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SCATTERING AND THE CONSTITUENT INTERCHANGE MODEL*

R. Blankenbecler

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

I. Introduction

I would like to describe some new results for inclusive scattering processes which are based on the constituent interchange model, or CIM, developed by J. Gunion, S. Brodsky and myself.[1,2] The idea that the dominant hadron-hadron force arises from constituent interchange has also been applied to discuss aspects of large transverse momentum processes by P. Landshoff and J. Polkinghorne[3] who used their covariant parton model approach. I shall not attempt to describe all the approaches to this problem but shall use the constituent interchange model and dimensional counting exclusively. The reasons for this is that the interchange model is simple with very definite predictions, has very few parameters as you will see, and gives a quantitative description of elastic and inclusive data from low energies, 5-10 GeV/c, to the highest ISR range. The energy and angular dependence of many elastic and quasi-elastic processes are given correctly by assuming that the simplest configuration in the hadron's wave function is given by the valence quark model.

The object of this paper is to describe the predictions of the CIM for exclusive scattering at fixed t and fixed angle,and inclusive scattering throughout the entire Peyrou plot.[4] This means that we will present a unified description of exclusive scattering at fixed t and fixed angle, the triple Regge region, the

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central region, and the region of large transverse momentum.[5] In addition, we will describe several correction terms to the triple Regge formula which are suggested by our model that should be important for both small and large missing mass. Let us now turn to a qualitative description of a general scattering process in the CIM which clarifies the physical reasons behind the smooth connection between simple and predictable power law behavior at large transverse momentum and the full complexities of Regge behavior,[6] and absorption effects, in the small momentum transfer limit.

Let us consider, in transverse impact space, the collision of two composite hadrons. At large momentum transfer, the relevant impact parameters will be as small as possible and the finite sized hadrons will overlap in impact space. It is natural to expect then that an important force will be that between the constituents of one hadron and the containment field of the other hadron. This will naturally give rise to constituent interchange as a dominant force between hadrons in this large t region. Indeed, the CIM assumes that any other interactions can be neglected.

As the momentum transfer decreases, the collision will not necessarily be between the incident hadrons but can occur between secondaries emitted by them (which must then be reabsorbed on the way out after the collision in an exclusive process). These secondaries will be predominantly hadrons since they are the lightest states and possess the longest compton wave lengths and have large coupling constants associated with their emission amplitude. Due to the finite size of the particles involved, these emissions and reabsorption processes occur at small average transverse momenta, and can become more and more important as the momentum transfer decreases. Furthermore, if the basic interchange process falls with increasing incident momenta, as most reasonable models suggest, then collisions between secondaries carrying only a small fraction of the incident momenta and hence small relative energy will dominate. This physical picture has therefore led us to a rather conventional explanation of the origin of Regge behavior in the subenergies — the interaction between "wee" components of the incident particles to use Feynman's term. Furthermore, note that if more than one pair of secondaries interact, it will give rise to multiple exchange contributions, absorption effects with all the requisite nonplanar graphs, etc.

The advantage of this picture of the interaction is that it forces us to recognize that the Regge behavior in the forward direction must join smoothly and continuously onto the fixed angle behavior. Since the backward Regge behavior must also join onto this same fixed angle behavior, there must also exist continuity relations between the forward Regge parameters and the backward Regge parameters. In practice, this leads to relations between the leading forward Regge trajectories and the leading backward Regge residues, and vice versa.[7]

If a model can be developed which will allow a calculation of the basic interchange interaction between hadrons, then by working sufficiently hard, one should be able to calculate the scattering amplitude at all momentum transfers. This is, of course, a gedanken calculation; one must eventually settle for a few terms in an expansion. However, even this allows a calculation of the manner in which the Regge functions approach their asymptotic limits as required by the fixed angle behavior.

The simplest model for the basic interaction that is naturally consistent with the large angle data is the constituent interchange model or CIM. In this model, one assumes a particular constituent model for the hadrons and their interaction and then calculates using any convenient theoretical apparatus. The dimensional counting approach of Brodsky and Farrar[8] will be utilized here for its simplicity and generality and it will be considerably extended. The model for the simplest configuration present in the pion wave function is the familiar quark-antiquark pair in a renormalizable theory. For the nucleon, two simple models are consistent with the present data. The lowest configuration in the nucleon is either three quarks interacting via a renormalizable interaction, or a quark and a "core" (which may be a tightly bound 2-quark state) which interact via a super-renormalizable interaction. This latter model was used in the original CIM papers but here I shall assume the former. Hopefully experiment will throw out one of them before it throws out both of them.

