SLAC-PUB-1575
March 1975

RECENT PROGRESS IN THE PHENOMENOLOGY OF LARGE TRANSVERSE MOMENTUM REACTIONS*

Stanley J. Brodsky

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

(Extracted from SLAC Summer Institute, SLAC Report 179, Volume II)

*Work supported by the U. S. Atomic Energy Commission.

I. INTRODUCTION

Hadronic reactions involving the production of particles at large transverse momentum have the exciting potential of being able to unravel the underlying structure of hadrons and the interactions of their constituents at very short distances. The recent measurements at the CERN-ISR[1] and FNAL[2] of hadron production at transverse momenta up to 9 GeV/c have now determined inclusive cross sections to momentum transfer squared $|t|$ up to 475 GeV/c^2. Although the underlying mechanisms involved in such reactions are bound to be more complex than the simple parton model description of deep-inelastic lepton-hadron scattering,[3] the enormous momentum transfers which are accessible and the richness of the phenomenology which can be studied, make large $p_\perp$ reactions one of the most important phenomenological areas of particle physics. An important first question is whether one can describe large $p_\perp$ processes in terms of simple underlying parton scattering mechanisms, and whether the quark model does in fact determine the essential degrees of freedom at short distances.

There are, however, other possible explanations of the large transverse momentum phenomena which do not rely on a constituent structure of the hadrons. The thermodynamic and hydrodynamic models[4] contain an essential energy scale related to a hadronic temperature and predict cross sections of the form $E\, d\sigma/d^3p \sim \exp(-E_{cm}/kT(s))$ where the specific dependence of T on s depends on the model. In the Fermi-type statistical model of Meng Ta-Cheng, which assumes a constant volume, one has $T \sim s^{1/8}$. This gives a good description of low $p_\perp$ data, but fails to account for the highest $p_\perp$ and highest energy data of the Chicago-Princeton group at FNAL.[5] An important point which we discuss later is that central fireball models with an isotropic distribution are incompatible with the coincidence measurements[6] which show a broad correlation in angle for the multiplicity distribution associated with a large $p_\perp$ reaction.[7]

A possible explanation of the data may be provided by the two-fireball models, wherein two heavy (usually assumed baryonic) systems are created, either of which subsequently decays to particles at large angles. Such models trace from the early work of Berman and Jacob,[8] Berger and Branson,[9] and others.[10] Although this depends on assumptions, many of the features of the single particle production cross sections are indistinguishible from the quark-parton model predictions; there are, however, contrasting predictions of the correlations and the distributions of particles accompanying a large transverse momentum reaction.[11]

One critical fact apparent from the Bjorken scale-invariant behavior of deep inelastic electron, muon, and neutrino scattering cross sections[12] is that the carriers of the currents within the hadrons have no discernible internal size ($\Lambda^{-1} \lesssim (10\text{ GeV})^{-1}$), but yet they carry a finite fraction of the hadronic momentum. Thus one expects that hadrons can scatter to large transverse momentum via hard, large-angle scattering processes involving their constituents. In such models this implies asymptotic inverse-power scaling laws for both exclusive and inclusive large $p_\perp$ processes.[13,14]

$$\frac{d}{dt}(A + B \to C + D) \to \frac{1}{s^N} f(\theta_{cm})$$
$$\frac{d\sigma}{d^3p/E}(A + B \to C + X) \to \frac{1}{(p_\perp^2)^N} f\left(\theta_{cm}, \frac{\mathcal{M}^2}{s}\right) . \qquad (1)$$

$$-\frac{t}{s} \simeq \frac{1}{2}(1 - \cos\theta_{cm}), \qquad p_\perp^2 = \frac{tu}{s}, \qquad \mathcal{M}^2 = (p_A + p_B - p_C)^2$$

where the functional dependence on the ratios of invariants t/s and $\mathcal{M}^2/s$ and the specific power N depends on the nature of the internal interactions and the external particles. In general, one can expect a sum of terms with different powers of N to be present. However, for fixed t/s and $\mathcal{M}^2/s$, the term with a minimum power of $p_\perp^{-2}$ will dominate at $p_\perp^2 \to \infty$.

We shall review some of the experimental evidence and some model-dependent predictions for the scaling laws (1) in the next section. A rather extraordinary feature of such laws is that the "universal" function

$$f(\theta_{cm}) = s^N \frac{d\sigma}{dt}(s, \theta_{cm}) \qquad (2)$$

for large $p_\perp^2$ is in fact predicted to be independent of energy, no matter how large s is. This is of course a literal interpretation of a point-like or scale-free theory and one would not be surprised to see logarithmic modifications a lá asymptotic freedom theories eventually to be important.

In this talk I shall review some of the experimental implications of the constituent picture of hadrons for large transverse momentum phenomena. The work described concerning dimensional counting[15] and the constituent interchange model[13] was done in collaboration with Glennys Farrar, Dick Blankenbecler, and Jack Gunion. I am grateful to them, and to J. Bjorken and Dennis Sivers for helpful suggestions. Many further details will be found in Dick Blankenbecler's lectures in these volumes and the references.[16] Also, an extensive review of large $p_\perp$ reactions is now in preparation.[17]

II. HARD-SCATTERING MODELS

All of the hard-scattering parton model descriptions of inclusive large $p_\perp$ particle production $A + B \to C + X$ are based on the schematic of Fig. 1. The fragments a and b of the incident particles scatter at large angles to particles c and d, one of which, in turn, fragments to the observed particle C. In general a sum of possible contributions with different "active" particles a, b, c, and d can be included; and in principle A, B or C can in fact be among the active particles. If we choose a Lorentz frame so that $|\vec{P}_A|$ and $|\vec{P}_B|$ are very large, then one can define a distribution function $G_{a/A}(x_a)$ which describes the probability of particle a in hadron A to have fractional longitudinal momentum x_a along $\vec{P}_A$, quite in analogy with the Weisacker-Williams spectrum in QED. For the case of deep inelastic electron scattering, the familiar quark model result is

$$\nu W_2(x) = \sum_a e_a^2\, x G_{q_a/p}(x)\Big|_{x=-q^2/2p\cdot q} \qquad (3)$$

Note that in Fig. 1, the effective collision energy is $s_{eff} = x_a x_b s$, where kinematically we must have $s_{eff} > 4p_\perp^2$. Clearly for fixed $p_\perp^2$, $s \gg 4p_\perp^2$ the extra energy can be "dumped" down the incident beam directions and the cross section becomes energy independent. This "hadronic" scaling is explicitly what happens when one uses the Feynman spectrum $G_{a/A} \sim 1/x_a$, $G_{b/B} \sim 1/x_b$ (for small x_a, x_b). In fact Regge behavior in $\sigma_{\bar{a}A} \sim s^{\alpha-1}$ demands that $G_{a/A} \sim x_a^{-\alpha}$ at $x_a \to 0$.

Thus, because of the Regge behavior of hadronic physics, high energy beams <u>alone</u> are not sufficient to study interactions at high energy or short distances. However, since $s_{eff} > 4p_\perp^2$, a production processes at high $p_\perp$ does in fact depend on high energy dynamics. Accordingly, the assumption of a single "violent" scattering, as well as the impulse approximation can be valid concepts for large $p_\perp$ phenomena--unlike the situation at low momentum transfer.

description. A detailed discussion of how this is accomplished in the CIM is given in Ref. 28. Teper's approach also leads to a smooth connection to a multiperipheral picture at low momentum transfer.

