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FIXED POLE RESIDUES AND THE DIFFRACTIVE LIMIT IN VIRTUAL COMPTON SCATTERING: A SIMPLE APPROACH*

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ABSTRACT

We propose as an ansatz the constancy of the ratio ξ of the non-diffractive pieces of νW_2^n and νW_2^p for $\omega \gtrsim 12$. This enables us to obtain a simple relation between the experimental data and the fixed pole residues R_p, R_n, as well as some constraints on the constant $\nu W_2^{\mathbb{P}} = \lim_{\omega \to \infty} \nu W_2^{p,n}$. For $\xi = 2/3$ and $R_p = 1$, we obtain $R_n = 0.37$ and $\nu W_2^{\mathbb{P}} \simeq 0.20$.

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INTRODUCTION

Some five years after its existence was conjectured[1], the J = 0 fixed pole in nucleon Compton scattering still presents a certain irritation to the phenomenologist. Let us present a brief review of the problem. The existence of a fixed pole at J = 0 in the Compton amplitude T_2 (where $\pi W_2 = \mathrm{Im} T_2$) receives strong support from analyses[2] based on light cone expansions and, more intuitively, from parton model calculations[3], where it reflects the existence of seagull (or seagull-like) terms due to the scattering of light from the partons.

Application of finite energy sum rule (FESR) techniques to the amplitude νT_2 leads in a standard way to the sum rule[4]

$$R(q^2) = \int_0^{\omega^*} d\omega \left[F_2(\omega, q^2)\ \theta(\omega-1) - F_2^{asymp}(\omega, q^2) \right] \tag{1}$$

where $\omega = 2m\nu/q^2$, $\nu = -p\cdot q/m$, $R(q^2) = m\beta_0(q^2)/q^2$, $F_2 = \nu W_2$, with the fixed pole appearing in T_2 as $T_2 \underset{\nu\to\infty}{\approx} -\beta_0(q^2)\nu^{-2}$ + other terms. The upper limit on the integration is such that for $\omega \gtrsim \omega^*$, $F_2(\omega, q^2) \approx F_2^{asymp}(\omega, q^2)$.

If now we suppose that for $q^2 \gtrsim 1\ \mathrm{GeV}^2$, $F_2(\omega, q^2) \to F_2(\omega)$ (Bjorken scaling[5]) then Eq. (1) implies that $R(q^2) \xrightarrow[q^2\to\infty]{} R$ = constant (which may be zero), and that $\beta_0(q^2) \leq q^2$ for large q^2. If indeed $\beta_0(q^2) \propto q^2$ (polynomial residue[6]) then we can compare R as obtained from Eq. (2) with the results of analyses of on-shell Compton scattering. For the proton[7], this implies $R_p \simeq 1.0$, while for the neutron[8] $R_n = 0 \pm 0.5$.

Even a qualitative examination of the present data shows that to obtain $R_p \simeq 1$, from Eq. (1) F_2^p must decrease considerably at large ω from its value ~ 0.33 in the range $\omega = 5$ to 10. The previous attempt[9] to present a quantitative discussion of this possibility involved a rather detailed analysis which made heavy use of a secondary trajectory with $\alpha(0) = -1/2$. It was concluded that it is possible to fit present data on νW_2^p, satisfy quark model sum rules, and have $R_p \simeq +1$. There was, however, very little constraint placed on R_n, and a value of $(\nu W_2)_{diffractive} \lesssim 0.12$ seemed to be favored.

In this note, we wish to present a simple scheme which will allow us to place rather narrow bounds on R_p, R_n and $F_2^{IP} \equiv \lim_{\omega\to\infty} \nu W_2^p = \lim_{\omega\to\infty} \nu W_2^n$. The basic approximation we shall make (next paragraph) will allow us to obtain these bounds without any assumptions as to the "asymptotic" <u>forms</u> of $F_2^{p,n}$ (beyond $\omega = 12$).

THE ANSATZ

Let us suppose that for $\omega \geq \omega^*$ (which we take as 12 in this paper), we may write

$$F_2^p(\omega) = F_2^{IP} + F_2^R(\omega)$$

$$F_2^n(\omega) = F_2^{IP} + \xi F_2^R(\omega) \qquad (2)$$

where $F_2^R \xrightarrow[\omega\to\infty]{} 0$.

