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# ON THE INTRINSIC TIME VARIANCE OF FAST TIMING CIRCUITS USING TUNNEL DIODES*

by

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## ABSTRACT

An analysis is presented for the intrinsic time variance originating from thermal noise and finite capacitances in high-speed timing circuits using tunnel diodes. The nonlinear differential equation of the transition is solved numerically for a wide range of circuit parameters, and limitations on the minimum attainable time variance are derived.

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In many high-speed timing circuits significant time variance is contributed by the triggering jitter of the tunnel-diode discriminator. Even when the input waveform to the circuit is deterministic, there always exists a superimposed thermal noise which results in a finite time variance if the tunnel diode has a finite capacitance. This variance is computed here for the circuit configuration of Fig. 2a.

A representative current vs voltage characteristic of the tunnel diode is shown in Fig. 1. The device is biased initially at $v = V_i$, $I_i = I_p - I_o$ and transition to $v > V_v$ will take place if the voltage across the diode is increased above $V_p$. The time variance of this transition will be analyzed under the following assumptions: (a) The dc i vs v characteristic of the tunnel diode in the vicinity of the peak current can be approximated by the quadratic $i = I_p - I_o \cdot (v - V_i - V_o)^2 / V_o^2$, where $V_o$ will be determined later. (b) The tunnel diode is imbedded in the circuit shown in Fig. 2a. (c) The tunnel diode can be represented by its dc i vs v characteristic of (a) parallel with a fixed capacitance C (see Fig. 2b). (d) The drive current $i_s$ is a ramp function: $i_s = I_i + V_i/R$ for $t \leq 0$ and $i_s = I_i + V_i/R + kt$ for $t > 0$.[1]

Writing the current sum for node X in Fig. 2b, one obtains the nonlinear differential equation

$$C\frac{dv}{dt} - I_o\left(\frac{v-V_i}{V_o}\right)^2 + 2I_o\left(\frac{v-V_i}{V_o}\right) + \frac{v-V_i}{R} - kt = 0. \tag{1}$$

Defining $S \equiv 1 + V_o/(2RI_o)$ as a measure of the source impedance, $x \equiv I_oSt/(CV_o)$ as the normalized time, $y \equiv (v - V_i)/(V_oS)$ as the normalized voltage, and $A \equiv kCV_o/(I_o^2S^3)$ as the normalized slope of $i_s$, Eq. (1) becomes

$$dy/dx - y^2 + 2y - Ax = 0. \tag{2}$$

This nonlinear differential equation was solved numerically for y and dy/dx with values of A ranging from 0.01 to 1000.[2] The results, summarized in Fig. 3, show the expected behavior.[3] In the limiting case of a slow-rising drive current $A \ll 1$, $y \approx 1 - (1 - Ax)^{\frac{1}{2}}$ for $0 \leq y < 1$, thus following the dc characteristic; while at $y = 1$, transition to $v > V_v$ takes place. In the other extreme, when $A \gg 1$, $y \approx Ax^2/2$ for all $x > 0$.[4] It was also found that

$$(dy/dx)_{y=1} \approx A^{\frac{2}{3}} \; (1 + A/8)^{\frac{1}{6}} \tag{3}$$

with an error of less than 5% for $0.01 \leq A \leq 1000$.

The time jitter of a regenerative circuit, although its origin is superimposed thermal noise, is independent of the temperature and depends only on the time derivative of the damping rate (characteristic root) $\alpha$. For the circuit of Fig. 2b,

$$\alpha = -1/(C \cdot dv/di)_{\alpha=0} \quad , \tag{4}$$

and the time spread can be written as[5]

$$\langle \Delta t^2 \rangle^{\frac{1}{2}} \equiv \sigma_j \approx 0.61 \; (d\alpha/dt)^{-\frac{1}{2}}_{\alpha=0} \quad . \tag{5}$$

Here $(d\alpha/dt)_{\alpha=0} = (d\alpha/dv)_{\alpha=0} \cdot (dv/dt)_{\alpha=0}$ .

