SLAC-PUB-1381
(T/E)
January 1974

ENERGY CONSERVATION AND FRAGMENTATION IN THE EIKONAL MODEL*

Shau-Jin Chang†

Department of Physics
University of Illinois, Urbana, Illinois 61801

and

Tung-Mow Yan††

Stanford Linear Accelerator Center
Stanford University, Stanford, Calif. 94305

ABSTRACT

A thermodynamic approach is proposed to enforce the energy conservation in the eikonal model. A generalized eikonal representation is given for the elastic and inelastic amplitudes when fragmentation takes place. Experimental consequences are discussed.

(Submitted for publication)

*Work supported in part by the U. S. Atomic Energy Commission and National Science Foundation GP 25303.

†Alfred P. Sloan Foundation Research Fellow, 1972-1974.

††On leave of absence from Cornell University, Ithaca, New York 14850.

Recent ISR and NAL data [1] indicate that the proton-proton total cross section rises significantly with energy and suggest that the average multiplicity may increase with energy faster than logarithmic. These two features emerge rather naturally from an eikonal model provided the eikonal function is associated with a connected piece with Regge intercept $\alpha(0)$ greater than unity. [2] Assuming the possibility that the eikonal model may become a useful alternative to organize the data, we reexamine and explore further certain aspects in an eikonal model with rising total cross sections. This work is partly motivated by our earlier calculation [3, 4] in which it was noted that when the coupling is sufficiently strong energy conservation is not automatically satisfied in the eikonal approximation.

In this paper a thermodynamic approach is proposed to enforce the energy conservation constraints. We also extend the eikonal model to include the fragmentation effects. A generalized eikonal representation is given for the elastic and inelastic amplitudes. This improved discussion of energy conservation not only confirms essentially all the results obtained previously by a more crude method, [3, 4] it also offers further insight and more detailed information.

Our calculations are based on studies of a certain class of Feynman graphs in specified kinematic regions in a ϕ^3 theory. To be definite, we will restrict ourselves to the graphs depicted in Fig. 1 and 2 for the elastic and inelastic amplitudes, respectively. The exchanged connected piece will be limited to the t-channel ladder. We will always consider the collision in the center of mass system. The lines labeled $a_1, \ldots, a_n$ ($b_1, \ldots, b_m$) are the fragments of particle a (particle b) which share the large momentum p_a (p_b). The exchanged objects denoted by blobs in Fig. 1 contain only particles in the pionization region. A fragment has a longitudinal momentum in the range

$$\eta\,|p_a| = \eta\,|p_b| \leq p_3 \leq |p_a| = |p_b| \qquad (0 < \eta << 1) \tag{1}$$

Particles with momenta outside this region are pionization particles.

The main results are:

(a) The inclusion of the fragmentation does not modify the characteristics of the inclusive one-particle distribution in the central region. It is given by

$$d\sigma^{(1)}(k) = \text{const.}\ f(\vec{k}_\perp)\ \frac{d^3k}{\epsilon} \left(\frac{E}{\mu}\,\ell n\,\frac{E}{\mu}\right)^{\frac{2\alpha-2}{2\alpha+3}} \left(\ell n\,\frac{E}{\mu}\right)^{\frac{4}{2\alpha+3}}$$
$$\times \frac{1}{\tau(0)\epsilon}\left[1 - e^{-\tau(0)\epsilon}\right] \tag{2}$$

where $\alpha = \alpha(t = 0)$, $E = \sqrt{s}$ and

$$\mu\,\tau(0) = \text{const.} \left[\left(\frac{E}{\mu}\right)^5 \ell n\,\frac{E}{\mu}\right]^{-\frac{1}{2\alpha+3}} \tag{3}$$

