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PROBLEMS WITH THE NORMALIZATION OF CROSS SECTIONS IN THE CONSTITUENT INTERCHANGE MODEL*

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ABSTRACT

It is in principle straightforward to estimate the size of differential cross sections at wide angle in parton models. We show in this note that such estimations in the framework of the Constituent Interchange Model disagree completely with experimental data. We comment about some attempts to understand this failure.

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From the time when large transverse momentum reactions have been experimentally studied, it has been widely recognized that the results were reasonably well explained in the framework of hard scattering models[1] based on a description of hadrons as a set of almost free partons. If the energy[2] and angular dependencies[3] predicted by these models have often been confronted with the experimental data, the absolute normalizations of the cross sections have been left aside.[4] However, the sizes of these processes are not arbitrary in these models and can be estimated by such simple arguments as the following: in any hard scattering model, one expects a matrix element for an elastic meson-baryon scattering at 90 degrees of the form

$$\mathcal{M} = Kg^2 F_M(-s/2) F_1(-s/2) ,$$

g being a "gluon-parton type" coupling constant, F_M and F_1 the electromagnetic form factors of the meson and the target, and K a numerical factor of order unity. However, experimentally the constant Kg^2 turns out to be large (of the order 500 for πp scattering). But the natural understanding of these models relies upon the relevance of asymptotically free theory for the description of strong interactions, in order to neglect higher order processes, so that we must conclude that K is far from being of order unity (typically $K \simeq 100$ for $g^2/4\pi \simeq 0.3$). In this note we want to precise somewhat this problem, in the framework of the Constituent Interchange Model (CIM) of Blankenbecler, Brodsky, and Gunion.[5] Although we do not believe that this must make us forget the successes of this description, we want to stress the urgency of curing this discrepancy. Some attempts will be made in conclusion but without success.

Estimation of the size of a differential cross section in the CIM. Let us examine, as an example, K^+p elastic scattering at 90 degrees in the CIM. In the impulse approximation, which is certainly valid in this case, the electromagnetic form factor of the target can be factorized,[6] as shown in Fig. 1. We are left with the estimation of a simpler process, namely, K^+-parton elastic scattering. The amplitude for this process can be written as

$$T = \int \frac{id^4k}{(2\pi)^4} \int \frac{id^4\ell}{(2\pi)^4} \psi^+_{p+r}(\ell)\, \zeta(p,r,\ell,k) \psi_p(k)$$

where ψ is the Bethe-Salpeter function of the bound state formed by the partons, ζ is the six-point amplitude, and the momenta are explained in Fig. 1; by iterating the Bethe-Salpeter kernel once, we can draw ζ as in Fig. 2. ζ can then be straightforwardly evaluated in the partition model, where the partons share equally the hadrons' momenta. We evaluate it in a $\lambda\phi^4$ scalar theory (Fig. 3), spin modifications being not expected to be important; the result is $6\lambda^2/u$ including a factor of 2 for the two possible choices of u-quark in the proton. T is then evaluated as

$$T = \zeta(p,r,\langle\ell\rangle,\langle k\rangle) \left| \int \frac{d^4k}{(2\pi)^4} \psi_{BS}(k) \right|^2$$

$$= \left| \psi_{BS}(x=0) \right|^2 \zeta(p,r,\langle\ell\rangle,\langle k\rangle) ;$$

then we can write

$$\frac{d\sigma}{dt}(K^+p \to K^+p)\Big|_{90^o} = \frac{\left|\psi_{BS}(0)\right|^4 \left|F_1(t)\right|^2 \left|\zeta\right|^2}{16\pi \left[s-(m_p-m_K)^2\right]\left[s-(m_p+m_K)^2\right]} .$$

