notation used is that of Harari and Leurer[7] who, along with Fritzsch and Plankl,[8] proposed this parametrization as a particular case of a form generalizable to an arbitrary number of "generations," as was also done by Botella and Chau.[9] Here, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, with i and j being "generation" labels, $\{i,j = 1,2,3\}$. In the limit $\theta_{23} = \theta_{13} = 0$, the third generation decouples, and the situation reduces to the usual Cabibbo mixing of the first two generations with θ_{12} identified with the Cabibbo angle.[2] The real angles θ_{12}, θ_{23}, θ_{13} can all be made to lie in the first quadrant by an appropriate redefinition of quark field phases. Then all s_{ij} and c_{ij} are positive, and $|V_{us}| = s_{12}c_{13}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}c_{13}$. As c_{13} deviates from unity only in the fifth decimal place (from experimental measurement of s_{13}), $|V_{us}| = s_{12}$, $|V_{ub}| = s_{13}$, and $|V_{cb}| = s_{23}$ to an excellent approximation. The phase δ_{13} lies in the range $0 \leq \delta_{13} < 2\pi$, with nonzero values generally breaking CP invariance for the weak interactions.

The values of individual K–M matrix elements can, in principle, all be determined from weak decays of the relevant quarks, or, in some cases, from deep inelastic neutrino scattering. Our present knowledge of the matrix elements comes from the following sources:

(1) Nuclear beta decay, when compared to muon decay, gives[10,11]

$$|V_{ud}| = 0.9747 \pm 0.0011 \quad . \tag{3}$$

(2) Analysis of K_{e3} decays yields[12] $|V_{us}| = 0.2196 \pm 0.0023$. The analysis of hyperon decay data has larger theoretical uncertainties because of first order $SU(3)$ symmetry breaking effects in the axial-vector couplings, but due account of symmetry breaking gives a consistent value[13] of $0.220 \pm 0.001 \pm 0.003$. The average of these two results is[3]

$$|V_{us}| = 0.2197 \pm 0.0019 \quad . \tag{4}$$

(3) The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. When the dimuon production cross sections of the CDHS group[14] are supplemented by more recent measurements of the semileptonic branching fractions and the production cross sections in neutrino reactions of various charmed hadron species, the value[15]

$$|V_{cd}| = 0.21 \pm 0.03 \tag{5}$$

is extracted.

(4) Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange quark density in the parton-sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an $SU(3)$ symmetric sea, leads to a lower bound,[14] $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of D_{e3} decay with the expression[16] that follows from the standard weak interaction amplitude. This gives:[3]

$$|f_+^D(0)|^2 |V_{cs}|^2 = 0.51 \pm 0.07 \quad .$$

With sufficient confidence in a theoretical calculation of $|f_+^D(0)|$ a value of $|V_{cs}|$ follows,[17,18] but even with the very conservative assumption that $|f_+(0)| < 1$ it follows that

$$|V_{cs}| > 0.66 \quad . \tag{6}$$

The constraint of unitarity when there are only three generations gives a much tighter bound (see below).

(5) The ratio $|V_{ub}/V_{cb}|$ can be obtained from the semileptonic decay of B mesons by fitting to the lepton energy spectrum as a sum of contributions involving $b \to u$ and $b \to c$. The relative overall phase space factor between the two processes is calculated from the usual four-fermion interaction with one massive fermion (c quark or u quark) in the final state. The value of this factor depends on the quark masses, but is roughly one-half. The lack of observation of the higher-momentum leptons characteristic of $b \to u\ell\bar{\nu}_\ell$ as compared to $b \to c\ell\bar{\nu}_\ell$ has resulted thus far only in upper limits which depend on the lepton energy spectrum assumed for each decay.[18,19,20] Using the lepton momentum region near the end-point for $b \to c\ell\bar{\nu}_\ell$ and taking the calculation[20] of the lepton spectrum that gives the least restrictive limit results in[21]

$$|V_{ub}/V_{cb}| < 0.20 \quad . \tag{7}$$

A lower bound on $|V_{ub}|$ can be established from the observation of an exclusive decay of a B meson into all nonstrange particles, such as the claimed ARGUS observation[22] of exclusive B decays into $p\bar{p}\pi$ and $p\bar{p}\pi\pi$ (which involve $b \to u + d\bar{u}$ at the quark level). This claim is disputed by the CLEO collaboration, however, on the basis of an enlarged data sample in which they see no evidence for these modes.[23] We will have to wait for a resolution of the difference between experimental results, and a quantitative measurement of V_{ub} probably depends on the eventual observation of the semileptonic decays $B \to \pi e\nu$ and $B \to \rho e\nu$.

(6) The magnitude of V_{cb} itself can be determined if the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual V–A interaction:[3]

$$|V_{cb}| = 0.046 \pm 0.010 \quad . \tag{8}$$

Most of the error quoted in Eq. (9) is not from the experimental uncertainty in the value of the b lifetime, but in the theoretical uncertainties in choosing a value of m_b and in the use of the quark model to represent inclusively semileptonic decays which, at least for the B meson, are dominated by a few exclusive channels. We have made the error bars larger than they are sometimes stated to reflect these uncertainties. They then also include the central values obtained for $|V_{cb}|$ by using a model for the exclusive final states in semileptonic B decay and extracting $|V_{cb}|$ from the absolute width for one or more of them.[18,20,24]

From Eqs. (3) through (8) plus unitarity (assuming only three generations), the 90% confidence limits on the magnitude of the elements of the complete matrix are:[3]

$$\begin{pmatrix} 0.9748 \text{ to } 0.9761 & 0.217 \text{ to } 0.223 & 0.003 \text{ to } 0.010 \\ 0.217 \text{ to } 0.223 & 0.9733 \text{ to } 0.9754 & 0.030 \text{ to } 0.062 \\ 0.001 \text{ to } 0.023 & 0.029 \text{ to } 0.062 & 0.9980 \text{ to } 0.9995 \end{pmatrix} \quad . \tag{9}$$

The ranges shown are for the individual matrix elements. The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of the others. The ranges given in Eq. (9) are consistent with the one standard deviation errors on the input matrix elements.

The data do not preclude there being more than three generations. Of course, the constraints deduced from unitarity are loosened when the K–M matrix is expanded to accommodate more generations. Still, the known entries restrict the possible values of additional elements if the matrix is expanded to account for additional generations. For example, unitarity and the known elements of the first row require that any additional element in the first row have a magnitude

$$|V_{ub'}| < 0.07 \quad , \tag{10a}$$

and the known elements of the first column require that

$$|V_{t'd}| < 0.15 \quad . \tag{10b}$$

Further information on the angles requires theoretical assumptions. For example, B_d–$\bar{B}_d$ mixing, if it originates from short distance contributions to ΔM_B dominated by box diagrams involving virtual t quarks, gives information on $V_{tb}\ V_{td}^*$ once hadronic matrix elements and the t quark mass are known.[25)] A similar comment holds for $V_{tb}\ V_{ts}^*$ and B_s–$\bar{B}_s$ mixing. Even at the present stage of knowledge, we may use the published ARGUS data claiming the observation of B–$\bar{B}$ mixing,[26)] which has now been confirmed by CLEO,[27)] to obtain a significant lower bound on $|V_{td}|$ within the three generation standard model. This is because the magnitude of the mixing depends on m_t, an hadronic matrix element, and $|V_{td}|$. Taking $m_t < 180$ GeV,[28)] and the relevant matrix element parametrized as $|B_B f_B^2|$ to be less than $(200\ \text{MeV})^2$, we obtain

$$|V_{td}| > 0.006 \quad . \tag{11}$$

This is a considerable improvement over the constraint provided by unitarity and the measured values of other matrix elements in Eq. (9).

Up to this point, we have discussed only information on magnitudes of K–M matrix elements. In principle, such measurements of magnitudes could tell us about the phase, δ_{13}, as well as the "rotation angles" θ_{12}, θ_{23}, and θ_{13} in Eq. (2). This is most easily seen for the case at hand, where the "rotation angles" are small, by using the unitarity of the K–M matrix applied to the first and third columns to derive that (c_{ij} have been set to unity):

$$1 \cdot V_{ub}^* - s_{12} \cdot V_{cb}^* + V_{td} \cdot 1 \approx 0 \quad . \tag{12}$$

This equation is represented graphically in Fig. 1 in terms of a triangle in the complex plane, the length of whose sides is $|V_{ub}^*|$, $|s_{12} \cdot V_{cb}^*|$, and $|V_{td}|$. After the measurement of the b lifetime in 1983, which assured us that all three mixing angles were small, such a triangle was implicit in any work which used the experimental constraints from τ_b and $b \to u$ to put limits on V_{td} and thence processes (like B–$\bar{B}$ mixing and various rare K decays which depend on it. It appears explicitly in Ref. 4, and has been commented on by many people,[29] but has been particularly emphasized by Bjorken.[30]

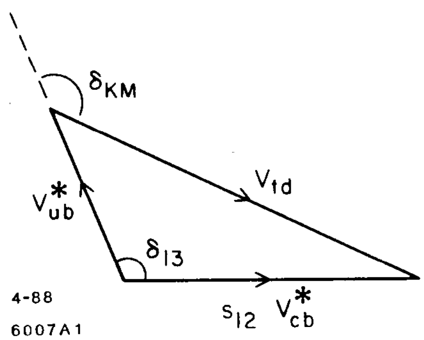

Figure 1: Representation in the complex plane of the triangle formed by the K–M matrix elements V_{ub}^*, $s_{12} \cdot V_{cb}^*$, and V_{td}.

With this representation of the unitarity of the K–M matrix, it is possible to see more directly the interplay of various pieces of experimental information. For example, an increase in the magnitude of the $b \to u$ transition obviously increases the side whose length is $|V_{ub}|$. The present upper bound on $|V_{ub}/V_{cb}|$ means that this side at most is as long as the side whose length is $|s_{12}V_{cb}^*|$. On the other hand, an increased magnitude for B_d–$\bar{B}_d$ mixing implies (keeping m_t and the appropriate hadronic matrix element, $B_B f_B^2$, fixed) stretching the side whose length is $|V_{td}|$. With the other sides set by independent measurements, the triangle gets flatter and flatter and eventually "breaks." At that point B–$\bar{B}$ mixing has become incompatible with other data plus assumed values of m_t and the hadronic matrix element. Hence the derivation of a lower bound on m_t from B–$\bar{B}$ mixing.[25,31)]

In principle, accurate measurement of the lengths of all three sides could show that the triangle can not exist (and we must go beyond the three-generation standard model), or cause the triangle to collapse to a line (and we must go beyond the standard model for an explanation of CP violation), or demand the existence of a nontrivial triangle with δ_{13} not equal to 0° or 180°. Unfortunately, given our present experimental knowledge and our limited theoretical ability to compute hadronic matrix elements, the three sides are not known with sufficient accuracy to discriminate between these situations, let alone determine the value of δ_{13}. To do this, we are forced to consider a CP-violating quantity and assume it can be understood within the three-generation standard model.