III. STRUCTURE OF THE HARD SCATTERING MODELS AND DIMENSIONAL COUNTING

The general structure of the cross section for any hard scattering model based on Fig. 1 is

$$\frac{d\sigma}{d^3p/E}(A + B \to C + X) \simeq \int_0^1 dx_a \int_0^1 dx_b \int_0^1 \frac{dx_C}{x_C^2} G_{a/A}(x_a)\, G_{b/B}(x_b)\, G_{C/c}(x_c) \times \delta(s' + t' + u')\, \frac{s'}{\pi} \frac{d\sigma}{dt'}(a + b \to c + d)\Bigg|_{\substack{s'=x_a x_b s \\ t'=(x_a/x_C)t \\ u'=(x_b/x_C)u}} \tag{4}$$

where the G's are the probability functions discussed in Section II. This formula was given by BBK[10] for specific cases, and has been derived and developed in various forms using infinite-momentum frame methods,[13] light-cone variables, covariant Sudakov analyses,[14] and generalizations of the multiperipheral model.[25] Note that in the case where A, B, or C is an active particle we can use $G_{A/A}(x) \propto \delta(1-x)$, etc. Transverse momentum fluctuations in the hadronic wave functions are integrated out here but can be reintroduced in order to predict coplanarity correlations.

As we have noted, there are many possibilities for the two-body subprocesses. In the view of Bander, Barnett, and Silverman,[29] and Ellis,[29] all three possibilities, A.-C., should be included; hadron-hadron scattering dominates for $x_\perp \sim 1$, quark-quark scattering for $x_\perp \sim 0$, and quark-hadron scattering covers the midrange. In order to compute the contribution of each type of subprocess, we will use the dimensional counting rules which are based on an underlying scale-invariant theory. First one counts the number of active fields participating in the large $p_\perp$ process

$$n_{active} = n_a + n_b + n_c + n_d \tag{5}$$

and the number of spectators or passive fields in A, B, and C:

$$n_{passive} = n(\bar{a}A) + n(\bar{b}B) + n(\bar{C}c) \tag{6}$$

Then following the guide of simple Born graphs in renormalizable field theories we can derive the following results

$$\frac{d\sigma}{d^3p/E} = \frac{1}{(p_\perp^2)^N} f(\theta_{cm}, \epsilon) \tag{7}$$

for $p_\perp^2 \gg M^2$, θ_{cm} and $\epsilon \equiv \mathcal{M}^2/s$, fixed, and

$$f(\theta_{cm}, \epsilon) \to f(\theta_{cm})\, \epsilon^F \qquad \text{for } \epsilon \to 0 \tag{8}$$

where[15]

$$N = n_{active} - 2 \tag{9}$$

and[30]

$$F = 2n_{passive} - 1 \,. \tag{10}$$

It is physically clear that N should increase as the number of fields forced to change direction increases, and that F (the degree of "forbiddeness") should increase as increasing number of spectators take away the available phase-space. The reader can readily check that the usual parton model prediction for deep inelastic processes are included as special cases here. For $ep \to eX$, $n_{active} = 4$ $(eq \to eq)$ and $n_{passive} = 2$ giving $\nu W_2(x) \sim (1-x)^3$ at $x \to 1$. For scattering on antiquarks in the proton, $n_{passive} = 4$, and

$\nu W_{2(\bar{q})}(x) \sim (1-x)^7$. This last result has been used by Gunion[30] and Farrar[31] as a simple parametrization of the nucleon antiquark distribution.

More generally, the spectator rule[30] gives for $x \to 1$

$$G_{A/B}(x) \sim (1-x)^{2n-1} , \qquad n = n(\bar{A}B) \tag{11}$$

for the fractional longitudinal distribution of hadron B in hadron A. Some examples are $G_{\bar{q}/\pi} \sim (1-x)$, $G_{\pi/p} \sim (1-x)^5$, $G_{K^-/p} \sim (1-x)^7$, $G_{\bar{p}/p} \sim (1-x)^{11}$, $G_{(qq)/p} \sim (1-x)$, $G_{(qqq)/p} \sim (1-x)^{-1}$, etc. We can use this result to predict the diffractive dissociation contribution to inclusive reactions in the triple Regge region.[30] We have $E\, d\sigma/d^3p \sim (1-x_L)^{1-2\alpha_{eff}(0)}$ $(s \to \infty,\ t \sim 0,\ \mathcal{M}^2/s \sim 1-x_L$ finite, $x_L \sim 1)$ where $\alpha_{eff}^{B\to A} = 1 - n(\bar{A}B)$. More generally, there are contributions from two step processes indicated in Fig. 2, which give $\alpha_{eff}^{A\to B} = \alpha^{C\to A} - n(\bar{C}B)$. A discussion and comparison of these results with available data is given in Dick Blankenbecler's lectures and Ref. 30. For the case of electromagnetic couplings, e.g. leptons or quarks in a photon, the corresponding rules for the Weisacker-Williams distribution and iterated electromagnetic processes are discussed in Ref. 32.

IV. DIMENSIONAL COUNTING AND EXCLUSIVE REACTIONS

The general result (7-10) is also applicable to exclusive processes ($n_{passive} \equiv 0$). We have[15,33]

$$\frac{d\sigma}{dt}(A + B \to C + D) \to \frac{1}{s^{n_a+n_b+n_c+n_D-2}} f_{A+B\to C+D}\left(\frac{t}{s}\right) \tag{12}$$

and for $e + H \to e' + H$, one predicts

$$F_H(t) \sim \frac{1}{t^{n_H-1}} \tag{13}$$

for the spin-averaged electromagnetic form factor. All of these results follow heuristically from the following argument: we partition each hadron's momentum among its constituents. The calculation of $M_{A+B\to C+D}$ then involves the computation of a corresponding amplitude M_n for n_{active} constituents with dimensions $[\text{length}]^{n-4}$. If there is no internal scale, then barring an infrared problem, we have $M_n \sim (\sqrt{s})^{4-n}$ for the asymptotic fixed angle behavior. Equation (12) follows from $d\sigma/dt \sim s^{-2}\,|M|^2$. Also, for an exclusive process at fixed invariant ratios we have using quark counting[15]

$$\Delta\sigma \sim \frac{1}{s^{1+N_M+2N_B}} \tag{14}$$

integrating over a fixed center-of-mass region with finite p_{CM}. This can be compared with the asymptotic falloff of exclusive channels measured at SPEAR provided resonance contributions are separated. We predict $\Delta\sigma(e^+e^- \to 4\pi) \sim s^{-5}$ and $\Delta\sigma(e^+e^- \to p\pi\pi) \sim s^{-4}$.

The prediction $d\sigma/dt \sim s^{-10}$ for $pp \to pp$ can be compared[34] with the data, as illustrated in Fig. 3. Although this appears to give a good representation of the cross section for $|t| \gtrsim 2\ \text{GeV}^2$, it should be noted that the points at the highest t values may be falling faster than s^{-10} at fixed angle. Further experiments, at higher energies, even integrated over a fixed cm region, are clearly very important. The prediction $d\sigma/dt\ (MB \to MB) \sim s^{-8}$ seems to be consistent with the available data.[35] A measurement at SLAC[36] of $\gamma p \to \pi p$ gives $n = 7.3 \pm 0.4$ compared to the dimensional counting prediction $n = 7$. A recent measurement of $p\bar{p} \to p\bar{p}$ at CERN at $p_{lab} = 10$ GeV shows prominent resonance behavior even at large angles, and is not consistent at present energies with a fixed angle scaling law.[37] It should also be noted that purely hadronic physics explanations are possible; e.g. a statistical treatment for non-exotic scattering cross sections at large t appears to be comparable with the data.

