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HADRONIC PRODUCTION OF HEAVY-QUARK BOUND STATES*

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ABSTRACT

We consider the hadronic production of heavy quark bound states $(Q\overline{Q})$ and also associated production of pairs of hadrons containing heavy quarks $(\overline{Q}q + \overline{q}Q)$. Calculations of total cross sections and of x_F spectra are presented for both the J/ψ and Υ. Processes involving fusion of quarks as well as amalgamation of gluons from the beam and target hadrons are important, but in both cases the production of $J^{PC} = 1^{--}$ bound states proceeds mainly via P-wave intermediate states. Processes involving gluons from the initial hadrons dominate at high energies for all beams and targets, and at all energies for proton and K^+ beams where quark fusion cannot proceed using only valence quarks.

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I. INTRODUCTION

There has been much discussion in print and elsewhere about mechanisms for the hadronic production of particles carrying new quantum numbers.[1-7] We here continue this discussion, and examine several of the proposed mechanisms in an attempt to synthesize several viewpoints. We find that no one process can dominate at all energies for all choices of initial particles and that given the amount of numerical work that can be done in a parameter free or parameter insensitive way, we feel it is possible to assert this statement fairly strongly.

Before proceeding, we should note that most of our remarks will be phrased in terms of charmonium and charmed mesons, but they can be straightforwardly extended to heavier states such as the Υ. Also, the belief that strong interaction dynamics are governed by QCD ("Quantum Chromodynamics") or something like it is implicit in most of our calculations. Indeed, from a theoretical viewpoint, part of the importance of studying heavy particle production in hadronic collisions resides in its ability to probe otherwise dormant degrees of freedom required by the strong interaction field theory, such as the gluonic components of the original hadrons.

One of our motives in reexamining hadronic J/ψ production was to reassess the importance of one particular model--the gluon-cascade or gluon fusion model[1,2]--in the light of recent data[8] showing much larger cross sections for $\bar{p}$ induced J/ψ production than for p induced production at $\sqrt{s}$ = 8.75 GeV. The data does indicate that other processes are important, at least at low energies. Our present study shows that processes involving quark fusion do play a dominant role in $\bar{p}p \rightarrow J/\psi + X$, but only at low energies. At higher energies the relative importance of this mechanism is small compared to processes involving gluon components within the initial hadrons.

A fairly complete list of the mechanisms that have been suggested is the following:

$gg \rightarrow$ intermediate state $\rightarrow J/\psi + \gamma$

$q\bar{q} \rightarrow$ intermediate state $\rightarrow J/\psi + \gamma$

$q\bar{q} \rightarrow J/\psi$

$c\bar{c} \rightarrow J/\psi$

$gg \rightarrow c\bar{c}$

$q\bar{q} \rightarrow g \rightarrow c\bar{c}$.

The symbols on the left-hand side represent the active constituents of the initial hadrons, and q stands for light quarks, c for charm quarks, g for gluons and γ for photons. We will use $\mathcal{N}$ to denote nucleons.

We begin by discussing $c\bar{c}$ $J^{PC} = 1^{--}$ bound states of heavy quarks production in Section II. We shall argue that the first two processes listed above are the dominant ones, with the intermediate states mainly those ^3P_J $c\bar{c}$ states which lie below threshold for decay into $c\bar{q} + \rho\bar{c}$. For the first process, all the necessary quantities, such as the parton wave function in the initial hadrons, the gluon coupling constant, and the heavy quark mass, are known or can be thought to be known. Hence an absolute calculation of this process is possible. For the second process listed above, one must estimate the coupling of the P-states to light quarks. A way of doing so that gives a reasonable result is discussed in Section II, and then the relative importance of the two processes can be calculated. The result is that the gluon-gluon mechanism dominates at all energies when the initial hadrons are $p\mathcal{N}$ or $K^+\mathcal{N}$ (i.e., where the quark annihilation process cannot proceed using only valence quarks), while for $\bar{p}\mathcal{N}$, $\pi^{\pm}\mathcal{N}$, or $K^-\mathcal{N}$, the quark annihilation process dominates at low energies and the gg dominates at high energies. For $\bar{p}\mathcal{N}$, our calculation shows the two processes are equal at $\sqrt{s}$ = 12 GeV for the case of J/ψ production. Our model is compared to the by now fairly copious data on $p\mathcal{N} \to J/\psi + X$ and $pp \to J/\psi + X$, and the results are very satisfactory.

In Section III we extend the discussion to the Upsilon specifically.

Section IV is a short section devoted to charm meson production. Calculations like those presented here have been reported recently.[9] Our motive is simply to emphasize which subprocess dominates, and also to provide results for a variety of incident particles. We shall approximate the cross section for charm meson production by that for free $c\bar{c}$ production and the only processes that contribute are the last two listed above. There are no quantities that are difficult to estimate and we find again that gluon-gluon processes dominate at all energies for p or K^+ beams on nucleons, but that for $\bar{p}$, $\pi^{\pm}$, or K^- beams there is a crossover with $q\bar{q}$ dominating at low energies and gg dominating at high energies. The crossover occurs at $\sqrt{s}$ = 30 GeV for $\bar{p}\mathcal{N}$, somewhat higher than for J/ψ production.

Section V contains comments and conclusions.

II. PRODUCTION OF J/ψ

The class of models that we study is distinguished by having color singlet states explicitly produced at each stage of the calculation. The key calculational step for most of the processes in this class lies in estimating accurately the coupling of light quarks or gluons with heavy quark bound states. In what follows we shall discuss in some detail the following processes (see Fig. (1)).

(a) gg → intermediate state → J/ψ + a

(b) $q\bar{q}$ → intermediate state → J/ψ + a

(c) $q\bar{q}$ → J/ψ

(d) $c\bar{c}$ → J/ψ

(a) The process gg → P → ψ + a, where P stands for a P wave $c\bar{c}$ bound state and a is usually a photon, has been previously considered by us[1] and by others.[2] The contributions from this process can be large because the fraction of the initial hadron's momentum carried by the gluons is large. Perturbation theory arguments, which ought to be applicable for large masses, suggest that two gluon amalgamation is favored over three gluon amalgamation processes. It is therefore more likely to produce charge conjugation C = + states than C = - states; the J/ψ is produced in the subsequent decay of the C = + state. The branching ratio for decay into J/ψ will be large only for those states which are below threshold for charm production and will be biggest when the C + states are the ^3P_j's. The limited number of intermediate states makes this model more attractive.

The total cross section for the process can be written in the form

$$\sigma_{gg}(AB \to J/\psi + X) = \frac{8\pi^2}{M_P^3} \Gamma_{eff} (Pgg) F_{AB}(\tau) \qquad (2.1)$$

where $\tau = M_P^2/s$ and $F_{AB}(\tau)$ is the "excitation function,"

$$F_{AB}(\tau) = \tau \int_\tau^1 \frac{dx}{x} f_g^A(x) f_g^B(\tau/x) \qquad (2.2)$$

and

$$\Gamma_{eff}(Pgg) = \sum_{j=0}^{2} (2j + 1) B(^3P_j \to 2g)\Gamma(^3P_j \to J/\psi + \gamma),$$

where B is a branching ratio. Also, the x_F spectrum of P-states is given by

$$\frac{d\sigma_{gg}}{dx_F} (AB \to {}^3P_j + X) = \frac{8\pi^2}{M_P^3} (2j + 1)\Gamma(^3P_j \to gg) \frac{x_+ f_g^A(x_+) x_- f_g^B(x_-)}{x_+ + x_-} \qquad (2.3)$$

where x_F is the longitudinal momentum fraction of the ^3P_j in the c.m. and

$$x_\pm = \frac{1}{2}\left[\sqrt{x_F^2 + 4\tau} \pm x_F\right] \qquad (2.4)$$

The J/ψ spectrum differs from the one calculated above only by a smearing due to the decay $^3P_j \to J/\psi + \gamma$:

$$\frac{d\sigma}{dx_F} (AB \to J/\psi + x) = \frac{8\pi^2}{M_P^3} \Gamma_{eff} \frac{M_P^2}{M_P^2 - M_\psi^2} \qquad (2.5)$$

$$\times \int_{x_L}^{x_U} \frac{dx}{x^2 + \tau} f_g^A(x) f_g^B(\tau/x)$$

with

$$x_L = \frac{M_\psi^2}{M_P^2} x_U = \frac{1}{2}\left[\left(x_F^2 + \frac{4M_\psi^2}{s}\right)^{1/2} + x_F\right] . \qquad (2.6)$$

Γ_{eff} is amenable to calculation in the usual sort of model that treats charmonium as two quarks moving non-relativistically in a potential. One has[11,12]

$$\Gamma(^3P_j \to J/\psi + \gamma) = \frac{4}{3}\,\alpha e_Q^2\, \omega^3 |\langle J/\psi|\vec{r}|^3P_j\rangle|^2$$

$$\Gamma(^3P_o \to gg) = \left[\frac{2}{3}\right] 9\, \alpha_g^2\, m_Q^{-4} |\frac{d\phi}{dr}(o)|^2 = \frac{15}{4}\,\Gamma(^3P_2 \to gg) \qquad (2.7)$$

$$\Gamma(^3P_1 \to gg) \simeq \frac{1}{15}\,\Gamma(^3P_o \to gg)$$

where ϕ is the radial wave function of the P-states, normalized by $\int|\phi(r)|^2 dr = 1$ and ω is the energy of the photon. We use the potential

$$V(r) = -\frac{4}{3}\frac{\alpha_g}{r} + \lambda r \qquad (2.8)$$

where $\lambda = 0.2029\ (\mathrm{GeV})^2$, $\alpha_g = 0.19$, and obtain

$$\Gamma(^3P_o \to J/\psi + \gamma) = 0.263\ \mathrm{MeV}$$

$$\Gamma_{eff}(P\ gg) = 2.76\ \mathrm{MeV} \qquad (2.9)$$

The matrix elements and $d\phi/dr$ are taken from a computer solution to the above potential.[13,14]

We shall next estimate the ratio of process (b) $[q\bar{q} \to P \to J/\psi + \gamma]$, to process (c) $[q\bar{q} \to J/\psi]$ and following this, estimate the couplings for process (c). This is easier than estimating the couplings for process (b) directly. The processes begin identically with $q\bar{q}$ states, but one goes directly to the J/ψ, which is a C = - state, and the other via an intermediate C = + state, which will again be dominated by ^3P_j states. The coupling of $q\bar{q}$ to a C = + $c\bar{c}$ state is presumably mediated by two gluons, while the coupling to a C = - state is mediated by three gluons. Hence the ratio of $q\bar{q}$ producing J/ψ via the intermediate state compared to $q\bar{q}$ producing J/ψ directly may be estimated by:

$$\frac{\Gamma\left({}^1S_o \to q\bar{q}\right)}{\Gamma\left(J/\psi \to q\bar{q}\right)} \simeq \frac{12}{5}\left(\frac{\pi}{\alpha_g}\right)^2 \simeq 100 \tag{2.10}$$

The 12/5 is a color factor, and this ratio is estimated from the simplest connected Born diagram. If we also assume

$$\frac{\Gamma\left({}^3P_o \to q\bar{q}\right)}{\Gamma\left({}^1S_o \to q\bar{q}\right)} \simeq \frac{\Gamma\left({}^3P_o \to gg\right)}{\Gamma\left({}^1S_o \to gg\right)} = 36\left|\frac{\phi_p{}'(o)}{m_Q\phi_s(o)}\right|^2 \simeq 0.5 \tag{2.11}$$

we conclude that the ratio of the two processes is

$$\begin{aligned} &\frac{\sum_j (2j+1)\,\Gamma\left({}^3P_j \to q\bar{q}\right)B\left({}^3P_j \to \psi + \gamma\right)}{\Gamma(J/\psi \to q\bar{q})} \\ &\simeq \sum_j \frac{12}{5}\,(2j+1)\left(\frac{\pi}{\alpha_g}\right)^2 B(P \to \psi + \gamma) \simeq 450 \end{aligned} \tag{2.12}$$

Here $B(P \to J/\psi + \gamma)$ is typically $\frac{1}{6}$, and there are 9 intermediate states. Hence the process with the intermediate state dominates even the quark-initiated process.