In this connection, note that the law of sines applied to the triangle gives:

$$\frac{\sin\delta_{KM}}{|s_{12}V_{cb}^*|} = \frac{\sin\delta_{13}}{|V_{td}|} \quad .$$

Setting cosines of small angles to unity and expressing V_{cb} as s_{23}, but V_{td} as $s_1 s_2$ in the original notation of Kobayashi and Maskawa,[1)] allows this equation to be converted to ($s_{12} \approx s_1$):

$$s_1^2 s_2 s_3 \sin\delta_{KM} \approx s_{12}s_{23}s_{13}\sin\delta_{13} \quad , \tag{13}$$

which is twice the area of the triangle.

3. CP VIOLATION IN THE K° SYSTEM

As noted in the previous section, the standard model allows for CP violation in the form of phases originating in the quark-mixing matrix when there are three or more generations of quarks and leptons. With just three generations, there is precisely one nontrivial CP-violating phase.[32)]

The computation of any difference of rates between a given process and its CP conjugate process (or of a CP-violating amplitude) always has the form

$$\Gamma - \bar{\Gamma} \;\propto\; s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta_{KM} \;=\; s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin\delta_{13} \quad , \tag{14}$$

where we express things first in the original parametrization of the quark-mixing matrix[1)] and then in the "new" parametrization used in the previous section. Our present experimental knowledge assures us that the approximation of setting the cosines to unity induces errors of at most a few percent. In that case, the combination of factors in Eq. (14), involving the invariant measure of CP violation,[33)] becomes the approximate combination in Eq. (13), which was recognized earlier as characteristic of CP-violating effects in the three-generation standard model.[4)] This combination of factors is (after removing s_1^2, whose value is accurately known)

$$s_2 s_3 \sin\delta_{KM} \;\equiv\; s_2 s_3 s_\delta \quad ,$$

where we have used the "old" parametrization.

An example of such a CP-violating quantity is provided by the one well-measured CP-violation parameter, ϵ, in the neutral K system. Assuming that ϵ arises from short-distance effects, *i.e.*, the box diagram with virtual c and t quarks shown in Fig. 2, gives the relation:

$$\begin{aligned} \epsilon \;\approx\; & \frac{e^{i\pi/4}}{\sqrt{2}} \frac{B G_F^2 f_K^2 m_K}{6\pi^2 \Delta M_K} \\ & \times s_1^2 s_2 s_3 s_\delta [-\eta_1 m_c^2 + \eta_2 s_2 (s_2 + s_3 c_\delta) m_t^2 + \eta_3 m_c^2 \, \ell n(m_t^2/m_c^2)] \quad . \end{aligned} \tag{15}$$

As stressed above, the factor $s_1^2 s_2 s_3 s_\delta$ must appear in Eq. (15), and it does. The quantities η_1, η_2, and η_3 are due to strong interaction (QCD) corrections. They are

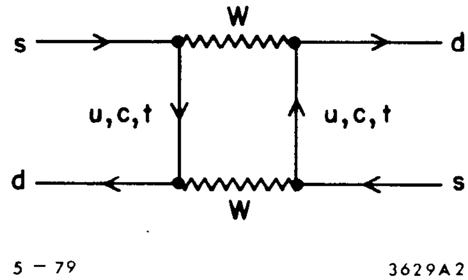

Figure 2: Box diagram whose imaginary part contributes to the CP-violation parameter ϵ in the neutral K system.

calculable and have the values 0.7, 0.6, and 0.4, respectively, given a renormalization scale of a few hundred MeV and typical quark masses.[34] Less well determined is the infamous parameter B, which is the ratio of the actual value of the matrix element between K^0 and $\bar{K}^o$ states of the operator composed of the product of two $V-A$ neutral, strangeness changing currents divided by the value of the same matrix element obtained by inserting the vacuum between the two currents. The parameter B has a long history of calculation and recalculation, but a reasonable range seems to be

$$1/3 \ < \ B \ < \ 1 \quad .$$

If we insert known experimental quantities, Eq. (15) becomes

$$|\epsilon| \ \approx \ \frac{0.314}{\mathrm{GeV}^2} \ B \ s_2 s_3 s_\delta \ \left[-\eta_1 m_c^2 + \eta_2 s_2 (s_2 + s_3 c_\delta) m_t^2 + \eta_3 m_c^2 \ \ell n (m_t^2/m_c^2) \right] \quad . \ (16)$$

Equations (15) and (16), as written, are strictly valid when $m_t^2 << M_W^2$, but their forms are representative of the general character of the full expression[35] which we use in the analysis that follows. Numerically, even for $m_t \approx M_W$, the changes in the coefficients of the last two terms in brackets are not large.

Viewed from the direction of making a prediction, can we understand the magnitude of ϵ, *i.e.*, why is CP violation in the neutral Kaon system so small? The answer is yes, we do understand its rough magnitude, for our present knowledge of the elements of the K–M matrix permits the placing of an upper bound on the quantity $s_2 s_3 s_\delta$ of about 2.5×10^{-3}. This plus the quark masses and other quantities on the righthand side of Eq. (16) make $|\epsilon|$ be in the ballpark of 10^{-3}.

From the opposite direction, we can constrain the mixing angles by using the measured magnitudes of ϵ and B–$\bar{B}$ mixing. These constraints will depend on what we assume for the quantities B and m_t, as well as on the still uncertain value for $b \to u/b \to c$.

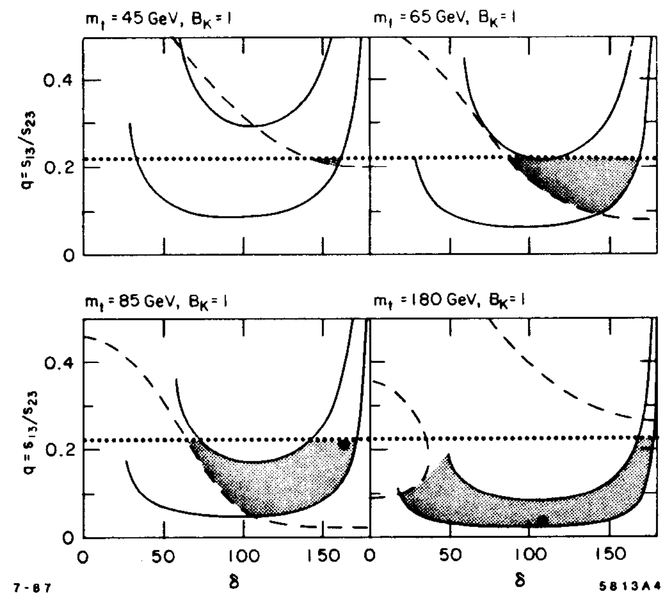

Figure 3: The allowed range (shaded) of the K–M matrix parameter $q = s_{13}/s_{23}$ versus that for δ_{13} for top quark masses of 45, 65, 85, and 180 GeV *when $B = 1$ as given in Ref. 36. The solid curves show the restrictions due to satisfying the constraint of imposing the experimental value of ϵ, while the dashed curves (the upper limit is off the scale of the first three subgraphs) do the same for B–$\bar{B}$ mixing. The horizontal dotted line gives the upper bound on q that follows from the bound on $b \to u$ using the inclusive lepton spectrum in B decay.*

- If $m_t = 45$ GeV, the magnitude of ϵ and the "large" observed[26)] B–$\bar{B}$ mixing push the K–M matrix elements into a corner: Such a "small" value of m_t together with the "large" mixing force V_{td} to be as large as possible. The long (V_{td}) side of the triangle in Fig. 1 is stretched as far as it will go, and in the process $|V_{ub}|$ is increased and the phase δ_{13} pushed toward 180°. The parameter B must be near the upper end of its allowed range as well, to obtain the experimental value of $|\epsilon|$ in Eq. (16). This is illustrated in Fig. 3 from

Nir,[37] which indicates that for $m_t = 45$ GeV and $B = 1$ there is just barely an allowed region. It is centered around a phase $\delta_{13} \approx 150°$ and $b \to u/b \to c$ (s_{13}/s_{23} in Fig. 3) near its maximum allowed value. Correspondingly,

$$0.75 \times 10^{-3} \lesssim s_2 s_3 s_\delta \lesssim 1.25 \times 10^{-3}$$

is rather narrowly constrained as well.

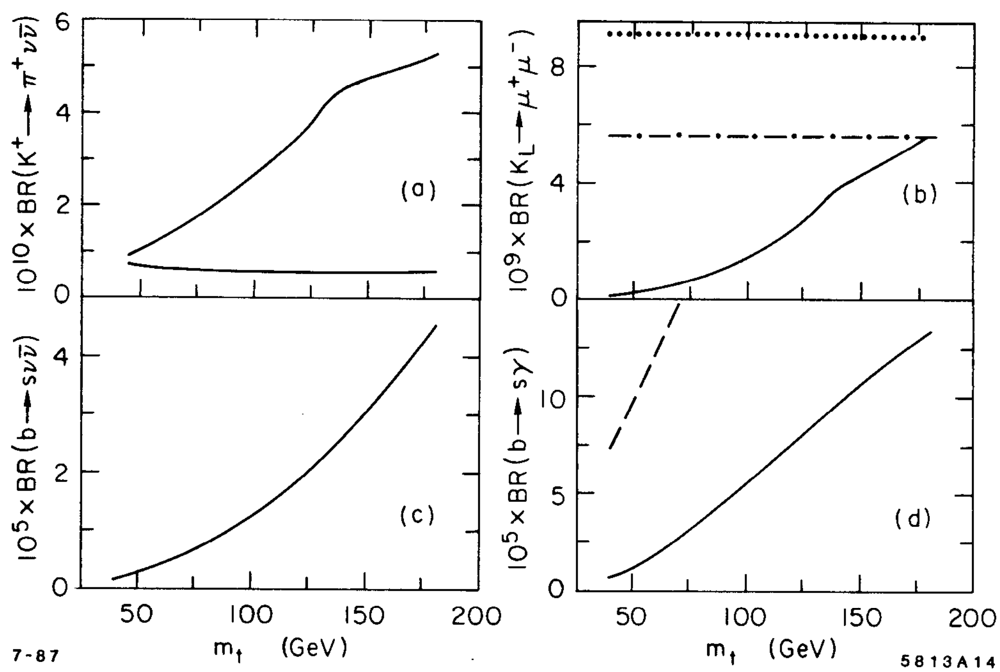

Figure 4: The range of branching ratios allowed for several rare decays as a function of m_t from Ref. 37: (a) Upper and lower limits on $B(K^+ \to \pi^+\nu\bar{\nu})$; (b) Upper limit (solid curve) on the short distance contribution to $B(K_L \to \mu^+\mu^-)$. The dotted line gives the experimental branching ratio and the dot-dashed line an upper limit on the short distance contribution once the (long distance) contribution from the two photon intermediate state is subtracted from the experimental rate; (c) The branching ratio $B(b \to s\nu\bar{\nu})$; (d) The branching ratio $B(b \to s\gamma)$ without (solid curve) and with (dashed curve) QCD corrections.