The experimental determination of the asymptotic behavior for $F_\pi(t)$ is crucial to the dimensional counting rules. The recent analysis of Bonneau et al,[38] using Frascati data for $e^+e^- \to \pi^+\pi^-$ indicates that if F_π falls on the average as a power t^{-n}, then n is less than 1.2 ± 0.3. Further measurements are essential, although the present e^+e^- storage ring energies may be too high to make measurements of $e^+e^- \to \pi^+\pi^-$ feasible. A possible alternative is measurement of $e^+e^- \to \pi^+\pi^-\gamma$ with a hard photon detected along the beam direction.

According to the Eq. (13), the asymptotic behavior of $F_{1p}(t)$ is t^{-2}; further using simple quark-gluon theories, one obtains $F_{2p} \sim t^{-3}$. Thus we predict G_E, $G_M \sim t^{-2}$. A plot[15] of the present data for $t^2 G_M(t)$ is shown in Fig. 4. We also predict $F_D(t) \sim t^{-5}$ for electron-deuteron scattering: $d\sigma/dt \sim 4\pi\alpha^2/t^2\, F_D^2(t) \sim t^{-12}$ ($s \gg t$). Thus one expects $F_D(t)/F_{1p}^2(t/4)$ to behave as $(1 - t/m^2)^{-1}$. Such a form should be testable in the new measurement of $eD \to eD$ scattering by Chertok et al. now in progress at SLAC.

A simple illustration of how the dimensional counting rule arises in the Bethe-Salpeter computation of the meson form factor is illustrated in Fig. 5. If we assume a falloff in the dependence of the Bethe-Salpeter wave function at large relative momentum (corresponding to a wave function which is finite at the origin in coordinate space), then the leading contribution to the asymptotic form factor comes from iterating the Bethe-Salpeter kernel wherever large relative momentum is required, as indicated in the diagram. A simple computation then gives $F_M(t) \sim t^{-1}$ (modulo a logarithm) assuming a scale-invariant kernel. The inverse factor of t^{-1} comes from the off-shell quark line. For an n-body state, n-1 quark lines are off-shell, giving the result (3). Notice that the minimum field description gives the leading asymptotic behavior.

The dimensional-counting rule for form factors has recently been looked at more carefully within the context of specific renormalizable field theories. In the case of asymptotic freedom theories, Appelquist and Poggio[39] have shown that the asymptotic behavior of the full Bethe-Salpeter kernel is effectively one logarithm more convergent at large momentum transfer than indicated by ladder approximation. They can then show that the quark-antiquark Bethe-Salpeter wave function falls with the required asymptotic dependence if both legs go off-shell at a constant rate; the coordinate-space wave function is finite at the origin up to a calculable logarithm, $\log^d(x)$. Assuming that the wave function has no anomalous infrared behavior when one leg goes on-shell (which seems to be a safe assumption for bound systems of finite size[15]), one predicts $F_\pi(t) \sim t^{-1}$ (modulo logarithms). Thus if F_π should turn out to fall faster than t^{-1}, either the $q\bar{q}$ description of the meson is incorrect, asymptotic freedom is inapplicable, or the infrared assumption is incorrect. A similar result holds when a renormalizable theory yields a small anomalous dimension.[40,15] In the case of quantum electrodynamics, the full Bethe-Salpeter kernel including radiative corrections undoubtedly falls faster than simple ladder approximation, again leading to a finite wave function at the origin. The true asymptotic dependence of the QED kernel to all orders in perturbation theory is not known, but to any finite order in perturbation theory the form factor of positronium obeys the dimensional counting rule--modulo powers of $\log(-t)$. It should be emphasized that the singular behavior which is derived from the Bethe-Salpeter ladder approximation (where the wave function singularity at $x_\mu \to 0$ depends on the coupling constant which in turn is restricted ad hoc by hermiticity) can be misleading for the determination of asymptotic behavior. This is discussed in detail in Ref. 15. Alabiso and Schierholz[41] have shown explicitly how the dimensional counting analysis goes through in the case of three-body bound states.

The general validity of the dimensional counting rules for exclusive reactions is complicated by the presence of the Glauber-like multiple-scattering contributions discussed by Landshoff.[42] In the case of p-p scattering, quarks in one hadron can scatter on-shell to the final direction on a quark of the other hadron with the same longitudinal fraction x. If the q-q scattering is scale-invariant, then the amplitude is only suppressed by a phase-space factor $(is^{-3/2})^{L-1}$ where L is the number of multiple scattering.[15] For pp scattering there are two such factors, giving the scaling law $d\sigma/dt \sim s^{-8}F(\theta_{cm})$, which is in conflict with the data. Clearly, any physical mechanism which suppresses scale-invariant on-shell quark-quark scattering eliminates the Landshoff graphs; e.g. an on-shell infrared suppression factor which has been proposed by Polkinghorne[43] and Appelquist and Poggio.[39] As we emphasize in Section V, such a suppression is required strictly on phenomenological grounds in absence of scale-invariance in the large $p_\perp$, $pp \to \pi X$ data.

Once the Landshoff diagrams are eliminated, then the other diagrams, including the quark rearrangement diagrams of the CIM can be shown to obey the dimensional counting rules (within logarithms) assuming a renormalizable theory and finite Bethe-Salpeter wave functions.[15]

V. APPLICATIONS TO INCLUSIVE REACTIONS

If we use the dimensional counting rules, then the quark model predicts a sum of terms in Eq. (7) with $n_{active} = 4, 6, 8, \ldots$ and $E\, d\sigma/d^3p \sim p_\perp^{-4}, p_\perp^{-8}, p_\perp^{-12}$ at fixed invariant ratios. The fact that a scale invariant term $\sim p_\perp^{-4}$ is not observed could be due to any of a number of possible reasons:

(a) The gluon coupling strength could be very weak--at least at short distances. Of course, at order α^2, electromagnetic and/or weak contributions to quark-quark scattering are expected,[18] but such contributions should not be important until $p_\perp \gtrsim 25$ GeV.

(b) The $p_\perp^{-4}$ term could be suppressed via the quasi-exclusive nature of $pp \to \pi X$, but still be present in the measurement of "jet" production $pp \to J + X$, where the jet is defined as a sum of hadrons with $p_\perp^{jet} = \Sigma_i\, p_\perp^i$ total transverse momentum measured on one side.[44] Accordingly, calorimeter-type measurements will be very interesting. The idea that the cross section is suppressed if a large fraction of the momentum of a scattered parton needs to be transferred to a single hadron is in apparent conflict with the simple Drell-Yan scaling predictions for semi-inclusive deep inelastic scattering processes $ep \to ehX$. However, in the case of asymptotic freedom theories at very large momentum transfers, the predicted deviation from scaling and suppression of the three structure functions $G_{q/A}$, $G_{q/B}$, $G_{C/q}$ (each required with $x \gtrsim 1/2$) could be sufficient to diminish the importance of the $qq \to qq$ term.[45] Measurements of $ep \to \pi x$ and $\mu p \to \pi X$ might help to clarify the situation. The conventional parton prediction has a scale-invariant contribution from $\ell q \to \ell q$ (if the lepton balances the pion momentum) and a contribution $p_\perp^{-6}(\log(s/m_e^2))$, from $\gamma q \to \pi q$ (if a hadron balances the pion momentum). If $G_{q/\pi} \sim (1-x)$, then the $p_\perp^{-4}$ coefficient should have ϵ^5 behavior for small ϵ.

(c) The process $q + q \to q + q$ may be suppressed when the quarks are effectively near their mass shells due to exponentiation of infrared factors; in the Fried and Gaisser model $p_\perp^{-4}$ behavior is expected only for very small $x_\perp$ (see Section II). As pointed out by Polkinghorne,[43] and by Appelquist and Poggio[39] one could still retain scale-invariance of the $qq \to qq$ interaction in the "light cone" region where all quark legs are off-shell, and thus preserve the dimensional counting rules for exclusive processes.