The partial width $\Gamma(J/\psi \to q\bar{q})$ can be estimated by comparing $\sigma(e^+e^- \to KK^*$ or $\rho\pi)$ on and off the J/ψ resonance. Off resonance the ρπ production proceeds with a virtual photon coupling to a $u\bar{u}$ or $d\bar{d}$ which is subsequently dressed into the ρπ. On resonance, it is a J/ψ that couples to the $u\bar{u}$ or $d\bar{d}$ and we shall ignore for the moment decays of the J/ψ via photons. When we take the ratio the strong final state corrections cancel, leaving us the ratio of the desired coupling to the quark electric charge,

$$\frac{\int \sigma(e^+e^- \to J/\psi \to \rho\pi)\ d(\sqrt{s})}{\sigma(e^+e^- \to \rho\pi)_{\text{off resonance}}} = \frac{M_\psi^2}{2g_\psi^2}\ \frac{\pi}{\Gamma_{tot}(J/\psi)} \left[\frac{g_{\psi uu} + g_{\psi dd}}{e_u + e_d}\right]^2 \qquad (2.13)$$

The strong J/ψ coupling[15] is $g_\psi^2/4\pi \simeq 11.5$. The numerator of the LHS is $(6\pi^2/s)\Gamma(J/\psi \to e^+e^-)B(J/\psi \to \rho\pi)$, which is experimentally measured.[16] For the denominator of the LHS we estimate 0.18 nb (using a measurement[17] at $\sqrt{s}$ = 2 GeV of about 2 nb, and scaling to 3 GeV assuming an s-dependence which is 1/s times a monopole form factor with a mass of 1 (GeV)). Then given $B\left(J/\psi \to \rho^\pm\pi^\mp\right) \simeq 1.0$ percent,[16] and assuming that the u and d couplings are the same, we get

$$g_{\psi q\bar{q}}^2/4\pi \simeq 10^{-6} \qquad (2.14)$$

This gives

$$\Gamma(J/\psi \to q\bar{q}) = 1\ \text{keV} \qquad (2.15)$$

Incidentally, the total $q\bar{q}$ contribution to the J/ψ width is the above summed over q = u, d, s, which gives only a fraction of the total width.[18] This is no surprise since the decay begins in the three-gluon mode, and one should not expect that the first step in the conversion of gluons into physical hadrons is the conversion into a quark pair; more likely, at least one of the individual gluons will fragment.

For the process involving $q\bar{q} \to J/\psi$ directly, the above width plus that due to $q\bar{q} \to J/\psi$ via photons is what we call Γ_{eff}. For the dominant process in this category, $qq \to P \to J/\psi + \gamma$, we can estimate

$$\Gamma_{eff}(P\ q\bar{q}) \simeq 600 \text{ keV} \tag{2.16}$$

and

$$\Gamma(P \to q\bar{q}) \simeq 70 \text{ keV}$$

The cross sections for this process are given by

$$\frac{d\sigma}{dx_F} \approx \frac{8\pi^2}{M_P^3} \frac{\Gamma_{eff}(P\ q\bar{q})}{x_+ + x_-} \frac{1}{9} \sum_{q\leftrightarrow\bar{q}} x_+ f_q^A(x_+)\ x_- f_{\bar{q}}^B(x_-) \tag{2.17}$$

(the smearing can be put in easily) and

$$\sigma_{tot} = \frac{8\pi^2}{M_P^3} \Gamma_{eff}(P\ q\bar{q}) \frac{1}{9} \sum_{q\leftrightarrow\bar{q}} \int_\tau^1 \frac{dx}{x} f_q^A(x)\ f_{\bar{q}}^B(\tau/x) \tag{2.18}$$

We now treat what seems to be the least likely process (d) [$c\bar{c} \to J/\psi$]. <u>A priori</u>, one should not dismiss the possible presence of a c-quark ocean in the wave function of ordinary hadrons.[6] In fact, at infinite q^2 a flavor symmetric ocean is expected, but it is far from being the case when probing hadrons at $q^2 \simeq M_\psi^2$. Work has been done,[19] using QCD and experimental data on electron scattering, to extract the distribution functions of the c-quark, and they are expectedly small. If the coupling $g^2(\psi c\bar{c})4\pi$ is of order unity, then the $c\bar{c} \to J/\psi$ process is numerically insignificant. This receives support from experiments which see no additional muons in conjunction with the[20] J/ψ; if $c\bar{c} \to J/\psi$ were significant, then a large fraction of the events should have extra muons.

We will therefore calculate the sum of processes (a) and (b).

Before proceeding to calculate the cross sections let us comment about the distribution functions used for nucleons and mesons. We begin with the quark distribution function for the proton. Following McElhaney and Tuan[21] we have chosen for the valence quarks:

$$u(x) = \frac{1.742}{\sqrt{x}} (1 - x)^3 (1 + 2.3x) + s(x)$$

$$d(x) = \frac{1.107}{\sqrt{x}} (1 - x)^{3.1} + s(x) \tag{2.19}$$

where s(x) is the distribution for each of the "ocean" quarks, $\bar{u}$, $\bar{d}$, $\bar{s}$ and s. We take this from recent work[22] to be

$$s(x) = \frac{0.26}{x} (1 - x)^9 \tag{2.20}$$

This distribution has been chosen because when used in the Drell-Yan calculation it gives a good fit to the dimuon continuum up to $M_{\mu\mu} \simeq 14$ GeV. Observe for a moment that these distributions at x = 0.1, where there is data at various Q^2, give a value of $\nu W_2 = 0.38$. This can be compared with another often quoted[23] ocean quark distribution $xs(x) = 0.15 (1 - x)^7$. The exponent, 7, is given by the counting rules[24] and the corresponding value for $\nu W_2(x = 0.1) = 0.33$. This value is in agreement with the SLAC data, while the former value, 0.38, is in agreement with the higher Q^2 FNAL experiments. We can say that the ocean distribution (2.20) simulates some of the effects of asymptotic freedom in attempting to fit the data better.

We have assumed SU(3) symmetry for the light quarks in the ocean only because a definite choice was needed. Any contributions to our cross sections from strange quarks are a small part of the total, which is mainly due to gluonic terms or valence-valence up and down quark-antiquark annihilation. It matters little what we choose for the strange quark distribution.

The neutron distributions are obtained from the proton ones by the isospin rotation ($u \leftrightarrow d$). We will do most of our calculations for I = 0 nuclei; therefore our nucleon $\mathcal{N}$ distribution will be

$$f^{\mathcal{N}}(x) = \frac{1}{2}\left(f^{p}(x) + f^{n}(x)\right) \qquad (2.21)$$

For the meson distribution, we have replaced data with art. The valence and ocean quark distribution used for most of our calculations is

$$v_{\pi}(x) = \frac{0.75}{\sqrt{x}}(1 - x) + s_{\pi}(x)$$

$$s_{\pi}(x) = \frac{0.1}{x}(1 - x)^{5} \qquad (2.22)$$

The exponents are gotten from counting rules,[24,25] and the normalization for the ocean distribution was obtained from the canonical argument[26] that the hadron-hadron scattering at high energy is governed by the ocean distribution at x = 0. This means that the cross section for hadron-hadron scattering is directly proportional to the number of "wee" quarks in the two hadrons, or

$$\lim_{x \to o} \frac{s_{\pi}(x)}{s(x)} = \frac{\sigma_{tot}(\pi\mathcal{N})}{\sigma_{tot}(p\mathcal{N})} \simeq \frac{2}{3} \qquad (2.23)$$

Since in the absence of data we are using the counting rule distributions for the mesons, we obtain the 0.1 above. The gluon distributions are untested for both mesons and nucleons. We shall use the counting rules to estimate $f_g(x)$;[3,27] i.e.,

$$f_g(x) = \frac{C_n}{x}(1 - x)^{n} \qquad (2.24)$$

with n = 5 for nucleons and n = 3 for mesons. For nucleons, experiments do tell us the overall normalization. Deep inelastic electron-nucleon data imply that only about 50 percent of the momentum is carried by the quarks. The quark distributions above are consistent with this data (more precisely, they give 52.6 percent of the momentum carried by quarks). The remaining 50 percent (or 47.4 percent) of the momentum must be carried by gluons. Hence $C_n = (n + 1)/16$. For the mesons, there is no data, so that the fraction of momentum carried by quarks is what we calculate from the above distributions. Amusingly, it comes out to exactly 50 percent so that the gluon normalization is the same for pions as for nucleons. Our results for J/ψ production are shown in the next group of figures. Proton-induced J/ψ production is always dominated by gg, and we show σ_{tot} and $d\sigma/dy|_{y=0}$ and the available data points in Fig. (2). For the gluon distribution quoted above, σ_{tot} varies with energy as $\ln(s/M^2)$ for large s. The agreement with the data is fairly impressive.

The $\bar{p}\mathcal{N} \to J/\psi + X$ total cross section is shown in Fig. (3) along with the gg and $q\bar{q}$ contributions separately. Quark annihilation dominates at low energies while gluon amalgamation dominates at high energy. The crossover occurs at $\sqrt{s} = 12$ GeV. The ratio of proton induced to antiproton induced J/ψ production is plotted in Fig. (4). There is one experimental point here,[8] at $\sqrt{s} = 8.75$ GeV,

$$\frac{\sigma(p\mathcal{N} \to J/\psi + x)}{\sigma(\bar{p}\mathcal{N} \to J/\psi + x)} = 0.15 \pm 0.08 \tag{2.25}$$

Pure gluon amalgamation models of course give unity for this ratio, but one should also note that pure $q\bar{q}$ gives a ratio well below the data.

Figure (5) shows our calculations for meson-nucleon J/ψ production. The calculated curves for $\pi^{\pm}\mathcal{N}$ and $K^{+}\mathcal{N}$ are identical and data for $\pi\mathcal{N}$ is also plotted. The cross section for $K^{-}\mathcal{N}$ lies lower. This feature is similar to the case of $\bar{p}p$ and pp: if the beam has valence anti-up or anti-down quarks, then quark annihilation can give significant contribution, especially at lower energies. Otherwise, gg production dominates.

It is also straightforward to calculate the x_F-spectrum, and we shall compare our calculations to some of the available data. Figure (6) shows data from the Chicago-Princeton group[28] at $\sqrt{s}$ = 20.6 GeV along with our calculation. The quark annihilation process is important to the spectrum in x_F even though it does not give a significant contribution to the total cross section. This is because it falls much more slowly in x_F, so that at large x_F it dominates and makes the spectrum flatter than it would be if only gg contributed to the cross section. Figures (6b) and (6c) show similar plots for some other energies and beams. For Fig. (6c) (only) we have adjusted the normalization in order to be able to check the shape of the curve.

Another class of models that we shall discuss only briefly involves the creation of free heavy quarks. The bound state production is calculated by integrating the free quark cross section over a small energy range around the mass of the bound state.[4] The reasoning behind these models relies upon the argument that if the energy of the $c\bar{c}$ pair is below the threshold for charm production there is little else the pair can do but produce bound states. The produced $c\bar{c}$ pair is in general not in a color singlet state. It is widely assumed that the color "evaporates" by soft gluon emission. However one should bear in mind that the matrix element for a transition from a color non-singlet to a color singlet state by soft gluon emission is proportional to the gluon energy, i.e., negligibly small in the infrared limit. The demonstration is straightforward and will be given in the Appendix. In addition, it should be noted that duality-like arguments, such as the integration referred to above, may be misleading when one considers a specific final state because one should not forget the interplay between the produced heavy quarks and the partons from the outgoing hadrons. This interplay can give more than enough energy to make $D\bar{D}$, ... states accessible, and makes us uneasy about calculations based on duality and color evaporation.

III. THE UPSILON

We shall devote this section to the upsilon while also making some remarks concerning still heavier quarks.