and $f(\vec{k}_\perp)$ is the standard inclusive one-particle distribution in the ϕ^3 multiperipheral model. The overall normalization in (2) depends on the details of the fragmentation effects. But apart from the factor $\frac{1}{\tau\epsilon}(1 - e^{-\tau\epsilon})$ which reduces to 1 when $\tau\epsilon << 1$, Eq. (2) has the same structure as Eq. (4) in Ref. 3. Equation (2) explicitly displays how the inclusive cross section approaches zero as ϵ reaches beyond its kinematic limit $\epsilon \geq \frac{1}{\tau(0)}$. Thus the inclusive one-particle distribution (2) is flat over a region in rapidity of width

$$D = 2\,\ell n\,\frac{1}{\mu\tau(0)} = \frac{5}{2\alpha+3}\,Y_0 \tag{4}$$

where $Y_0 = 2\,\ell n\,\frac{E}{\mu}$ is the total rapidity available. The height of the central region grows with Y_0 (or E),

$$H \propto \left(Y_0\,e^{1/2\,Y_0}\right)^{\frac{2\alpha-2}{2\alpha+3}} = \left(2\,\frac{E}{\mu}\,\ell n\,\frac{E}{\mu}\right)^{\frac{2\alpha-2}{2\alpha+3}} \tag{5}$$

In our model the central region does not cover the whole range of available rapidity Y_0 ($D < Y_0$). There is a rapidity gap $\frac{2(\alpha-1)}{2\alpha+3} Y_0$ between the pionization and fragmentation regions. This gap has not been observed experimentally, but it may be very difficult to detect in practice if α is only slightly greater than unity. The existence of this rapidity gap is required by energy conservation, since according to (2) the average multiplicity grows as a power of energy

$$< n > \propto \left(\frac{E}{\mu}\right)^{\frac{2\alpha-2}{2\alpha+3}} \tag{6}$$

within logarithmic corrections. This result also agrees with what we obtained in Ref. 3 and 4.

(b) The one-particle inclusive cross section in the fragmentation region $1/\sigma_T \; \epsilon \; d^3\sigma/d^3k$ satisfies the Feynman scaling [5] or Benecke, Chou, Yang and Yen's limiting distribution [6].

(c) The ratio of elastic to total cross section lies between one-half and zero. In the previous models without fragmentation, [3, 4, 7] the total cross section σ_T saturates the Froissart bound and is twice the elastic cross section σ_E. When fragmentation or diffraction channels are included the general result is [8]

$$\sigma_E + \sigma_D = \frac{1}{2} \sigma_T \tag{7}$$

where σ_D is the partial cross section in which all the additional particles produced are in the fragmentation region. In principle, σ_D/σ_T need not vanish. This is welcome since the ratio σ_E/σ_T stays practically unchanged in the ISR data and is significantly less than 1/2.

The investigation leading to the above conclusions will now be briefly described. The details will be published elsewhere.

To study the constraint of energy conservation in the eikonal approximation, we first neglect completely the fragmentation events. Our model then reduces to the model proposed in Ref. 3 and 4. We shall refer the reader to these references for detail. In particular, the inelastic cross section due to the opening of N-ladders can be written as (see Eq. (3.11), Ref. 4).

$$d\sigma_N = \int d^2b\, e^{-2A(s,b)} \prod_{i=1}^{N} \left| \frac{1}{2s} \widetilde{M}(s,b;\{k^{(i)}\}) \right|^2 \times \prod_{\text{all } j} \frac{d^3k_j}{(2\pi)^3 2\epsilon_j} \tag{8}$$

where $\widetilde{M}(s,b;\{k^{(i)}\})$ is the Fourier transform of the multiperipheral amplitude and $A(s,b)$ is the appropriate eikonal function. All the momenta $k_j^{(i)}$ belong to particles in the pionization region.