(d) In the massive quark model of Preparata,[22] scale-invariance only occurs in the case of double-fireball production $\bar{q}q \to F\bar{F}$. This is suppressed by the $\bar{q}$ distribution in the nucleon, $G_{\bar{q}/p} \sim (1-x)^7$, as well as by a possible large mass scale. Similarly $qq \to qq$ may be suppressed relative to $q\bar{q} \to q\bar{q}$

if one uses duality as a guide.[46] The latter gives a contribution of order $p_\perp^{-4}\,\epsilon^{13}$ for $pp \to \pi X$, since $n_{passive} = 7$. Such a term could well be hidden by the CIM contributions at NAL and ISR energies. Furthermore, the Landshoff contributions to exclusive scattering are absent, except for $p\bar{p}$ scattering.

(e) Another possibility, suggested by Gunion,[4] is that the coherent sum of <u>all</u> gluon exchange contributions generates the Pomeron contribution to $qq \to qq$. Single gluon exchange would thus be suppressed at high $p_\perp$.

In the constituent interchange model,[13] an explicit quark-quark interaction is never introduced. This idea was originally motivated by the fact that the observed angular dependence $f_{A+B\to C+D}(\theta_{cm})$ for exclusive processes can be rather simply explained in terms of quark-exchange diagrams. Quark-interchange is analogous to rearrangement collisions in atomic and molecular physics. This also seems to be a natural way for hadrons to scatter in the "bag" models. In any event, the quark-hadron amplitude must exist just by the existence of the hadronic wave function. A review of the applications of the CIM to effective trajectories and θ_{cm} dependence in the interchange model may be found in Dick Blankenbecler's lectures and Ref. 13.

The leading processes for $pp \to \pi X^-$ in the CIM derive from quark-hadron interactions. The minimum $n_{active}(= 6)$ terms correspond to $q + M \to q + M$ and $q + q \to B + \bar{q}(\to \bar{q} + M)$ or their crossing variants. An excellent fit to the CCR-ISR data for $pp \to \pi^0 X^-$ (but not the NAL data) can be obtained from Eq. (4) with $G_{q/\pi} \sim (1-x)^5$, $G_{q/p} \sim (1-x)^3$ giving $E\,d\sigma/d^3p \sim (p_\perp^2 + M^2)^{-4}\epsilon^9$ since $n_{passive} = 5$. Similar fits have been given by Ellis[29] and by Barnett and Silverman.[29] The relative importance of the two $n_{active} = 6$ contributions can be settled by measurements of quantum-number correlations.[30]

However, for the Chicago-Princeton-NAL data,[2] which involves $x_\perp > 0.4$, a $\sim p_\perp^{11}$ scaling law is observed. This data, which is closer to the exclusive limit, indicates that other terms involving a larger number of active particles must be involved as ϵ becomes smaller. This is perhaps not unnatural: in general, as one approaches the exclusive limit, $\epsilon \to 0$, we can expect that more active quarks are required in order to produce a hadron with a sizable share of the available center-of-mass energy. In the CIM, the terms which contribute to $pp \to \pi X$ with $n_{active} = 8$ and minimal $n_{passive}(= 3)$ derive from the subprocesses $q + (qq) \to B^* + \pi$ or $p + q \to B^* + q(\to q + \pi)$ and give $p_\perp^{-12}\epsilon^5$. This suggested fits to the data of the form[48]

$$E\,\frac{d\sigma}{d^3p} = \frac{A}{(p_\perp^2 + m_8^2)^4}\,\epsilon^9 + \frac{B}{(p_\perp^2 + m_{12}^2)^6}\,\epsilon^5 \tag{15}$$

The fits are quite good and even are consistent with data at BNL energies. For the ISR data the $p_\perp^{-12}$ term is negligible; the $p_\perp^{-8}$ and $p_\perp^{-12}$ are of comparable importance in the NAL range, with the $p_\perp^{-12}$ term dominant at large $p_\perp$ due to its slower falloff in ϵ. Further details on these fits are discussed in R. Blankenbecler and J. Cronin's contributions to this conference. It is interesting to note that $p_\perp^{-12}$ contribution derives from subprocesses in which a baryonic system balances the large transverse momentum of the detected pion. For any subprocess, however, one expects resonance contributions (i.e. clusters) and a single particle in the recoil system is not likely. The effect of two contributions in $E\,d\sigma/d^3p$ implies that the effective power $p_\perp^{-n}$ will vary from 8 to 12 as $p_\perp$ increases across the NAL range, but remains close to 8 for the ISR data. However, at small $p_\perp < 2$ GeV the effective value of n will drop due to the mass terms and also the non-asymptotic behavior of the effective trajectory $\alpha(t)$. This last effect corresponds to the Reggeization due to the emission and absorption of hadronic bremsstrahlung softens the falloff of the subprocess in t.[28]

Clearly there are a myriad number of contributions from subprocesses in which more and more constituents participate in the large $p_\perp$ subprocesses. In order to make a simple classification, we can utilize the correspondence principle of Bjorken and Kogut[49] which assures a smooth connection between the form of the inclusive cross section for $\epsilon = \mathcal{M}^2/s \to 0$ and a corresponding exclusive cross section. This is a generalization of Bloom-Gilman duality which has been proposed for deep inelastic lepton scattering. Thus if a contribution to the inclusive cross section for $A + B \to C + X$ at fixed θ_{cm} is to join smoothly for $\epsilon \to 0$ to an exclusive cross section for $A + B \to C + D$, we have

$$\int^{\bar{M}^2} d\mathcal{M}^2 \frac{d\sigma}{dt\, d\mathcal{M}^2} (A + B \to C + X) = \int^{\bar{M}^2} d\mathcal{M}^2 \frac{\pi}{s} \frac{1}{(p_\perp^2)^N} \epsilon^F f^{incl}(\theta_{cm})$$

$$\sim \frac{1}{s^{P_{excl}}} f_{A+B\to C+D}(\theta_{cm}) \qquad (16)$$

We thus have[15,50]

$$N + F + 1 = N + 2n_{passive}$$
$$= P_{excl} = n_A + n_B + n_C + n_D - 2 \qquad (17)$$

and the identification

$$f^{incl}(\theta_{cm}) \propto f_{A+B\to C+D}(\theta_{cm}) (\sin^2\theta_{cm})^N \qquad (18)$$

Thus, generally speaking, for large $p_\perp$ and small ϵ, one would expect contributions from those allowed subprocesses $(a + b \to c + d)$ which correspond to the minimum number of hadrons in the related exclusive channel to dominate. Note further that all of the contributions which yield the same P_{excl}, i.e. are dual to the same exclusive channel, may be summed in the form[51]

$$\frac{d}{d^3p/E} \sim \frac{1}{(p_\perp^2)^N} \epsilon^F \left[1 + O\left(\frac{M^2}{p_\perp^2\epsilon}\right)^2 + \cdots + O\left(\frac{M^2}{p_\perp^2\epsilon}\right)^{F+1}\right] f^{incl}(\theta_{cm})$$

where the first term dominates for $p_\perp^2\epsilon \gg M^2$, and where the subsequent terms correspond to allowing additional passive spectator quarks to become active participants in the large momentum transfer reaction. The last term gives the exclusive channel contribution. Note that the corrections to the leading term are of the same form as that obtained by expanding $(p_\perp^2)^{-N}\epsilon^{-1}(\epsilon')^{F+1}$ where $\epsilon'^2 = \epsilon^2 + O(M^4/p_\perp^4)$. This is analogous to the corrections to scaling introduced by the Bloom-Gilman variable $\omega = -(p\cdot q + M^2)/q^2$ in the analysis of deep inelastic scattering. Note that the ω' correction terms for $ep \to eX$ automatically includes the non-scaling contribution from the subprocess $e(qq) \to e(qq)$.