Let us now turn to a discussion of the motivation behind the dimensional counting rules.

II. Even Tempered Operators

In order to introduce the conceptual ideas behind dimensional counting, we will first consider the matrix elements of a single particle operator Q_q that brings in a large momentum transfer q (the current operator is a familiar example) to a composite system (spin will be ignored)

$$M(q^2) = \langle p + q | Q_q | p \rangle , \tag{1}$$

which is illustrated in Fig. 1a. If the states are vertex functions with amputated legs, then, for example, $Q_q = G_1 j_1^q G_1 G_2 \dots G_N$ in an N-body state. For definiteness, assume N = 2 and assume that the vertex function satisfies a Bethe-Salpeter type wave function with an interaction kernel K. Then we can also write

$$M(q^2) = \langle p + q | \overline{Q}_q | p \rangle \equiv \langle p + q | G_1 G_2 K Q_q | p \rangle \tag{2}$$

so that $\overline{Q}_q$ has the same matrix element as Q_q. However, $\overline{Q}_q$ is a connected operator and hence it is straightforward to estimate its magnitude, especially in the asymptotic limit of large q^2.

It is easy to see that for a renormalizable interaction K, M becomes (for the current operator, for example, with point constituents)

$$M_\mu \sim (2p+q)_\mu \left\langle \frac{x}{q^2(1-x)} \right\rangle \left[\int G_1 G_2 |p> \int <p+q| G_1 G_2 \right] , \tag{3}$$

where brackets mean the average value of the usual infinite momentum frame variable x. If the integrals over the wave functions are finite (this is a crucial assumption about the short-distance structure of the bound state), then the form factor falls as $1/q^2$. For an N-body bound state, the kernel K must be iterated (N - 1) times, and one finds the asymptotic behavior of the connected operator and hence the form factor to be

$$F_N(q^2) \sim (q^2)^{1-N} .$$

One can now proceed in the same manner to discuss scattering processes illustrated in Fig. 1b. The general result at fixed angle $z = \cos\theta_{cm}$, for the process $A + B \to C + D$ can be expressed as

$$\frac{d\sigma}{dt} = s^{2-n_A-n_B-n_C-n_D} f_{ABCD}(z) , \tag{4}$$

where n_A is the minimum number of constituents in particle A, etc. It is necessary to examine the contributing graphs carefully in order to extract the behavior of the effective Regge functions and hence separate the s, t, and u behavior, but in certain cases it is quite simple.

One of the fundamental assumptions in the CIM is that the constituents of one hadron have a negligible direct interaction with the constituents of another hadron.

Discarding these graphs then lead to interchange terms of the form given in Fig. 1b. One then finds the general asymptotic results

$$\alpha_{AC}(-\infty) = \frac{1}{2}(4 - n_A - n_B - n_I)$$

$$\beta_{BD}(t) \sim (-t)^{\frac{1}{2}(n_I - n_B - n_D)}, \tag{5}$$

where n_I is the minimum number of exchanged quarks that are required to transport the quantum numbers of interest. Some sample predictions are for meson exchange trajectories $\alpha_{\pi\pi}(-\infty) = -1$, $\alpha_{pp}(-\infty) = -2$, and for baryon exchange trajectories $\alpha_{\pi p}(-\infty) = -2$, and for exotic trajectories $\alpha_{\pi^+\pi^-}(-\infty) = -2$, $\alpha_{K^-p}(-\infty) = -3$, and $\alpha_{p\bar{p}}(-\infty) = -4$. It would be very useful to have enough large energy data available at large momentum transfers to allow these predictions to be checked in a way which is independent of the particular Regge form that one uses to extract the effective trajectories. They are in agreement with the pp and π^-p elastic data[7] and in reasonable agreement with the low energy, large angle charge exchange data that exists.[9] Note that in the original CIM model of core plus quark, $\alpha_{pp}(-\infty) = -3$, but the other trajectories involving mesons are unchanged. The effective trajectory in pp scattering is shown in Fig. 2. The details of its extraction from the data is given in Ref. (7).