The Bethe-Salpeter function of the kaon at the origin is determined by the elastic magnetic form factor of the meson by

$$F_K(t) = \frac{2\lambda\left|\psi_{BS}(0)\right|^2}{t} .$$

It is currently believed that strong interactions are asymptotically free[7] and that the effective hadron's fine structure constant $\alpha_s = g^2/4\pi \simeq \lambda/4\pi$ is of the order of 0.3 at relatively large momenta such as those committed here. We have such a rough estimation of the size of the K^+p elastic cross section at 90 degrees

$$\frac{d\sigma}{dt}(K^+p \to K^+p)\Big|_{90^o} \simeq 10^2/s^8 \text{ mbGeV}^{-2}$$

where we have used the following approximate experimental results

$$F_K(t) \simeq 1/t \qquad\qquad F_1(t) \simeq 1/t^2 \quad .$$

Experimentally[8]

$$\frac{d\sigma}{dt}(K^+p \to K^+p)\Big|_{90^o} \simeq 10^5/s^8 \text{ mbGeV}^{-2} \quad .$$

We observe a discrepancy of a factor 1000 between our theoretical estimation and the actual value of the cross section. Similar calculations show that such discrepancies occur in all other exclusive hadronic processes and in photoproduction at large momentum transfer, the magnitude of the discrepancy often being worse than in our example.

Let us now estimate the order of magnitude of K^+ inclusive production at large transverse momentum. We will focus on the characteristic subprocess shown in Fig. 4. The invariant cross section is given by[1]

$$E\frac{d\sigma}{d^3p}(pp \to K^+X) = \int_0^1 dz \int_0^1 dy\, G_{K^+/p}(z) 2G_{q/p}(y)\delta(s'+t'+u')\, \frac{s'}{\pi}\frac{d\sigma}{dt'}(K^+q \to K^+q)$$

where the factor 2 accounts for the two possible choices of up-quark in the proton, and the subprocess cross section is evaluated at $s' = zys$, $t' = zt$ and $u' = yu$. Dimensional counting[1,2] leads to the following forms for the structure functions

$$G_{q/p}(x) \sim (1-x)^3/x \qquad\qquad G_{K^+/p}(x) \sim (1-x)^5/x \; ;$$

their normalizations are fixed by considering the average momentum of a quark (or a kaon) in a proton. We will assume here:

$$\int_0^1 xG_{q/p}(x)\,dx \simeq \int_0^1 xG_{K^+/p}(x)\,dx \simeq 0.2 \; .$$

The subprocess cross section has been determined earlier to be $100/16\pi s'^2 u'^2$. The integration is then straightforward and leads to

$$\frac{Ed\sigma}{d^3p}(K^+p \to K^+X)\Big|_{90^0} = \frac{(1-x_T)^9}{p_T^8}\ \mathrm{mbGeV}^{-2}$$

with $x_T = 2p_T/\sqrt{s}$. When compared to experimental data, this estimation turns out to have the correct p_T and $(1-x_T)$ behavior but to be about one order of magnitude too small. However our result is likely to be enhanced if one considers K^* decay and other subprocesses (such as the fusion subprocess $q+q \to K^+K^-$).

Thus the estimated inclusive cross section in contradiction to the exclusive case turns out to be of the right order of magnitude. We recover here the problem noticed by Bjorken and Kogut[9] concerning their correspondence arguments.

Attempts to explain the discrepancy. We review here some possibilities to cure the present problem:

1. We can think that the experiments have not yet attained the energy region where the estimates will fit the data. However, if this is the case one has to wonder why the energy dependence is so well described at present energies by the dimensional scaling laws in the CIM.
2. In asymptotically free theory, one expects logarithmic modifications to our estimations to occur. It is hard to believe that they could fill the whole gap we have exhibited.

3. Initial or final state interactions could change the magnitude of the process without disturbing its shape. However such a big effect is not expected. Moreover, it has been shown[10] in some cases and under rather general assumptions that the net effect of such processes is null.

4. The Landshoff diagrams,[11] if taken into account, could enhance our estimation (with also changing slightly the energy dependence). It has been argued[12] that these diagrams are unlikely to contribute. Phenomenologically their inclusion would raise other difficulties.[13]

Finally we may think of a coincidental conspiracy between different effects due to spin insertion, final and initial state interactions, and logarithmic corrections. Since this conspiracy has not been shown to occur in a definite plausible model, one cannot rely without reluctance on such an explanation.