Thus the leading contributions in the CIM can be classified according to their dual exclusive channel (which determines $N + F$) and the distribution of active and passive quarks. To obtain the CIM candidates we only need to exclude the basic subprocesses $qq \to qq$ and $q(qq) \to q(qq)$. A list of various contribution subprocesses for the inclusive processes involving meson and baryons, using meson and baryon or electromagnetic beams is discussed in Ref. 30.

VI. PARTICLE RATIOS AT LARGE $p_\perp$.[31]

The study of particle ratios at large transverse momentum is particularly important since the way in which the quantum numbers of the incident particles feed through to the produced hadrons can be a sensitive discriminant of the contributing quark hard-scattering mechanisms. An important caution is that uncertain nuclear physics effects may be quite important for the particle ratio data obtained from the Chicago-Princeton-NAL experiment.[2] The predictions of the interchange model rely on the fact that different hadrons will be produced with different threshold factors ϵ^F at small ϵ. Note that from Eq. (17), terms with the same power $(p_\perp^2)^{-N}$ yield a particle ratio[51] (e.g. for the B-term in Eqn. (15))

$$\frac{E\, d\sigma/d^3p\ (A + B \to C + X)}{E\, d\sigma/d^3p\ (A + B \to C' + X)} \sim \epsilon^{N_{excl}(C)-N_{excl}(C')} \tag{19}$$

at small ϵ. Thus for p-p or p-n collisions we predict a ratio $\sim \epsilon^0$ for π^+/π^-, π^0/π^-, and K^+/π, whereas K^-/K^+ is suppressed by ϵ^2 because the minimum final state for K^- production-which conserves strangeness-contains two mesons plus two baryons, compared to a one meson-two baryon state which is allowed for K^+ or π. Similarly, one predicts the ratio of cross sections for $\bar{p}/\pi \sim \epsilon^4$ on account of the four baryon final state required for $\bar{p}$ production. These results do seem to be consistent with the Chicago-Princeton-NAL data when the contributions to the $p_\perp^{-8}$ and $p_\perp^{-12}$ terms are separated out. The suppression of K^- relative to K^+ is a critical consequence of the quantum number constraints implicit in quark-parton models and provides a key contrast with the predictions of statistical or fireball models. It seems unlikely that the observed suppression of K^- relative to K^+ is due to kinematic mass differences.

There are serious difficulties in understanding the observed proton production cross section using the CIM. The NAL data does not have the form of a cross section $\sim p_\perp^{-12}\epsilon^3$ predicted from the "leading diagram" based on the subprocess $q + p \to q + p$ (see Fig. 6); the measured $p_\perp^{-12}$ contribution falls faster than ϵ^3 at small ϵ and there could well be important $p_\perp^{-16}$ contributions.

VII. APPLICATIONS TO PHOTON PROCESSES[30,51]

It is particularly interesting to apply the CIM to inclusive photoproduction measurements at large transverse momentum. Measurements of $\gamma p \to \pi^\pm X^-$ by Bojarski et al.[52] and $\gamma p \to \pi^0 X$ by Eisner et al.[53] have been performed at SLAC at transverse momenta beyond 2 GeV. Because of the SLAC energy limit, this demands that ϵ will be quite small. Nonleading terms in $p_\perp$ which fall-off slowly for $\epsilon \to 0$ (with a minimum number of spectators) thus can dominate the cross section. The characteristic contributing terms for $p_\perp$ large or ϵ small, respectively, are

$$E\frac{d\sigma}{d^3p}(\gamma p \to \pi X) \sim \frac{\epsilon^3}{(p_\perp^2 + m^2)^3}\ ,\ \frac{\epsilon^{0,1}}{(p_\perp^2 + m^2)^6}$$

from the subprocess $\gamma q \to \pi q$ and $p + q \to \pi + (qq)$ (see Fig. 7). The second term is "inverted" in that the "target" is the photon. The choices ϵ^0 and ϵ^1 correspond to assuming a direct $\gamma \to q\bar{q}$ or vector-meson dominated structure function for the photon. The experimentalists have tried fits of the form $E\, d\sigma/d^3p \sim (p_\perp^2 + m^2)^{-N}\epsilon^F$ to their data and find[52,53]

$$\gamma p \to \pi^0 X : F = 0.6 \pm 0.3,\ N = 5.8 \text{ to } 7.6,\ m^2 = .5 \text{ to } 1.2\ \text{GeV}^2$$

$$\gamma p \to \pi X : F \cong 1,\ N \cong 6,\ m^2 = 1\ \text{GeV}^2$$

Similarly, in the case of the Bjorken-Paschos process, deep inelastic Compton scattering, the characteristic leading terms at large $p_\perp$ or small ϵ are

$$E\frac{d\sigma}{d^3p}(\gamma p \to \gamma X) \sim \frac{\epsilon^3}{(p_\perp^2 + m^2)^2}\ ,\ \frac{\epsilon^{0,1}}{(p_\perp^2 + m^2)^5}$$

from $\gamma + q \to \gamma + q$, and $p + q \to \gamma + (qq)$, respectively. The Santa Barbara group fit is[53]

$$\gamma p \to \gamma X : F \cong 0.5,\ N \cong 4.5,\ m^2 \cong 0.8\ \mathrm{GeV}^2 .$$

Further, the ratio of $\gamma p \to \gamma X$ to $\gamma p \to \pi X$ does seem to be consistent with the predicted $\sim p_\perp^2$ behavior; despite the extra power of α, the cross sections should eventually become of comparable magnitude. Note that if $\gamma p \to \gamma X$ is measured at large ϵ --away from the edge of phase space--we still expect the scale-invariant parton model prediction to hold at large $p_\perp$.

VIII. OTHER FEATURES OF THE CONSTITUENT INTERCHANGE MODEL[13,28]

One of the most important constraints on a theory of large transverse momentum reactions is that it smoothly connect with the known phenomenology of low momentum transfer reactions. In the CIM, Reggeization occurs naturally from the t-channel iteration of the basic interchange kernal which describes fixed angle processes. Low t processes then have the character of multi-perpheral graphs with hadronic intermediate states. At large t the iterations are unimportant, Regge trajectories approach negative integers and one can prove the validity of the impulse approximation at large momentum transfers. The results at large t are consistent with dimensional counting--with the quark degrees of freedom--and give reasonable parametrizations of the angular distributions. Using the CIM for inclusive cross sections, one obtains smooth connections to the exclusive limit at all t and u and the formulas reduce to the standard triple-Regge and double-Regge (central region) cross sections at low momentum transfers. One thus attains a unity of the physics throughout the Peyrou plot. Further, because of the power-law analytic behavior of the CIM cross sections, there is a simple continuation from one Regge region to another. This is evidenced by the central region contributions to the effective triple-Regge trajectory discussed in Section III. Further one finds that the usual triple-Regge formulae should have an extra factor of $[\mathcal{M}^2/(\mathcal{M}^2-t)]^n$ analogous to $(1-x)^n$ from $F_2(x)$ in deep inelastic scattering. This ensures the inclusive/exclusive connection at any fixed t. Further discussions may be found in Ref. 30.

