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BREAKDOWN OF THE DRELL-YAN RELATION IN A BOUND STATE MODEL DUE TO SPIN COMPLICATIONS*

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ABSTRACT

The influence of spin factors on the validity of the Drell-Yan relation is investigated in the framework of the Drell-Lee bound state model for the nucleon. Calculations are performed after introducing an infinite momentum frame parametrization. It turns out that in one of the two spin combinations looked into the relation has to be modified.

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I. INTRODUCTION

Drell and Yan[1] made the conjecture that a behavior of the scaling function $\nu W_2(\omega)$ proportional $\left(1 - \frac{1}{\omega}\right)$ near threshold — with $\omega = -\frac{2p\cdot q}{q^2}$ — is up to logarithmic factors linked to an elastic form factor $F_1(q)$ behaving like $\left[1/(q^2)\right]^{n+1/2}$ in the limit of large q^2. In contrast to this conjecture the bound state model for the nucleon of Drell and Lee[2] yields a structure function

$$\nu W_2(\omega) \propto \left(1 - \frac{1}{\omega}\right)^2 \tag{I.1}$$

and an elastic form factor

$$F_1(q) \propto \frac{1}{q^4}\left(\log \frac{q^2}{m^2}\right)^2 \tag{I.2}$$

for the case of a charged spin-0 constituent of the proton.

More generally it was established in a parton model[3] that due to crossing symmetry the scaling function for a nucleon current is of the form

$$\nu W_2(\omega) = C_N \left(1 - \frac{1}{\omega}\right)^{2n+1} \qquad n = 0, 1, 2 \ldots \tag{I.3}$$

and for a pion current

$$\nu W_2(\omega) = C_\pi \left(1 - \frac{1}{\omega}\right)^{2n} \qquad n = 0, 1, 2, \ldots \tag{I.4}$$

Further it is known[4,5] that in a Bethe-Salpeter type bound state model with a covariant potential of the form

$$V(q) = \int \frac{\sigma(\kappa^2)\, d\kappa^2}{q^2 + \kappa^2} \tag{I.5}$$

with

$$\int \sigma(\kappa^2)\, d\kappa^2 = \text{finite} \tag{I.6}$$

— that means a wave function $\phi(x)$ which is finite for $x \to 0$ — the elastic form factor $F_1(q)$ must at least fall off like $O(q^{-4})$ up to logarithms.

This is compatible with the Drell-Lee model, but not with Drell-Yan's conjecture. To determine at what point their suggestion has to be modified we first translate the covariant model of Drell and Lee into the infinite momentum frame parton language of Drell and Yan. Besides providing calculational advantages this form lends itself to a comparison with the bound state model of Gunion, Brodsky, and Blankenbecler.[6]

It turns out that in the nucleon current case the spin factors do not affect the Drell-Yan relation. In the pion current case a modified relation holds.

II. THE DRELL-LEE BOUND STATE MODEL

To establish the feature of scaling in a relativistic field theory Drell and Lee proposed a bound state model for the nucleon.[2] It is assumed that the proton is composed of a spin-1/2 particle and a spin-0 particle. Either of the two can couple point-like to the electromagnetic field. Radiative corrections are neglected.

Let the proton of mass m and four-momentum p contain a charged constituent of mass m_1 and four-momentum $\mathbb{P}$ and an uncharged constituent of mass m_2 and four-momentum $\mathbb{X}$ (see Fig. 1). Then

$$p = \mathbb{P} + \mathbb{X} \tag{II. 1}$$

The ansatz for the bound state wave function $\Phi_p(\kappa^2)$ with

$$\kappa = (m_1+m_2)^{-1} (m_2 \mathbb{P} - m_1 \mathbb{X}) \tag{II. 2}$$

takes its inspiration from the ladder approximation of the Bethe-Salpeter equation. κ is the reduced relative momentum. $\Phi_p(\kappa^2)$ is written in the form

$$\Phi_p(\kappa^2) = g(\kappa^2)\gamma_5 u_p \tag{II.3}$$

where u_p is a free Dirac spinor of four-momentum p and mass m. $g(\kappa^2)$ is assumed to be a scalar. Drell and Lee show that it is a scalar at least in the limit $\kappa^2 \to \pm\infty$.

To make calculations easy we will need the stronger assumption that even for small κ^2 the spinor structure of $\Phi_p(\kappa)$ is given by $\gamma_5 u_p$. If then $g(\kappa^2)$ was set a constant, the graphs of the theory would turn into the graphs of a standard pseudoscalar-coupling meson theory. The asymptotic behavior of $\Phi_p(\kappa^2)$ as $\kappa^2 \to \pm\infty$ is

$$\Phi_p(\kappa^2) \propto 0\,(\kappa^{-2}) \tag{II.4}$$

for a potential as in (I.5).