It seems well accepted that the Υ is a $^3S_1$ bound state $Q\bar{Q}$ of a new heavy quark. An important difference from psions is that more than two $^3S_1$ states and more than one set of ^3P states may be stable against OZI allowed strong decay. After examining the spectrum of the upsilon system, we calculate its production following the J/ψ calculations of the previous section. A brief partial report of this work has been given previously.[30,31]

The potential. We shall work first with the potential

$$V(r) = -\frac{4}{3}\frac{\alpha_g}{r} + \lambda r \tag{3.1}$$

which works quite well for the psion system. Asymptotic freedom effects will decrease α_g to 0.15, but we shall continue with $\lambda = 0.2029\ (\mathrm{GeV})^2$. The fact that λ is the same as for psions follows from a belief that it is a property of the binding force and not of the specific quarks being bound. This can be shown to result from various kinds of bag or string models. In the string model $\lambda^{-1} = 2\pi\alpha'$, with α' the universal Regge slope;[32] in the bag model, the linear potential arises for large quark separations from the interplay of the bag pressure and color electrostatic energy and both terms are quark-mass independent.[33]

For the charm system, the quark mass is fixed by the $J/\psi - \psi'$ mass difference and the widths of the J/ψ. The widths have not yet been measured for the Υ, so that we shall determine the quark mass as well as the spectrum following Eichten and Gottfried.[34] The mass of the lowest $^3S_1$ state can be written as

$$M = 2m_Q + E_o(m_Q) + \Delta(m_Q) \qquad (3.2)$$

where $E_o(m_Q)$ is the lowest Schrodinger energy for the above potential. We approximate the dependence of Δ upon m_Q by

$$\Delta(m_Q) = \frac{m_c}{m_Q}\,\Delta(m_c) \qquad (3.3)$$

and explicit calculation for the charm system shows that $\Delta(m_c) = -0.22$ GeV. We can now apply Eq. (3.2) to the Υ and find that $m_Q = 4.60$ GeV. Then the lowest-lying meons states $Q\bar{u}$ or $u\bar{Q}$ are at

$$M(Qu) = m_D - m_c + m_Q + \frac{3}{4}\left(1 - \frac{m_c}{m_Q}\right)\delta = 5.19 \text{ GeV}$$

where $\delta = M_{D*} - M_D$. There are $Q\bar{Q}$ S-states at 9.41, 9.85, and 10.17 GeV and P states at 9.71, 10.05, and 10.34 GeV. The margin by which the 3^3P state is bound is smaller than the uncertainties of the calculation.

The experiment[35] shows an Υ - Υ' splitting of about 600 MeV, similar to the J/ψ - ψ' splitting and larger than that predicted using the above for psions. Possible ways out have been suggested.[36] One method

for charmonium. Possible ways out have been suggested.[36] One method consists in adjusting the parameters α_g, and λ and m_c in (3.1) so as to fit both the charmonium system and get the experimentally observed Υ' - Υ splitting. That this can be accomplished is easy to understand; the contribution to the mass difference arising from the linear potential decreases with m_Q while the Coulomb term increases with m_Q. A value of $\alpha_g = 0.42$ is needed, and other parameters and masses are given in Table I. The second alternative used was to choose a purely logarithmic potential $V(r) = \lambda \log (r/b)$, which gives mass splittings[37] that are independent of

m_Q. This potential fits surprisingly well the known properties of charmonium, and will no doubt be verified or negated soon since it predicts two S states in the region of 4.1 GeV while the linear plus coulomb potentials predict only one. The masses gotten from this potential are also in Table I. A drawback common to both alternatives is that m_c is fairly low so that a non-relativistic treatment is less secure for charmonium, although quite good for the Υ's. The low m_c mass is also responsible for the relatively high calculated $(Q\bar{u})$ meson mass, which in turn leads to having four rather than three bound P-states. This result should perhaps be taken with a grain of salt.

Whatever the potential, the fine structure splittings are much smaller for the Υ system than the psion system. For example, if we have a pure linear potential one can work out scaling laws to show that the overall level spacing (e.g., Υ to Υ') goes like $m_Q^{-1/3}$ but the fine structure splitting goes like $m_Q^{-5/3}$. The corresponding spacings in a logarithmic potential are independent of m_Q and proportional to m_Q^{-1} respectively. Hence we do not need to worry about the fine structure splittings of the P-states in the Υ system or its effect upon the phase space for E1 decay; this effect is considerable for the psions.

Calculation of total rates. We shall now discuss the $p\mathcal{N}$ case, where gluons dominate the production of Υ's, and following the experiment that observed the Υ, we will quote $\Sigma B_\mu\, d\sigma/dy|y=0$. The cross section for producing an $n\,^3S$ state (the Υ, Υ', Υ'', ...) from a given set of $m\,^3P_j$ states is

$$\left.\frac{d\sigma}{dy}\right|_{y=0} = \frac{8\pi^2}{M_P^3}\ \Gamma_{eff}(mP \to nS)\ \tau\ f_g^2\ (\sqrt{\tau}) \tag{3.4}$$

where

$$\Gamma_{eff}(mP \to nS) = \sum_j (2j + 1) \frac{\Gamma(m\ ^3P_j \to 2g)\ \Gamma(m\ ^3P_j \to n\ ^3S_1 + \gamma)}{\Gamma_{tot}(m\ ^3P_j)} \quad (3.5)$$

The above just considers T's produced by one intermediate step and does not include, for example, a T produced by making a T' followed by $T' \to T + 2\pi$. These will be discussed shortly. There is a formidable variety of decays within the T system, as illustrated in Fig. (7).

The observed sum of T production, $\Sigma B(T^i \to \mu\bar{\mu})\sigma(T^i)$, is independent of the cascade rates $\Gamma(T^i \to T^i + x)$ provided that the ratio of widths, $\Gamma(T^i \to \mu\bar{\mu})/\Gamma(T^i \to 3g)$ is independent of i. This proviso follows from the lowest-order formulas,

$$\Gamma(T^i \to \mu\bar{\mu}) = \frac{4\alpha^2}{M_T^2} e_Q^2 |\phi_i(0)|^2$$

and

$$\Gamma(T^i \to 3g) = \frac{16}{9\pi}(\pi^2 - 9)\frac{5}{18}\frac{{\alpha_g}^2}{M_T^2}|\phi_i(0)|^2 \quad (3.6)$$

A proof of our assertion is straightforward and we illustrate is considering just the two levels T and T' and the decay $T' \to T + x$. Let the T' widths be

$$\Gamma'_\mu = \Gamma(T' \to \mu\bar{\mu})$$

$$\Gamma'_h = \Gamma(T' \to 3g) + R\Gamma(T' \to \mu\bar{\mu})$$

$$\Gamma'_c = \Gamma(T' \to T + x) \quad (3.7)$$

denote the corresponding branching ratios by B'_μ, B'_n, B'_c, and use analogous notation for the T. Then if σ_o and σ'_o are the T and T' production cross sections before any cascade, we have

$$\rightarrow \quad B_\mu \sigma + B'_\mu \sigma' = B_\mu(\sigma_o + B'_c \sigma'_o) + B'_\mu \sigma'_o = B_\mu \sigma_o + B_{eff}\sigma'_o \tag{3.8}$$

Here

$$\begin{aligned} B_{eff} &= B'_\mu + B_\mu B'_c \\ &= (3\Gamma'_\mu + \Gamma'_h + \Gamma'_c)^{-1}(3\Gamma'_\mu + \Gamma'_h)^{-1}[\Gamma'_\mu(3\Gamma_\mu + \Gamma_h) + \Gamma_\mu \Gamma'_c] \\ &= B_\mu \end{aligned} \tag{3.9}$$

where the last result follows using $\Gamma'_\mu \Gamma_h = \Gamma_\mu \Gamma'_h$. In general we have

$$\sum_i B(T^i \rightarrow \mu\mu)\sigma(T^i) = B(T \rightarrow \mu\mu)\sum_i \sigma_o(T^i) \tag{3.10}$$

To extend the theorem to show that the summed cross section is independent of the cascade decays $P^i \rightarrow P^j + x$ would require that the branching ratio of the P-states into upsilons plus photons be independent of i. We have no argument to show this is true in general. However, it is roughly true numerically, and we will ignore the P-state cascades.

The calculations are now simple. The E1 widths and gg widths for the first potential are given in Table II, and the summed cross section times branching ratio is 0.45 pb for $e_Q = -1/3$ and 2.59 pb for $e_Q = 2/3$. (The numbers increase by about 60% if we use n = 4 instead of n = 5 in the gluon distribution, and by another 60% if we use n = 3.) The experimental value[35] is 0.25 pb. Clearly e = -1/3 is favored.

The corresponding cross sections are 0.67 pb for $e_Q = -1/3$ and 3.83 pb for $e_Q = 2/3$ for the modified linear plus coulomb potential; for the logarithmic potential we have 0.66 pb and 3.87 pb.[38]

Decays among the T's. In order to predict the relative rate of T' and T production, observed to be 0.07 pb/0.18 pb ≃ 0.4, one needs to know the branching ratios among the Υ states, the most important being $B(\Upsilon' \to \Upsilon + X)$.

However the rate $\Upsilon' \to \Upsilon + X$ cannot be calculated. The other main decays of the Υ' into leptons, hadrons, or P-states plus photons can be calculated or, perhaps better, scaled from the measured widths of the ψ'. (One can readily examine the Schrodinger equation with linear potential and see how various quantities will change if m_Q is changed. For example, lengths $\propto m_Q^{-1/3}$.) To estimate $\Gamma(\Upsilon' \to \Upsilon + X)$, we can crudely suppose that the rate is proportional to just the phase space (perhaps taking into account the deviations from phase space decay observed in the ψ' case). Since the dominant channel is $\Upsilon' \to \Upsilon + 2\pi$, we have three-body phase space, which is proportional to the fifth power of the available energy. Thus the rate estimates are very sensitive to the $\Upsilon - \Upsilon'$ mass difference. If the mass difference were 440 MeV, a low value of $B(\Upsilon' \to \Upsilon + X) \simeq 0.30$ would result. But taking the experimental mass difference yields a huge branching ratio.

To consider the problem more fully, let us again consider just the Υ' and Υ levels, supposing for a moment that the ^3P state is not bound. Then

$$\frac{B(\Upsilon' \to \mu\mu)\sigma(\Upsilon')}{B(\Upsilon \to \mu\mu)\sigma(\Upsilon)} = \frac{(1 - B_c)\sigma'_o}{\sigma_o + B'_c\sigma'_o} \qquad (3.11)$$

where σ_o are again the cross sections before any cascade and $B_c = B(\Upsilon' \to \Upsilon + X)$ Our calculations show $\sigma'_o = 0.7\ \sigma_o$, so that $B \simeq 0.24$ is needed to fit the experiment. This is a low value of B, and other effects, such as including the $2P \to 1P$ decays and the 3P state will enhance the Υ more than the T'. However, we should emphasize again that the absolute rate of $T' \to T + x$ is not understood, and the problems of the high T' rate can be resolved if the $T' \to T\pi\pi$ amplitude is a property decreasing function of m_Q (e.g., if it decreased like m_Q^{-2}, the rates could be understood nicely).[39]

Rates for Pions. Proceeding along the same lines as in Section II, we can calculate the $\pi\mathcal{N} \to T + x$. Quark annihilation, via P wave intermediate states, does in fact dominate the upsilon production cross section for $P_{lab} \simeq 450$ GeV/c. This effect is relevant for FNAL energies. In particular, for $P_{lab} \simeq 225$ GeV, we have

$$\frac{\sum_i B_\mu(T^i)\sigma(\pi\mathcal{N} \to T^i + x)}{\sum_i B_\mu(T^i)\sigma(p\mathcal{N} \to T^i + x)} \simeq 14 \qquad (3.12)$$

In Fig. (8) we show the x_F distribution. For completeness, in Figs. (9) and (10) we quote the total cross sections, as a function of energy for $\mathcal{N}\mathcal{N}$ and $\pi\mathcal{N}$. The total as well as separate contributions from the two dominant graphs (Figs. (1a) and (1b), with $J/\psi \to T$) are given.