IX. GENERAL CORRELATION FEATURES OF HARD-SCATTERING MODELS

The characteristic angular and energy distribution and quantum number features of the events containing a large transverse momentum hadron can provide important clues to the nature of the underlying subprocesses in the hard scattering models. As emphasized by Bjorken,[18] a measurement of the double jet cross section at large $p_\perp$ allows us to "look back" and measure the "parton-parton" cross section at $s_{eff} \cong 4p_\perp^2$. However, it now seems clear from the broad coplanarity distributions observed at the ISR, that jets, if present, have very broad decay distributions, with little sign of a significant low transverse momentum cutoff. This also matches with the lack of a $1 + \cos^2\theta$ distribution predicted for $e^+e^- \to$ hadrons in a quark-jet model.

The predicted jet structure of the hard parton models reflects the assumed two-body nature of the underlying large angle process. In the CIM, the subprocess $q + M \to q + M$ produces a quark jet on the opposite side of the triggered meson with characteristics similar to the hadronic system which balances the leptonic momentum in $ep \to eX$. The meson M could be a resonance or "cluster." In both the CIM and q-q scattering models, one expects an increasing multiplicity on the opposite side, and a minimal increase in multiplicity on the same side as a detected high $p_\perp$ hadron. Because of the rapid fall-off of the cross section in $p_\perp$, it does not pay significantly to increase s_{eff} above $4p_\perp^2$. Nevertheless the correlation function

$$\frac{dN/(d^3p_1/E_1)\ (d^3p_2/E_2)}{dN/(d^3p_1/E_1)\ (dN/d^3p_2/E_2)} - 1$$

is large and positive in models with power-law behavior, because it is more probable to have correlated large $p_\perp$ production (from momentum conservation, or from a cluster or resonance) than to have two uncorrelated events. The correlation should be stronger for opposite side events. A very useful correlation quantity to measure is dN/dx, which gives the distribution of the momentum fraction $x = p/p_\perp^{max}$ of one hadron in the recoil system. The φ^3 model of Amati, et al.[25] and the Landshoff-Polkinghorne[14] quark-fusion model $q\bar{q} \to M\bar{M}$ predict a strong enhancement of $N(x)$ at $x \sim 1$, but this is not apparent from the data. In some double fireball models, $N(x)$ is s-dependent. This is an important question which can readily be settled by experiment.

The correlations predicted in hard-scattering models can be easily obtained from simple Monte Carlo programs which use the probability functions $G_{a/A}(x,\vec{k}_\perp)$ to weight the incident particles, and then bin the events in proportion to $d\sigma/dt\ (a + b \to c + d)$. Ellis[54] and Gunion[16] have recently begun such calculations. As mentioned above, the measured[6] non-coplanarity distributions are large, compared to which would be expected from the convolution of up to four gaussians in transverse momentum with a 300 MeV cutoff.[56] This is somewhat accounted for if one employs the power-law falloff in $k_\perp^2$—characteristic of form factorlike hadronic wave functions,[13] but the angular decay of a jet is still larger than what had been expected.

The correlation in θ_{cm}, or the rapidity variable $\eta = \log \tan(\theta_{cm}/2)$ between opposite side particles reflects both the angular dependence of the active subprocess and the distribution of momentum in $G_{a/A}(x)$, $G_{b/B}(x)$, $G_{C/c}(x)$, and $G_{D/d}(x)$. A subprocess with an isotropic distribution is already ruled out by the data, since it produces much too narrow an angular correlation, compared to the $\Delta\eta \sim 3.5$ correlation width measured by the Stony Brook group at $\sqrt{s} = 52$ GeV with one particle at $\theta_{cm} = 90°$ and $p_\perp > 3$ GeV. Angular dependences such as $d\sigma/dt \sim t^{-4}$ or u^{-4} which might be expected in modified gluon exchange models, or the forms $d\sigma/dt \sim 1/su^3$, u/s^5, $1/s^2u^2$, which are possible for $q + \pi \to q + \pi$ are not inconsistent[56] with the present data measured at $x_\perp \sim 0.1$. However, at larger $x_\perp$, the predicted differences between the various models for $d\sigma/dt$ are very distinct. Multiparticle correlations also should be able to discriminate between the models.

Recent experiments have also determined the correlation in η as a function of the cm angle of the detected large $p_\perp$ particle. If two different distributions $G_{a/A}(x)$ and $G_{b/B}$ occur, as in $\pi + q$, then the event tends to be "thrown" in the direction of the "heavier" of the particles a and b. In the case of an isotropic $d\sigma/dt$, one expects events to have an "anti-back-to-back" correlation, i.e., the particles on the opposite side of the detected high $p_\perp$ particle should have the same sign of p_L^{cm}. However, if $d\sigma/dt\ (a + b \to c + d)$ is forward or backward peaked, then the above effect can be negated, and a back-to-back correlation can occur. Future measurements of these correlations, especially at higher momentum transfers and with complete momentum determination will be very useful discriminants of the models. Generally, the features of the correlations are expected to sharpen as $p_\perp$ increases.

Finally, we note that the distinctive characteristics of the various models for the large angle subprocesses, especially in the CIM, lead to dramatic correlations in quantum numbers. In some cases, the recoil system for meson production at large $p_\perp$ is predicted to be a baryonic resonance. The quantum number correlations in meson-baryon collision are clearly very important tests of the hard-parton scattering theories

X. SUMMARY

Large transverse momentum reactions have now given us important clues concerning the internal structure and dynamics of hadrons. The quark-parton hard scattering models seem to provide at least a semi-quantitative description of the present data, including the scaling behavior of the cross sections, most features of the correlation data, and the relative suppression of π vs K^- vs $\bar{p}$ as one approaches the boundary of the Peyrou plot. The postulate that quark-hadron scattering is a dominant interaction at short distances give a simple description of the angular dependence of exclusive reactions at large t and u, and allows one to understand the behavior of the exclusive cross section scaling laws. An important phenomenological observation is that scale-invariant interactions between quarks of different hadrons are relatively unimportant. This poses an extraordinarily interesting theoretical problem that is undoubtedly connected with the quark confinement problem. An attractive possibility is that in bag models without strong gluon interactions, the scattering of hadrons takes place by quark rearrangements, thus supplying a theoretical underpinning for the constituent interchange model.

The CIM also provides new insights into Regge behavior at large momentum transfer, and the connections between large t and small t physics. The prediction that the trajectories $\alpha(t)$ approach a negative integer[13,57] (perhaps modulo logarithms) places important new constraints on Regge phenomenology.

The dimensional counting rules apparently give a reasonable description of the asymptotic behavior of form factors and power dependence of exclusive cross sections at fixed angle--at least within logarithmic accuracy. These results, together with Bjorken scaling for deep inelastic lepton scattering, provide an almost compelling proof of the finite-compositeness of hadrons with the degrees of freedom of the quark model. A mathematically rigorous basis for this connection would be extremely interesting; the work of Appelquist and Poggio[39] and Ezawa and Polkinghorne[43] for asymptotic freedom and scale-free theories lays the foundations for such a proof.

There are, however, phenomenological difficulties with the parton description of large $p_\perp$ reactions (aside from the CEA-SPEAR data for $e^+e^- \to$ hadrons) especially in regard to the normalization of various contributing forms, and the very broad angular correlations.

Further data at large $p_\perp$, especially correlation measurements where the energy and quantum numbers are determined, and the cross sections for π, K, p, and γ beams will be extremely important for further clarifying the features of the underlying mechanisms of hadronic interactions.