The matrix element of the electromagnetic current J between proton states of four-momentum p and p+q=p' respectively, as well as the structure functions can be derived readily.

Two cases shall be distinguished. Either the spin-1/2 constituent or the spin-0 constituent couples to the electromagnetic field. The bound state wave function is the same in both cases, as the charge of constituents does not affect the Bethe-Salpeter equation.

The Bethe-Salpeter graph of Fig. 2 translates into

$$\begin{aligned} \langle p|J_\nu|p'\rangle &= -e(2\pi)^{-4} \int d^4\mathbb{P}\, \bar{\phi}_{p'}(\kappa')\, \frac{1}{\not{\mathbb{P}}+\not{q}-m_1}\,\gamma_\nu \cdot \frac{1}{\not{\mathbb{P}}-m_1} \cdot \frac{1}{\mathbb{X}^2-m_2^2} \cdot \phi_p(\kappa) \\ &= -e(2\pi)^{-4} \int d^4\mathbb{P}\, \frac{1}{\mathbb{X}^2-m_2^2} \cdot \frac{1}{\mathbb{P}^2-m_1^2} \cdot \frac{1}{(\mathbb{P}+q)^2-m_1^2} \cdot G_\nu^I \end{aligned} \tag{II.5}$$

with

$$G_\nu^I = \bar{\phi}_{p'}(\kappa')\,(\not{\mathbb{P}} + \not{q} + m_1)\,\gamma_\nu\,(\not{\mathbb{P}} + m_1)\,\phi_p(\kappa) \tag{II.5a}$$

for the case of a charged spin-1/2 constituent and into

$$\langle p|J_\nu|p'\rangle = -e(2\pi)^{-4}\int d^4\mathbb{P}\,\bar{\phi}_{p'}(\kappa')\,\frac{1}{\not{\mathbb{X}} - m_2}\,\frac{1}{(\mathbb{P}+q)^2 - m_1^2}\cdot(2\mathbb{P}_\nu + q_\nu)\cdot\frac{1}{\mathbb{P}^2 - m_1^2}\,\phi_p(\kappa)$$

$$= -e(2\pi)^{-4}\int d^4\mathbb{P}\;\frac{1}{\mathbb{X}^2 - m_2^2}\cdot\frac{1}{\mathbb{P}^2 - m_1^2}\cdot\frac{1}{(\mathbb{P}+q)^2 - m_1^2}\,G_\nu^{II} \tag{II.6}$$

with

$$G_\nu^{II} = \bar{\phi}_{p'}(\kappa')\,(\not{\mathbb{X}} + m_2)\,\phi_p(\kappa)\,(2\mathbb{P}_\nu + q_\nu) \tag{II.7}$$

for the case of a charged spin-0 constituent. $\bar{\Phi}_p(\kappa)$ is the conjugate solution to $\bar{\Phi}_p(\kappa)$. Further

$$\kappa' = \kappa + \frac{m_2}{m_1 + m_2}\,\mathbb{P} \tag{II.8}$$

The structure functions W_1 and νW_2 are given through (Fig. 3):

$$W_{\mu\nu}(q^2, p\cdot q) = e^2(2\pi)^{-8}\int d^4\mathbb{X}\,d^4\mathbb{P}\;\theta(\mathbb{X}_0)\,\theta(\mathbb{P}_0)\,\delta\left(\mathbb{X}^2 - m_2^2\right)\delta\left(\mathbb{P}^2 - m_1^2\right)\cdot$$

$$\cdot\,\delta^4(p - \mathbb{X} - \mathbb{P})\,\frac{1}{\left(\mathbb{P}^2 - m_1^2\right)^2}\begin{Bmatrix} H_{\mu\nu}^I \\ H_{\mu\nu}^{II}\end{Bmatrix} \tag{II.9}$$

inserting

$$H_{\mu\nu}^I = \mathrm{Tr}\left[\bar{\Phi}_p(\kappa)\,(\not{\mathbb{P}} + m_1)\,\gamma_\mu\,(\not{\mathbb{P}} + \not{q} + m_1)\,\gamma_\nu\,(\not{\mathbb{P}} + m_1)\,\Phi_p(\kappa)\right]_{\substack{\text{spin}\\ \text{averaged}}} \tag{II.10}$$

if the spin-1/2 constituent carries the charge and

$$H_{\mu\nu}^{II} = \mathrm{Tr}\left[\bar{\Phi}_p(\kappa)\,(\not{\mathbb{X}} + m_2)\,\Phi_p(\kappa)\right]_{\substack{\text{spin}\\ \text{averaged}}}\quad(2P_\mu + q_\mu)\,(2P_\nu + q_\nu) \tag{II.11}$$

if the spin-0 constituent carries the charge.