- As we go to larger values of m_t, a bigger range of angles is allowed. This is shown in Fig. 3, where the allowed range is shaded. The corresponding figures for other values of B show the same effect.[37]

- Very roughly, larger values of m_t generally favor smaller values of $s_2 s_3 s_\delta$, as the constraint imposed by the experimental value of $|\epsilon|$ inserted into Eq. (16) forces them to move in compensating directions if the other parameters (like B and m_c) are fixed.
- The region of K–M angles allowed by the constraints translates into a corresponding range for the amplitudes of processes induced at one loop which involve a virtual t quark. A sample of the resulting theoretical predictions is shown[37] in Fig. 4.

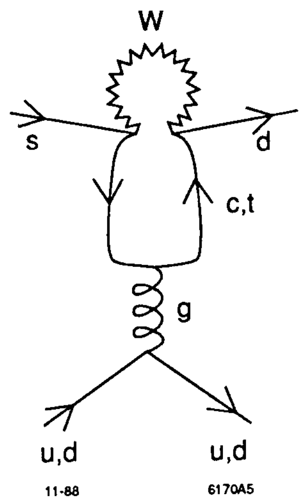

Figure 5: "Penguin" diagram whose imaginary part contributes to the CP-violation parameter ϵ' in the neutral K system.

A second quantity is provided by the parameter ϵ', which measures CP violation in the K decay amplitude itself, and arises in the standard model from diagrams involving heavy quarks in loops, the so-called "penguin" diagrams, an example of which is shown in Fig. 5. By inserting experimentally measured quantities, the contribution to ϵ' from the "penguin" operator contribution to $K \to \pi\pi$ can be written[38)]

$$\epsilon'/\epsilon \;=\; 6.0\; s_2 s_3 s_\delta \left(\frac{Im\widetilde{C}_6}{-0.1}\right) \left(\frac{<\pi\pi|Q_6|K^0>}{1.0\; GeV^3}\right) \left(1 - \Omega_{\eta,\eta'} + \Omega_{em}\right) \quad , \qquad (17)$$

where Q_6 is the "penguin" operator in the short distance expansion of the strangeness-changing weak Hamiltonian,[39)] $Im\widetilde{C}_6$ is the imaginary part of the corresponding

Wilson coefficient with the K–M factor taken out, and $\Omega_{\eta,\eta'}$ and Ω_{em} are corrections due to $\pi^0 - \eta$ and $\pi^0 - \eta'$ mixing, and to "electromagnetic penguins," respectively. These latter two contributions tend to cancel; the factor $(1 - \Omega_{\eta,\eta'} + \Omega_{em})$ may result[40] in anything between a $\sim 30\%$ decrease and a small increase in ϵ'/ϵ. The value of -0.1 for $Im\widetilde{C}_6$ is relatively stable from calculation to calculation if the renormalization scale is taken as a few hundred MeV, since the imaginary part depends on momentum scales from m_c to m_t where the short distance expansion is well justified. The value of the matrix element of Q_6 is much less certain. If it is large enough to explain the experimental magnitude of $A(K \rightarrow \pi\pi)$, *i.e.*, roughly 1 to 2 GeV^3, then, combined with the value of $s_2 s_3 s_\delta$ needed to fit $|\epsilon|$ (see above), it yields the prediction that ϵ'/ϵ is positive and of order 10^{-2}. This was the basic observation in Ref. 39.

Over the past ten years, there have been many calculations of ϵ'/ϵ. The prediction depends directly on the value of the matrix element of Q_6, but also on that of m_t. The latter dependence is direct in that $Im\widetilde{C}_6$ depends on m_t. In the calculations done until now, there is also an indirect dependence on m_t which is even more important: the constraint involving $|\epsilon|$ is used to determine $s_2 s_3 s_\delta$. One needs to be very careful in comparing the predictions from different calculations as the fashionable value of m_t has drifted higher and higher over the years. Sometimes nothing more fundamental than a change in the favorite value of m_t is the cause of a large change in the predictions. As m_t has risen, the predictions for ϵ'/ϵ have correspondingly gone down.[41]

As the last year has unfolded, the standard model "explanation" of CP violation has looked better and better. In particular, there have been two important new experimental results for ϵ'/ϵ. First came the result from a test run of the Fermilab experiment[42] which has been updated to:

$$\epsilon'/\epsilon \;=\; 3.2 \pm 2.8 \pm 1.2 \;\times 10^{-3} \quad .$$

The full data set is now being analyzed. The result from the NA31 experiment[43] at CERN,

$$\epsilon'/\epsilon \;=\; 3.3 \pm 1.1 \;\; \times 10^{-3} \quad ,$$

is the first significant indication for CP violation in the decay amplitude itself. They are running again. Both experiments have the capability of eventually decreasing both their statistical and systematic error bars below the 10^{-3} level. We will have to wait and see if the central value of ϵ'/ϵ remains nonzero by many standard deviations when the combined error bars shrink to this level.

While we wait, we can ask in any case whether the present central value, if it persists, is consistent with the standard model. The answer is yes, particularly if the value of m_t is large. One perspective on this is obtained by taking whatever knowledge we have of the matrix element, m_t, and $s_2 s_3 s_\delta$ and predicting ϵ'/ϵ in the usual way.[44] A different perspective is gained by turning the situation around and assuming a value for ϵ'/ϵ, and then asking what combined Wilson coefficient, "penguin" matrix element, and electromagnetic corrections would produce such a result. In the future, when the experimental situation settles down with small error bars, this will be more typical; we will take the experimental value of ϵ'/ϵ as an input and use it to measure the magnitude of the "penguin" operator contribution to K decay. Hopefully, the theory will have progressed sufficiently that there will then be a significant comparison between this and lattice gauge theory calculations of the same quantity.

So, let us assume that $\epsilon'/\epsilon = 3.5 \times 10^{-3}$. When $m_t = 45$ GeV, there is not too much room to maneuver and still satisfy the constraint of getting the correct value of $|\epsilon|$. Our previous discussion, together with Eq. (17), makes

$$\left(\frac{Im\widetilde{C}_6}{-0.1}\right)\left(\frac{<\pi\pi|Q_6|K^0>}{1.0\ \mathrm{GeV}^3}\right)\left(1-\Omega_{\eta,\eta'}+\Omega_{em}\right) \;=\; 0.47$$

for the biggest value allowed for $s_2 s_3 s_\delta$, and

$$\left(\frac{Im\widetilde{C}_6}{-0.1}\right)\left(\frac{<\pi\pi|Q_6|K^0>}{1.0\ \mathrm{GeV}^3}\right)\left(1-\Omega_{\eta,\eta'}+\Omega_{em}\right) \;=\; 0.78$$

for the smallest. The corresponding values for $m_t = 100$ GeV are 0.4 and 2.1, respectively.

The outcome of this exercise, recalling that a value for the matrix element of the "penguin" operator of 1 to 2 GeV^2 is large enough to make it a plausible explanation for the $\Delta I = 1/2$ rule, is that the "penguin" contribution to the $K \rightarrow \pi\pi$ amplitude is unlikely to be negligible when compared with an "interesting" value. "Penguins" could well play an important part in K decay.

Another decay in which it is possible to observe CP violation in the K° system is $K_L \rightarrow \pi^0 e^+ e^-$. If we define K_1 and K_2 to be the even and odd CP eigenstates, respectively, of the neutral K system, then

$$K_2 \rightarrow \pi^0 \, \gamma_v \rightarrow \pi^0 e^+ e^- \tag{18}$$

is CP violating, while

$$K_1 \rightarrow \pi^0 \, \gamma_v \rightarrow \pi^0 e^+ e^- \tag{19}$$

and

$$K_2 \rightarrow \pi^0 \, \gamma\gamma \rightarrow \pi^0 e^+ e^- \tag{20}$$

are CP conserving. Therefore $K_L \rightarrow \pi^0 e^+ e^-$ has three contributions:

- Through the small (proportional to ϵ) part of the K_L which is K_1 due to CP violation in the mass matrix. We call this "indirect" CP violation.
- Through the large part of the K_L which is K_2 due to CP violation in the decay amplitude. We call this "direct" CP violation.
- Through a two-photon intermediate state. This is higher order in α, but is CP conserving.

The question before us is the relative magnitude of these three contributions. Let us take them one at a time.

- We may estimate the contribution to the decay rate from the amplitude induced by "indirect" CP violation by using the identity:

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} \equiv B(K^+ \to \pi^+ e^+ e^-) \times$$
$$\frac{\tau_{K_L}}{\tau_{K^+}} \times \frac{\Gamma(K_1 \to \pi^0 e^+ e^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)} \times \frac{\Gamma(K_L \to \pi^0 e^+ e^-)_{\text{indirect}}}{\Gamma(K_1 \to \pi^0 e^+ e^-)} \quad . \tag{21}$$

Experimental values[45] of 2.7×10^{-7} and 4.2 may be inserted for the first two factors on the righthand side. The last factor is $|\epsilon|^2$ by the definition of what we mean by "indirect" CP violation in the convention where $A_0(K \to \pi\pi)$ is real. The third factor can be measured directly one day. For the moment, it is the subject of model-dependent theoretical calculations, with a value of 1 if the transition between the K and the π is $\Delta I = 1/2$. This is the case for the short-distance amplitude which involves a transition from a strange to a down quark. For $\Delta I = 3/2$, the corresponding value is 4. With both isospin amplitudes present and interfering, any value is possible.[46] Using a value of unity for this factor makes

$$B(K_L \to \pi^0 e^+ e^-)_{\text{indirect}} = 0.58 \times 10^{-11} \quad . \tag{22}$$