Scaling Laws. We feel that it is instructive to note that fairly reliable "back of the envelope" calculations can be done by using

$$B_\mu\ \sigma(\text{"onium"},\tau) = N_P\left(\frac{e_Q}{e_c}\right)^4\left(\frac{m_Q}{m_c}\right)^{-14/3} \times B_\mu\ \sigma(J/\psi,\tau) \qquad (3.13)$$

where N_P stands for the number of bound P waves. Eq. (3.13) results from Eq. (3.1), using the universality of the "excitation function" AB (we scale $p\mathcal{N}$ production from $p\mathcal{N}$ for J/ψ, and similarly for other beams and targets), and also uses the approximation

$$\Gamma_{eff} \simeq \Gamma(P \to S + \gamma) \tag{3.14}$$

This width scales like $e_Q^2\, m_Q^{-5/3}$ for a linear potential (or $e_Q^2\, m_Q^{-1}$ for a logarithmic potential), B_μ scales like e_Q^2. Combining these facts with Eq. (2.1) one is led to Eq. (3.13). The exponent -14/3 in Eq. (3.13) becomes -4 for the logarithmic potential.

Applying Eq. (3.13) gives the result that for a linear potential the Υ production (times branching ratio) is 1500 times smaller than J/ψ production at the same τ, for $e_Q = 1/3$. The available experimental data observes Υ at τ = 0.118. At this τ, $B_\mu\, \sigma(J/\psi) \simeq 0.5$ μb, in good agreement with the Υ cross section for $e_Q = -1/3$.

These recipes can be useful for making qualitative estimates for the production of still heavier $Q\bar{Q}$ bound states.

Heavier Quarks. Still heavier quarks may be found. If the quark in the Υ is the "bottom" quark with charge -1/3, then one expects a "top" quark with charge 2/3 to appear also (perhaps at m_Q = 15 GeV!).

Sooner or later, weak decays of the heavy quarks will become significant.[40] The weak decay $Q\bar{Q} \to (q l \nu)\bar{Q}$ will have the width

$$\Gamma_{wk} \simeq \frac{G_F^2}{192\pi^3}\, m_Q^5 \tag{3.15}$$

and we can approximate the $\mu\bar{\mu}$ decay via a virtual photon by

$$\Gamma_{\mu\mu} \simeq \frac{\alpha^2 \lambda e_Q^2}{m_Q} \tag{3.16}$$

which is the result for a purely linear potential. The above two widths are the same when

$$m_Q^6 = \frac{192\pi^3}{G_F^2}\, \alpha^2 \lambda e_Q^2 \tag{3.17}$$

or

$$m_Q = 22 \text{ GeV}$$

for $e_Q = -1/3$. This mass is still low enough that the linear part of the potential is more important than the coulomb part. Beyond this mass, the $\mu\mu$ branching ratio of the $Q\bar{Q}$ system will get smaller and smaller and it may become hard to pick the signal of a new resonance out of the $\mu\bar{\mu}$ continuum.

IV. ASSOCIATED PRODUCTION OF CHARM

In this section we will discuss the production of $c\bar{c}$ pairs. This is believed to give a good estimate of the total charm-production cross section.

A few notes of caution are in order. The types of arguments[41] that allow us to replace a sum over physical hadron states by free quark states work better as energies rise higher above threshold. However, in hadronic collisions the subprocesses that produce the $c\bar{c}$ occur close to charm threshold no matter what the energy of the initial hadrons, as one can see by examining the integrands in the explicit formulas below. It is possible that final state interactions that we are neglecting here could have a noticeable effect on the charm cross section. We can also point out that much of the inclusive charm production will end up as $D\bar{D} + X$ or $F\bar{F} + X$ states since any heavier charmed mesons decay strongly into lighter charmed mesons, but only weakly if charm is not conserved. Of course, not all of the charm production yields charmed mesons, because charmed baryons are not very much heavier and they can also be produced in quantity.

The two processes that we consider are shown in Fig. (11). The first process (Fig.(11)) is identical to the Drell-Yan process, except that one replaces the photon by the gluon. (The same diagram with a photon intermediate state is down by two orders of magnitude in the cross section.)

The total cross section for this process can be readily written as:

$$\sigma_I(AB \to c\bar{c}\ X) = \sum_{q\leftrightarrow\bar{q}} \int_{4M_D^2}^{s} \frac{ds'}{s'}\, \sigma(s',\, q\bar{q} \to c\bar{c}) \times \tau \int_{\tau}^{1} \frac{dx}{x}\, f_q^A(x)\ f_{\bar{q}}^B(\tau/x) \qquad (4.1)$$

with

$$\sigma(s',\, q\bar{q} \to c\bar{c}) = \left[\frac{2}{3}\right] \frac{4\pi\alpha_g^2}{3s'} \cdot \beta \left[1 + \frac{2m^2}{s'}\right] \qquad (4.2)$$

where $\tau = s'/s$, f_q represents the quark distribution function for hadron A; m is the mass of the c quark, $\sqrt{s'}$ is the invariant mass of the $c\bar{c}$ pair, $\beta = (1 - 4m^2/s')^{1/2}$. Finally, the factor 2/3 is due to color.

The contribution of the second process can be written as (Fig. (10a), (b), and (c))

$$\sigma_{II}(AB \to c\bar{c}\ X) = \int_{4M_D^2}^{s} \frac{ds'}{s'}\, \sigma(s', gg \to c\bar{c}) \tau \int_{\tau}^{1} \frac{dx}{x}\, f_g^A(x)\ f_g^B(\tau/x) \qquad (4.3)$$

where f_g is the gluon distribution function and[5,9]

$$\sigma(s',\, gg \to c\bar{c}) = \frac{\pi\alpha_g^2}{s'} \left[12A + \frac{16}{3} B + 6C - \frac{2}{3} D\right] \qquad (4.4)$$

$$A = \left(\frac{2}{3} + \frac{1}{3}\gamma\right)\beta$$

$$B = (4 + 2\gamma)\ \ln \frac{1 + \beta}{1 - \beta} - 4(1 + \gamma)\beta$$

$$C = 2\gamma\ \ln \frac{1 + \beta}{1 - \beta} - 4(1 + \gamma)\beta$$

$$D = 2\gamma(1 - \gamma)\ \ln \frac{1 + \beta}{1 - \beta}$$

where β is the same as before and $\gamma \equiv 4m^2/s'$. The coefficients of A, B, C, and D are color factors, and we have written the contributions of the individual diagrams in Fig. (12) so that A comes from the square of the first diagram (12a), B is the sum of the squares of (12b) and (12c), C is the cross term between (12a) and the pair (12b) and (12c) and D is the cross term between (12b) and (12c).

There is the question of whether the $c\bar{c}$ state needs to be a color singlet. Of course, it is not the $c\bar{c}$ which needs to be colorless, but rather the physical hadrons made from them (e.g., $c\bar{u} + u\bar{c} + \ldots$). We shall not make any color selection of $c\bar{c}$ in presenting the results here, but shall trust to the final state interactions, which include effects due to the c and $\bar{c}$ combining with other final state quarks as well as soft gluon bremsstrahlung, to give us color singlet hadrons.

If it were necessary that the $c\bar{c}$ be a color singlet, then none of the diagrams with a one-gluon intermediate state, Figs. (11) and (12a), could contribute. We would then have the equation for $\sigma(gg \to c\bar{c})$ above without the A and C terms,[3] and with both the remaining color factors changed to + 2/3. This would be just the Dirac calculation[42] of $\gamma\gamma \to \mu\mu$ multiplied by 2/3.

Our results are shown in Figs. (13) through (15). In Fig. (13) we give the total cross sections as a function of s for $p\mathcal{N}$ and $\bar{p}\mathcal{N}$. Figure (14) gives explicitly the separate contributions from gluon-gluon and quark annihilation processes for both $p\mathcal{N}$ and $\bar{p}\mathcal{N}$. Finally, in Fig. (15) we plot the total cross sections for $K^+\mathcal{N}$, $\pi^{\pm}\mathcal{N}$, and $p\mathcal{N}$. Observe that for I = 0 nuclei the $\pi^{\pm}\mathcal{N}$ and $K^-\mathcal{N}$ cross sections are all the same. Our calculations agree with results in Ref. (8).

We close this section with a few comments regarding these results. First, we have no adjustable parameters. Second, in all cases, gluon-gluon processes dominate at high enough energies. As may be expected, when the projectiles contain up or down valence antiquarks, the $q\bar{q} \to g \to c\bar{c}$ dominates at low energies (because valence quarks are more likely to carry a large fraction, x, of the initial hadron's momentum, and large x is needed to produce heavy particles in the final state at low energies). The crossover occurs at NAL energies, at about $\sqrt{s}$ = 29 GeV for $p\mathcal{N}$ and $\sqrt{s} \simeq$ 13 GeV for $\pi\mathcal{N}$.

V. COMMENTS AND CONCLUSIONS

We have studied the subject of onium production in hadronic collisions. We have discussed, from the theoretical point of view, the different alternatives, i.e., direct coupling of the constituents to the 1^{--} systems (J/ψ, Υ, etc.) vis-à-vis cascade mechanisms. We have also separated the contributions from the quark constituents from that due to gluons. How may we ascertain experimentally among these different alternatives? The separations of direct versus indirect production could be made obvious if a careful search for photons in conjunction with the J/ψ or Υ is attempted. Useful information can be obtained by studying the $\langle p_\perp \rangle_{\ell\bar{\ell}}$ as a function of mass, it should be bigger on resonance than off resonance. (We are assuming that the continuum is mediated by a Drell-Yan mechanism.) The difference in $\langle p_\perp \rangle_{\ell\bar{\ell}}$ is due to the cascade decay.

The second question, whether quark annihilation or gluon amalgamation is more important can also be answered. At low energies (M^2/s large) the answer is straightforward. The ratio of say $\bar{p}/p$ production on nucleons should be much larger for a quark-initiated process than for a gluon-initiated process. In the very high energy regime, (M^2/s small), this test becomes less relevant, given that this ratio should approach one even for quark initiated processes. (It is always equal to one for gg collisions, neglecting threshold effects.) This is due to the fact that at high enough energies the quarks from the ocean are the ones that become effective in producing the bound state and their wave function is the same for particle and antiparticle. A second test is useful at low and high energies, and that is related to the multiplicity of the residual hadrons in reactions where $Q\bar{Q}$ are produced. If the reaction has proceeded by quark annihilation, the residual partons of the beam and target will be in 3 and $\bar{3}$ color states, and will produce physical hadrons with a certain multiplicity dependent upon the total energy possessed by those partons. If, on the other hand, a gluon has been removed from each of the initial particles, the blobs of residual partons are each in an 8-color state. Their color "charge" is bigger than before, and they will produce hadrons with a higher multiplicity, a factor 9/4 higher according to the work of Brodsky and Gunion.[43]

This may also give some explanation of why it is hard to see charmed particles in hadronic collisions. There are simply very many other particles in the same final state. Let us, by way of example, suppose we are producing a $D\bar{D}$ pair at p_{lab} = 400 GeV, which is $\sqrt{s}$ = 27 GeV, and that the $D\bar{D}$ pair takes up just 4 GeV energy, leaving $\sqrt{s}$ = 23 GeV for the residual blobs. Then in the case that these blobs are 3 and $\bar{3}$ color states, they give on the average $\log(s) \simeq 6$ particles. If, optimistically, both the D and $\bar{D}$ undergo a two-body decay, we have 10 particles in the final state. Now if we do the same example for color octet residual blobs, we find 18 particles in the final state! Thus when the two-gluon process dominates, a charm search would be more likely to be a success if high-multiplicity events could be clearly studied. Or, if background is more of an experimental difficulty than beam luminosity, it may be better to look for charmed particles in $\bar{p}\mathcal{N}$ rather than $p\mathcal{N}$ collisions.