With Eq. (4) and assumption (a),

$$(d\alpha/dv)_{\alpha=0} = -(1/C) \cdot (d^2i/dv^2)_{\alpha=0} =$$

$$= (1/C) \cdot (2I_o/V_o^2) \approx (1/C) \cdot (I_p/V_1^2) , \tag{6}$$

where $V_1$ is a property of the tunnel diode: for germanium devices $V_1 \approx 70$ mV. From Eq. (3) with the definitions of x and y,

$$\left[\frac{dv}{dt}\right]_{\alpha=0} = \frac{S^2 I_o}{C} \left[\frac{dy}{dx}\right]_{y=1} = \frac{S^2 I_o}{C} A^{\frac{2}{3}} \left(1 + \frac{A}{8}\right)^{-\frac{1}{6}} . \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (8) and using the definition of A

$$\sigma_j = 0.61 \cdot 2^{-\frac{1}{6}} \left( \frac{C^2 V_1^2}{k I_p} \right)^{\frac{1}{3}} \cdot \left( 1 + \frac{A}{8} \right)^{\frac{1}{12}} . \tag{8}$$

If, as is the case in many high-speed timing circuits, $A \ll 8$, then for germanium tunnel diodes ($V_1 \approx 70$ mV)

$$\sigma_j \approx 0.1 \cdot \left( \frac{C^2}{k I_p} \right)^{\frac{1}{3}} . \tag{9}$$

Thus it may be concluded that in order to have a minimum contributed time spread $\sigma_j$, one should have a fast-rising drive current (large k) and a small $C^2/I_p$. It is interesting, although perhaps not too surprising, that for today's fastest germanium and gallium-arsenide tunnel diodes, $C^2/I_p$ is in the vicinity of a constant 0.3 $pF^2/mA$ (or $C_1 V_1^2 / I_p \approx 1.5 \times 10^{-24}$ Coul. sec) for peak currents ranging from $I_p = 1$ mA to 50 mA.[6] The source of this limitation was not identified.

## NOTES AND REFERENCES

1. A current step with a finite risetime can be approximated by such a ramp if the voltage across the tunnel diode reaches $V_p$ in a time less than the risetime. The time of $v = V_p$ is given by the $y = 1$ point of Fig. 3.

2. Newton's method was used in an Algol program on the Burroughs B5500 digital computer of Stanford University.

3. For details of the computations and for an application, see A. Barna, "A Subnanosecond Coincidence Circuit," SLAC Report No. 52, Stanford Linear Accelerator Center, Stanford University (December 1965).

4. This limiting case has been discussed by C. Infante and F. Pandarese, "The Tunnel Diode as a Threshold Device: Theory and Application," Nuclear Electronics, Proceedings of the Belgrade Conference, 1961, Vol. III, 29-40, and by F. Lacour, "Utilisation des Diodes Tunnel en Electronique Impulsionelle," Nuclear Electronics, Proceedings of the Belgrade Conference, 1961, Vol. III, 179-193.

5. For a derivation of Eq. (5) and for references, see Chu-Sun Yen, "On the Trigger-Time Jitter of a Regenerative Pulse Circuit," IEEE Convention Records, Vol. II, Part 2, 1963.