III. Distribution Functions

For later use, it is convenient to introduce the function $G_{H/A}(z)$ which is the probability of finding an (offshell) particle H in the incident particle A with a fraction z of its momenta (in the infinite momentum frame of A). Now the full distribution function for quarks in A can clearly be written as the convolution

$$G_{q/A}(x) = \int_x^1 \frac{dz}{z} \sum_H G^I_{q/H}(x/z)\, G_{H/A}(z) \ , \tag{6}$$

where the sum is over all hadron states H that contains a quark of type q, and the superscript I means hadron irreducible (this obviously avoids double counting). The deep inelastic structure function can be written in terms of G's and the quark charge λ_q as

$$F_{2B}(x) = \sum_q \lambda_q^2 \, x \, G_{q/B}(x)$$

or

$$= \int_x^1 dz \sum_H F_{2H}^I(x/z) \, G_{H/B}(z) \ . \tag{7}$$

Using an extension of the arguments used in the previous section for the form factors, one easily derives the threshold behavior (see Ref. 4 for details)

$$G_{q/B}(x) \sim G_{q/B}^I(x) \sim (1-x)^{g(q/B)} \tag{8}$$

where

$$g(q/B) = 2n(\bar{q}B) - 1$$

and $n(\bar{q}B)$ is the minimum number of quarks in the state $(\bar{q}B)$. This simply reflects the difficulty of stopping $n(\bar{q}B)$ quarks and giving all the momenta to quark q. This is consistent with and extends the Drell-Yan-West relation.[10] Using the above equations, the only consistant behavior for hadrons H and B is

$$G_{H/B}(z) \sim (1-z)^{g(H/B)}$$

where

$$g(H/B) = 2n(\bar{H}B) - 1 \tag{9}$$

and $n(\bar{H}B)$ is the minimum number of quarks in the hadronic state $(\bar{H}B)$. This simple result depends only on constituent counting and the assumption of an underlying scale-invariant theory on the quark level.

Sample values of $g(H/B)$ are easily computed: $g(A/A) = -1$ or 3, $g(\pi^{\pm,0}/p) = 5$, $g(K^+/a) = 5$, $g(K^-/p) = 9$, $g(\bar{p}/p) = 11$. Note also the familiar results $g(q/\pi) = 1$, $g(\bar{q}/\pi) = 1$, $g(q/p) = 3$ and $g(\bar{q}/p) = 7$.[11,12]

The Regge behavior of G is reflected in its power behavior for small z and we shall assume throughout our discussions that

$$G_{H/A}(z) = z^{-\alpha_A(0)} (1 - z)^{g(H/A)} , \tag{10}$$

but this could be multiplied by any smooth function of z without modifying our results in any essential way.

It is amusing to note that the above discussion can be extended to the decay of highly unstable states,[4] and one finds, for example, for $(\bar{N}N \to \pi X)$

$$\frac{d\Gamma}{d\omega} \cong (1 - \omega)^{g(\pi/\bar{N}N) - 1} , \quad \text{as } \omega \to 1 ,$$

where $\omega = 2E_\pi/4M$, $g(\pi/\bar{N}N) = 3$, and the $\bar{N}N$ system is at rest. This is in agreement with the data as has been shown by Pelaquier and Renard.[13]

IV. Inclusive Scattering

These distribution functions can be used to implement a dynamical calculational scheme using the picture of the scattering process described in the introduction and depicted in Fig. 3. The incident particles A and B bremsstrahlung particle a and b (with negligible transverse momentum) which interact at lower relative energy and produce the final observed particle C with finite p_T and the recoil spray d*. The subprocess $a + b \to C + d^*$ is hadron-irreducible; that is, no other hadrons are emitted before the basic interaction occurs. The total inclusive cross section can be written as

$$E \frac{d\sigma}{d^3p} (A+B \to C+X) = \sum_{a,b} \int dx dy \, G_{a/A}(x) \, G_{b/B}(y) \left[E \frac{d\sigma^I}{d^3p} (a+b \to C+d^*) \right] \tag{11}$$

and the basic irreducible process $(a + b \to C + d^*)$ is evaluated at $s' = xys$, $t' = xt$, $u' = yu$ and hence $p'^2_T = p^2_T$.