Two assumptions have gone into Eq. (2):

(1) $\lim_{\omega\to\infty} F_2^p = \lim_{\omega\to\infty} F_2^n = F_2^{IP}$ and

(2) for $\omega > \omega^*$, $F_2^{R,n}/F_2^{R,p} \approx \text{const} = \xi$.

The first assumption is canonical in either Regge or parton model lore. The second assumption, our ansatz, is true in a simple Regge pole model with degenerate f^o and A_2 trajectories. If there is a lower effective trajectory which is important for $\omega > \omega^*$, then to conform to this assumption its F/D ratio must be similar to that of the 2^+ octet. For orientation, a totally anti-symmetric (F) coupling (or quark model additivity) would imply $\xi = 2/3$.

BASIC EQUATIONS

If these assumptions hold, then we can find from Eq. (2) a linear combination which is purely diffractive beyond $\omega = \omega^*$:

$$F_2(\omega, \xi) \equiv F_2^n - \xi F_2^p \underset{\omega > \omega^*}{\approx} (1 - \xi) F_2^{IP} \equiv F_2^{asymp}(\omega, \xi) \tag{3}$$

Since the FESR (1) is a linear relation, we may write

$$R_n - \xi R_p = \left(I_n(\omega^*) - \xi I_p(\omega^*)\right) - \int_0^{\omega^*} d\omega \; F_2^{asymp}(\omega, \xi) \tag{4}$$

where $I_{p,n} = \int_1^{\omega^*} d\omega \, F_2^{p,n}(\omega)$.

According to Eq. (3), $F_2^{asymp}(\omega, \xi) = \text{constant} = F_2^n(\omega^*) - \xi F_2^p(\omega^*)$ and hence Eq. (4) becomes

$$\begin{aligned} R_n &= \tilde{I}_n(\omega^*) + \xi (R_p - \tilde{I}_p(\omega^*)) \\ &= (\tilde{I}_p(\omega^*) - \tilde{I}_{pn}(\omega^*)) + \xi (R_p - \tilde{I}_p(\omega^*)) \end{aligned} \tag{5}$$

where $\tilde{I}_{p,n}(\omega^*) = I_{p,n}(\omega^*) - \omega^* F_2^{p,n}(\omega^*)$

and $\tilde{I}_{pn} = \tilde{I}_p - \tilde{I}_n$.

The diffractive limit F_2^{IP} is obtained from Eq. (3), evaluated at $\omega = \omega^*$:

$$F_2^{IP} = F_2^p(\omega^*) - \left[F_2^{pn}(\omega^*) / (1 - \xi)\right] \qquad (6)$$

where $F_2^{pn} = F_2^p - F_2^n$.

Eqs. (5) and (6) will provide the basis for our discussion.

DATA

The quantities $\tilde{I}_p(\omega^*)$, $\tilde{I}_{pn}(\omega^*)$ are in principle determined by experiment. The former is rather well constrained. With a change in variable to $\omega' = \omega + M_N^2/q^2$, we can use a recent compilation by Bodek[10] to obtain

$$I_p(12) = 3.33 \pm 0.08$$

$$F_2^p(12) = 0.33 \pm 0.01$$

and hence[11]

$$\tilde{I}_p(12) = -0.63 \pm 0.12$$

As might be imagined, $\tilde{I}_{pn}$ is much more problematic. We present our results for two different sets of data: (a) The recent 6^o and 10^o SLAC-MIT data[12]. This is plotted in Fig. 1. (b) A compilation by Bodek[10] for $0.15 \leq x' < 1$ ($1 \leq \omega' \leq 6.67$), and some 4^o SLAC data reported[13] at the Bonn Conference for $7 \leq \omega' \leq 12$. This set is plotted in Fig. 2.

For Data Set (a), we obtain

$$I_{pn}(12) = 0.55 \pm 0.12$$

$$F_2^{pn}(12) = 0.038 \pm 0.01$$

$$\tilde{I}_{pn}(12) = 0.09 \pm 0.12$$

For Data Set (b),

$$I_{pn}(12) = 0.70 \pm 0.12$$

$$F_2^{pn}(12) = 0.051 \pm 0.01$$

$$\widetilde{I}_{pn}(12) = 0.09 \pm 0.12$$

It should be noticed that the value of $\widetilde{I}_{pn}(12)$ is insensitive to one's choice between these sets of data. Thus, from Eq. (5), the relation between R_n, R_p and ξ is the same whichever data set is used.