- The amplitude for "direct" CP violation comes from penguin diagrams with a photon or Z boson replacing the usual gluon and also from box diagrams with quarks (of charge $2e/3$), leptons (neutrinos) and W bosons as sides, shown in Fig. 6. For values of $m_t << M_W$, it is the "electromagnetic penguin" that gives the dominant short-distance contribution to the amplitude, which behaves like $\ell n(m_t^2/m_c^2)$. A full analysis, including QCD corrections, has been carried out in the case of six quarks,[47] building upon work done with four quarks.[48] The CP-violating amplitude from the "electromagnetic penguin" is summarized in the Wilson coefficient of the appropriate operator,

$$Q_7 = \alpha \left(\bar{s}\gamma_\mu(1-\gamma_5)d\right)\left(\bar{e}\gamma^\mu e\right) \quad ,$$

with the K–M factor $s_2 s_3 s_\delta$ factored out. This coefficient is $Im\widetilde{C}_7$ in the notation of Ref. 47, and typically has a value of order 0.1 to 0.2 for $m_t^2 <<$

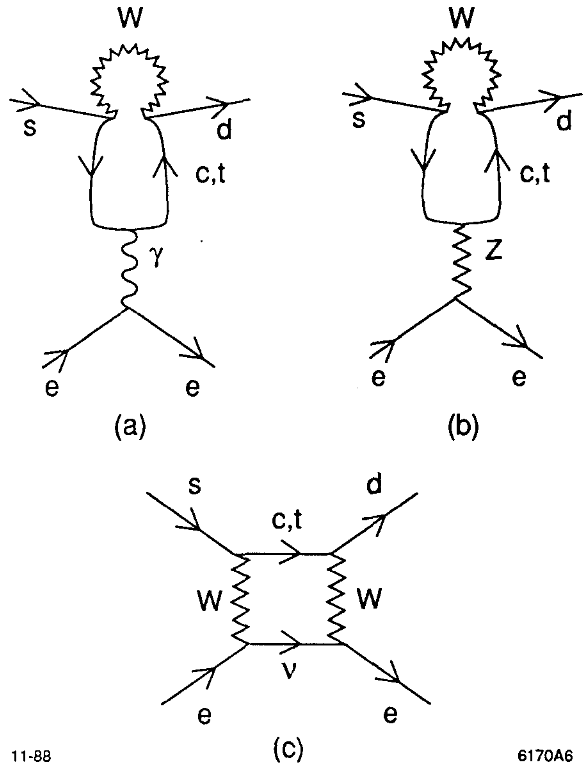

Figure 6: Three diagrams giving a short-distance contribution to the process $K \to \pi e^+ e^-$: (a) the "electromagnetic penguin"; (b) the "Z penguin"; (c) the "W box."

M_W^2, with QCD corrections included. The Z penguin and W box graph contributions are suppressed in amplitude by a power of m_t^2/M_W^2. This is no longer a suppression when we contemplate values of $m_t \sim M_W$. In fact, the "Z penguin" and "W box" contributions add another operator involving $\bar{e}\gamma_\mu\gamma_5 e$ and together become comparable to that of the "electromagnetic penguin" in this region. We may relate the hadronic matrix element of the relevant operator and the phase space to that which occurs in K_{e3} decay. Then we find that[49]

$$B(K_L \to \pi^0 e^+ e^-) \;=\; 1.0 \times 10^{-5}\,(s_2 s_3 s_\delta)^2\,[(Im\widetilde{C}_7)^2 + (Im\widetilde{C}_{7A})^2] \quad . \quad (23)$$

With QCD corrections and m_t between 50 and 200 GeV, the last factor ranges[49] between about 0.1 and 1.0, so that the corresponding branching ratio induced by this amplitude alone for $K_L \to \pi^0 e^+ e^-$ is around 10^{-11}. It appears that the contribution from the "direct" CP-violating amplitude is at least comparable to that from the "indirect" amplitude, reinforcing an earlier conclusion[47)] that they gave comparable contributions (at a time when the favorite values used for m_t were 15 and 30 GeV).

- The CP-conserving amplitude is interesting, if only for its checkered history of theoretical ups and downs. There are two invariant amplitudes[50)] for the CP-conserving subprocess $K_2 \to \pi^0\ \gamma\gamma$. If we take the momenta as p, p', q_1 and q_2, respectively, and define $x_{1,2} = p\cdot q_{1,2}/p\cdot p$, then they may be expressed in a gauge invariant way as:

$$\begin{aligned} < \pi\gamma\gamma|K_2 > = A(x_1,x_2)\big[q_2\cdot\epsilon_1\ q_1\cdot\epsilon_2 - q_1\cdot q_2\ \epsilon_1\cdot\epsilon_2\big]\ + \\ B(x_1,x_2)\big[p^2\ x_1x_2\epsilon_1\cdot\epsilon_2 + q_1\cdot q_2\ p\cdot\epsilon_1\ p\cdot\epsilon_2/p^2 \\ - x_1\ q_2\cdot\epsilon_1\ p\cdot\epsilon_2 - x_2\ q_1\cdot\epsilon_2\ p\cdot\epsilon_1\big] \end{aligned} \tag{24}$$

with $\epsilon_{1,2}$ the polarization vectors of the two photons. When joined with the QED amplitude for $\gamma\gamma \to e^+e^-$ to form the amplitude for $K_2 \to \pi^0 e^+e^-$, the contribution from the A amplitude gets a factor of m_e in front of it. This is not hard to understand, as the total angular momentum of the $\gamma\gamma$ system that pertains to the A amplitude is zero; the same is then true of the final e^+e^- system. However, the interactions, being electroweak, always match (massless) lefthanded electrons to righthanded positrons and *vice versa*, causing the decay of a J = 0 system to massless electrons and positrons to be forbidden. Hence, the factor of m_e in the overall amplitude for $K_2 \to \pi^0\ e^+e^-$, so that the A amplitude provides a negligible contribution. A corollary of this theorem applies to the $K_2 \to \pi^0\ \gamma\gamma$ amplitude calculated using current algebra low energy theorems. In the limit of vanishing pion four-momentum, a nonvanishing A amplitude is predicted. The factor of m_e found[51)] in the resulting amplitude for $K_2 \to \pi^0\ e^+e^-$ is then no surprise. On the other hand, the contraction of $\gamma\gamma \to e^+e^-$ with the B amplitude produces no such factor of m_e. B does, however, contain a coefficient with two more powers of momentum, and one might hope for its contribution to be suppressed by angular momentum barrier factors. Because of the extra powers of momentum, in chiral perturbation theory this amplitude is put in by hand with its coefficient not predicted. An order-of-magnitude estimate may be obtained by pulling out the known dimension-full factors in terms of powers of f_π, and asserting that the remaining coupling strength should be of order one.[50] The branching ratio for $K_2 \to \pi^0\ e^+e^-$ is then of order 10^{-14}. If so, the CP-conserving amplitude makes a negligible contribution to the decay rate. However, an old-fashioned vector dominance, pole model predicts[52] comparable A and B amplitudes and a branching ratio of order 10^{-11}, comparable to that from the CP-violating amplitudes. The applicability of such a model, however, can be challenged on the grounds that the low-energy theorems and Ward identities of the overall theory are not being satisfied.[53] The consistent implementation of vector dominance with all the other constraints leads to extra powers of momentum in some of the couplings, and possibly to a smaller prediction than in the old-fashioned model.

The dust is not yet settled. The burden is still on the theorists to show that the CP-conserving amplitude is very much smaller than the CP-conserving one, so that the experimental observation of the decay $K_L \to \pi^0\ e^+e^-$ could be interpreted as another example of CP violation in the neutral K system, this time with comparable effects from the mass matrix and the decay amplitude itself.

4. CP VIOLATION IN B DECAY

When we form a CP-violating asymmetry, we divide a difference between the rate for a given process and the rate for its CP conjugate by their sum:

$$\text{Asymmetry} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \quad . \tag{25}$$

If we do this for K decays, the decay rates for the dominant hadronic and leptonic modes all involve a factor of s_1^2, *i.e.,* essentially the Cabibbo angle squared. A

CP-violating asymmetry will then have the general dependence on K–M factors:

$$\text{Asymmetry}_{K\ Decay} \propto s_2 s_3 s_\delta \ . \tag{26}$$

The righthand-side is of order 10^{-3}. This is both a theoretical plus and an experimental minus. The theoretical good news is that CP-violating asymmetries in the neutral K system are naturally at the 10^{-3} level, in agreement with the measured value of $|\epsilon|$. The experimental bad news is that, no matter what the K decay process, it is always going to be at this level, and therefore difficult to get at experimentally with the precision necessary to sort out the standard model explanation of the origin of CP violation from other explanations.

Note also that because CP violation must involve all three generations, while the K has only first- and second-generation quarks in it (and its decay products only involve first-generation quarks), CP-violating effects must come about through heavy quarks in loops. There is no CP violation arising from tree graphs alone.

This is not the case in B decay (or B mixing and decay). First, the decay rate for the leading decays is very roughly proportional to s_2^2, which happens to be much smaller than the corresponding quantity (s_1^2) in K decay. But more importantly, we can look at decays which have rates that are K–M suppressed by factors of $(s_1 s_2)^2$ or $(s_1 s_3)^2$, just to choose two examples. By choosing particular decay modes, it is even possible to have asymmetries which behave like

$$\text{Asymmetry}_{B\ Decay} \propto s_\delta \ . \tag{27}$$

With luck, this could be of order unity! Note, though, that we have to pay the price of CP violation somewhere. That price, the product $s_1^2 s_2 s_3 s_\delta$, is given in the CP-violating difference of rates in Eq. (25). The K–M factors either are found in the basic decay rate, resulting in a very small branching ratio, or they enter the asymmetry, which is then correspondingly small. This is a typical pattern: the rarer the decay, the bigger the potential asymmetry. The only escape from this pattern comes from outside of K–M factors. A good example of this is provided by B–$\bar{B}$ mixing, which can be big because of a combination of the values of a hadronic matrix element and m_t, as well as a particular combination of K–M matrix elements.

The fact that asymmetries in K and B decay can be different by orders of magnitude is part and parcel of the origin of CP violation in the standard model. It "knows" about the quark mass matrices and can tell the difference between a b quark and an s quark. This is entirely different from what we expect in general from explanations of CP violation that come from very high mass scales, as in the superweak model or in left-right symmetric gauge theories. Then, all quark masses are negligible compared to the new, very high mass scale. Barring special provisions, there is no reason why such theories would distinguish one quark from another; we expect all CP-violating effects to be roughly of the same order, namely that already observed in the neutral K system.