We propose to make the energy conservation constraint explicit by rewriting (8) as

$$\sigma_N = \frac{1}{N!} \int d^2b\, \frac{\Pi_N(s,b)}{\Pi(s,b)} \tag{9}$$

where $(E \equiv \sqrt{s})$

$$\Pi_N(s,b) = \int \prod_{\text{all } i} \frac{d^3k_i}{(2\pi)^3 2\epsilon_i} \prod_{i=1}^{N} \left| \frac{1}{2s} \widetilde{M}(s,b;\{k^{(i)}\}) \right|^2 \times \theta\left(E - \sum_{\text{all } j} \epsilon_j\right) \tag{10}$$

and

$$\Pi(s,b) = \sum_{N=0}^{\infty} \frac{1}{N!} \Pi_N(s,b)\,, \qquad \Pi_0 = 1 \tag{11}$$

The step function in (10) destroys the factorizability of the individual ladder amplitudes. However, the factorizability is regained by making a Laplace transform:

$$\widetilde{\Pi}_N(\tau, b) = \int_0^\infty dE\, e^{-\tau E}\, \Pi_N(s, b)$$

$$= \frac{1}{\tau}\left[2\widetilde{A}(\tau, b)\right]^N \tag{12}$$

and

$$\widetilde{\Pi}(\tau, b) = \frac{1}{\tau}\, e^{2\widetilde{A}(\tau, b)} \tag{13}$$

The function $2\widetilde{A}$ is given by, in ϕ^3 ladders,

$$2\widetilde{A}(\tau, b) = \frac{1}{s^2}\, \frac{\widetilde{\beta}}{8\pi c \mu^2 \ln \frac{1}{\mu^2 \tau^2}} \left(\frac{1}{\tau^2 \mu^2}\right)^{\alpha+1} e^{-\frac{b^2}{4 c \ln \frac{1}{\mu^2 \tau^2}}} \tag{14}$$

where $\widetilde{\beta}$ and c are independent of τ and b. From (9) - (11) we can calculate the inclusive particle distributions, multiplicity and its higher moments, etc. The one-particle inclusive distribution will be worked out to illustrate the technique of Laplace transform and to exhibit its thermodynamic interpretation. The inclusive single particle distribution is given by

$$d\sigma^{(1)}(k) = \int d^2b\, \frac{L^{-1}\left[\widetilde{\Pi}(\tau, b)\, \widetilde{B}_1(\tau, b, k)\right]_{E-\epsilon}}{L^{-1}\left[\widetilde{\Pi}(\tau, b)\right]_E}\, \frac{d^3k}{\epsilon} \tag{15}$$

where

$$\widetilde{B}_1(\tau, b, k) = 2\widetilde{A}(\tau, b)\, f(\vec{k}_\perp) \tag{16}$$

The symbol L^{-1} signifies the inversion of the Laplace transform. The subscripts denote the arguments of the inverse Laplace transforms. The ratio of the inverse Laplace transforms can be easily determined noting that at high energies, method of steepest descent is applicable to integrals such as (15). Thus

$$d\sigma^{(1)}(k) = \int d^2b \; \frac{\Pi(E-\epsilon, b)}{\Pi(E,b)} \; \widetilde{B}\; \tau(E,b),\; b,\; k \;\; \frac{d^3k}{\epsilon} \tag{17}$$

with $\tau(E,b)$ determined by the standard relation

$$E = -\frac{\partial}{\partial\tau} 2\widetilde{A}(\tau, b) = -\frac{\partial}{\partial\tau} \ln \widetilde{\Pi}(\tau, b) \tag{18}$$

and

$$\frac{\Pi(E-\epsilon, b)}{\Pi(E,b)} = e^{-\tau(E,b)\,\epsilon} \tag{19}$$

Equation (18) is recognized to be the familiar connection between the energy E and the partition function $\widetilde{\Pi}(\tau, b)$. Equation (19) is the well-known Boltzmann factor found in statistical mechanics. The Laplace transform variable τ appears as the inverse temperature and $\widetilde{\Pi}$ the partition function. Many physical questions can be answered from thermodynamics considerations. Equation (2) follows from (17)-(19). [9]