The basic irreducible subprocess can be directly computed from Fig. 4. For example, the process depicted first in Fig. 4, in which the projectile a interacts with a quark in hadron b, can be shown to be

$$\pi E \frac{d\sigma^I}{d^3p}(s,t,u) = \frac{s}{s+u} \sum_q x\, G_{q/b}(x) \left. \frac{d\sigma}{dt}(a + q \to C + q') \right|_{\substack{s' = xs \\ t' = t}} \tag{12}$$

where $\qquad x \equiv -t/(s+u)$.

As t becomes small, one must also include the virtual bremsstrahlung diagrams of the second graph in Fig. 4 which builds up the normal Regge trajectories $\alpha_{aC}(t)$ for the quasi-elastic process $a + q \to C + q'$. The behavior of this process for fixed angle is simply given by the dimensional counting rules and has the form

$$\frac{d\sigma}{dt} \sim (s')^{-N} f(z') \ ,$$

where the value of N can be computed by using Eq. (4). In the last term in Fig. 4, the production of C via the decay of a resonance c is illustrated.

Clearly, in the exclusive limit in which $\mathcal{M}^2/s \to 0$, one must obtain the strong interaction analogue of Bloom-Gilman duality — that is, a smooth connection between the inclusive reaction and the exclusive channels calculated in the CIM. Thus we see that the constituent quark description, which is necessary in order to understand and to compute the power law behavior of the basic processes and the distribution functions, joins smoothly onto the conventional hadronic description at small momentum transfers by virtue of hadronic bremsstrahlung.

There are other important basic processes that must be considered, such as the fusion process, $q + \bar{q} \to M + M^*$, inverse fusion, and the process $q + q \to B + \bar{q}$

which may or may not exist in the model (depending on the basic assumptions) and may or may not be suppressed because the "binding" field or "bag" must be created out of the vacuum. Experiment should be able to answer this question quite easily.

A. Triple Regge Region

As the missing mass decreases, so that $(\mathcal{M}^2/s)$ is small compared to one, the amount of bremsstrahlung that can be emitted is suppressed due to phase-phase considerations if nothing else. If in addition $(t/\mathcal{M}^2)$ is sufficiently small, one is in the triple Regge regime where the bremsstrahlung from particle A is supposed to be totally suppressed. Let us explore that contribution and expected corrections to it. In this case $G_{a/A} \propto \delta_{aA}\,\delta(1-z)$, and the inclusive cross section achieves the form

$$\pi E \frac{d\sigma}{d^3p} = \frac{s}{s+u} F_{2B}(x) \frac{d\sigma}{dt} (A + q \to C + q')\Big|_{\substack{s'=xs \\ t'=t}} ,$$

where x is the Bjorken scaling variable, relevant in deep inelastic scattering, $x = -t/(s+u) = -t/(\mathcal{M}^2 - t)$. The scattering subprocess $A + q \to C + q'$ develops Regge behavior by virtue of the second diagram shown in Fig. 4, and the final result has of the typical form

$$E \frac{d\sigma}{d^3p} = \frac{-u}{s}^2 \frac{\gamma^2(t)}{(p_T^2 + M^2)} F_{2B}(x) \left(\frac{\mathcal{M}^2 - t}{s(p_T^2 + M^2)}\right)^{1-2\alpha_{AC}(t)} \tag{13}$$

$$\simeq \beta(t)\, F_{2B}(0) \left(\frac{\mathcal{M}^2 - t}{s}\right)^{1-2\alpha_{AC}(t)} \tag{14}$$

in the triple Regge limit. Notice that in this limit, the structure function $F_{2B}(x)$ is evaluated at $x = 0$ and its variation is lost in this limit. However, its threshold dependence is very important for small $\mathcal{M}^2$ and in fact, the limit of the exact formula agrees with the exclusive scattering cross section (calculated in the CIM) in both the fixed t and fixed angle limits. The triple Regge formula does not have the proper limit — the Regge residues are wrong.

To summarize this discussion, an important correction to the triple Regge formula for small missing mass has been identified by virtue of the fact that the CIM predicts the form of the amplitude throughout the entire Peyrou plot. It might be expected that the double bremsstrahlung terms will produce important corrections at large missing mass. This is the case, and we will return to this point after a discussion of the central region cross section.