References

1. F.W. Büsser et al., Phys. Letters 46B, 471 (1973).
2. J.W. Cronin et al., Phys. Rev. Letters 31, 1426 (1973); also EFI preprint, University of Chicago (1974), Production of Hadrons at Large Transverse Momentum at 200, 300 and 400 GeV; review talk, this conference.
3. J.D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969); R. Feynman, Phys. Rev. Letters 23, 1415 (1969); R. Feynman, Photon-Hadron Interactions, W.A. Benjamin, Reading, Mass., 1972.
4. E. Fermi, Progr. Theor. Phys. 5, 570 (1950); Phys. Rev. 81, 683 (1951); Meng Ta-Cheng, Freie Universität Berlin, preprint FUB HEP 74/2 (1974); L. Heiko, Université Catholique de Louvain, preprint (1974).
5. R. Sumner, private communication.
6. G. Finocchiaro et al., CERN preprint (1974), Charged Particle Multiplicities Associated with Large Transverse Momentum Photons at the ISR.
   B. Betev et al., CERN preprint (1974), Observation of Proton-Proton Interactions with π^0 of Large Transverse Momentum at the ISR.
   See also G. Finocchiaro et al., CERN preprint (1974), Measurement of Charged Particle Multiplicities Associated with Large Transverse Momentum Photons in Proton-Proton Collisions; R. Kephart et al., CERN preprint (1974), s-Dependence of Charged Particle Multiplicities Associated with Large Transverse Momentum Photons at the ISR;
   F.W. Büsser et al., CERN preprint (1974), Correlations between Large Transverse Momentum π^0 Mesons and Charge Particle or π^0 Mesons at the CERN ISR.
7. This point has been emphasized by J. Bjorken.
8. S.M. Berman and M. Jacob, Phys. Rev. Letters 25, 1683 (1970).
9. E.L. Berger, and D. Branson, Phys. Rev. Letters 45B 57 (1973).
10. S. Sakai, Tokyo University preprint TUETP-73-6 (1973).
   A. Bouquet, University of Paris, Thesis (1974); A. Bouquet, V. Letessia and A. Tounsi, preprint PAR-LPTHE 74/1, IPNO 74-20 (1974).

11. T. Uematsu, Kyoto University preprint 1974.
12. For a review see F. Gilman, Review talk given at the International Conference on High Energy Physics, London (1974).
13. R. Blankenbecler, S.J. Brodsky and J.F. Gunion, Phys. Rev. D6, 2652 (1972), Phys. Letters 42B, 461 (1973), Phys. Letters 39B, 649 (1972), Phys. Rev. D8, 287 (1973).
14. P.V. Landshoff and J.C. Polkinghorne, Phys. Rev. D8, 4159 (1973), Phys. Letters 45B, 361 (1973) and preprint DAMTP 73/31 (1973); Phys. Rev. D8, 927 (1973).
15. S. Brodsky and G. Farrar, Phys. Rev. Letters 31, 1153 (1973) and SLAC-PUB 1473 (1974).
16. For other recent reviews of this topic, see P. Landshoff, S. Ellis, and J. Gunion, Proceedings of the London Conference (1974); S.D. Ellis and R. Thun, IXe Recontre de Moriond (1974), CERN preprint TH.1874 (1974); S.D. Ellis, Vth International Symposium on Many Particle Hadrodynamics, Eisenach-Leipzig (1974); S.J. Brodsky, Proceedings of the Stony Brook Conference and SLAC-PUB-1329 (1973); R. Blankenbecler, IXe Recontre de Moriond (1974); R. Blankenbecler, preprint SLAC-PUB-1438 (1974), talk presented at the IXth Balaton Symposium on Particle Physics, Hungary (1974); J. Bjorken, Aix Conference (1974).
17. R. Blankenbecler, S. Brodsky, D. Sivers, Physics Reports (in preparation).
18. S.M. Berman, J.D. Bjorken and J. Kogut, Phys. Rev. D4, 3388 (1971), J.D. Bjorken, Phys. Rev. D8, 4098 (1973).
19. S.D. Ellis and M.B. Kislinger, Phys. Rev. D9, 2027 (1974).
20. D. Cline, F. Halzen and H. Woldrop, Nuclear Phys. B55, 157 (1973).
21. H. Fried, B. Kirby, T. Gaisser, Phys. Rev. D8, 3210 (1973); T. Gaisser, Phys. Rev. D2, 1337 (1970); H. Fried, Brown University preprint (1974).
22. G. Preparata, CERN preprint TH 1859 (1974), and references therein.
23. M. Teper, Westfield College preprints (1974).
24. I would like to thank Dr. M. Creutz for discussions on this point.
25. D. Amati, L. Caneschi and M. Testa, Phys. Letters 43B, 186 (1973) and CERN preprint TH.1644 (1973).
D. Amati and L. Caneschi, CERN preprint TH.1854 (1974).
26. See I. Halliday, J. Huskins, and C. Sachrajda, Imperial College preprints (1973).
27. A hadron bootstrap approach is discussed by J. Harte, Nuclear Physics, B50, 301 (1972).
28. R. Blankenbecler, S.J. Brodsky, J.F. Gunion and R. Savit, SLAC-PUB-1294 (1973), to be published in the Physical Review.
29. M. Bander, R.M. Barnett and D. Silverman, Phys. Letters 48B, 243 (1974); S.D. Ellis, Phys. Letters 49B, 189 (1974).
30. R. Blankenbecler and S. Brodsky, SLAC-PUB-1430 (1974), to be published in Phys. Rev.; J. Gunion, Phys. Rev. 10, 242 (1974).
31. G. Farrar, Caltech preprint (1974).
32. M. Chen and P. Zerwas, to be published.
33. V. Matveev, R. Muradyan and A. Tavkhelidze, Nuovo Cimento Letters 7, 719 (1973); see also Z.F. Ezawa, preprint DAMTP 74/5 (1974) (to be published in Nuovo Cimento).
34. P. Landshoff and J. Polkinghorne, Phys. Letters 44B 293 (1973).
D. Horn and M. Moshe, Phys. Letters 1357, 139 (1973); F. Halzen, University of Wisconsin preprint (1973).
35. See, e.g., G.W. Brandenberg, et al., Phys. Letters 38B, 441 (1972); A. Lundby, Proceedings of the Stony Brook Conference (1973).
36. R. Anderson, et al., Phys. Rev. Letters 30, 627 (1973).
37. Further comparisons and references may be found in Ref. 15, and R. Blankenbecler lectures. For a review, see D. Sivers, SLAC-PUB (1974).
38. G. Bonneau, G. Crunbers, F. Martin, T. Phan, T. Truong, Ecole Polytechnique preprint (1974).

39. T. Appelquist and E. Poggio, Harvard University preprint (1974).

40. Z.F. Ezawa, preprint DAMTP 74/5 (1974) (to be published in Nuovo Cimento); P. Menotti, SLAC-PUB 1485 (1974).

41. C. Alabiso and G. Schierholz, SLAC-PUB 1395 (1974).

42. P.V. Landshoff, preprint DAMTP 73/56 (1973) (to be published in Phys. Rev.); see also Ref. 40, and P. Cvitanovic, Phys. Rev. D10, 338 (1974).

43. J.C. Polkinghorne, Phys. Letters B49, 277 (1974). More recently, Z. Ezawa and J. Polkinghorne, preprint DAMTP 74/20 have shown rigorously that an asymptotically scale-free quark-quark interaction (scale-invariant for off-shell quark legs) of a certain type does yield the dimensional counting rules.

44. This point has been particularly emphasized by S. Ellis, Ref. 16.

45. J. Kogut and L. Susskind, to be published.

46. This was suggested by M. Green (private communication).

47. J. Gunion, Ref. 16.

48. See D. Sivers and J. Newmeyer, SLAC-PUB 1423 (1974).

49. J. Bjorken and J. Kogut, Phys. Rev. D8, 1371 (1973).

50. D.M. Scott, preprint DAMTP 73/37 (1973).

51. J. Gunion, R. Blankenbecler, S. Brodsky, to be published.

52. A. Bojarski, et al., to be published; D. Sheridan (private communication)

53. A. Eisner, et al., Santa Barbara preprints (1974).

54. S.D. Ellis, in the Proceedings of the Vth International Symposium on Many Particle Hadrodynamics, Eisenach-Leipzig (1974).

