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TRANSVERSE MOMENTUM LIMITATION IN INCLUSIVE ELECTROPRODUCTION PROCESSES*

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ABSTRACT

We show that if the longitudinal phase space is populated in a statistically independent manner (Poisson-like distribution), in an interval between the limits fixed by the rapidity of the leading particles, one being virtual photon, then the average transverse momentum must be limited in the Bjorken limit, and this limit is scale invariant. As a result we further show that the average density of particles (the rate of increase of average multiplicity with ln s) has a universal upper bound independent of dynamical variables.

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Recently there has been increasing theoretical [1] interest in deep-inelastic lepton scattering processes in which one or more of the final hadron is detected in coincidence with the scattered lepton. The latest results of the experiments done at SLAC, Fermilab, Cornell and DESY are reported at the 1975 SLAC conference [2]. New results, sometimes conflicting each other, keep coming [3].

Some time ago, the present author [4] and independently others [5] made an attempt to study the problem by adopting the formalism developed by Mueller [6] for purely hadronic production processes. This approach based on very simple and plausible assumption that processes involving highly virtual photons are also dominated by Regge trajectory exchanges, Pomeron being the highest, at high energies. The price paid by not having any fictitious entities like partons or quarks, is that predictions in this approach are not as detailed as that of parton models.

The processes under study is inclusive virtual photoproduction off a nucleon target

$$\gamma(q) + N(p) \rightarrow h(k) + \text{anything } (p_X) \tag{1}$$

where h is a pion unless otherwise specified. We make the following definitions:

$$s = (q+p)^2 , \quad W^2 = (q+p-k)^2 , \quad p^2 = m^2 , \quad k^2 = \mu^2$$

It was noted in ref. [4] that as long as the $k_\perp$ behavior is damped, the total multiplicity is proportional to the length of the phase space (fig. 1), $\ln\left[(1-\omega)\frac{s}{m^2}\right]$, and this is in excellent agreement with experiment (fig. 2) [2]; here $\omega = Q^2/2q\cdot p$ is the Bjorken scaling variable. This behavior is strikingly similar to what we expect from the purely hadronic production processes. However (1) differs dramatically from the latter in the following sense: For (1) the major contribution to the multiplicity comes from the photon fragmentation region [4] whereas in the hadronic case it was coming from the central region. Also it was noted that the contribution of the central region scales in ω.

It is well known that for the purely hadronic case the logarithmic growth of the average multiplicity arises from populating the longitudinal phase space dk_3/k_0, in a statistically independent manner [7]. As a result the average transverse momentum is limited [8, 9]. In this note, we point out that for this type of longitudinal distribution, the average transverse momentum must be limited in the general case, like the one in hand, also where there are highly off-shell virtual photons involved. Therefore the assumption that the $k_\perp$ behavior should be damped, which was necessary in proving that the average multiplicity is proportional to the longitudinal phase space, is not really an assumption.

First let us observe that the absolute kinematical limits imposed on the rapidity of the detected hadron (rapidity specifies the longitudinal Lorentz transformation that relates the lab frame to the rest frame) by the energy momentum conservation is [10]

$$\ln\left(\frac{\mu_\perp}{m}\right) + \ln\left(\frac{1}{1-\omega}\right) \lesssim y \lesssim \ln\left(\frac{m}{\mu_\perp}\right) + \ln\left(\frac{s}{m^2}\right) \tag{2}$$

So the allowed region in the longitudinal phase space for the detected hadron is

$$Y = \ln\left[(1-\omega)\,\frac{s}{\mu_\perp^2}\right] \tag{3}$$

The phase space, specified by the target, and the projectile has a length

$$Y_{12} = y_1 - y_2 = \ln\left[\left(\frac{1}{1-\omega}\right)\,\frac{s}{mQ}\right] \tag{4}$$

Because they look different it looks as if there is a gap in the phase space which is not populated by the secondaries:

$$\Delta y = Y_{12} - Y = \ln\left[\left(\frac{m}{\mu_\perp}\right)(1-\omega)^2\right] + \ln\left[\frac{Q}{\mu_\perp}\right] \tag{5}$$

When $Q^2 \simeq \mu_\perp^2$ (which means $\omega \simeq 0$) each term is separately small; so is Δy. As Q^2 gets large, the second term gets large, but fortunately the first term also gets large, and negative, so they cancel. Therefore Δy is always of order ~1. This proves that the phase space is completely filled by the secondaries; there is no gap for any value of Q^2. In other words phase space actually extends only between limits fixed by the rapidities of the leading particles.