In conclusion, we want to stress that the successes of the CIM are strong enough for the problem shown here to be carefully examined.

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References

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mass m has to be given a rather small value (30-50 MeV) for the data to be fitted.

6. In some processes this factorization may be questionable. For example, in pp elastic scattering one can think of a process where a diquark is exchanged. However in our example this assumption seems very reliable.
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13. G. R. Farrar and C. C. Wu, Nucl. Phys. B 85, 50 (1975).

Figure Captions

1. The factorization of the form factor of the target as seen in the CIM.
2. The six-point amplitude ζ in lowest order in g.
3. The six-point amplitude in a $\lambda\phi^4$ scalar theory.
4. The subprocess $K^+q \to K^+q$ for the inclusive reaction $pp \to K^+X$ as seen in the CIM.

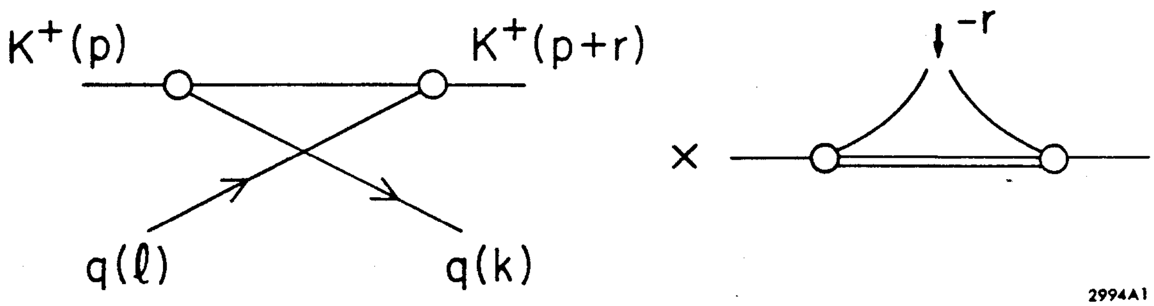

Fig. 1

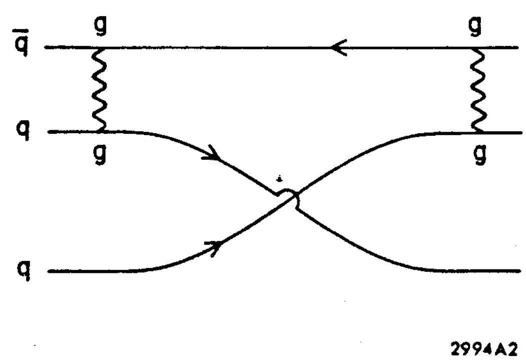

Fig. 2

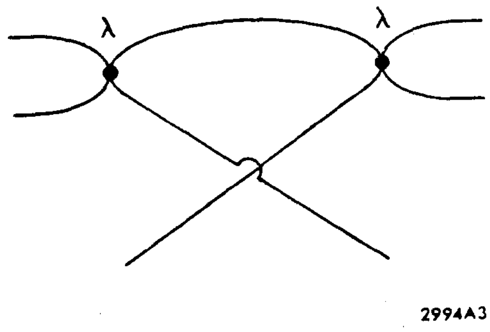

Fig. 3

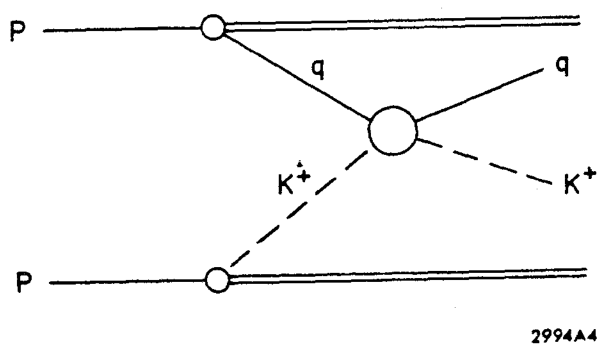

Fig. 4