There are other tests that can separate quark-initiated from gluon-initiated processes. They are somewhat more model-dependent. The x_F

distribution of the gluon should be more peaked than that of valence quarks, and this should be reflected in differences between the x_F spectra seen in $\bar{p}p$ and pp at low energies. Also, as noted by Ioffe,[18] the angular distribution of the dilepton is different for the different processes considered here.

We end by summarizing our most important conclusions.

1. (J/ψ, Υ, ...) production observed in hadronic collisions comes mainly from production via an intermediate state. There should be photons associated with these of the observed events. The γ-J/ψ, and also the $\gamma\Upsilon$ spectrum should show peaks at the appropriate P wave masses.

2. Production processes involving the gluons within the beam and target dominate at high energies for all beams and targets, and at all energies for $p\mathcal{N}$ and $K^+\mathcal{N}$. This is also true for free $c\bar{c}$ production.

3. For the case of J/ψ production, there is considerable data for us to compare our calculation to. Our agreement with the data is impressive, particularly for the total cross section at high energies.[44] Also, the spectra $d\sigma/dx_F$ that we have calculated agree tolerably well with the data. We shall remark that we have no parameters to vary, so the cascade process we are considering cannot be made to go away. One can consider additional processes but not alternate processes.

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Note Added in Proof:

After this paper was submitted for publication we learned about the CERN experiment by J. H. Cobb et al., Phys. Lett. 72B, 497 (1978). They report on a search for additional photons accompanying J/ψ particles; as suggested by our model. They report that (43±21)% of the J/ψ are produced via the photonic decay of one of the $\chi(3.5)$ states. These results give strong support to our model.

APPENDIX

We shall calculate the transition between a color octet and a color singlet $c\bar{c}$ via the emission of one gluon. The gluon can be emitted by either the c or $\bar{c}$, and the calculation is similar to calculation of the matrix element for an E1 radiative transition.

A color octet state is

$$|\psi_c(P)\rangle = \frac{1}{\sqrt{2}} (\lambda_c)_{ij} \int (dp)\ \psi(p) \left|c_i(\tfrac{1}{2} P + p)\ \bar{c}_j(\tfrac{1}{2} P - p)\right\rangle$$

where $(dp) = d^3p/(2\pi)^3$; a color singlet state is

$$|\psi(P')\rangle = \sqrt{\frac{1}{3}}\ \delta_{kl} \int (dp')\ \psi(p')\ |c_k(\tfrac{1}{2} P' + p')\ \bar{c}_l(\tfrac{1}{2} P' - p')\rangle$$

and the current is

$$j_{\mu b} = \sum_{\text{flavors}} \bar{q}\ \gamma_\mu \frac{\lambda_b}{2} q \quad .$$

One may work out directly the matrix element,

$$\langle\psi_2|j_{\mu b}|\psi_{1a}\rangle = \sqrt{\frac{2}{3}}\ \delta_{ab} \int (dp)\psi_2(p + \tfrac{1}{2}q)\psi_1(p)\bar{u}(\tfrac{1}{2} P + p + q)\gamma_\mu u(\tfrac{1}{2} P + p)$$

where $q = (\omega,\vec{q})$ is the gluon four momentum. For small ω, the spatial components of j_μ will dominate, and

$$\langle\psi_2|\vec{j}_b|\psi_{1a}\rangle \simeq \sqrt{\frac{2}{3}}\ \delta_{ab} \int (dp)\ \psi_2(p)\ \psi_1(p)\vec{p}$$

$$= im\sqrt{\frac{2}{3}}\ \delta_{ab} \int (dp)\ \psi_2(p)[H,\vec{r}]\psi_1(p)$$

$$= i\sqrt{\frac{2}{3}}\ \delta_{ab} \times m\,\omega\,\vec{r}_{21}$$

$$\rightarrow 0 \text{ as } \omega \rightarrow 0$$

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11. The last result should be taken with caution, i.e., $\Gamma(^3P_1 \to gg) = 0$ in lowest order, by Yang's theorem. It arises from $^3P_i \to gg \to g\,q\bar{q}$. This (α_g^3) term is enhanced by a log(R) coefficient, where R is the dimension of the bound state. The presence of the infrared logarithm makes the perturbative result suspect. See R. Barbieri, R. Gatto and E. Remiddi, Phys. Lett. 61B, 465 (1976), and L. Okun and M. Voloshim Preprint [ITEP-95-1976].

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13. We thank N. De Takacsy for providing us with the program appropriate for this computation.

14. This is a larger number than used in our previous work (Ref. (1)). When we wrote Ref. (1) the P-states had not been disentangled, so we used $M_P = 3.44$ GeV for all states. However, the data shows that $M(^3P_2) = 3.55$ GeV, and this leads to a significantly larger ω in the calculation of $^3P_2 \to J/\psi + \gamma$, which in turn is responsible for most of the change in Γ_{eff}.

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32. Some weak mass dependence may be present when, for example, vibrational modes are excited, as in R. C. Giles and S.-H. Tye, Phys. Rev. Lett. 37, 1175 (1976).

33. See Peter Hasenfretz and J. Kuti, in Proceedings of the Tenth Rencontre de Moriond, Meribel-les-Allues, France, 1975, ed. by J. Tran Thanh Van (Inst. de Physique Théorique et Particules Elementaires, Paris, France), Vol. II, p. 209.

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37. The parameter b is really just an additive constant in V(r) and has no effect in the level spacing or the wave functions. It does, however, have an indirect effect on the value we obtain for m_Q, since varying b induces a change in $\Delta(m_c)$. We have chosen b to make $\Delta(m_c) = 0$, or $b = 0.179$ fm $= 0.907$ GeV^{-1}.

38. For both the logarithmic and modified linear plus Coulomb potential we have included only three P-states. If a fourth P-state is included, the answer should be increased by ca. 20%.

39. For a complementary view to the problem see R. Cahn and S. Ellis, Preprint [Report number RLO-1388-734, November 1977].

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44. In our previous reports on J/ψ production (Ref. (1)) we were systematically low. Our understanding of the spectrum (see Footnote 14) and the quality of the data has increased substantially, making the agreement between theory and experiments striking.

Table 1

Spectrum and Parameters for Upsilon

Potential	Linear + Coul I	Linear + Coul II	Logarithmic
$\alpha_g(m_Q)$	0.17	0.42	--
λ	.2029 GeV^2	.1529 GeV^2	.735 GeV
m_c (GeV)	1.37	1.065	1.085
$\Delta(m_c)$ (")	-.22	-.60	.0
m_Q(")	4.60	4.87	4.50
Upsilon			
S-states masses (GeV)	9.41	9.41	9.41
	9.85	9.98	10.00
	10.17	10.30	10.32
P-states masses (GeV)	9.71	9.87	9.85
	10.05	10.22	10.22
	10.34	10.48	10.47
Lowest $Q\bar{u}$ state (GeV)	5.19	5.75	5.36

Table II

Relevant Widths for Upsilon

	E1 Widths (keV)			2g Width of $^3P_0$ (keV)
	1S	2S	3S	
1P	19.7	--	--	201
2P	3.69	14.1	--	297
3P	1.76	2.5	14.85	368

$B(\Upsilon \to \mu\bar{\mu}) = 2.75\%$

FIGURE CAPTIONS

1. Processes for A + B → J/ψ + anything.
2. Total cross section and $d\sigma/dy|_{y=0}$ (per nucleon) for p$\mathcal{N}$ → J/ψ + anything. The data are total cross sections; if only $d\sigma/dy|_{y=0}$ was available, we scaled to σ_{tot} using the curves shown here. The individual points are[28] ● Aubert et al., ∇ Antiprov et al., o Branson et al., Δ Cobb et al., □ Snider et al., ■ Büsser et al., ▼ Amaldi et al., x Bamberger et al.
3. Individual contributions and σ_{tot} for $\bar{p}\mathcal{N}$ → J/ψ + anything.
4. The ratio of $\bar{p}$ induced to p induced J/ψ production. The data point is from Ref. (8).
5. Total cross section (per nucleon) for meson + $\mathcal{N}$ → J/ψ + anything. The data points are: x Bushnin et al.;[29] ● Corden et al.;[8] o Branson et al.
6. Some x_F spectra. The data are from Branson et al.[28] for Figs. (6a) and (6b), and from Ref. (8) and (29) for Fig. (6c).
7. Decays among upsilonium states. For clarity, not all possible decays are drawn.
8. Longitudinal momentum x_F distribution for $\pi\mathcal{N}$ → Υ + X at p_{lab} = 225 GeV.
9. $B_\mu \sigma_{total}$ for $\mathcal{N}\mathcal{N}$ → Υ + X, solid line includes contributions from Figs. 1(a) and 1(b). Dashed line includes Fig. 1(b) only. The data point is from Ref. 35.
10. $B_\mu \sigma_{tot}$ for $\pi\mathcal{N}$ → Υ + X, solid line includes contributions from Figs. 1(a) and 1(b). Dashed line includes Fig. 1(a) only.

11. A process contributing to $c\bar{c}$ production.

12. Other processes contributing to $c\bar{c}$ production.

13. Total cross sections for $\bar{p}\mathcal{N}$ and $p\mathcal{N}$ producing $c\bar{c}$ pairs.

14. The individual contribution of Figs. (11) and (12) to $\bar{p}\mathcal{N}$ and $p\mathcal{N}$ producing $c\bar{c}$ pairs. The gg process (11) is the same for $p\mathcal{N}$ and $\bar{p}\mathcal{N}$.

15. Total cross section for $\pi^{\pm}\mathcal{N}$ and $K^{+}\mathcal{N}$ producing $c\bar{c}$ pairs. A K^{-} beam gives the same cross section as a $\pi^{\pm}$ beam, and $p\mathcal{N} \to c\bar{c}$ + anything is included for comparison.