If we now make the assumption that the transverse momentum is independent of rapidity (phase space is populated in a statistically independent manner), we can treat the longitudinal phase space as a tube of gas—"Feynman gas"—secondaries playing the role of gas atoms, the ends of the tube being determined by the leading particles [7]. Let us calculate the mass, M, of the Feynman gas of length Y. Let us first translate the gas in y by a Lorentz transformation so that it is centered at y=0, and call the average density of particles integrated over $k_\perp$, c, i.e., $d\bar{n}/dy = c$. Since $k_3 = \mu_\perp \sinh y$, with the gas centered at y=0, its total momentum is zero; therefore its mass is its total energy (here $\mu_\perp^2 = k^2 + \mu^2$). With the assumption that $\bar{k}_\perp$ is independent of y, a simple calculation gives

$$M = 2c\,\bar{\mu}_\perp \sinh\left(\frac{Y}{2}\right) \simeq c\,\bar{\mu}_\perp\, e^{Y/2} \tag{6}$$

Substituting here in the expression for Y, given by up to an ambiguity of order ~1, we get

$$M = c\,\bar{\mu}_\perp \sqrt{1-\omega}\left(\frac{s}{m^2}\right)^{1/2} \tag{7}$$

Since M cannot exceed the total available center-of-mass energy,

$$M \lesssim E_{tot}^{c.m.} \simeq \sqrt{s}$$

We finally obtain, in the Bjorken limit

$$\frac{\bar{\mu}_\perp}{m} \lesssim \frac{1}{c}\left(\frac{1}{1-\omega}\right) \tag{8}$$

From (8) it is seen that, if c is to be of order ~1 as is experimentally (from fig. 1, $c \simeq 1$), then for $\omega \simeq 0$ (deep Regge region) $\bar{\mu}_\perp$ is constrained to be of the order of a typical hadron mass or less, the same behavior we see in hadronic production processes [9]. As $\omega \to 1$, i.e., as Q^2 gets very large, (8) predicts large deviation from this typical hadronic behavior. For fixed and large s, $\bar{\mu}_\perp/m$ increases linearly with Q^2 slowly. This slow linear increase with Q^2, for fixed s, is consistent with the data [2]: From fig. 3, we get 0.47 for the left-hand side, and 1.08 for the right-hand side.

Equation (8) also gives a rough upper limit on the value of c, because always $\mu \leq \bar{\mu}_\perp$. These upper limits are

$$c_{max}^{(\pi)} \simeq \frac{m_N}{m_\pi} \simeq 7 , \qquad c_{max}^{(N)} \simeq \frac{m_N}{m_N} = 1 \tag{9}$$

These further give the following relations:

$$c_{max}^{(\pi)}/c_{max}^{(N)} \simeq m_N/m_\pi \simeq 7 \quad .$$

If the central plateau was two plateaus, hadronic plateau and the current plateau, lying between the current and target fragmentation regions, and separated by the hole fragmentation regions [11], the same result holds again, as long as these different plateaus are of the same height; in other words our result is insensitive to the character of the plateaus.