As the Drell-Yan relation connects the $(1 - 1/\omega)$-behavior of the structure function νW_2 in the scaling limit and the q-behavior of the elastic form factor F_1, one has to project out these quantities from the ones given above. This becomes particularly simple in the infinite momentum frame of the proton, given by[1]

$$p_\mu = \left(P + \frac{m}{2P}\,,\ 0_\perp\,,\ P\right) \tag{II. 12a}$$

and

$$q_\mu = \left(\frac{\nu}{P}\,,\ \vec{q}_\perp\,,\ 0\right) \tag{II. 12b}$$

with $P \to \infty$ and $\nu = p \cdot q$.

For the elastic form factor

$$\nu = \frac{\vec{q}_\perp^{\,2}}{2} \tag{II. 13}$$

Taking the zero- (or three-) component of $<p'|J_\nu|p>$ and averaging over the proton spin, $\frac{1}{2}\left(<\uparrow|J_0|\uparrow> + <\downarrow|J_0|\downarrow>\right)$, one obtains F_1 up to a constant factor.

$$F_1(q^2) = \frac{m}{P}\,\overline{\sum} <p'|J_0|p> \tag{II. 14}$$

νW_2 is calculated either from W_{00} or W_{33}.

$$\nu W_2 = \nu\,\frac{m^2}{P^2}\,W_{33} = \nu\,\frac{m^2}{P^2}\,W_{00} \tag{II. 15}$$

III. CHANGE OF PARAMETRIZATION

The expressions for F_1 and νW_2 are easier to handle if one chooses a particular parametrization for the four-momenta involved.[7]

$$\begin{aligned} \mathbb{P}_\mu &= \left(xP + \frac{\mathbb{P}^2 + \vec{k}_\perp^{\,2}}{2xP}\,,\ \vec{k}_\perp\,,\ xP\right) \\ \mathbb{X}_\mu = p_\mu - \mathbb{P}_\mu &= \left((1-x)P + \frac{\mathbb{X}^2 + \vec{k}_\perp^{\,2}}{2(1-x)P}\,,\ -\vec{k}_\perp\,,\ (1-x)\,P\right) \end{aligned} \tag{III. 1}$$

The loop-integration can then be written as

$$\int d^4\mathbb{P} = \int d^2\vec{k}_\perp \int_{-\infty}^{\infty} \frac{dx}{2|x|} \int_{-\infty}^{\infty} d\mathbb{P}^2 \tag{III. 2}$$

After carrying out the integration over $\mathbb{P}^2$, with this choice of variables the new expressions can be compared to the corresponding quantities in time-ordered perturbation theory in the infinite momentum frame.

For F_1 the integration over $\mathbb{P}^2$ can be done as a Cauchy contour integral closed with a semicircle at infinity. Therefore the pole-structure of the integrand in the complex $\mathbb{P}^2$-plane must be known. If one replaces the wave function by a constant, all poles originate from the propagators and one gets the usual time-ordered perturbation expansion in the infinite momentum frame. With a wave function in the integrand a specific form has to be assumed for it to be able to do the integration.

As the behavior of $\Phi_p(\kappa)$ for $\kappa^2 \to \pm\infty$ is $0(\kappa^{-2})$ we use an ansatz

$$\Phi_p(\kappa) = \frac{1}{\kappa^2 - \lambda^2 + i\epsilon} \gamma_5 u_p \tag{III. 3}$$

approximating all singularities of $\Phi_p(\kappa)$ by a single pole at λ^2. The value of λ turns out to be irrelevant, what matters is the asymptotic behavior. For both cases of charge distribution F_1 takes the form

$$F_1(q^2) = -e(2\pi)^4 \int d^4\mathbb{P} \frac{1}{\mathbb{X}^2 - m_2^2 + i\epsilon} \frac{1}{\mathbb{P}^2 - m_1^2 + i\epsilon} \frac{1}{(\mathbb{P}+q)^2 - m_1^2 + i\epsilon}$$

$$\cdot \frac{1}{\kappa^2 - \lambda^2 + i\epsilon} \frac{1}{\kappa'^2 - \lambda^2 + i\epsilon} \cdot \text{spin factor} \cdot \frac{2m\mathbb{P}_0}{P} \tag{III. 4}$$