The possibilities for observation of CP violation in B decays are much richer than for the neutral K system. The situation is even reversed, in that for the B system the variety and size of CP-violating asymmetries in decay amplitudes far overshadows that in the mass matrix.[54]

To start with the familiar, however, it is useful to consider the phenomenon of CP violation in the mass matrix of the neutral B system. Here, in analogy with the neutral K system, one defines a parameter ϵ_B. It is related to p and q, the coefficients of the B° and $\bar{B}^\circ$, respectively, in the combination which is a mass matrix eigenstate by

$$\frac{q}{p} = \frac{1-\epsilon_B}{1+\epsilon_B} \quad .$$

The charge asymmetry in $B^\circ\bar{B}^\circ \to \ell^\pm\ell^\pm + X$ is given by[55]

$$\frac{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) - \sigma(B^\circ\bar{B}^\circ \to \ell^-\ell^- + X)}{\sigma(B^\circ\bar{B}^\circ \to \ell^+\ell^+ + X) + \sigma(B^\circ\bar{B}^\circ \to \ell^-\ell^- + X)} = \frac{|\frac{p}{q}|^2 - |\frac{q}{p}|^2}{|\frac{p}{q}|^2 + |\frac{q}{p}|^2} \tag{28}$$

$$= \frac{\mathrm{Im}(\Gamma_{12}/M_{12})}{1+\frac{1}{4}|\Gamma_{12}/M_{12}|^2} \quad , \tag{29}$$

where we define $< B^\circ|H|\bar{B}^\circ >= M_{12} - \frac{i}{2}\Gamma_{12}$. The quantity $|M_{12}|$ is measured in

B–$\bar{B}$ mixing and we may estimate Γ_{12} by noting that it gets contributions from B° decay channels which are common to both B° and $\bar{B}^\circ$, *i.e.,* K–M suppressed decay modes. This causes the charge asymmetry for dileptons most likely to be in the ballpark of a few times 10^{-3}, and at best 10^{-2}. For the foreseeable future, we might as well forget it experimentally.

Turning now to CP violation in decay amplitudes, in principle, this can occur whenever there is more than one path to a common final state. For example, let us consider decay to a CP eigenstate, f, like ψK_s°. Since there is substantial B°–$\bar{B}^\circ$ mixing, one can consider two decay chains of an initial B° meson:

$$\begin{array}{lcl} B^\circ \to B^\circ & \searrow & \\ & & f \quad , \\ B^\circ \to \bar{B}^\circ & \nearrow & \end{array}$$

where f is a CP eigenstate. The second path differs in its phase because of the mixing of $B^\circ \to \bar{B}^\circ$, and because the decay of a $\bar{B}$ involves the complex conjugate of the K–M factors involved in B decay. The strong interactions, being CP invariant, give the same phases for the two paths. The amplitudes for these decay chains can interfere and generate nonzero asymmetries between $\Gamma(B^\circ(t) \to f)$ and $\Gamma(\bar{B}^\circ(t) \to f)$. Specifically,

$$\Gamma(\bar{B}^\circ(t) \to f) \ \sim \ e^{-\Gamma t} \left(1 - \sin[\Delta m \ t] Im\left(\frac{p}{q}\rho\right)\right) \tag{30a}$$

and

$$\Gamma(B^\circ(t) \to f) \ \sim \ e^{-\Gamma t} \left(1 + \sin[\Delta m \ t] Im\left(\frac{p}{q}\rho\right)\right) \quad . \tag{30b}$$

Here we have neglected any lifetime difference between the mass matrix eigenstates (thought to be very small) and set $\Delta m = m_1 - m_2$, the difference of the eigenstate masses, and $\rho = A(B \to f)/A(\bar{B} \to f)$, the ratio of the amplitudes, and we have used the fact that $|\rho| = 1$ when f is a CP eigenstate in writing Eqs. (30a) and (b).

From this, we can form the asymmetry:

$$A_{\text{CP Violation}} = \frac{\Gamma(B) - \Gamma(\bar{B})}{\Gamma(B) + \Gamma(\bar{B})} = \sin[\Delta m\ t] Im\left(\frac{p}{q}\rho\right) \quad . \tag{31}$$

In the particular case of decay to a CP eigenstate, the quantity $Im\left(\frac{p}{q}\rho\right)$ is given entirely by the K–M matrix and is independent of hadronic amplitudes. However, to measure the asymmetry experimentally, one must know if one starts with an initial B° or $\bar{B}^\circ$, *i.e.,* one must "tag."

We can also form asymmetries where the final state f is not a CP eigenstate. Examples are $B_d \to D\pi$ compared to $\bar{B}_d \to \bar{D}\bar{\pi}$; $B_d \to \bar{D}\pi$ compared to $\bar{B}_d \to D\bar{\pi}$; or $B_s \to D_s^+ K^-$ compared to $\bar{B}_s \to \bar{D}_s^- K^+$. These is a decided disadvantage here in theoretical interpretation, in that the quantity $Im\left(\frac{p}{q}\rho\right)$ is now dependent on hadron dynamics.

It is instructive to look not just at the time-integrated asymmetry between rates for a given decay process and its CP conjugate, but to follow the time dependence,[56] as given in Eqs. (30a) and (b). As a first example, Figs. 7, 8, and 9 show[57] the time dependence for the process $\bar{b} \to \bar{c}u\bar{d}$ (solid curve) in comparison to that for $b \to c\bar{u}d$ (dashed curve). At the hadron level, this could be, for example, $B_d \to \bar{D}^- \pi^+$ in comparison to $\bar{B}_d \to D^+\pi^-$. The direct process is very much K–M favored over that which is introduced through mixing, and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much greater than unity. Figures 7, 8, and 9 show[58] the situation for $\Delta m/\Gamma = 0.2$ (at the high end of theoretical prejudice before the ARGUS result, Ref. 26, for B_d mixing), $\Delta m/\Gamma = \pi/4$ (near the central value from ARGUS), and $\Delta m/\Gamma = 5$ (roughly the minimum value expected for the B_s in the three-generation standard model, given the central value of ARGUS for B_d). In none of these cases are the dashed and solid curves distinguishable within "experimental errors" in drawing the graphs. This is simply because $|\rho|$ is so large that even with "big" mixing the second path to the same final state has a very small amplitude, and hence not much of an interference effect.

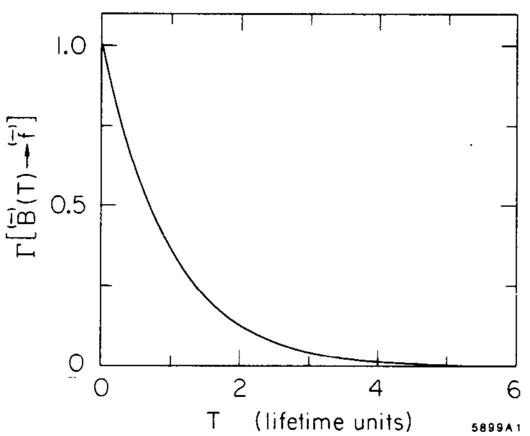

Figure 7: The time dependence for the quark level process $\bar{b} \to \bar{c}u\bar{d}$ (solid curve) compared to that for $b \to c\bar{u}d$ (dashed curve). At the hadron level this could be, for example, $B_d \to \bar{D}^-\pi^+$ compared to $\bar{B}_d \to D^+\pi^-$. $\Delta m/\Gamma = 0.2$.

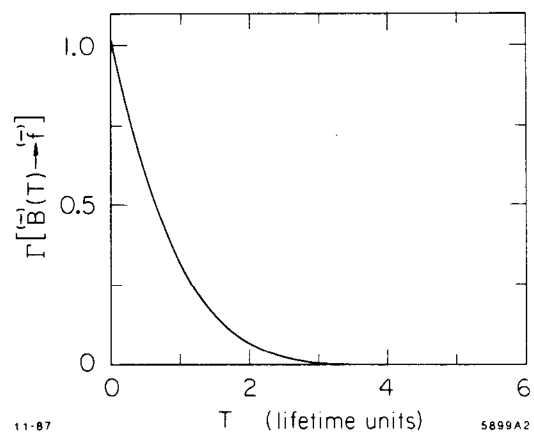

Figure 8: Same as Fig. 7, but with $\Delta m/\Gamma = \pi/4$.

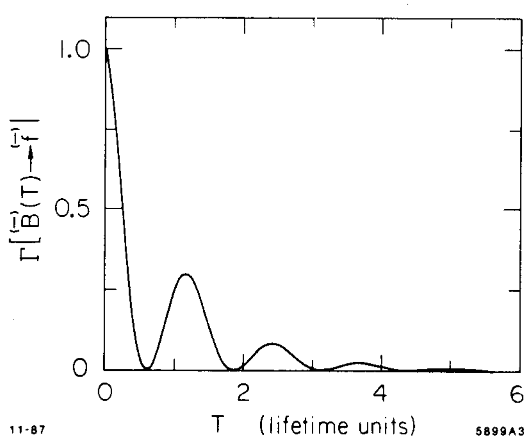

Figure 9: Same as Fig. 7, but with $\Delta m/\Gamma = 5$.

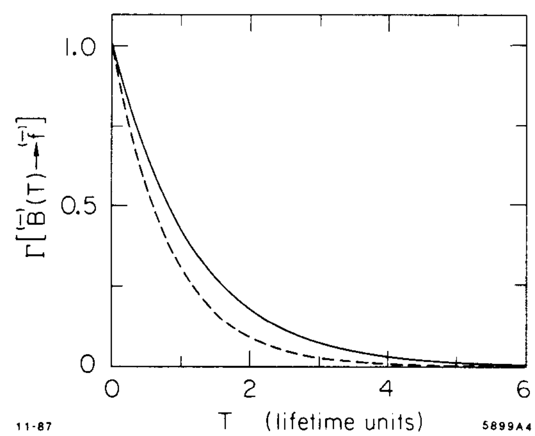

Figure 10: The time dependence for the quark level process $\bar{b} \to \bar{c}c\bar{s}$ (solid curve) compared to that for $b \to c\bar{c}s$ (dashed curve). At the hadron level this could be, for example, $B_d \to \psi K_s^o$ (dashed curve) compared to $\bar{B}_d \to \psi K_s^o$ (solid curve). (The curves are interchanged for the ψK_s^o final state because it is odd under CP.) $\Delta m/\Gamma = 0.2$.

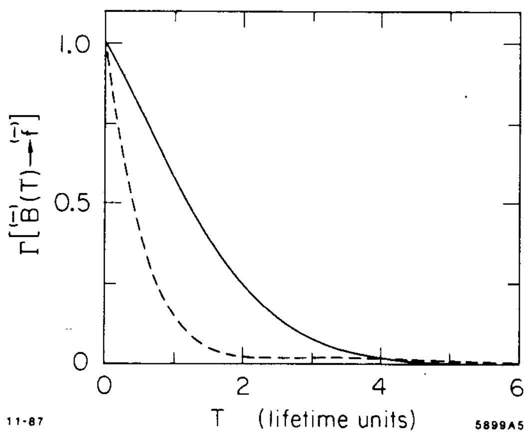

Figure 11: Same as Fig. 10, but with $\Delta m/\Gamma = \pi/4$.