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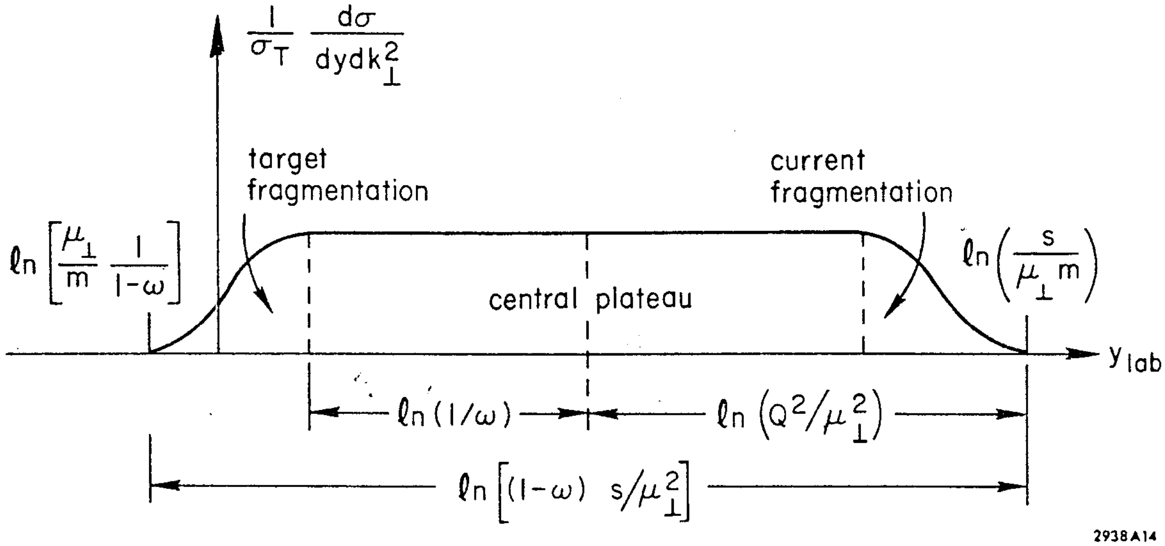
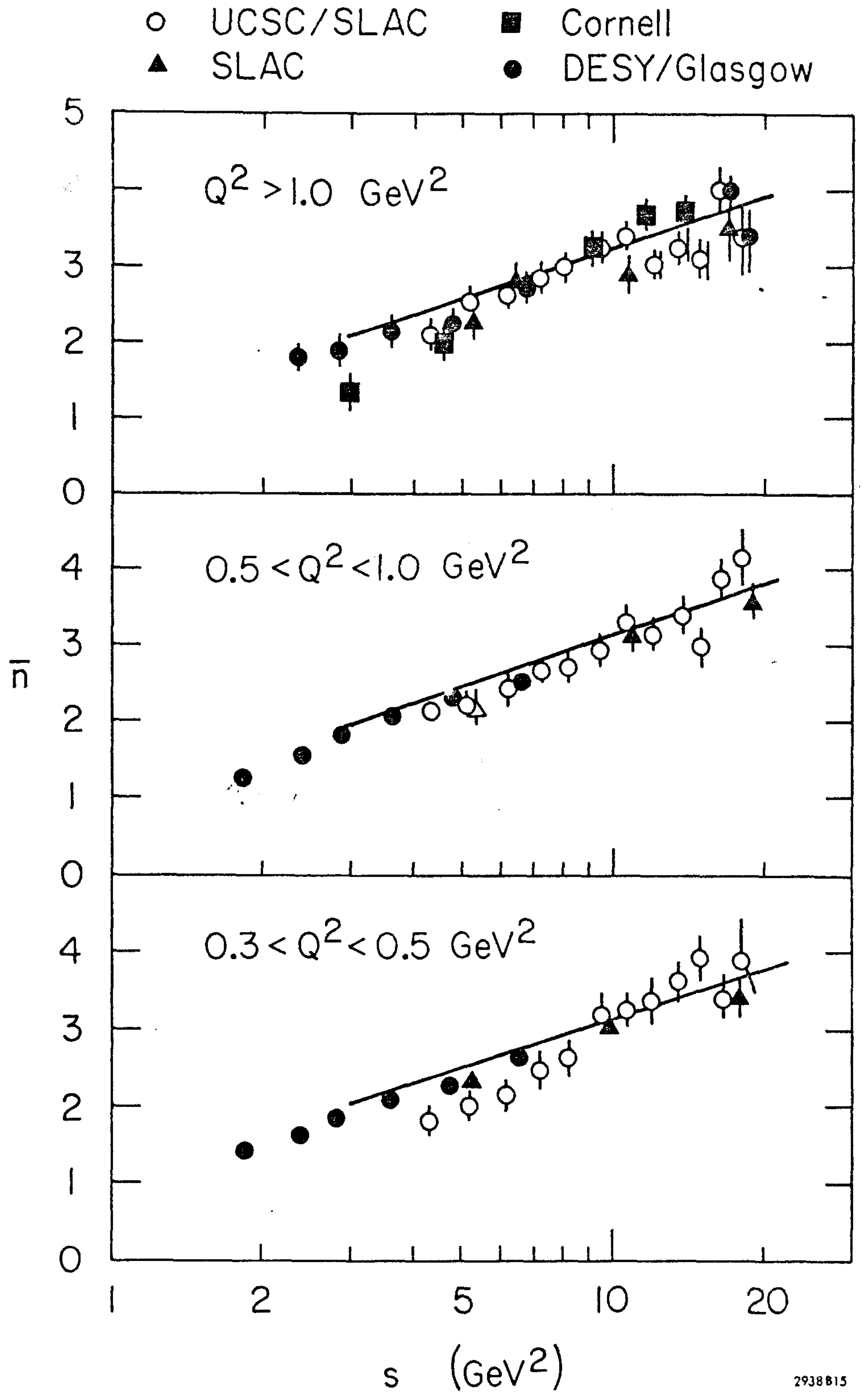
FIGURE CAPTIONS

1. Longitudinal phase space.
2. Average charged multiplicity versus s for different Q^2 intervals [2].
3. $\bar{k}_\perp^2$ versus Q^2 (GeV^2) for the processes $\gamma_V + p \rightarrow \pi^- +$ anything [2].