For a charged spin-1/2 constituent the spin factor is

$$S_{1/2} = \frac{1}{2\mathbb{P}_0} \overline{\sum} \bar{u}_{p'} \gamma_5 (\not{\mathbb{P}} + \not{q} + m_1) \gamma_0 (\not{\mathbb{P}} + m_1) \gamma_5 u_p \tag{III. 5}$$

and for a charged spin-0 constituent it is

$$S_0 = \overline{\sum} \bar{u}_{p'} \gamma_5 (\not{\mathbb{X}} + m_2) \gamma_5 u_p \tag{III. 6}$$

In the parametrization (III. 1) the spin factors transform into

$$
\begin{aligned}
S_{1/2} &= -\frac{1}{x}\left((m_1 - xm)^2 + \vec{k}_\perp^{\,2} + (1-x)\,\vec{q}_\perp \cdot \vec{k}_\perp\right) \\
S_0 &= -\frac{1}{1-x}\left((m_2 - (1-x)m)^2 + \vec{k}_\perp^{\,2} + (1-x)\,\vec{q}_\perp \cdot \vec{k}_\perp + \mathbb{X}^2 - m_2^2\right)
\end{aligned}
\qquad \text{(III. 7)}
$$

They do not change the pole structure of the integrand of (III. 4) in the $\mathbb{P}^2$-plane. It is important to note that the poles coming from the wave functions are <u>both</u> taken with a $+i\epsilon$. For a discussion that $\bar{\Phi}$ is <u>not</u> the time reverse of Φ see Drell and Lee.[2] The interesting quantities, deciding upon the position of the poles in the $\mathbb{P}^2$-plane are $\mathbb{X}^2$, $(\mathbb{P}+q)^2$, κ^2, and κ'^2. From

$$
(\mathbb{X}+P)^2 = p^2 = m^2 \qquad \text{(III. 8)}
$$

one finds

$$
\begin{aligned}
\mathbb{X}^2 &= -\frac{(1-x)}{x}\,\mathbb{P}^2 - \frac{1}{x}\,\vec{k}_\perp^{\,2} + m^2(1-x) \\
(\mathbb{P}+q)^2 &= \mathbb{P}^2 - 2\vec{k}_\perp \cdot \vec{q}_\perp - (1-x)\,\vec{q}_\perp^{\,2} \\
\kappa^2 &= -\frac{(\beta-x)}{x}\,\mathbb{P}^2 - \frac{\beta}{x}\,\vec{k}_\perp^{\,2} + m^2(\beta-x)\beta \\
\kappa'^2 &= \kappa^2 - 2(1-\beta)\,\vec{q}_\perp \cdot \vec{k}_\perp - (1-\beta)(1-x)\,\vec{q}_\perp^{\,2}
\end{aligned}
\qquad \text{(III. 9)}[8]
$$

with

$$
\beta = \frac{m_2}{m_1 + m_2} \qquad \text{(III. 10)}
$$

In the order in which the 5 propagator-type poles are written down in (III. 4), the first pole lies in the upper half of the $\mathbb{P}^2$-plane for $0 \le x \le 1$ and in the lower half-plane for all other x, the second and third pole lie always in the lower half-plane, the fourth and fifth pole lie in the upper half-plane for $0 \le x \le \beta$ and in the lower half-plane for all other x. Therefore if the integration-contour is closed in the upper half-plane, residues will contribute for $0 \le x \le 1$ only. The integral

(III.4) becomes formally

$$F_1 \propto \int d^2\vec{k}_\perp \int_{-\infty}^{\infty} \frac{dx}{2|x|} \int dP^2 \cdot \text{Integrand}\left(P^2, x, \vec{k}_\perp\right) =$$

$$= \int d^2\vec{k}_\perp \int_0^1 \frac{dx}{x} \text{ Residue of Integrand at } \mathcal{X}^2 = m_2^2$$

$$+ \int d^2\vec{k}_\perp \int_0^\beta \frac{dx}{2x} \text{ Residue of Integrand at } \kappa^2 = \lambda^2$$

$$+ \int d^2\vec{k}_\perp \int_0^\beta \frac{dx}{2x} \text{ Residue of Integrand at } \kappa'^2 = \lambda^2 \qquad \text{(III. 11)}$$

Being interested only in the q-behavior of F_1 at large $\vec{q}_\perp^{\,2}$ and assuming m_1 and m_2 nonzero, the $\vec{q}_\perp$-behavior of the second and third term on the right-hand side of (III. 11) can be determined by taking the limit $\vec{q}_\perp^{\,2} \to \infty$ inside the integral. Both terms contribute as $0\left(q_\perp^{-4}\right)$. The $q_\perp$-behavior of the first term on the right-hand side of (III. 11) cannot be determined as easily because the limit $\vec{q}_\perp^{\,2} \to \infty$ and the integration cannot be interchanged in this case, i.e., after taking the limit inside the integral one is left with a divergent expression. Therefore this term has to be looked at more carefully.