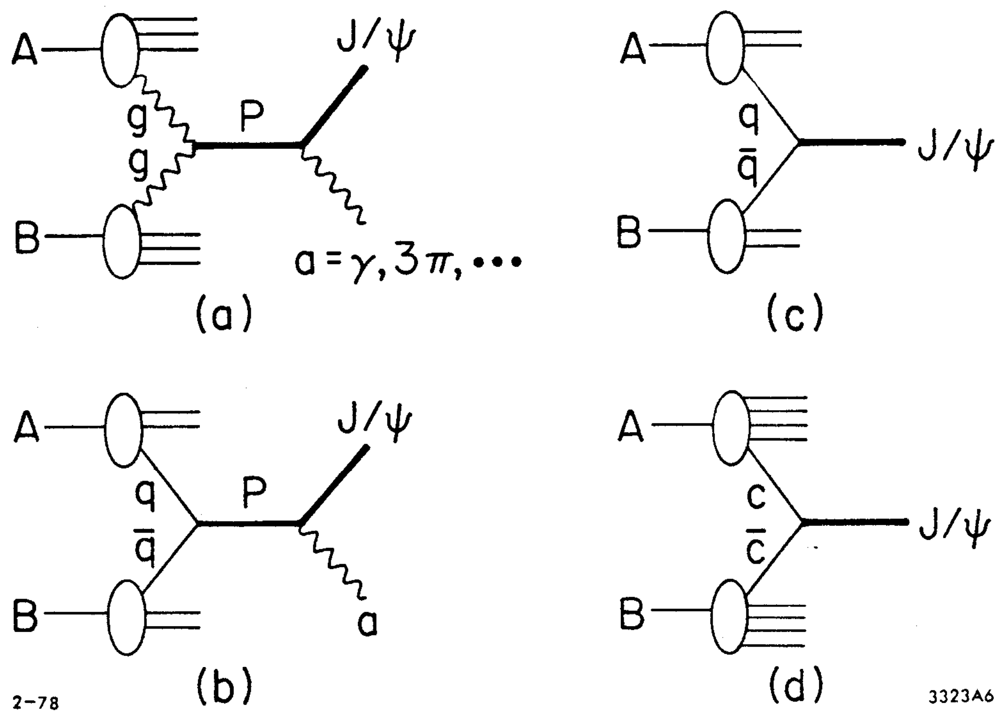


Fig. 1

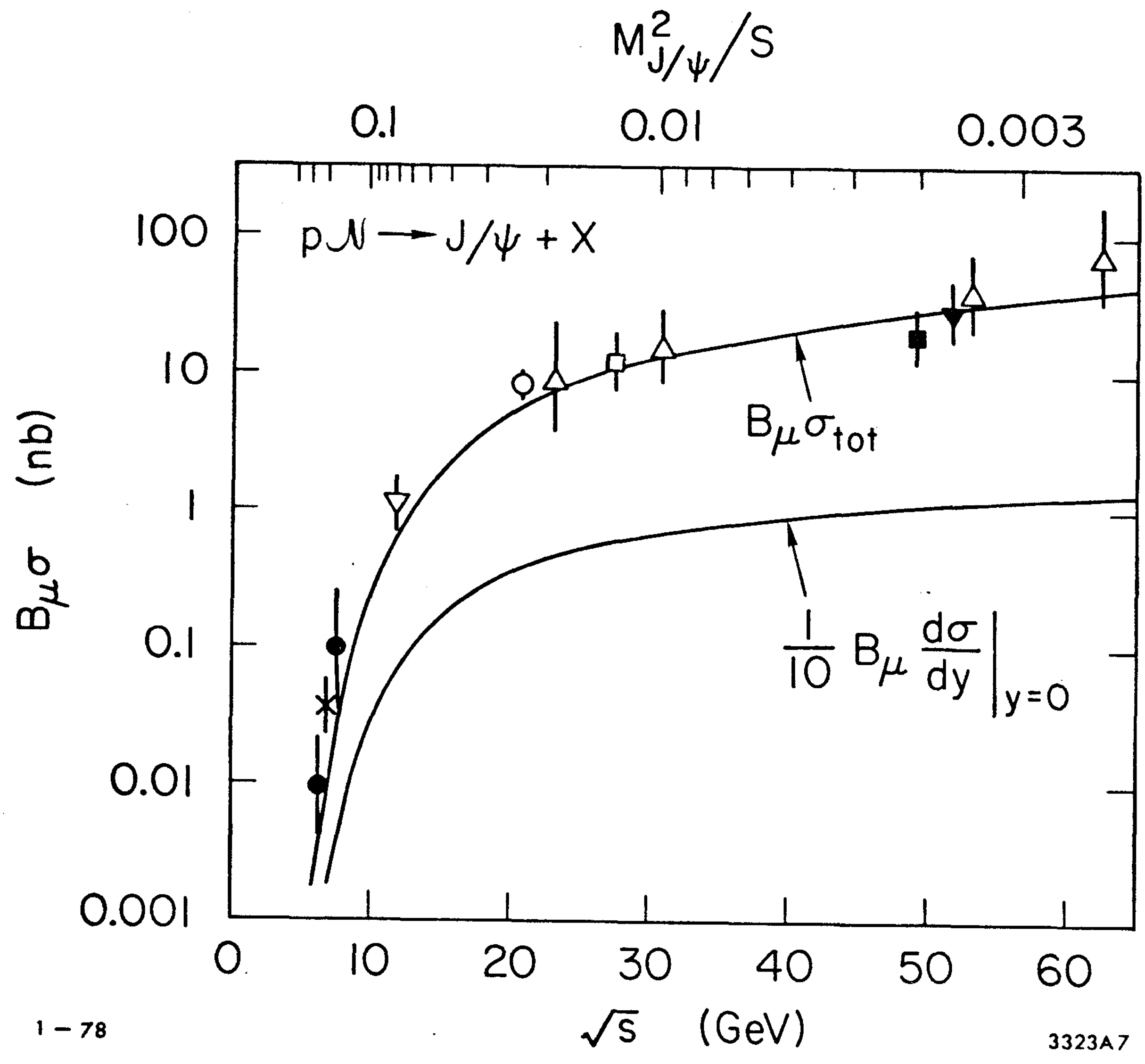
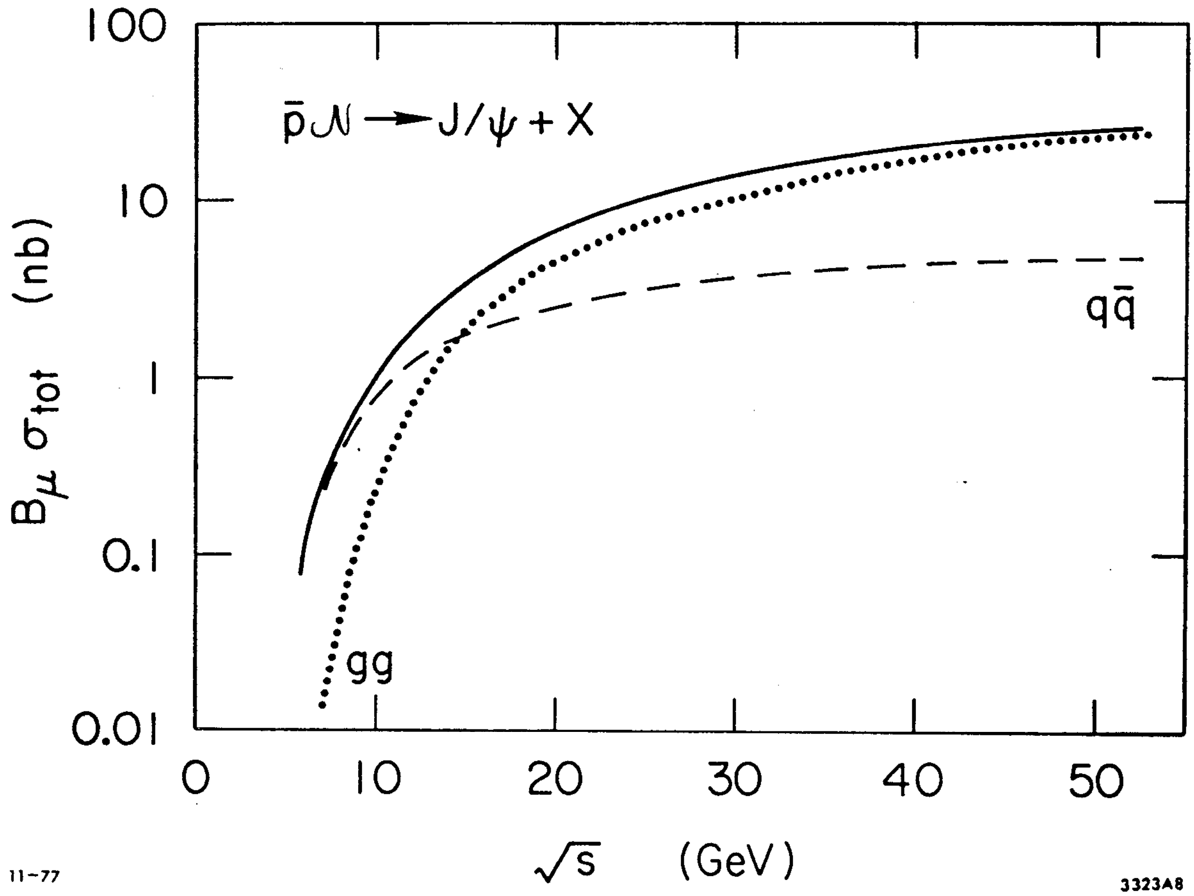

$M^2_{J/\psi}/S$
0.1
0.01
0.003
100
10
1
0.1
0.01
0.001
$p\mathcal{N} \to J/\psi + X$
$B_\mu \sigma_{tot}$
$\frac{1}{10} B_\mu \frac{d\sigma}{dy}\Big|_{y=0}$
$B_\mu \sigma$ (nb)
0
10
20
30
40
50
60
$\sqrt{s}$ (GeV)
1 – 78
3323A7

Fig. 2



100
10
1
0.1
0.01
p̄𝒩 ⟶ J/ψ + X
$B_\mu\, \sigma_{tot}$ (nb)
qq̄
gg
0
10
20
30
40
50
$\sqrt{s}$ (GeV)
11-77
3323A8