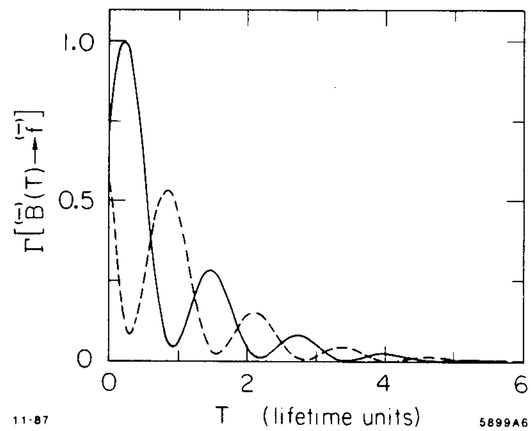

Figure 12: Same as Fig. 10, but with $\Delta m/\Gamma = 5$.

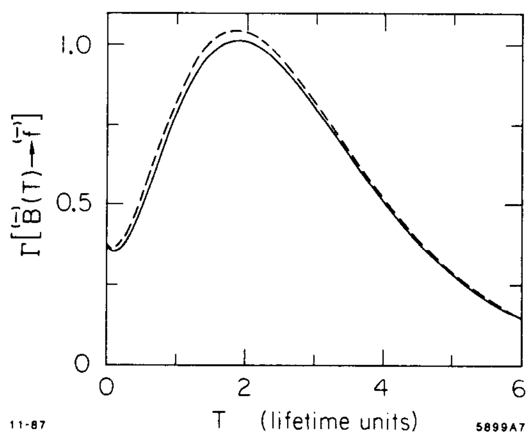

Figure 13: The time dependence for the quark level process $\bar{b} \to \bar{u}c\bar{d}$ *(solid curve) compared to that for* $b \to u\bar{c}d$ *(dashed curve). At the hadron level this could be, for example,* $B_d \to D^+\pi^-$ *compared to* $\bar{B}_d \to \bar{D}^-\pi^+$ $\Delta m/\Gamma = 0.2$.

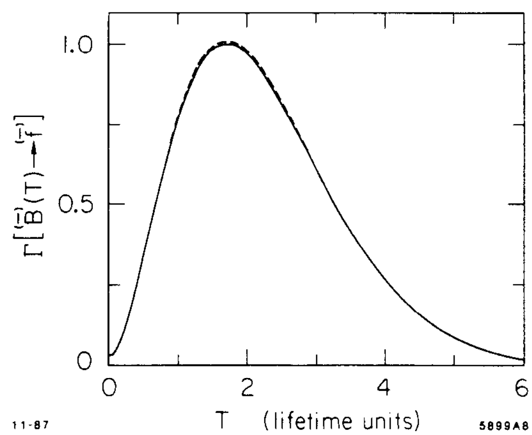

Figure 14: Same as Fig. 13, but with $\Delta m/\Gamma = \pi/4$.

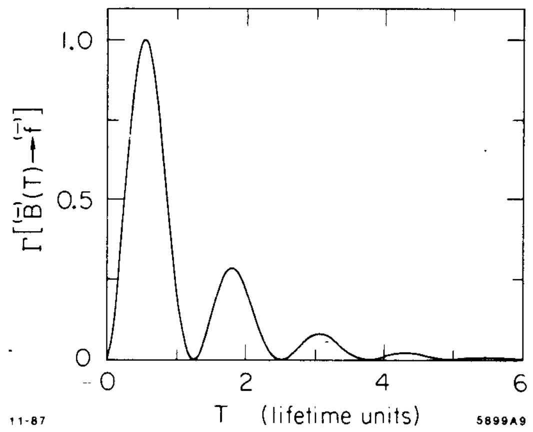

Figure 15: Same as Fig. 13, but with $\Delta m/\Gamma = 5$.

A much more interesting case is shown in Figs. 10, 11, and 12 for the time dependence at the quark level for the process $\bar{b} \rightarrow \bar{c}c\bar{s}$ (solid curve) in comparison to that for $b \rightarrow c\bar{c}s$ (dashed curve). At the hadron level, this could be, for example, B_d in comparison to $\bar{B}_d$ decaying to the same, (CP self-conjugate) final state, ψK_s^o. As discussed before, $|\rho| = 1$ in this case. The advantages of having $\Delta m/\Gamma$ for the B_d^o system as suggested by ARGUS (Fig. 11) rather than previous theoretical estimates (Fig. 10) are very apparent. When we go to mixing parameters expected for the B_s^o system (Fig. 12), the effects are truly spectacular.

Figures 13, 14, and 15 illustrate the opposite situation to that in Figs. 7–9: mixing into a big amplitude from a small one. We are explicitly comparing the quark level process $\bar{b} \rightarrow \bar{u}c\bar{d}$ (solid curve) to $b \rightarrow u\bar{c}d$ (dashed curve). At the hadron level, this could be, for example, $B_d \rightarrow D^+\pi^-$ in comparison to $\bar{B}_d \rightarrow \bar{D}^-\pi^+$. The direct process is very much K–M suppressed compared to that which occurs through mixing and hence the magnitude of the ratio of amplitudes, $|\rho|$, is very much less than unity. Here we have an example where too much mixing can be bad for you! As the mixing is increased (going from Figs. 13 to 15), the admixed amplitude comes to completely dominate over the original amplitude, and their interference (leading to an asymmetry) becomes less important in comparison to the dominant term.

A more likely example of the situation for B_s mixing is shown[59)] in Fig. 16(c). The oscillations are so rapid that even with a very favorable difference in the time dependence for an initial B_s versus an initial $\bar{B}_s$, the time-integrated asymmetry is quite small. Measurement of the time dependence becomes a necessity for CP-violation studies.

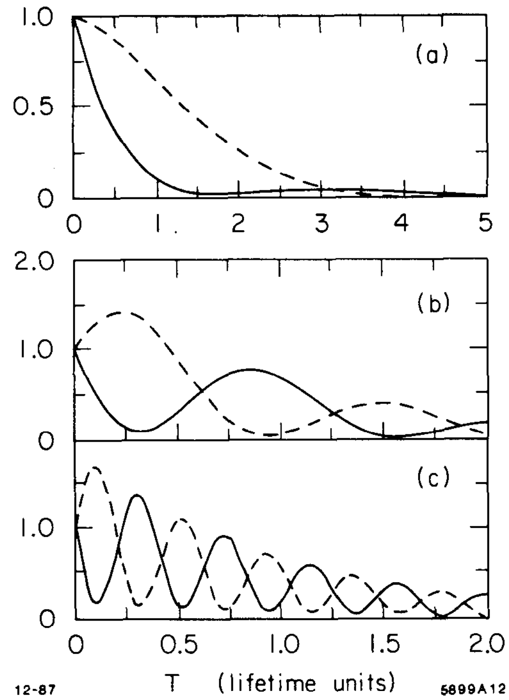

Figure 16: The time dependence for the quark level process $\bar{b} \to \bar{u}u\bar{d}$ *(dashed curve) compared to that for* $b \to u\bar{u}d$ *(solid curve). At the hadron level, this could be, for example,* $B_s \to \rho K_s^\circ$ *(solid curve) compared to* $\bar{B}_s \to \rho K_s^\circ$ *(dashed curve) (the curves are interchanged for the* ρK_s° *final state because it is odd under CP) for values of (a)* $\Delta m/\Gamma = 1$*, (b)* $\Delta m/\Gamma = 5$*, and (c)* $\Delta m/\Gamma = 15$*, from Ref. 58.*

A second path to the same final state could arise in several other ways besides through mixing. For example, one could have two cascade decays that end up with the same final state, such as:

$$B_u^- \to D^\circ K^- \to K_s^\circ \pi^\circ K^-$$

and

$$B_u^- \to \bar{D}^\circ K^- \to K_s^\circ \pi^\circ K^- \quad .$$

Another possibility is to have spectator and annihilation graphs contribute to the same process.[60] Still another is to have spectator and "penguin" diagrams interfere. This latter possibility is the analogue of the origin of the parameter ϵ' in neutral K decay, but as discussed previously, there is no reason to generally expect a small asymmetry here. Indeed, with a careful choice of the decay process, large CP-violating asymmetries are expected.

Note that not only do these routes to obtaining a CP-violating asymmetry in decay rates not involve mixing, but they do not require one to know whether one started with a B or $\bar{B}$, *i.e.,* they do not require "tagging." These decay modes are in fact "self-tagging" in that the properties of the decay products (through their electric charges or flavors) themselves fix the nature of the parent B or $\bar{B}$.

Even with potentially large asymmetries, the experimental task of detecting these effects is a monumental one. When the numbers for branching ratios, efficiencies *etc.* are put in, it appears that 10^7 to 10^8 produced B mesons are required to end up with a significant asymmetry (say, 3σ), depending on the decay mode chosen.[54] This is beyond the samples available today (of order a few times 10^5) or in the near future ($\sim 10^6$). The exciting prospect of being able to do this physics, but needing at least an order of magnitude more B's to have even a reasonable chance to see a statistically significant effect, has led to a series of studies (and even proposals) of high-luminosity electron-positron machines ("B factories"), of detectors for hadron colliders, and of the possibilities in fixed target experiments.[61]

I look at the next several years as being analogous to reconnaissance before a battle: We are looking for the right place and manner to attack CP violation in the B meson system. We need:

- Information on branching ratios of "interesting" modes down to the $\sim 10^{-5}$ level in branching ratio. For example, we would like to know the branching ratios for $B_d \to \pi\pi, p\bar{p}, K\pi, \psi K, D\bar{D}$ + three body modes+ ... and for $B_s \to \psi\phi, K\bar{K}, D\pi, \rho K$ ....
- Accurate $B\bar{B}$ mixing data, first for B_d, but especially verification of the predicted large mixing of B_s.
- A look at the "benchmark" process of rare decays, $B \to K\mu\bar{\mu}$.
- Experience with triggering, secondary vertices, tertiary vertices, "tagging" B versus $\bar{B}$, distinguishing B_u from B_d, distinguishing B_d from B_s ....

- Various "engineering numbers" on cross sections, x_F dependence, B versus $\bar{B}$ production in hadronic collisions ....

Many of these things are worthy, lesser goals in their own right, and may reveal their own "surprises." But the major goal is to observe CP violation. With all the possibilities, plus our past history of getting some "lucky breaks," over the next few years we ought to be able to find some favorable modes and a workable trigger and detection strategy. While the actual observation of CP violation may well be five or more years away, this is a subject whose time has come.