6. Thus for a drive current slope of $k = 10$ mA/nsec, $\sigma_j \approx 30$ psec.

## List of Figures

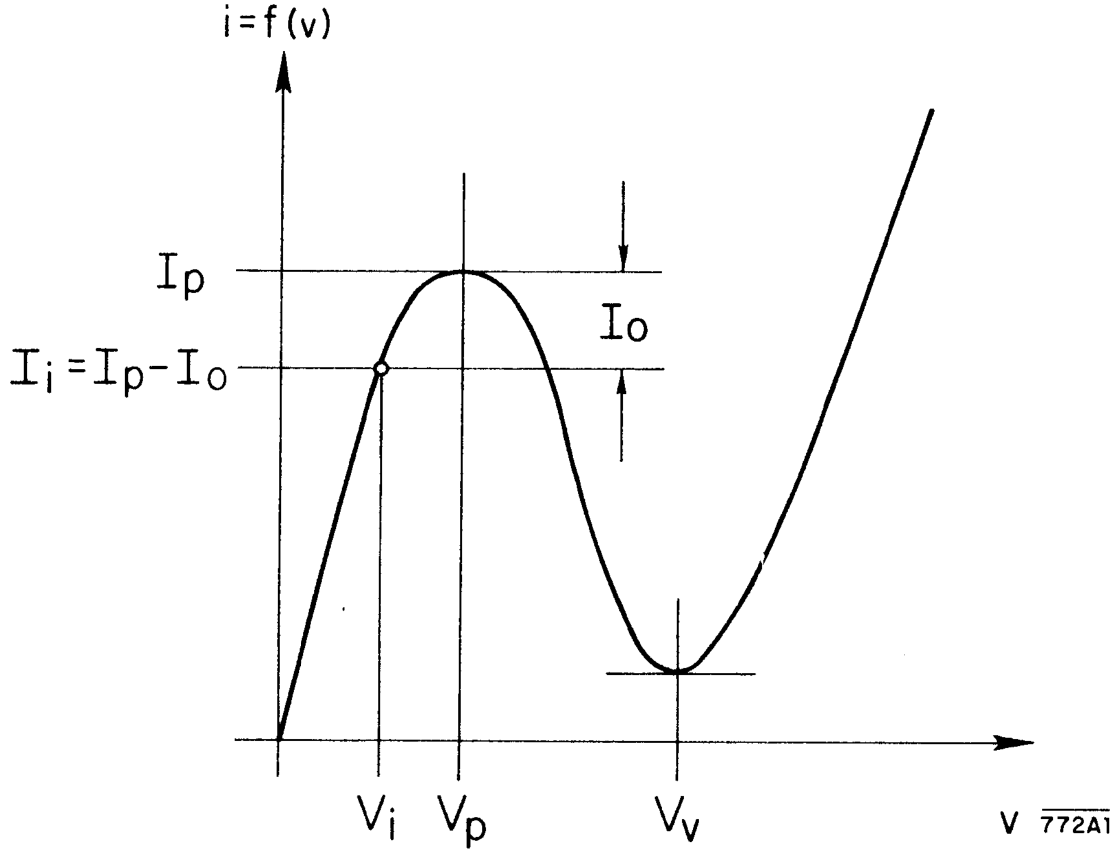