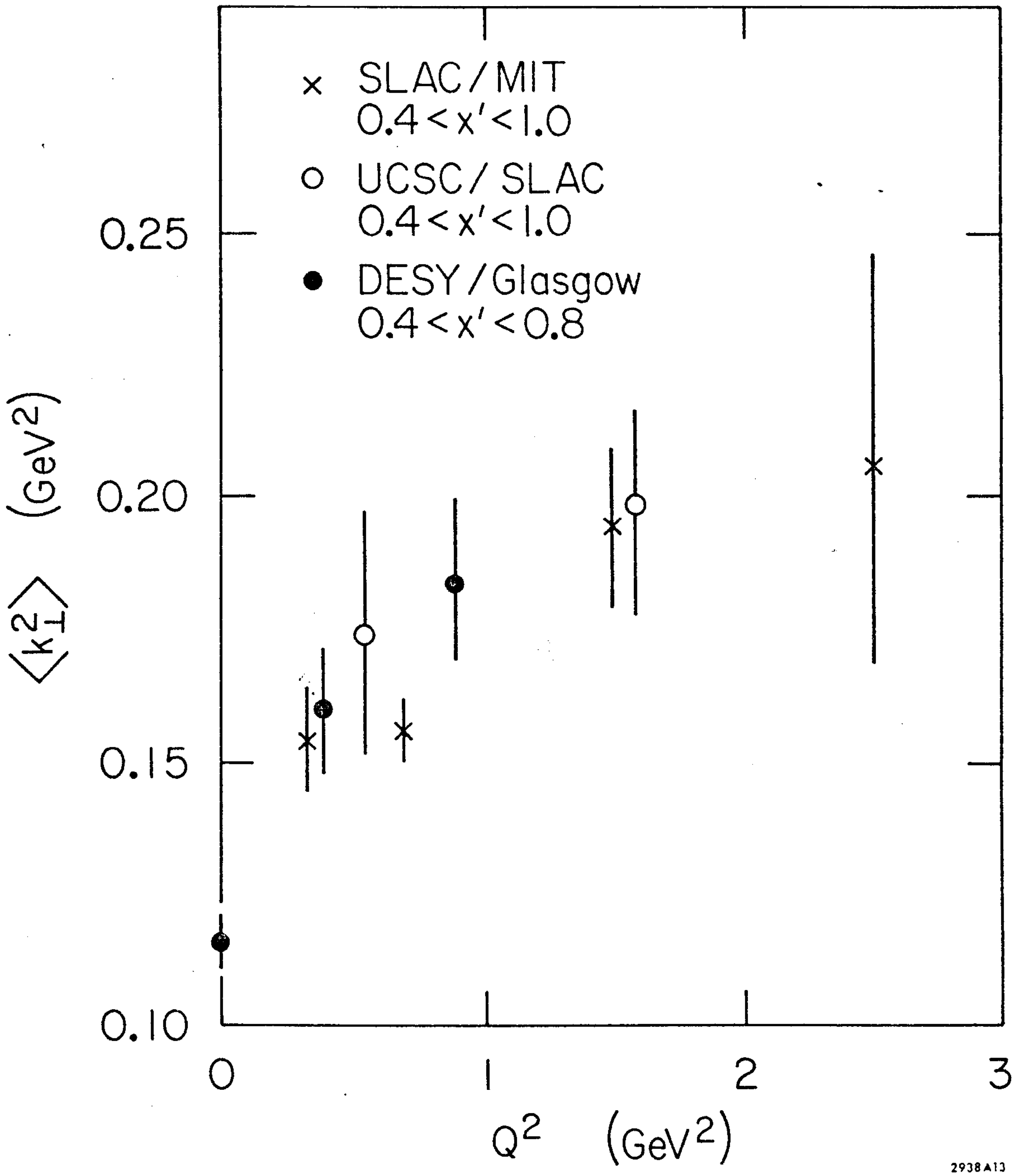
$\frac{1}{\sigma_T} \frac{d\sigma}{dy dk_\perp^2}$
target fragmentation
current fragmentation
central plateau
$\ell n \left[\frac{\mu_\perp}{m} \frac{1}{1-\omega}\right]$
$\ell n \left(\frac{s}{\mu_\perp m}\right)$
y_{lab}
$\ell n (1/\omega)$
$\ell n \left(Q^2/\mu_\perp^2\right)$
$\ell n \left[(1-\omega)\ s/\mu_\perp^2\right]$
2938A14

Fig. 1



UCSC/SLAC
SLAC
Cornell
DESY/Glasgow
$Q^2 > 1.0$ GeV^2
$0.5 < Q^2 < 1.0$ GeV^2
$0.3 < Q^2 < 0.5$ GeV^2
$\bar{n}$
s (GeV^2)
2938B15

Fig. 2



× SLAC / MIT
0.4 < x' < 1.0
○ UCSC / SLAC
0.4 < x' < 1.0
● DESY / Glasgow
0.4 < x' < 0.8
0.25
0.20
0.15
0.10
⟨$k_\perp^2$⟩ (GeV^2)
0
1
2
3
Q^2 (GeV^2)
2938A13

Fig. 3