Fig. 3

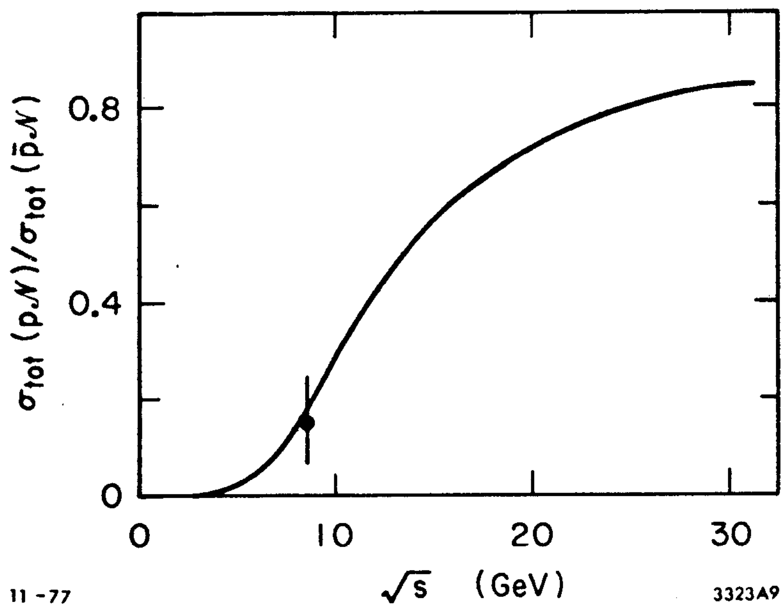


Fig. 4

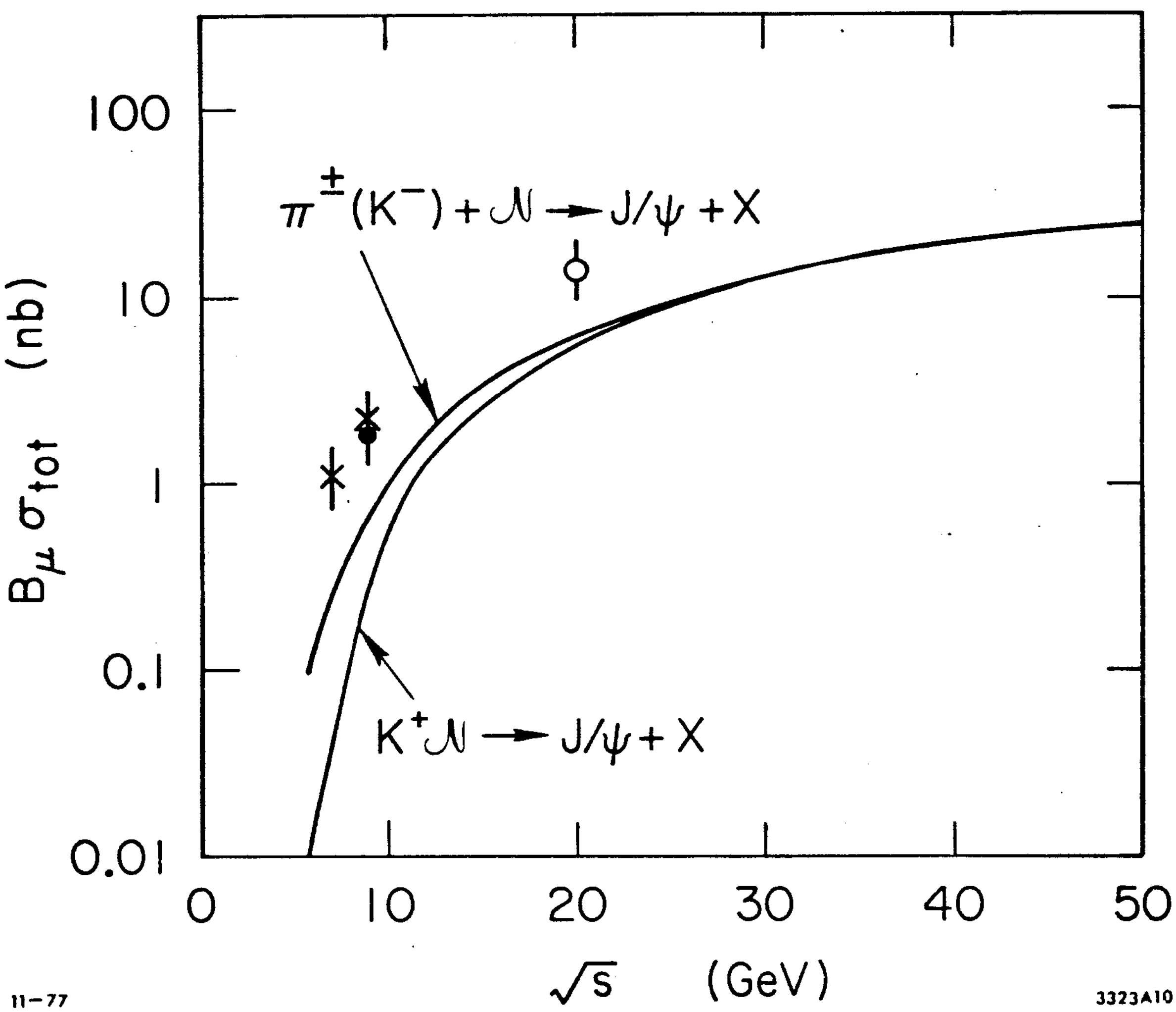
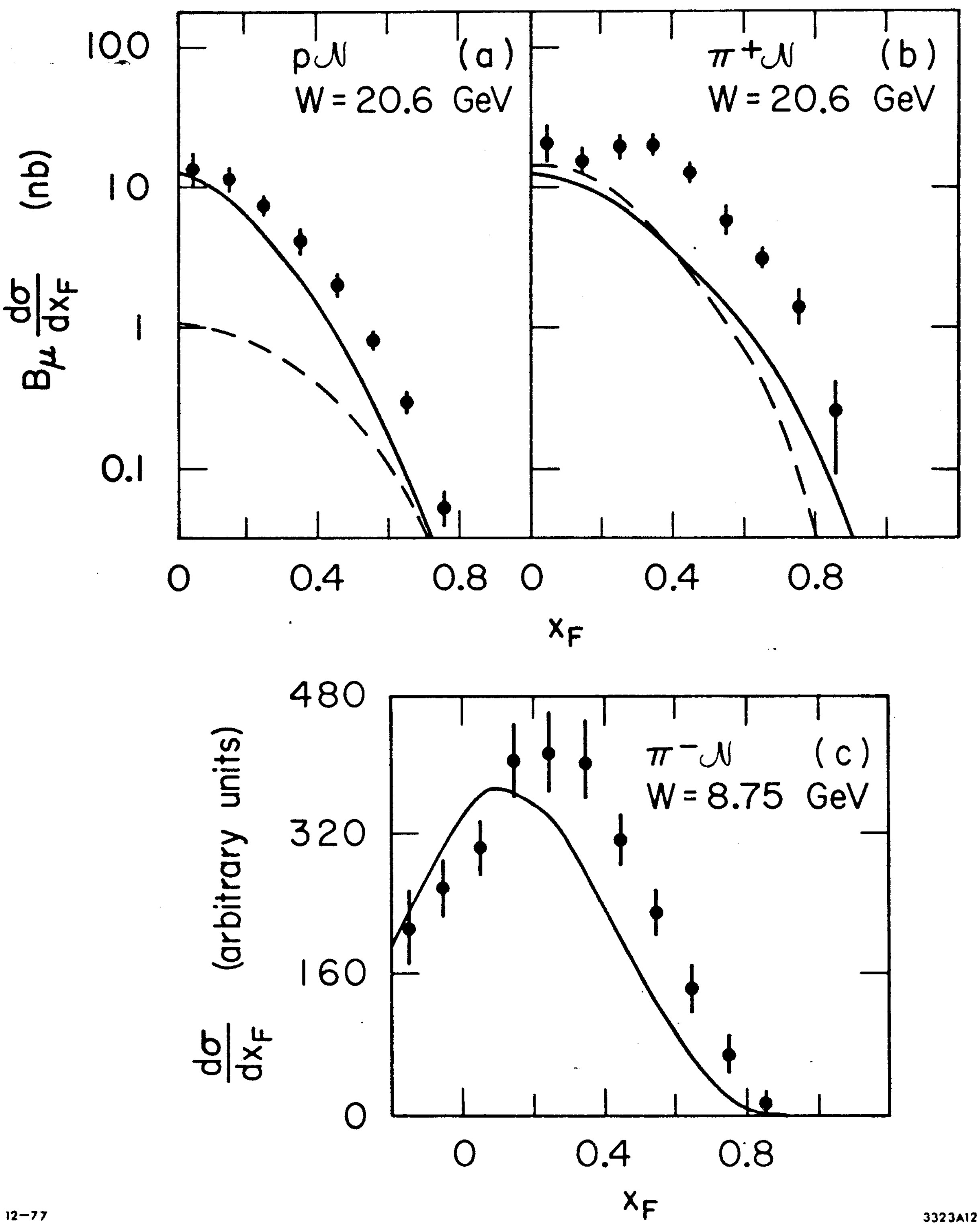

100
10
1
0.1
0.01
0
10
20
30
40
50
$B_\mu \, \sigma_{tot}$ (nb)
$\sqrt{s}$ (GeV)
$\pi^{\pm}(K^-) + \mathcal{N} \to J/\psi + X$
$K^+\mathcal{N} \to J/\psi + X$
11-77
3323A10

Fig. 5



pN (a)
W = 20.6 GeV
π⁺N (b)
W = 20.6 GeV
π⁻N (c)
W = 8.75 GeV
Bμ dσ/dx_F (nb)
dσ/dx_F (arbitrary units)
x_F
100
10
1
0.1
0
0.4
0.8
480
320
160
12-77
3323A12