Fig. 1

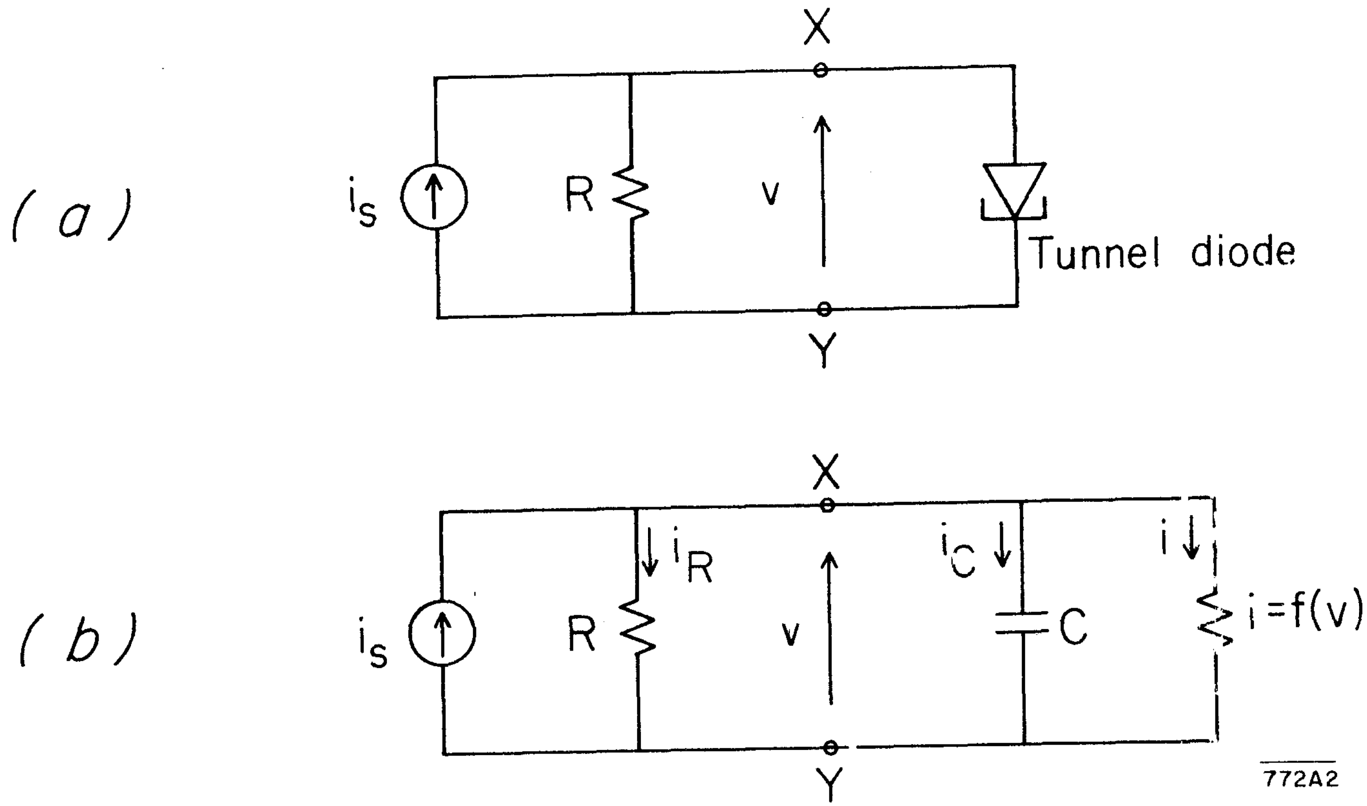


Fig. 2

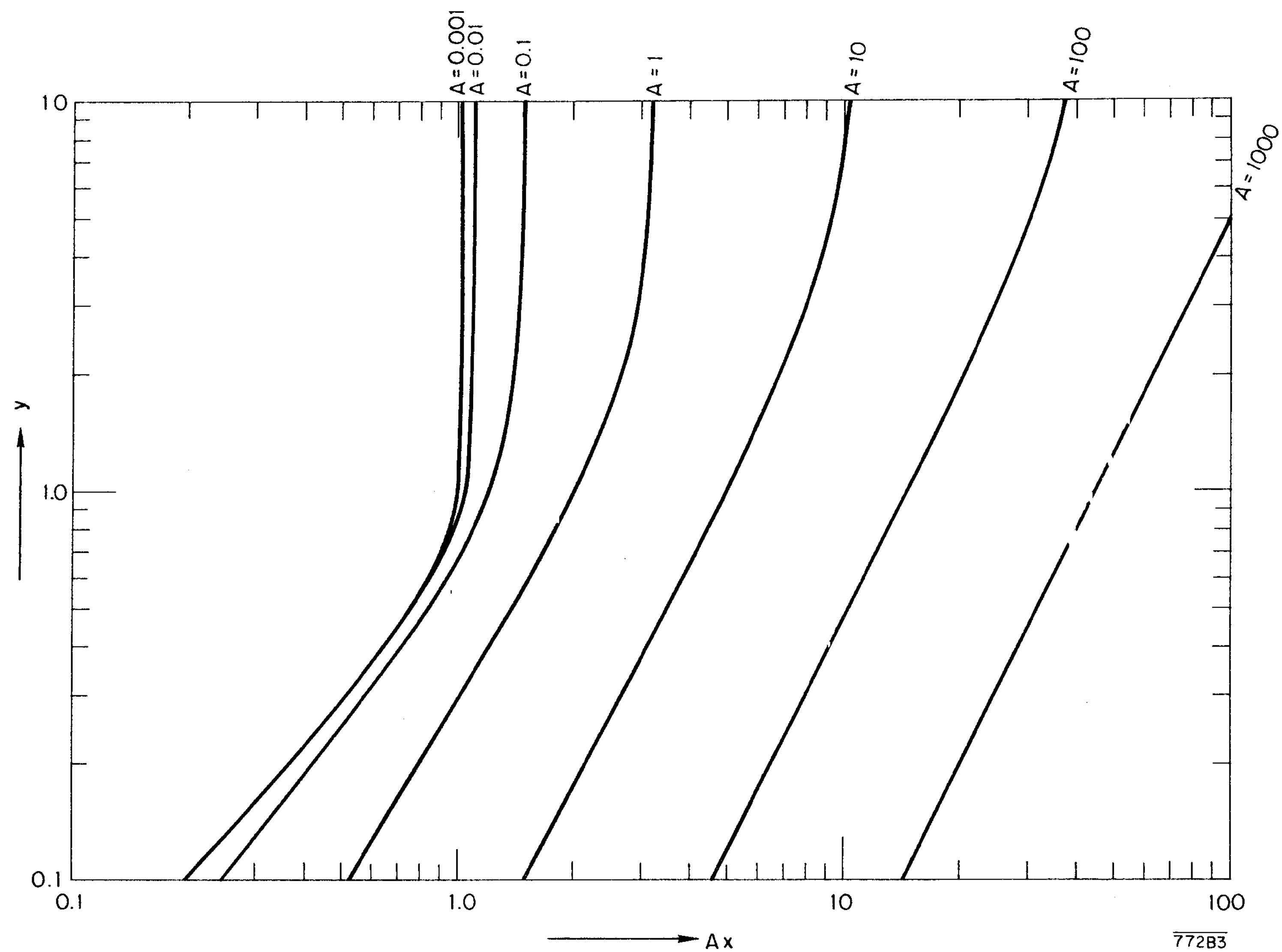


Fig. 3