BOUNDS ON R_n

In order to narrow the range of parameters, we shall now set $R_p = 1$ and examine the possibilities which ensue. In view of the comment at the end of the previous section, we can then reduce Eq. (5) to the numerical form

$$R_n = (1.63 \pm 0.12)\xi - (0.72 \pm 0.17) \qquad (7)$$

for either data set.

The value of ξ is (in principle) determined by plotting $F_2(\omega, \xi)$ ((Eq. (3)) vs. ω for various values of ξ, and determining the value of ξ for which $F_2(\omega, \xi)$ is flattest in the region $\omega > \omega^*$. The data, of course, is nowhere nearly accurate enough to make this procedure possible. Instead, we resort to a study of R_n (and in the next section, F_2^{IP}) with ξ treated as a parameter.

Eq. (7) is plotted (without delimiting the errors) in Fig. 3. We make the following comments:

(i) For $\xi = 2/3$ (pure octet, F coupling) we obtain $R_n = 0.37$. A slightly different value of R_n ($\simeq 0.5$) is obtained by taking ξ from the previously mentioned[8] fit to on-shell γd data ($\xi \simeq 0.77$). Either of the values is consistent with a linear residue for the neutron fixed pole.

(ii) Eq. (7) is inconsistent with the naive quark model choice $R_n/R_p = 2/3 = \xi$.

(iii) To achieve $R_n = 0$, $R_p = 1$ (the Thomson limit) would require a value $\xi \simeq 0.45$. This is outside the bounds tolerated by the on-shell analysis[8]. (It implies an A_2/f^o ratio of 0.38).

(iv) Higher values of R_n (such as $R_n = 1$) are eliminated when one considers the value of F_2^{IP} implied by such a possibility. To this we turn next.

BOUNDS ON F_2^p

Eq. (6) forms the basis of the discussion in this section. The dashed lines (a and b) in Fig. 3 are graphic representations of Eq. (6) for the two values of $F_{pn}(\omega^*)$ obtained from the two data sets (a) and (b) described in the previous section. The error spread on each curve is about equal to the distance between the curves. It is clear that even if we were given a value of ξ, the resulting F_2^{IP} is only roughly determined. However, with reference to Fig. 3, we can still make some interesting quantitative observations:

(i) The condition $F_2^{IP} \geq 0$ is seen to imply $\xi \leq 0.9$. This in turn implies $R_n \leq 0.7$ (for $R_p = 1$). Thus a solution with $R_p = R_n = 1$ is excluded.

(ii) With $\xi = 2/3$, we obtain $F_2^{IP} = 0.22 \pm 0.02$ for Data Set (a), $F_2^{IP} = 0.18 \pm 0.02$ for Data Set (b). In either case, we can see that F_2^{IP} must fall considerably below its present value for the consistency of our results. This conclusion we share with all authors. However, our values of F_2^{IP} are somewhat higher than those obtained in Ref. 9.

(iii) The quark model distinctly favors Data Set (b), since the quark charge sum rule[14] $\int_0^1 (dx/x)(F_2^p - F_2^n) = 1/3$ seems only a distant possibility with Data Set (a). In that case (and with $R_p = 1$), we can see from Fig. 3 that F_2^{IP} is restricted to be $\leq 0.24 \pm 0.02$ by the condition $R_n \geq 0$. (The latter is practically imperative in the seagull-parton interpretation[3] of the fixed pole term.)

RESUME

We have attempted to present a simplified framework for the examination of the fixed-pole residues in virtual Compton scattering. By dealing simultaneously with the neutron and proton, and by postulating that for $\omega' \gtrsim 12$, the non-diffractive pieces of F_2^n and F_2^p bear a constant ratio (ξ) to each other, we have been able to set up two simple equations ((5) and (6)) relating R_p, R_n, ξ, and the diffractive piece F_2^{IP} to various pieces of experimental data. With great insensitivity to the data for $\nu W_{2p} - \nu W_{2n}$, one can restrict R_n to values $\lesssim 0.5$ if $R_p = 1$. The diffractive piece F_2^{IP} depends more strongly on $\nu W_{2p} - \nu W_{2n}$, but a value of $\sim 0.20 \pm 0.04$ seems to be indicated.

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FIGURE CAPTIONS

Fig. 1 $\nu W_{2p} - \nu W_{2n}$ vs. ω', taken from J. Poucher, et. al., Ref. 12. Errors shown are statistical only. The interpolating curve is an eyeball fit to the data, and was used as the basis of obtaining the values of $I_{pn}(12)$, $F_2^{pn}(12)$ given in the text under Data Set (a).