B. Central Region

The previously given expressions for the irreducible subprocesses can be inserted into the full expression for $A + B \to C + X$ and the inclusive cross section can be evaluated. Since the most important region of the $(\text{energy})^2 = s'$ of the subprocess is small, and near the kinematic limit required for the production of C with its given momentum, the subprocess is thereby needed at a fixed angle and its p_T^2 or $(\text{energy})^2$ dependence is completely specified by quark counting. The threshold behavior of the G's control the behavior near the end-points of the x and y integration, and one finds that the contribution of the state $a + b$ is characterized by the behavior

$$E \frac{d\sigma^{ab}}{d^3p} = (p_T^2)^{-N} (\mathcal{M}^2/s)^P \, I(u/s, \mathcal{M}^2/s) , \tag{15}$$

where

$$P = 2(n(\bar{a}A) + n(\bar{b}B)) - 1 \tag{16}$$

and $N + 2$ is the number of fields involved in the basic subprocesses $a+b \to C + d^*$:

$$N = n_a + n_b + n_C + n_{d*} - 2 \ . \tag{17}$$

The factor I(x,y) is a smooth function (which can be read off from the above Eqs.) and it does not vanish when either of the arguments go to zero. It will be ignored in our discussion of the central region, where (u/s) is small but its variation should be taken into account in detailed comparisons with data, especially if (u/s) is finite. We have assumed that the momentum transfer is large so that Regge effects are unimportant. They will be included later. The two numbers N and P have a simple physical interpretation. The power N measures the number of fundamental fields in the basic subprocess that produces a particle of type C with its large transverse momenta. The power P measures the forbiddeness, that is, the number of fields that must be radiated by the incident systems A and B in order to arrive at the given subprocess.

Some numerical examples will be given later, but first one should note that the familiar Mueller-Regge variable ($\mathcal{M}^2/s$) can be written in the equivalent forms

$$\mathcal{M}^2/s \equiv \epsilon = 1 - |\vec{p}|/|\vec{p}|_{max}$$

$$= 1 - (x_T^2 + x_L^2)^{1/2} \quad , \tag{18}$$

where

$$x_T = 2p_T/s^{1/2} \quad , \qquad\qquad x_L = 2p_L/s^{1/2} \quad .$$

These forms demonstrate the smooth connection in our theory between the Feynman scaling region ($x_T \to 0$, x_L fixed) and the large angle deep region ($x_L \to 0$, x_T fixed). The above form, with I = constant, has been used by Carey et al[14] to fit $pp \to CX$ for $1 \lesssim p_T \lesssim 3$ and $C = \pi, K^+, K^-, \bar{p}$. They find an excellent fit with the values N = 4.5, P = 4,4,5,7, respectively. These values would not seem to fit the larger p_T value data, however.

For the process $B + B \to M + X$, there are three main contributions which arise from the subprocesses (1) $q + M \to q + M$, (2) $\bar{q} + q \to M + M^*$, and (3) $q + (qq) \to \pi + B^*$. For the particular cases of $pp \to \pi^{\pm,0}X, K^+X, \rho^{\pm}X$, for example, the inclusive cross section is of the form

$$E\,\frac{d\sigma}{d^3p} = (p_T^2 + M^2)^{-4}\left[h_1\,\epsilon^9 + h_2\epsilon^{11}\right] + (p_T^2 + M^2)^{-6}\,h_3\epsilon^5 + \ldots \qquad (19)$$

The constants h_1, h_2 and h_3, of course, vary from process to process. For the reaction $pp \to K^-X$ on the other hand, since K^- has no common quarks with the initial state, more bremsstrahlung is necessary and one finds

$$E\,\frac{d\sigma}{d^3p}\,(K^-) = (p_T^2 + M^2)^{-4}\left[h_1\epsilon^{13} + h_2\epsilon^{11}\right] + (p_T^2 + M^2)^{-6}\,h_3\epsilon^9 + \ldots \qquad (20)$$

Therefore, this process will fall off more rapidly as $\epsilon \to 0$ than the previous "allowed" reactions by a factor ϵ^2 or ϵ^4, depending upon the values of the constants h_i and the value of p_T^2. This agrees with the qualitative behavior of the Chicago-Princeton data.[15]

A rough fit to the particle ratios measured at NAL has been carried out by J. Gunion, and he finds substantial agreement with the predicted forms. Nuclear effects probably have a strong effect on the particle ratios at small x_T and a detailed fit is not warranted until hydrogen data becomes available. Also let me stress the importance of measuring the associated final states in large p_T events. The terms h_1, h_2 and h_3 have quite similar dependence on the kinematic variables so it is somewhat difficult to tell them apart in purely inclusive measurements. However, the associated final states are quite different: the h_1 term final state looks like deep inelastic electroproduction at $q^2 = -p_T^2$, the h_2 term has a recoiling meson or meson resonance system, the h_3 term has a recoiling baryon or baryon resonance system.