55. J. Bjorken, private communication.

56. J. Gunion, to be published. The calculations of J. Ellis, Refs. 54 and 16 are for $x_\perp = .2$

57. See also the recent work of T. Kinoshita, et al., Kyusho University preprints (1974); V. Matveev, R. Muradyan and A. Tavkhelidze, Joint Institute for Nuclear Research Report No. E3-8048 (1974); P.G.O. Freund and S. Nandi, University of Chicago preprint EFI 74/33 (1974).

Figure Captions

Fig. 1. The general mechanism for the large transverse momentum process $A + B \to C + X$ in the hard scattering models. A sum over all possible subprocesses $a + b \to c + d$ is understood. The subprocess $a + b \to c + d$ is irreducible in that no further bremsstrahlung from the beam fragments a or b is allowed. In general, the particles a, b, c, d can be quark, diquark, or systems with the quantum numbers of hadrons.

Fig. 2. The two-step contribution to particle production in the fragmentation region of incident particle B. The counting rules predict $E\, d\sigma/d^3p \sim (1-x_L)^F$, with $F = [2n(\bar{C}B) - 1] + 2(1 - \alpha_{CA})$ when $n(\bar{C}B)$ is the number of spectators bremsstrahlunged by B.

Fig. 3. The Landshoff-Polkinghorne fit[34] to pp scattering, $d\sigma/dt = s^{-n} f(\theta_{cm})$, with $n = 9.7$.

Fig. 4. Plot of $t^2 G_M(t)$ from Ref. 15.

Fig. 5. Computation of pion form factor in the weak binding limit. The wave function is explicitly iterated whenever large relative momenta are required.

Fig. 6. "Leading particle" contribution to $pp \to pX$ at large momentum transfers. The contribution does not possess Feynman scaling.

Fig. 7. (a) Contributions to $\gamma + p \to \pi^- X$ at large transverse momentum. The last diagram gives the "inverted" contribution where the photon acts as the "target" and the proton scatters to the pion on its antiquark constituent. Because there is only one quark spectator, the contribution can dominate near the exclusive edge of phase space.

(b) Contributions to $\gamma + p \to \gamma + X$ at large transverse momentum. The first two diagrams give the standard $p_\perp^{-4}$ scaling contributions of Bjorken and Paschos. The last diagram shows the "inverted" process.

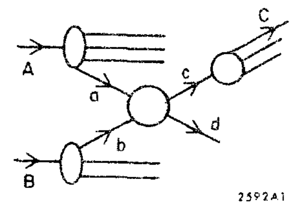

Figure 1

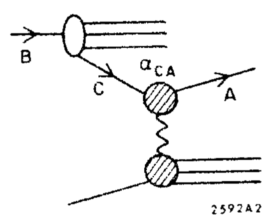

Figure 2

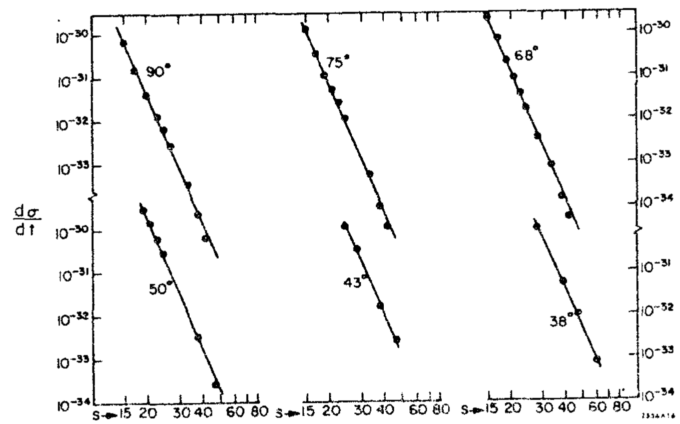

Figure 3

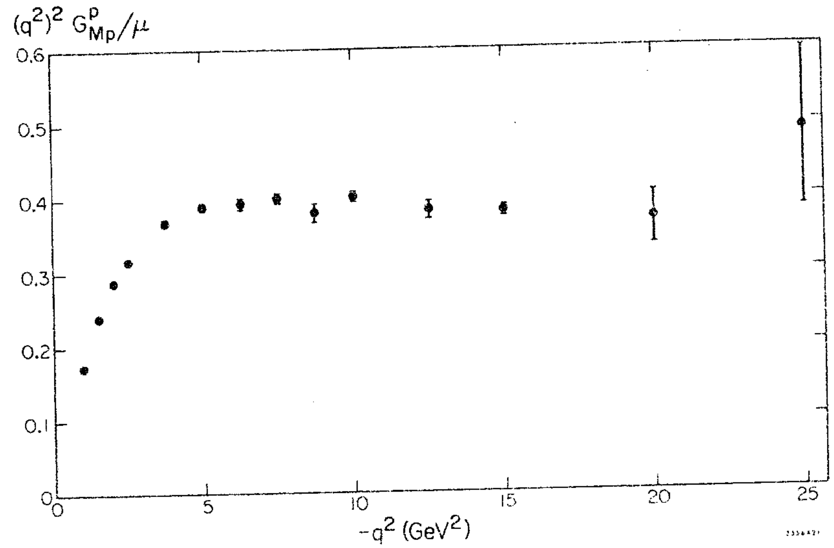

Figure 4

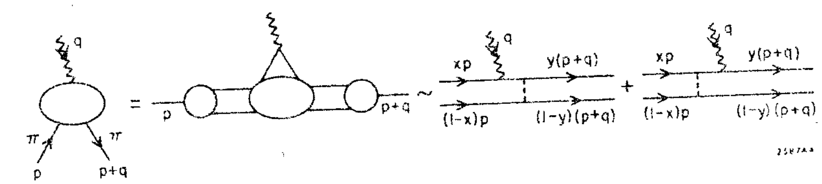

Figure 5

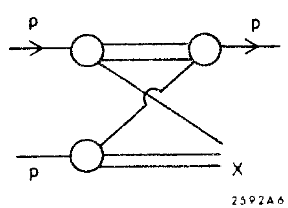

Figure 6

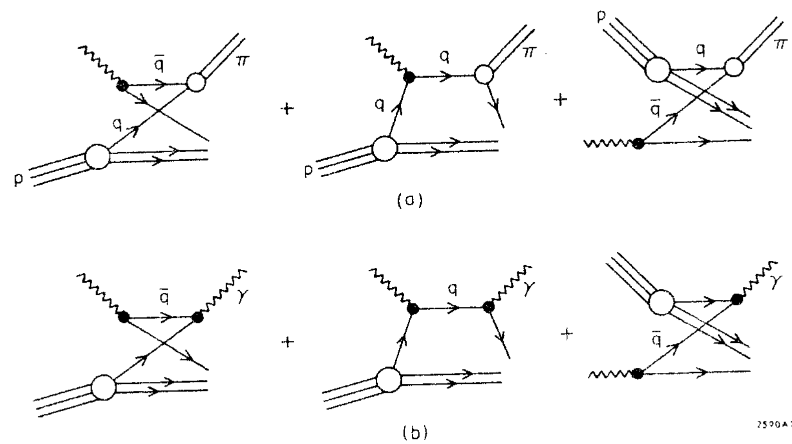

Figure 7