Now, we study the effect of the fragmentations. To simplify the discussion, we assume that only particle a fragments into particles 1 and 2. The elastic amplitude $T_E^{(2)}$ and the diffractive amplitude $T_D^{(2)}$ due to these fragments are

$$T_E^{(2)}(p_a') = 2is \int d^2b_1 d^2b_2 \, e^{-i(\vec{p}_a' - \vec{p}_a)\cdot\vec{b}} \int \frac{dx}{4\pi x(1-x)} \; |\psi(x, b_{12})|^2 \times \left[e^{-A(x,\, s,\, b_1, b_2)} - 1\right] \tag{20}$$

and

$$T_D^{(2)}(p_1', p_2') = 2is \int d^2b_1 d^2b_2 \, e^{-i(\vec{p}_1' - x\vec{p}_a)\cdot\vec{b}_1 - i(\vec{p}_2' - (1-x)\vec{p}_a)\cdot\vec{b}_2} \times \psi(x, b_{12}) \left[e^{-A(x,\, s,\, b_1, b_2)} - 1\right] \tag{21}$$

respectively, where x is the fractional longitudinal momentum of p_1 ($p_1 = x p_a$), $\vec{b} \equiv x\vec{b}_1 + (1-x)\vec{b}_2$, $\vec{b}_{12} \equiv \vec{b}_1 - \vec{b}_2$, $\psi(x, \vec{b}_{12}) \sim K_0(\sqrt{1 - x + x^2}\,\mu b_{12})$ is the infinite momentum frame wave function obtained by the Fourier transform of the energy denominator, ($\vec{p}_{12} = (1-x)\,\vec{p}_1 - x\vec{p}_2$),

$$\psi(p_{12}) = \frac{g}{2\,E_a\left[E_1(p_1) + E_2(b_2) - E_a\right]} = \frac{x(1-x)g}{\vec{p}_{12}^{\,2} + \mu^2(1 - x + x^2)} \tag{22}$$

and $A(x\ ,\ s\ ;\ b_1,\ b_2)$ is the appropriate (3-body) eikonal function. Equations (20) and (21) can be generalized when more complicated fragments are introduced. The inelastic amplitudes can be computed in a similar way.

From amplitudes (20) and (21) we can compute various cross sections,

$$\sigma_D^{(2)} = \int d^2b_1 d^2b_2 \int \frac{dx}{4\pi x(1-x)}\ |\psi(x,\ b_{12})|^2 (1 - e^{-A(b_1, b_2)})^2 \tag{23}$$

$$\sigma_I^{(2)} = \int d^2b_1 d^2b_2 \int \frac{dx}{4\pi x(1-x)}\ |\psi(x,\ b_{12})|^2\ (1 - e^{-2A(b_1, b_2)}) \tag{24}$$

and

$$\sigma_D^{(2)} + \sigma_I^{(2)} = -\ \frac{1}{s}\ \mathrm{Im}\ T_E^{(2)}\ (\vec{p}_a^{\,\prime} - \vec{p}_a = 0) \tag{25}$$

In the strong coupling case, the range of b_1 and b_2 in $A(b_1,\ b_2)$ increases like $\ln s$ while the range of $b_{12}(= b_1 - b_2)$, controlled by $\psi(b_{12})$ is finite. Thus, in this limit, we have $b_1 \approx b_2 \approx b$, $A(b_1, b_2) \approx A(b, b)$, and

$$\sigma_I^{(2)} = \sigma_D^{(2)} = \frac{1}{2}\,\sigma_T^{(2)} \tag{26}$$

When all possible diffractive channels are considered, we get the result (7). As mentioned earlier, the inclusion of fragmentation events alters only the overall normalization of Eq. (2), but not its structure.

Feynman's scaling of the one-particle inclusive distribution $1/\sigma_T \, \epsilon \, d\sigma^{(1)}/d^3k$ in the fragmentation region follows from Eq. (21) and its generalizations to other diffractive channels combined with the approximation of using the single impact parameter representations.

One of us (S.J.C.) wishes to thank Professors K. Gottfried and E. Berger for many stimulating discussions. One of us (T.M.Y.) wishes to thank Aspen Center for Physics and Prof. S.D. Drell of SLAC respectively for the hospitality during the summer of 1973 and the year 1973-74.