Let me now show you some rough fits to the data that I have carried out. These are not optimum fits in any sense, the parameters were simply varied until the theoretical curves looked something like the data for $pp \rightarrow \pi^- X$. The procedure used was as follows. I arbitrarily set $h_2 = 0$ even though by retaining it, a better fit could be achieved at intermediate values of p_T. The constants h_1 and h_3 were fit to the NAL data at large p_T (~ 5-6 GeV/c) and then the mass parameters associated with the p_T^2 denominators were chosen to agree with the data for $p_T \sim 1$ GeV/c. The resultant curves for the 200, 300, and 400 GeV/c data is shown in Fig. 5 along with the experimental points. Roughly speaking, the p_T^{-8} and the p_T^{-12} terms are comparable throughout this regime but the p_T^{-12} term always wins at large p_T due to its slower falloff in ϵ. One should take note of the fact that there are important nuclear effects in the data which affect the lower p_T range and primarily the magnitude of h_3.

In the upper ISR range of energies, the p_T^{-12} term is negligible ($\epsilon > 0.6$ for this data), and the agreement with the data (16) is excellent for $\sqrt{s} \gtrsim 30.6$ GeV. An important question is whether low energy accelerator data is exploring the same physics as the ultra high energy data discussed above. The answer seems to be in the affirmative but the low energy data is mostly nonexistent and much more is needed. In Fig. 6, the predictions of the theory using the same parameters as determined above are shown as dotted lines and compared with the data of Allaby, et al.[17] These curves check two aspects of the theory, the overall normalization (and its (scaling) energy dependence) and the behavior away from $x_L = 0$. The agreement is much better than could be expected. For increasing $x_L \gtrsim 0.5$, triple Regge and leading particle effects come in as expected and the agreement rapidly worsens. In fitting this data, the function I(x,y) is quite important in determining the $x_L \neq 0$ behavior. Finally, the predictions at

69 GeV/c are in quite good agreement with the recent results of the Saclay-Serphukov collaboration[18] for $p_T < 1.25$ GeV/c and $x_L = 0$. For a discussion of possible tests of the CIM using the predicted general features of the associated production as contracted with gluon models see Newmeyer and Sivers.[19]

One can also discuss the more complicated processes $pp \to px$, $pp \to \bar{p}x$, etc. and the most important contribution to them probably have $N(p) = 6$, $P(p) = 5$ and $N(\bar{p}) = 6$, $P(\bar{p}) = 15$ respectively, although other subprocesses will indeed contribute. These values are in reasonable agreement with the trends of the data for large x_T. These reactions are more complicated only because many subprocesses can contribute; a detailed fit to the data is now being undertaken by J. Gunion, S. Brodsky and myself, and will be published soon.

As a further example, let us consider the process $\pi p \to mX$. Since there are antiquarks present in the initial state, and since the pion probability function falls slower than the nucleons as $z \sim 1$, these cross sections should not fall as rapidly as in the case of the proton incident beam. The CIM predicts that the Feynman scaling terms are of the form

$$E \frac{d\sigma}{d^3p} (\pi p \to mX) = (p_T^2 + M^2)^{-4} k_1 \epsilon^5 + (p_T^2 + M^2)^{-6} k_2 \epsilon^3 + \ldots \tag{21}$$

for $m = \pi^{\pm,0}$, $K^{\pm}$, but for K^-, the second term should be less important. Furthermore, the recoiling system d* is a meson or meson resonance if the first term dominates and is a baryon or baryon resonance if the second term dominates.

Finally, I would like to remark that photoprocesses are particularly important in checking the predictions of the CIM since the point like nature of the constituents should be manifest, and the basic assumptions of the model are most clearly exposed. A preliminary analysis of a 19 GeV/c inclusive photo-pion production experiment performed at SLAC is consistent with the type of predictions given here,[20] and one finds $N \sim 6$, $P \sim 1$, $N \sim 6$-7, $P \sim 1$-2 are consistent for p_T's in the range 1-2 GeV/c. However, a more extensive analysis of more extensive data is required for detailed comparison with and critique of the theory.