Fig. 2 $\nu W_{2p} - \nu W_{2n}$ vs. ω'. Points designated by circles are taken from A. Bodek, Ref. 10, those designated by triangles are from Ref. 13. The errors are statistical only. The interpolating curve is an eyeball fit to the data, and was used as the basis of obtaining the values of $I_{pn}(12)$, $F_2^{pn}(12)$ given in the text under Data Set (b).

Fig. 3 Solid curve: "Reduced" Neutron Fixed Pole Residue R_n (see definition following Eq. (2)) vs. $\xi \equiv (\nu W_2^n - \nu W_2^{\mathrm{IP}}) / (\nu W_2^p - \nu W_2^{\mathrm{IP}})$, for $R_p = 1$. Dashed curves: Diffractive (Pomeranchuk) limits $\nu W_2^{\mathrm{IP}} = \lim_{\omega \to \infty} \nu W_2^p = \lim_{\omega \to \infty} \nu W_2^n$ vs. ξ. Curve a (b) based on Data Set a (b) (see text).

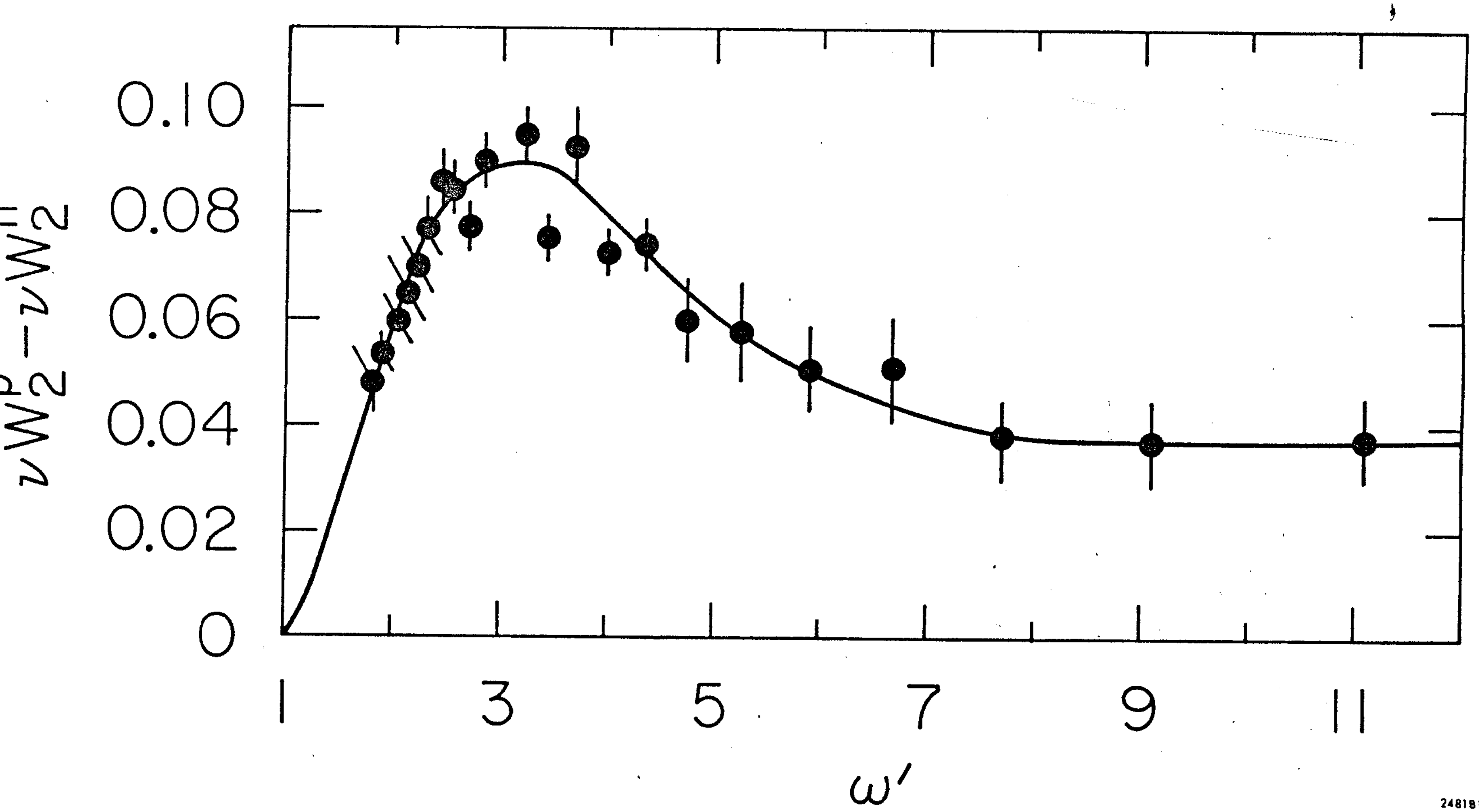

Fig. 1

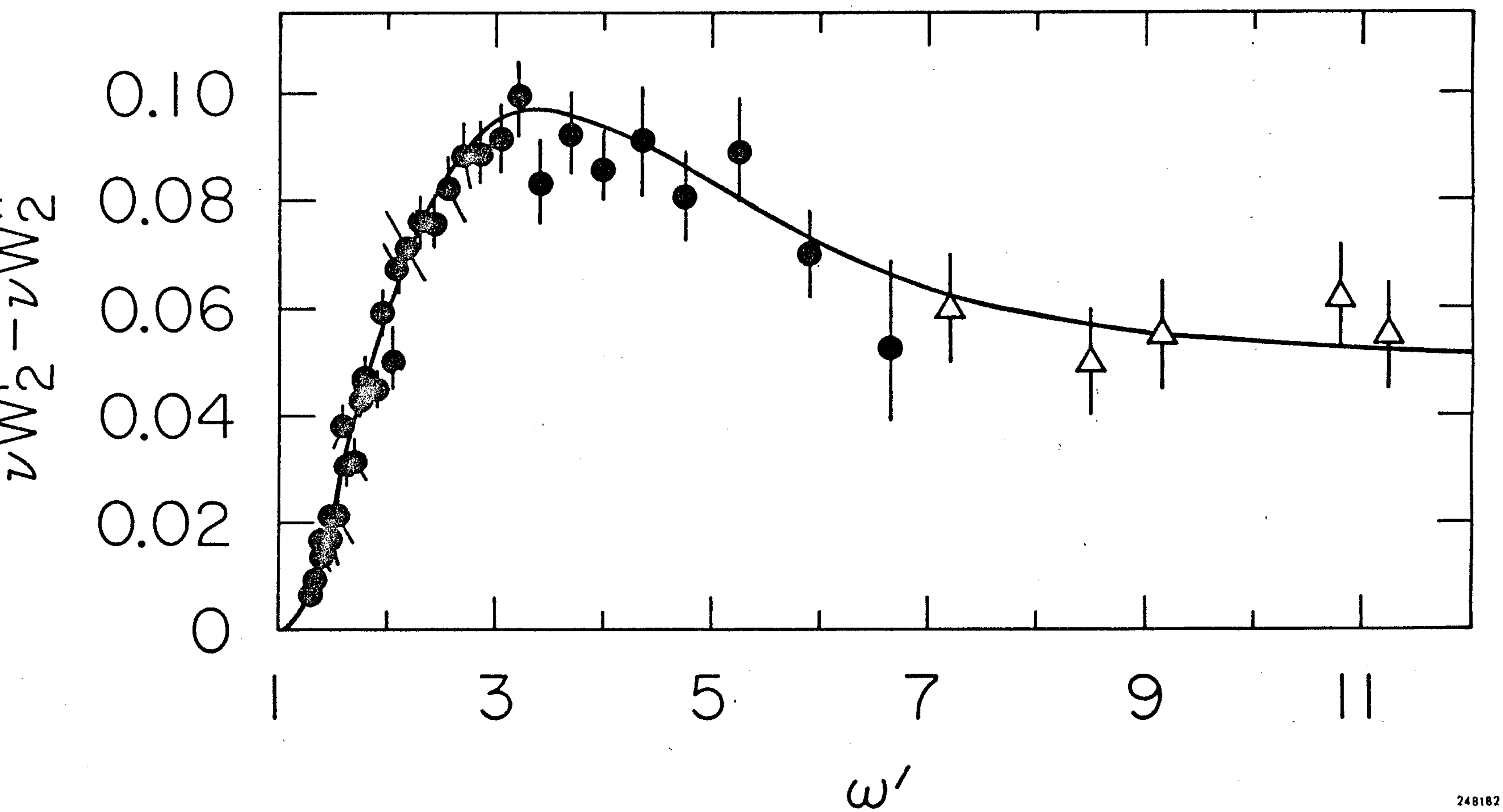

Fig. 2

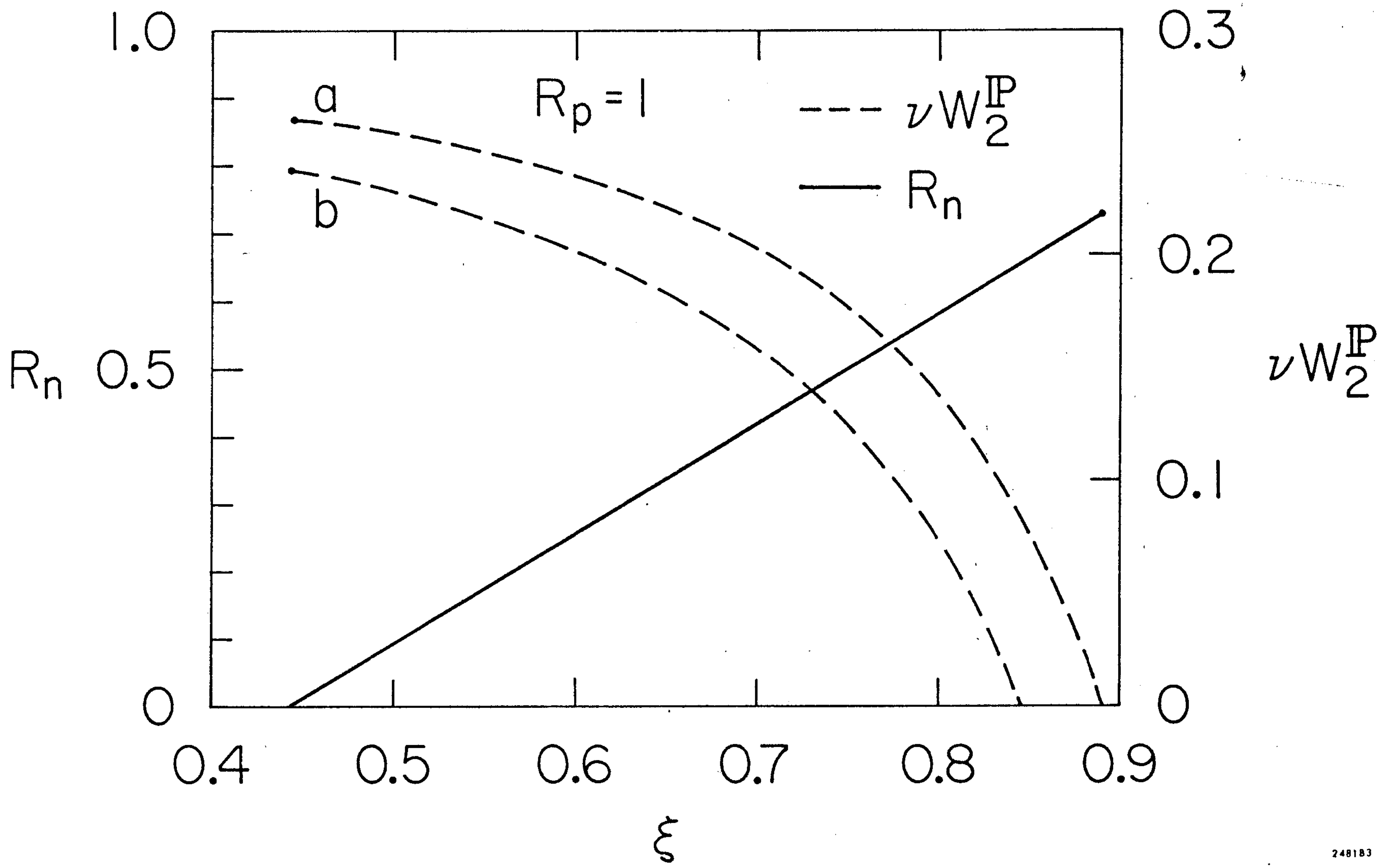
1.0
0.5
0
R_n
0.3
0.2
0.1
0
$\nu W_2^{\mathbb{P}}$
0.4
0.5
0.6
0.7
0.8
0.9
ξ
a
b
$R_p = 1$
$\nu W_2^{\mathbb{P}}$
R_n
248183

Fig. 3