5. THE t QUARK

We define the t quark to be the partner of the lefthanded b quark in a weak isospin doublet. Such a partner must exist, *i.e.*, the b quark cannot be in a singlet, because of the nonzero front-back asymmetry exhibited in the reaction $e^+e^- \to \bar{b}b$ in experiments at PEP and PETRA.[62)] As this asymmetry is generated by an interference of the vector couplings of the photon with the axial-vector couplings of the Z, its magnitude can be interpreted in terms of the coupling, $g_{A,b}$, of the Z. This, in turn, is proportional to $I_{3,b}$. The data[62)] indicates that this is nonzero, and to nobody's surprise, consistent with the value $-1/2$ that corresponds to the lower member of a weak isospin doublet. Therefore, the b quark has nonzero weak isospin; it must share the same weak multiplet with another quark. An earlier argument[63)] to the same end had shown that if the b was in a weak singlet, then there would be flavor-changing neutral currents inducing processes like $b \to se^+e^-$ at tree level, in contradiction to experiment.

The discovery and elucidation of the properties of the t quark and its bound states is interesting from a number of aspects, aside from just being the completion of the task of finding all the fermions of the three-generation standard model. In accordance with the reasons for studying heavy flavor physics which we outlined at the beginning of these lectures:

- The t quark mass and the K–M matrix elements connecting it to other quarks will allow us to check for possible relations between masses and mixing angles which follow from various proposals advanced until now. Perhaps it will suggest a new one.

- Its properties could also be an indication for physics beyond the standard model, for example by indicating mixing with a fourth generation or by decaying into a charged scalar.

- Its mass is a key input into many of the one-loop calculations which we have discussed and an important part of the present uncertainty in theoretical predictions for the magnitude of these amplitudes will thereby be eliminated.

- The top quark is a very useful tool for the discovery of other particles. For example, a heavy Higgs boson should have a prominent decay to $t\bar{t}$ and toponium, if light enough, should have an appreciable decay to a photon plus a light Higgs boson.

- The weak decays of the top quark will involve the electroweak and the strong interactions in a regime where the strong coupling is truly small. This simpler context may well permit a quantitative, perturbative understanding of weak nonleptonic decays of the t quark, which is interesting in itself, and which can be extrapolated back to lower mass scales and a better understanding of the b and c quark decays.

- The strong interaction spectroscopy of toponium probes the effective nonrelativistic potential between quarks as the distance between them becomes very small. This is a region where we might hope perturbative QCD would give us information, with the potential eventually behaving as $-(4/3)\alpha_s/r$ as $r \to 0$ from one gluon exchange. In this case, the spectroscopy is sensitive to other possible short distance effects due to new physics as well.

What is our present knowledge on the mass of the top quark? First, the t quark mass is constrained to be above 27.3 GeV from TRISTAN,[64] above 44 GeV from UA1,[65] and above about 50 GeV from theoretical considerations[25,31] based on the ARGUS result[26] for B–$\bar{B}$ mixing. In fact, the B–$\bar{B}$ mixing results, interpreted within the standard model and with nominal values for the relevant K–M and hadronic matrix elements, would have one entertain t quark masses in the vicinity of 100 GeV.

On the other end, there is an upper limit on the t quark mass from the ρ parameter, defined as

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \quad . \tag{32}$$

The value of ρ is constrained to be unity when $SU(2)_L$ symmetry is exact. If we do not define $\cos\theta_W$ by imposing[66] Eq. (32) with $\rho = 1$ and the physical W and Z masses, then the value of ρ will deviate from unity due to the one-loop contributions of quark-antiquark pairs to the vector boson masses, once the quarks in an $SU(2)_L$ doublet no longer have the same mass. In particular, the most relevant doublet is that consisting of the t and b quarks. Since the analysis[67,68] of the experimental data shows that[67]

$$\rho = 0.998 \pm 0.009 \quad ,$$

the splitting between the t and b quark masses cannot be arbitrarily large. Specifically, Ref. 67 finds that $m_t < 180$ GeV. While one may quibble with some of the input or analysis that leads to either the lower to upper limits, a range of about 40 to 200 GeV now seems to be the relevant hunting ground for t quarks.

Where and how can we expect to find such a t quark? At $p\bar{p}$ colliders, the lowest-order production processes are from creation of a W followed by the decay $W \to t\bar{b}$ (if the t is light enough) and from gluon-gluon fusion, $gg \to t\bar{t}$. At the CERN collider the total cross section is about 1 nb for $m_t = 50$ GeV, and comes dominantly from W decay.[65] At the TEVATRON collider, the cross sections are bigger—something like 3 nb for a 50 GeV top mass, but dropping to roughly 100 pb when $m_t = 100$ GeV.[69] The data planned for collection in the coming year should suffice for detection of the t quark if its mass is less than about 100 GeV. To get truly spectacular cross sections we can go to the SSC, where even a 200 GeV top quark, the limiting case, is expected to be pair produced at the 10 nb level.[70]

At electron-positron colliders, the asymptotic cross section (for $\sqrt{s} >> m_t, M_Z$) is

$$\sigma(e^+e^- \to t\bar{t}) \sim 2.1\sigma_{\mathrm{pt.}}(s) \quad ,$$

where $\sigma_{\mathrm{pt.}}(s) = 4\pi\alpha^2/3\ s$ is the cross section for $e^+e^- \to \mu^+\mu^-$ with just the one photon intermediate state. If m_t is as low as 40 GeV, so that $Z \to t\bar{t}$, the electron-positron annihilation cross section jumps at the Z peak to more than a nanobarn (more than a hundred times $\sigma_{\mathrm{pt.}}$ at that energy).

The actual detection of the t quark is somewhat similar for both high-energy $p\bar{p}$ and e^+e^- colliders. One looks for one t quark to decay in a semileptonic mode, (*i.e.* the real or virtual W decays to $\ell\bar{\nu}_\ell$), while the other decays in a hadronic mode (*i.e.* quark jets). The semileptonic decay can result in an isolated lepton with both high momentum and high transverse momentum with respect to the associated quark jet, giving a distinctive signature. One tries to "reconstruct" the W's, and then the t and $\bar{t}$. The demand that both the t and the $\bar{t}$ in the event yield the same mass is a nontrivial constraint which can be used to reject background.

How does the t quark decay? In the three-generation standard model, a great deal of the physics of t decays is fixed. Let us consider the semileptonic decay of t to b:

$$t \to b + W^+ \to b\ e^+\nu_e \quad ,$$

with the W^+ being either real for virtual, depending on the t mass. The tree-level width, for any value of m_t, is given by[71)]

$$\Gamma(t \to b\ e^+\nu_e) =$$

$$\frac{G_F^2 m_t^5}{24\pi^3} \int_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4\ |\vec{Q}\,|}{(Q^2 - M_W^2)^2 + M_W^2\Gamma_W^2} \left[2|\vec{Q}\,|^2 + 3\ Q^2(1 - \frac{Q_0}{m_t})\right] \quad , \tag{33}$$

where Γ_W is the total width of the W and the integration variable Q^2 is the square of the four-momentum which it carries, with the associated quantities $Q_0 = (m_t^2 +$

$Q^2 - m_b^2/2m_t)$ and $|\overrightarrow{Q}|^2 = Q_0^2 - Q^2$. In general, the righthand side of Eq. (33) should contain the square of the relevant K–M matrix element, $|V_{tb}|^2$. In the case of three generations, this is 0.997, *i.e.*, equal to one to high accuracy.

In the limit that $m_t << M_W$, the momentum dependence of the W propagator can be neglected and the expression simplifies to

$$\begin{aligned}\Gamma(t \to b\, e^+\nu) =& \frac{G_F^2 m_t^5}{24\pi^3} \int_0^{(m_t-m_b)^2} dQ^2\ |\overrightarrow{Q}| \Big[2|\overrightarrow{Q}|^2 + 3\ Q^2(1 - Q_0/m_t)\Big] \\ =& \frac{G_F^2 m_t^5}{6\pi^3} \int_0^{(m_t-m_b)^2} dQ^2\ |\overrightarrow{Q}|^3 \\ =& \frac{G_F^2 m_t^5}{192\pi^3} \Big[1 - 8\Delta^2 + 8\Delta^6 - \Delta^8 - 24\Delta^4\ ln\Delta\Big]\ ,\end{aligned} \tag{34}$$

where $\Delta = m_b/m_t$.

In the other limit, where $m_t >> M_W$, we may integrate over the Breit–Wigner for producing a "real" W, and using

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\pi\sqrt{2}}\ , \tag{35}$$

rewrite Eq. (33) as

$$\Gamma(t \to b + W \to b\, e^+\nu_e)\ =\ B(W \to e\nu)\cdot \frac{G_F|\overrightarrow{Q}|}{2\pi\sqrt{2}}\Big[2|\overrightarrow{Q}|^2 + 3M_W^2(1 - \frac{Q_0}{m_t})\Big]\ , \tag{36}$$

where now $Q^2 = M_W^2$ so that $Q_0 = (m_t^2 + M_W^2 - m_b^2/2m_t)$ and $|\overrightarrow{Q}|^2 = Q_0^2 - M_W^2$. For very large values of m_t, the width in Eq. (36) behaves as $B(W \to e\nu)\cdot G_F m_t^3/8\pi\sqrt{2}$, to be contrasted with Eq. (34).

The finite width of the W determines the behavior of the rate as we cross the threshold for producing a real W. Once we are several full widths of the W above threshold, the much larger width given in Eq. (36) for producing a "real" W

dominates the total t decay rate. This is seen in Fig. 17, where[72] the $t \to be^+\nu_e$ decay rate is plotted versus m_t. The dashed curve is the result in Eq. (36) which would hold for production of a real, infinitely narrow W, while the solid curve gives the result of integrating Eq. (33) numerically.[73] For smaller values of m_t, the width is less than $G_F^2 m_t^5/192\pi^3$ because of the finite value of m_b [here taken to be 5 GeV; see Eq. (34)], but then is enhanced by the W propagator as m_t increases. Even for values of $M_t \approx 50$ GeV, the finite mass of the W results in a $\approx 25\%$ increase in the t decay width over the value calculated with the point (infinite M_W) Fermi interaction; The exact result quickly matches that for an infinitely narrow W once we are several W widths above threshold. The finite W width simply provides a smooth interpolation as the decay rate jumps by over an order of magnitude in crossing the threshold.