Let us now turn to an evaluation of the corrections to the triple Regge formula which are due to a "spillover" from the central region. These should become important in the large missing mass part of the triple Regge region.

Using our fundamental formula, the contribution of the irreducible subprocess $a + b \to C + d^*$ for $x_L > 0$ has the form

$$\approx (\mathcal{M}^2/s)^{g(a/A) + g(b/B) + 1} \left(\frac{p_T^2 + \bar{y}\mathcal{M}^2}{s}\right)^{1-g(b/B)-2\bar{\alpha}} (p_T^2 + M^2)^{2(\bar{\alpha}-1)} f, \tag{22}$$

where we have concentrated only on the most important factors, f is a slowly varying function, and $\bar{y}$ is some number between 0 and 1 (the mean value theorem has been used to evaluate the integral). The trajectory $\bar{\alpha} = \alpha_{aC}(\bar{z}t)$ is evaluated using some average value of z and the asymptotic value $\bar{\alpha}$ for large momentum transfer can be computed from Eq. (5). The parameters $\bar{y}$ and $\bar{z}$ can be easily estimated from the formula.

In the region of large p_T^2, the important dependence on $(\mathcal{M}^2/s)$ is given by the first term, and this formula agrees with the ones given in the previous paragraphs. For small $p_T^2 \ll \mathcal{M}^2$ however, it can be interpreted as a triple Regge formula with an effective trajectory given by

$$\alpha_{eff}(t) = \alpha_{aC}(\bar{z}t) - \frac{1}{2}\left[1 + g(a/A)\right] , \tag{23}$$

where $\bar{z} \approx x_L + y(1 - x_L)$. This effective trajectory formula seems to be important in understanding the results of the analysis of Chen, Wang and Wong.[21] Therefore we have identified a correction to the triple Regge formula which should become important as $(\mathcal{M}^2/s)$ increases and which serves to smoothly join onto (a) the central region cross section and (b) the large p_T cross section.

V. Conclusions

In summary, the inclusive cross section in the triple Regge region can be written in the schematic form

$$E\,\frac{d\sigma}{d^3p} = \beta(p_T^2)\left(\frac{\mathcal{M}^2-t}{s}\right)^{1-2\alpha(t)}\Bigg[F_{2B}\left(-t/(\mathcal{M}^2-t)\right) + \sum_a K^a(p_T^2)\left(\frac{\mathcal{M}^2-t}{s}\right)^{2(\alpha-\overline{\alpha})}\left(\frac{\mathcal{M}^2}{s}\right)^{1+g(a/A)} + (A \leftrightarrow B)\Bigg], \tag{24}$$

where $\alpha = \alpha_{aC}(t)$, $\overline{\alpha} = \alpha_{aC}(\overline{z}t)$ for large p_T^2,

$$\beta(p_T^2) \simeq (p_T^2 + M^2)^{2(\alpha-1)}$$

$$K^a(p_T^2) \simeq (p_T^2 + M^2)^{2(\overline{\alpha}-\alpha)} \quad . \tag{25}$$

Interference terms have been ignored, and only a single triple Regge contribution has been retained for simplicity. There is some evidence from an analysis of inclusive data[22] that there are important contributions that increases rapidly as $\mathcal{M}^2/s$ $(=1-x_L)$ increases $(x \lesssim 0.7$ say). These do not seem to be easily accounted for from the PPR, PPR, RRP, RRR, $\pi\pi$P and $\pi\pi$R terms. It would be interesting to see if these additional terms extrapolate to small x_L and to large p_T with the proper normalization required by the data in these regions.

The inclusive cross sections for the reactions $Ap \to CX$, where $A, C = \pi, p, \overline{p}$, K, , ϕ,... are expected to be of the form

$$E\,\frac{d\sigma}{d^3p} = (p_T^2 + M^2)^{-4}\, H_1(\mathcal{M}^2/s,\ \theta_{cm}) + (p_T^2 + M^2)^{-6}\, H_2(\mathcal{M}^2/s,\ \theta_{cm})\ , \tag{26}$$

where the functional dependence of H_1 and H_2 are given, and their threshold behavior as $\mathcal{M}^2/s \to 0$ can be computed by simple quark counting. Therefore, it is extremely important to measure these inclusive reactions at large p_T with differing beam particles to test the basic assumptions of our model and the very simple and unifying form of the inclusive cross section predicted by it.

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