Fig. 6

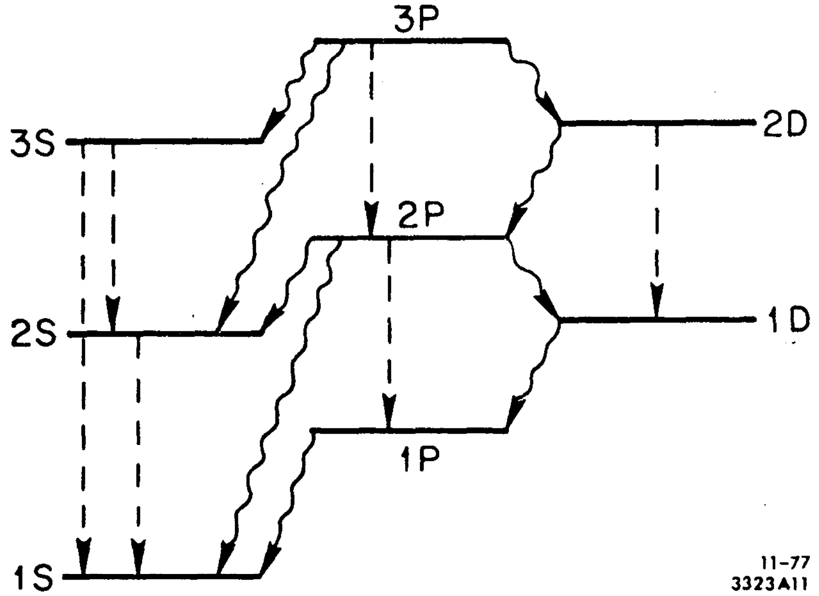


Fig. 7

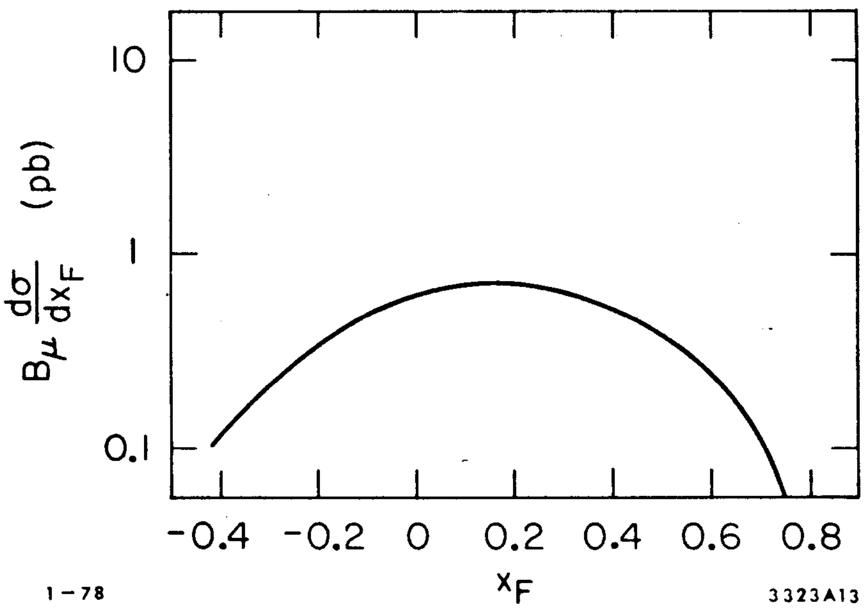


Fig. 8

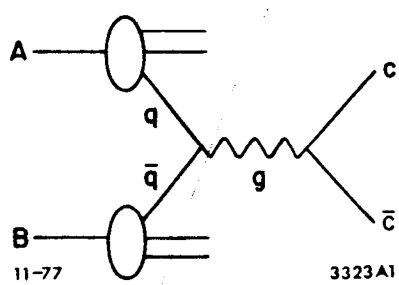


Fig. 9

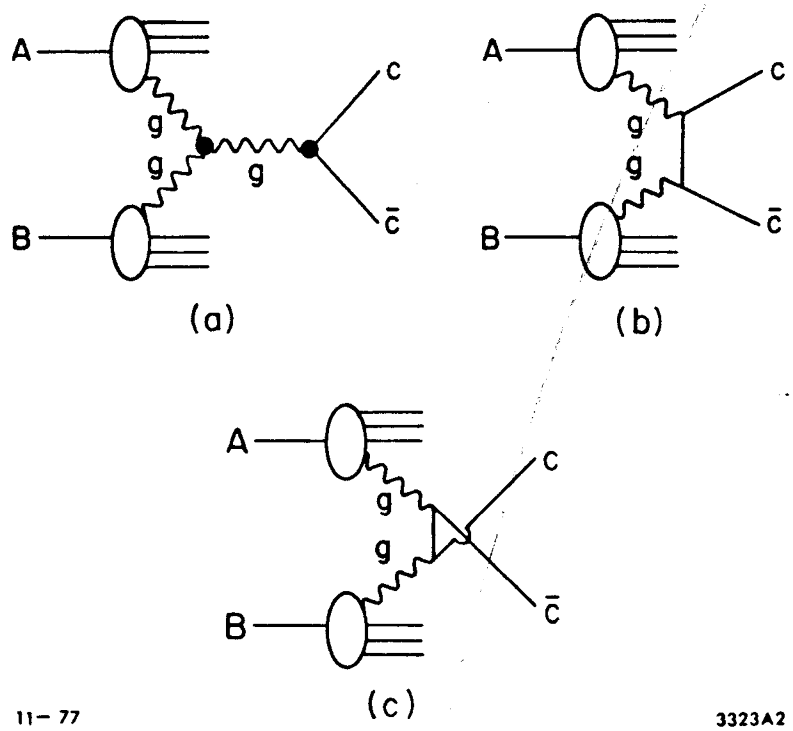
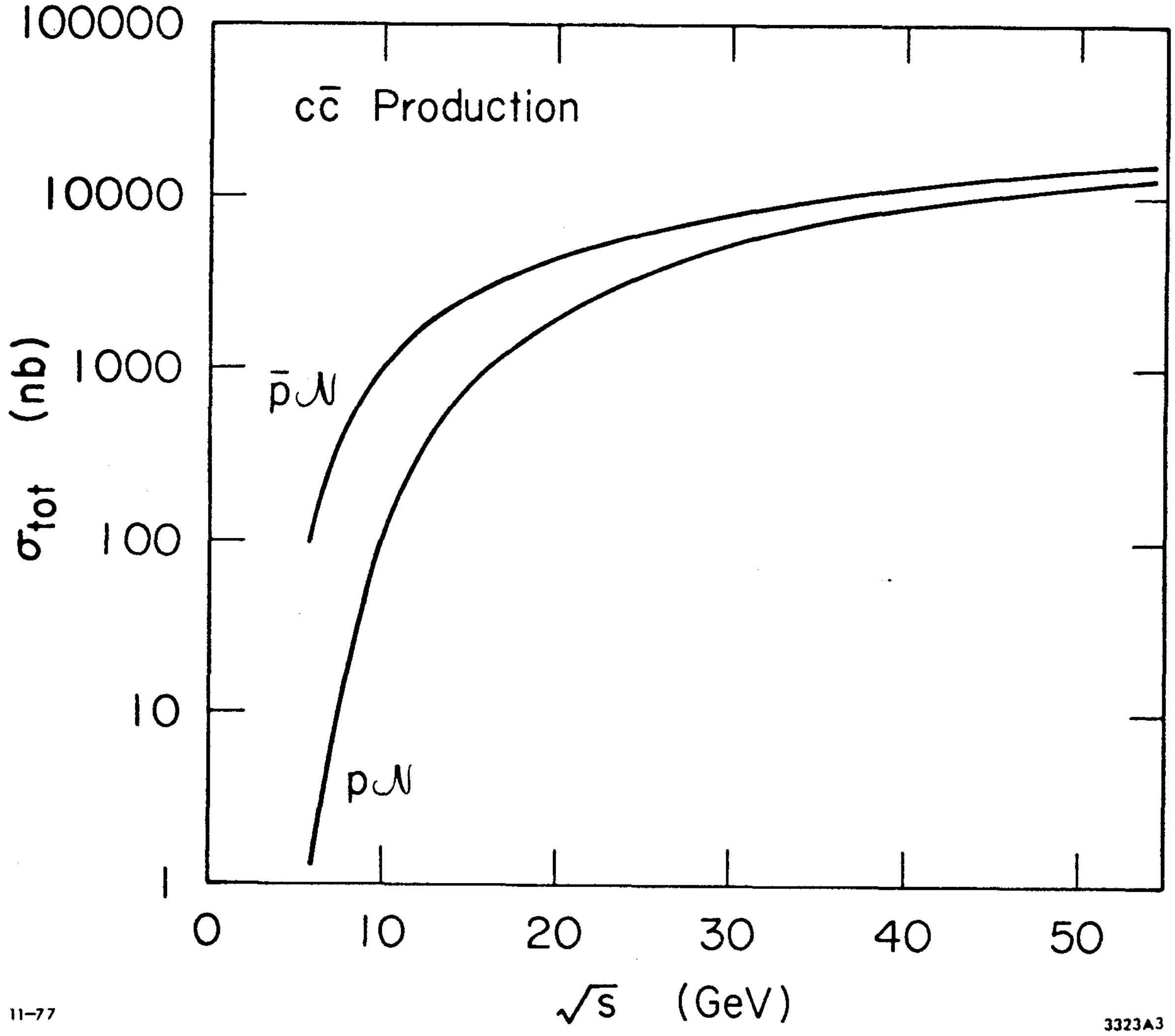


Fig. 10



100000
10000
1000
100
10
1
cc̄ Production
$\bar{p}\mathcal{N}$
$p\mathcal{N}$
σ_{tot} (nb)
0
10
20
30
40
50
$\sqrt{s}$ (GeV)
11-77
3323A3

Fig. 11

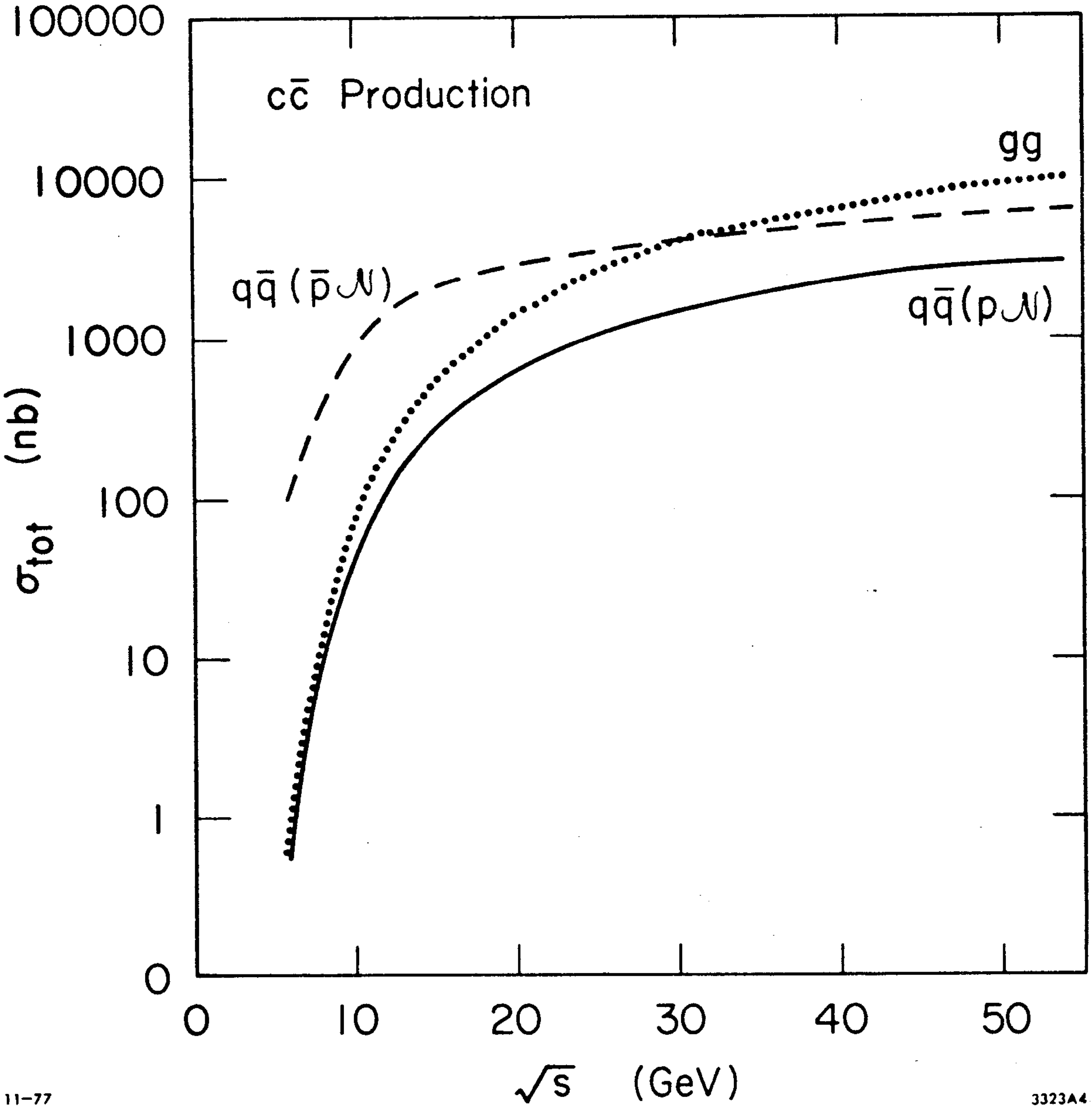

cc̄ Production
gg
qq̄ (p̄𝒩)
qq̄ (p𝒩)
100000
10000
1000
100
10
1
0
σtot (nb)
0
10
20
30
40
50
√s (GeV)
11-77
3323A4

Fig. 12

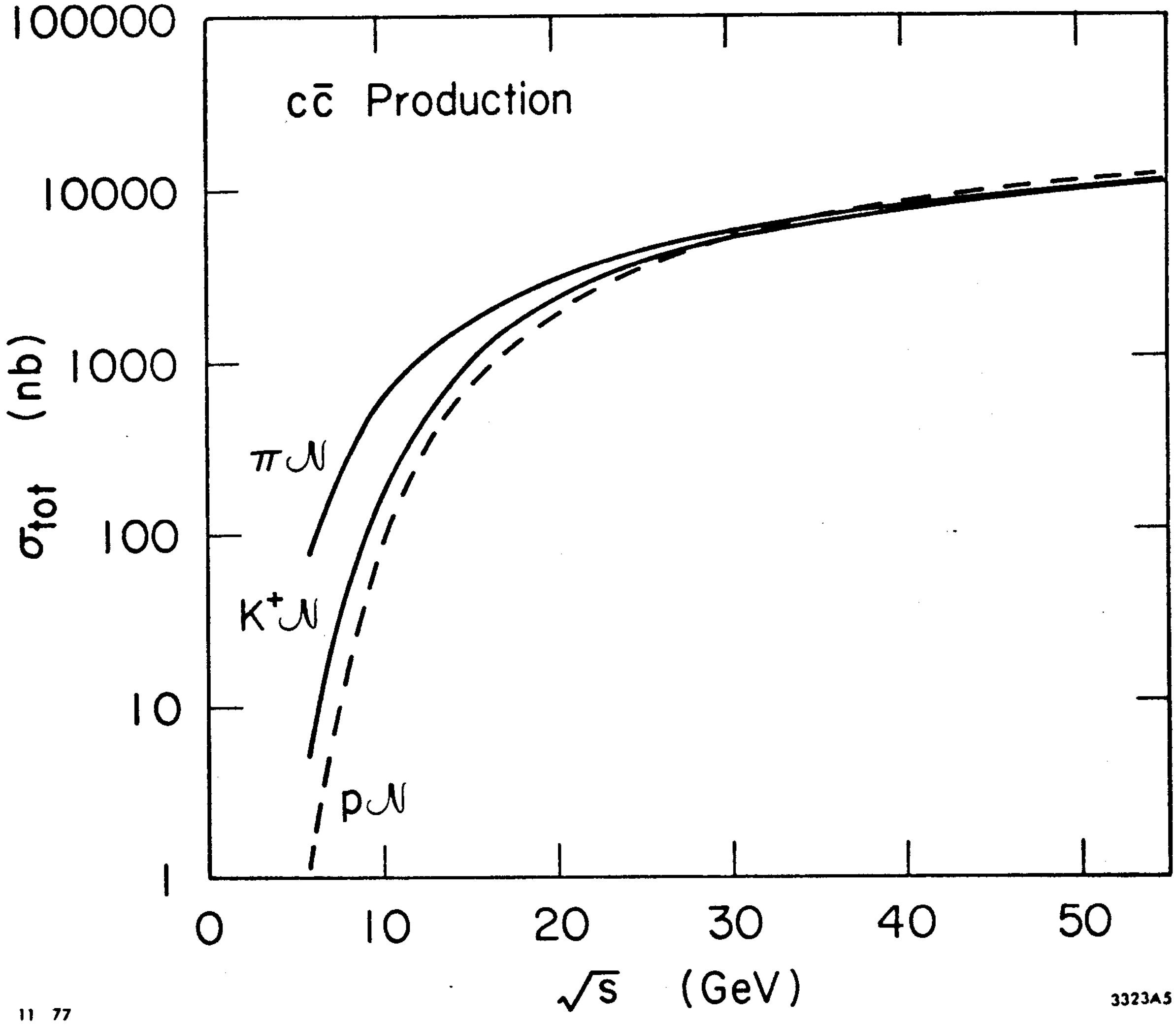

100000
10000
1000
100
10
1
cc̄ Production
σ_tot (nb)
πN
K⁺N
pN
0
10
20
30
40
50
√s (GeV)
11 77
3323A5

Fig. 13