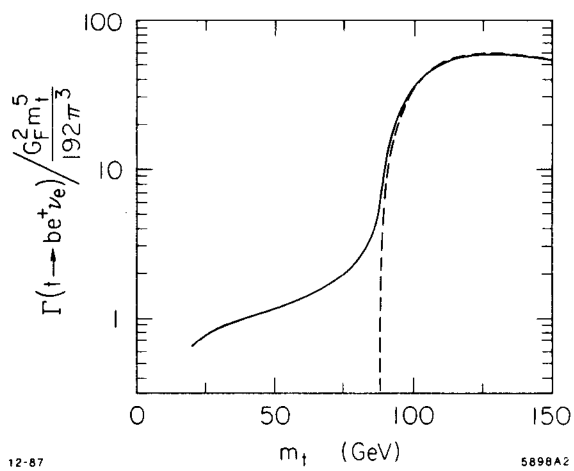

Figure 17: $\Gamma(t \to b\, e^+\nu_e)/(G_F^2 m_t^5/192\pi^3)$ *as a function of* m_t *from the full expression in Eq. (1) for* $M_W = 83$ GeV, $\Gamma_W = 2.25$ GeV *and* $m_b = 5$ GeV *(solid curve), and from Eq. (4) for decay into a real, infinitely narrow W (dashed curve).*

When m_t is in the present experimentally acceptable range, the rate for weak decay of the constituent t quarks within possible hadrons becomes comparable with that for electromagnetic and strong decays. Weak decays become a major fraction of, for example, the decays of the $J^P = 1^-$ toponium ground state, and even for the $T^*(t\bar{q})$ vector meson, weak decays can dominate the radiative magnetic dipole transition to its hyperfine partner, the T meson $J^P = 0^-$ ground state.[74]

By the stage that $m_t = 100$ GeV, the total decay width of the t quark is ≈ 80 MeV. The weak decays of the constituent t and $\bar{t}$ quarks completely overshadow the usual electromagnetic and strong interaction decays of toponium. In fact, with a slightly higher mass the t decays so fast that it disappears before hadronic bound states can form.[75)]

We now examine the transition region where $m_t \approx m_b + M_W$ in more detail.[72)] Ordinarily, the weak transition $t \to s$ is suppressed relative to $t \to b$ by the ratio of the relevant K–M matrix elements squared, $|V_{ts}|^2/|V_{tb}|^2 \approx 1/500$. However, we have seen that $\Gamma(t \to b\ e^+\nu_e)$ increases sharply as m_t crosses the W threshold, changing from being proportional to G_F^2 to being proportional to G_F. Thus we expect $\Gamma(t \to se^+\nu_e)$ to be enhanced relative to $\Gamma(t \to be^+\nu_e)$ when m_t lies between the two thresholds: $M_W + m_s < m_t < M_W + m_b$. The question is whether the threshold enhancement "wins" over the K–M suppression.

To examine this quantitatively, we consider the ratio of the widths with the K–M factors divided out:

$$\frac{\Gamma(t \to b\ e^+\nu_e)/|V_{tb}|^2}{\Gamma(t \to se^+\nu_e)/|V_{ts}|^2} \quad .$$

Either well below or well above threshold for a "real" W, this ratio should be near unity. For an infinitely narrow W, the denominator is strongly enhanced, but the numerator is not, when $M_W + m_s < m_t < M_W + m_b$. The ratio indeed drops dramatically near $t \to s + W$ threshold, as shown in Fig. 18, for $\Gamma_W = 0.0225$ GeV (dotted curve) and even for $\Gamma_W = 0.225$ GeV (dashed curve). However, the expected W width of 2.25 GeV (solid curve) smears out the threshold effect over a mass range that is of the same order as $m_b - m_s$, and gives only a modest dip (to ≈ 0.6) in the ratio. This is hardly enough to make $t \to s$ comparable to $t \to b$.

We now turn to the possible exclusive decay channels of hadrons which contain a t quark. In decays of heavy flavor mesons the branching ratios for typical exclusive channels scale like $(f/M_Q)^2$, where f is a meson decay constant (like f_π or f_K), of order 100 MeV, and M_Q is the mass of the heavy quark. For D mesons individual channels have branching ratios of a few percent; for B mesons they are

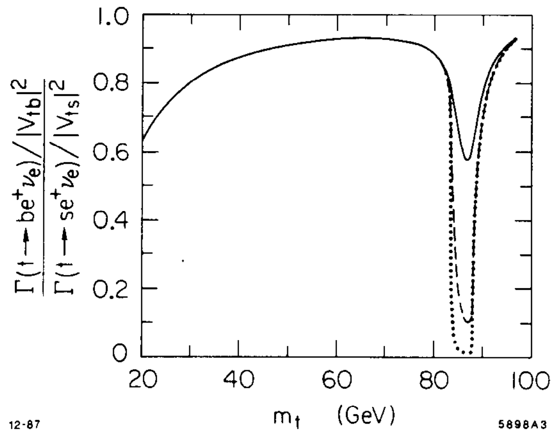

Figure 18: The ratio of decay rates with K–M factors taken out, $\left(\Gamma(t \rightarrow b\, e^+\nu_e)/|V_{tb}|^2\right)/\left(\Gamma(t \rightarrow s e^+\nu_e)/|V_{ts}|^2\right)$ *with* $m_b = 5$ GeV *and* $m_s = 0.5$ GeV *and* Γ_W *equal to fictitious values of* 0.0225 GeV *(dotted curve) and* 0.225 GeV *(dashed curve), and the expected* 2.25 GeV *(solid curve).*

roughly ten times smaller; and for T (or T^*) mesons they should be a hundred or more times smaller yet. It should be possible to treat T decays in terms of those of the constituent t quark, $t \rightarrow b + W^+$, with the b quark appearing in a b jet not so different from those already observed at PEP and PETRA.

There is one possible exception to these last statements, and that is when $m_t \approx m_b + M_W$, the situation we are examining in more detail here. In this case there is a premium on giving as much energy to the W as possible, *i.e.,* keeping as far above threshold for "real" W production as possible, and hence on keeping the invariant mass of the hadronic system containing the b quark small. Then we expect the T and T^* to decay dominantly into a few exclusive channels: a "real" W plus a B or a "real" W plus a B^*.

Furthermore, this is one place where the use of the nonrelativistic quark model is *a priori* well justified. The t quark and final W are very heavy. When $m_t \approx m_b + M_W$, the final heavy b quark is restricted to have a few GeV or less of kinetic energy if the W is to be as "real" as possible. The accompanying light quark in the T hadron is very much a spectator which simply becomes part of the final B or B^* hadron.

We need only match up the matrix elements of the weak currents taken between hadron states (and expressed in terms of form factors) with the matrix elements of those same currents taken between quark states in the appropriate spin and flavor configurations found in the hadrons. The details of all this are found in Ref. 72.

Within the scenario of discovery of the top quark at a hadron collider, it would be useful to have several handles on the value of m_t. An indirect method would be to measure a quantity in top decays which depends strongly on the top mass. For m_t in the vicinity of $M_W + m_b$, such a quantity is the ratio of the production of longitudinal W's to that of transverse W's in top decay.

The decay widths into longitudinal and transverse W's are defined by decomposing the numerator of the W propagator as

$$g_{\mu\nu} - Q_\mu Q_\nu / M_W^2 = \sum_\lambda \epsilon_\mu(\lambda)\epsilon_\nu^\star(\lambda) = \epsilon_\mu^{(+)}\epsilon_\nu^{(+)\star} + \epsilon_\mu^{(0)}\epsilon_\nu^{(0)\star} + \epsilon_\mu^{(-)}\epsilon_\nu^{(-)\star} \quad , \qquad (37)$$

where the superscripts give the helicity of the W, whether virtual or real. In calculating the t decay rate in Eq. (33), we define $\Gamma_L = \Gamma^{(0)}$, originating from W's with helicity zero, and $\Gamma_T = \Gamma^{(+)} + \Gamma^{(-)}$, originating from W's with helicity ± 1. Separating in this way the portions of Eq. (33) that originated from longitudinal and transverse W's, we find

$$\Gamma_L = \frac{G_F^2 m_t^5}{24\pi^3} \int_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4 \, |\overrightarrow{Q}|}{(Q^2 - M_W^2)^2 + M_W^2 \Gamma_W^2} \left[2|\overrightarrow{Q}|^2 + Q^2 (1 - \frac{Q_0}{m_t}) \right] \quad , \qquad (38a)$$

$$\Gamma_T = \frac{G_F^2 m_t^5}{24\pi^3} \int_0^{(m_t-m_b)^2} dQ^2 \frac{M_W^4 \, |\overrightarrow{Q}|}{(Q^2 - M_W^2)^2 + M_W^2 \Gamma_W^2} \left[2\, Q^2 (1 - \frac{Q_0}{m_t}) \right] \quad . \qquad (38b)$$

In the case $m_t << M_W$, the integrals can be done with the result

$$\frac{\Gamma_L}{\Gamma_T} = 2 \quad . \qquad (39)$$

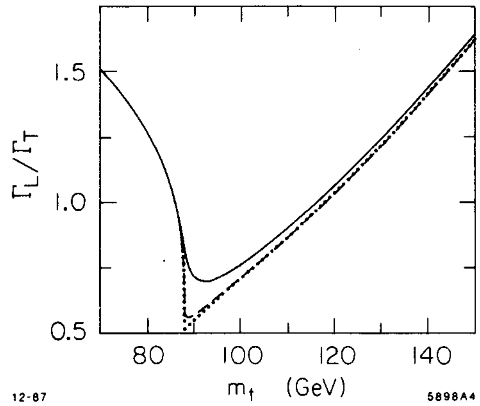

Figure 19: The ratio Γ_L/Γ_T of $t \to b+W \to b\ e^+\nu_e$ decay widths into longitudinal compared to transverse W's as a function of m_t for Γ_W equal to fictitious values of 0.0225 GeV *(dotted curve) and* 0.225 GeV *(dashed curve), and the expected* 2.25 GeV *(solid curve).*

Sufficiently far above the W threshold, we need only calculate the relative production of longitudinal and transverse real W's:

$$\frac{\Gamma_L}{\Gamma_T} = \frac{1}{2} + \frac{m_t|\vec{Q}_W|^2}{E_b M_W^2} \quad . \tag{40}$$

Precisely at threshold, where we have an s-wave decay with two transverse W polarization states and one longitudinal state, $\Gamma_L/\Gamma_T = 1/2$. The value of Γ_L/Γ_T near the threshold is shown in Fig. 19 for $\Gamma_W = 0.0225$ GeV (dotted curve), 0.225 GeV (dashed curve), and the expected 2.25 GeV (solid curve). In this case, we see that even for the expected value of Γ_W the ratio varies rapidly with m_t, especially just below the threshold.

The ratio of longitudinal to transverse W's is reflected in the angular distribution of the electrons[76] from its decay. With the final b quark direction as a polar axis,

$$\frac{d\Gamma}{d\cos\theta} = 1 + \alpha \cos^2\theta \quad , \tag{41}$$

where

$$\alpha = \frac{\Gamma_T - \Gamma_L}{\Gamma_T + \Gamma_L} \quad . \tag{42}$$

Thus, a measurement of α gives a value for Γ_L/Γ_T and indirectly a value for m_t. In particular, α changes rapidly and becomes positive only a few GeV below the threshold; this may provide a useful lower bound on m_t.

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