FOOTNOTES AND REFERENCES

[1] For a review of the present experimental situation, see M. Jacob, the Rapporteur's talk at Chicago-NAL Conference (1972), and U. Amaldi, 1973 Erice School Lectures.

[2] For a review of this subject, see R. Blankenbecler, J.R. Fulco, and R.L. Sugar, Stanford Linear Accelerator Center Report No. SLAC-PUB-1281 (1973).

[3] S.J. Chang and T.M. Yan, Phys. Rev. Letters 25 (1970) 1586.

[4] S.J. Chang and T.M. Yan, Phys. Rev. D4 (1971) 537.

[5] R.P. Feynman, Phys. Rev. Letters 23 (1969) 1415.

[6] J. Benecke, T.T. Chou, C.N. Yang and E. Yen, Phys. Rev. 188 (1969) 2159.

[7] H. Cheng and T.T. Wu, Phys. Rev. Letters 24 (1970) 1456.

[8] This result is obtained independently by Blankenbecler, Fulco and Sugar in Ref. 2. These authors discuss the conditions under which $\sigma_D/\sigma_T \neq 0$. These conditions are satisfied in our model.

[9] The analog of Eq. (2) for the massive vector gluon model in the region $\epsilon\tau(0) << 1$ has been obtained by Cheng and Wu [Phys. Letters 45 B, (1973) 367] , by a method similar to that used in Ref. 3 and 4.

FIGURE CAPTION

1. (a) Elastic amplitudes considered in our model.

   (b) Inelastic amplitudes considered in our model.

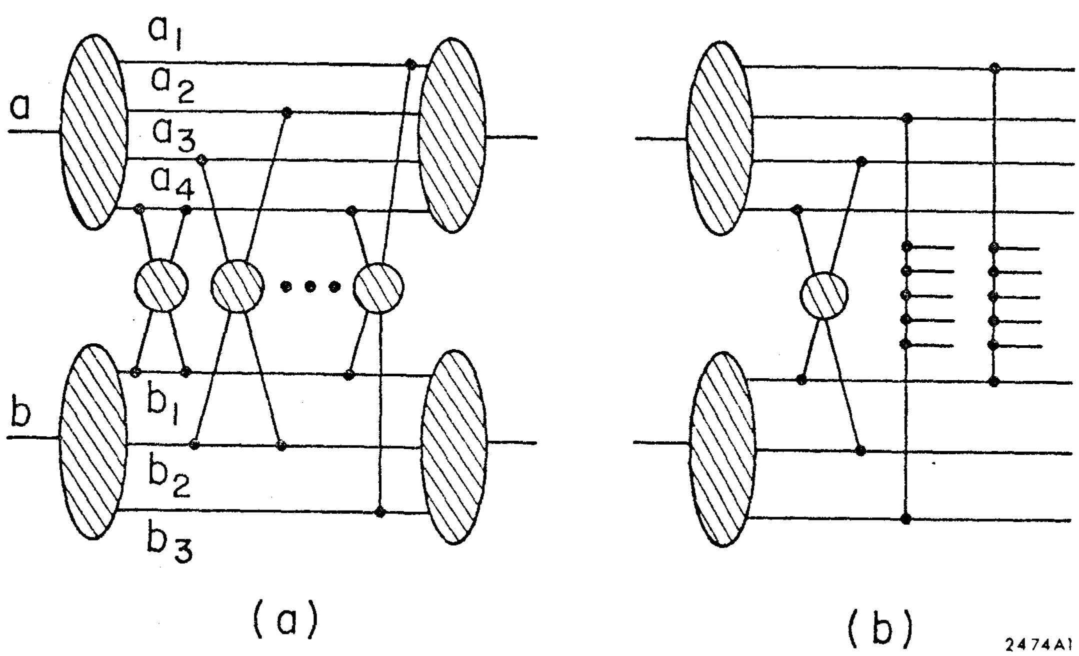

Fig. 1