$$F_1\left(\vec{q}_\perp^{\,2}\right) \propto 0\left(\frac{1}{q_\perp^4}\right) + \tilde{F}_1\left(\vec{q}_\perp^{\,2}\right) \qquad \text{(III. 12)}$$

with

$$\tilde{F}_1\left(\vec{q}_\perp^{\,2}\right) = \int d^2\vec{k}_\perp \int_0^1 \frac{dx\, x}{(1-x)} \frac{(1-x)}{\left(-\vec{k}_\perp^{\,2} - m_x^2 + m^2 x(1-x)\right)} \cdot \frac{(1-x)}{\left(-\left(\vec{k}_\perp + (1-x)\vec{q}_\perp\right)^2 - m_x^2 + m^2 x(1-x)\right)} \cdot$$

$$\cdot\, g(\vec{k}_\perp)\, g^*\left(\vec{k}_\perp + (1-x)\vec{q}_\perp\right) \cdot \begin{Bmatrix} S_{1/2} \text{ at } X^2 = m_2^2 \\ S_0 \quad \text{at } X^2 = m_2^2 \end{Bmatrix} \qquad \text{(III. 13)}$$

where

$$m_x^2 = m_1^2(1-x) + m_2^2 x \tag{III. 14}$$

$$g(\vec{k}_\perp) = \frac{(1-x)}{\left(-\vec{k}_\perp^{\,2} - m_2^2 \frac{x-\beta}{1-\beta} + m^2(x-\beta)(1-x) - (1-x)\frac{\lambda^2}{1-\beta}\right)}$$

and

$$\begin{aligned} S_{1/2}\left(\mathbb{X}^2 = m_2^2\right) &= -\frac{1}{x}\left((m_1 - xm)^2 + \vec{k}_\perp^{\,2} + (1-x)\,\vec{q}_\perp \cdot \vec{k}_\perp\right) \\ S_0\left(\mathbb{X}^2 = m_2^2\right) &= -\frac{1}{1-x}\left(\left(m_2 - (1-x)\,m\right)^2 + \vec{k}_\perp^{\,2} + (1-x)\,\vec{q}_\perp \cdot \vec{k}_\perp\right) \end{aligned} \tag{III. 15}$$

The important contributions from the $\vec{k}_\perp$ -integration to the integral (III. 13) come from $\vec{k}_\perp \approx 0$ and $\vec{k}_\perp \approx -(1-x)\,\vec{q}_\perp$, giving the same contribution in both regions due to the symmetry of the integrand. The dominant contributions from the x-integration come from the region $x \lesssim 1$.

Therefore $\widetilde{F}$ and g can be brought into the form

$$\widetilde{F}_1\left(\vec{q}_\perp^{\,2}\right) = \int d^2\vec{k}_\perp \int_0^1 \frac{dx}{(1-x)} \frac{(1-x)^2}{\left(-\vec{k}_\perp^{\,2} - m_2^2\right)\left(-\left(\vec{k}_\perp + (1-x)\,\vec{q}_\perp\right)^2 - m_2^2\right)} \cdot$$

$$\cdot\, \widetilde{g}(\vec{k}_\perp)\, \widetilde{g}^*\left(\vec{k}_\perp + (1-x)\,\vec{q}_\perp\right) \cdot \begin{Bmatrix} \widetilde{S}_{1/2} \\ \widetilde{S}_0 \end{Bmatrix} \tag{III. 16}$$

with

$$\widetilde{g}(k_\perp) = \frac{(1-x)}{\left(-\vec{k}_\perp - m_2^2\right)} \tag{III. 17}$$

and

$$\begin{aligned} \widetilde{S}_{1/2} &= -\left((m_1 - m)^2 + \vec{k}_\perp^{\,2} + (1-x)\,\vec{k}_\perp \cdot \vec{q}_\perp\right) \\ \widetilde{S}_0 &= -\frac{1}{1-x}\left(m_2^2 + \vec{k}_\perp^{\,2} + (1-x)\,\vec{k}_\perp \cdot \vec{q}_\perp\right) \end{aligned} \tag{III. 18}$$

Up to factors of x and terms of the sums in the denominators proportional to (1-x), which are inessential in the important integration region, this expression for F_1 coincides with the one of Gunion, Brodsky, and Blankenbecler.[6]

If one takes the integral (III. 16) separately for the terms appearing in the spin factors, the q-behavior of each can be read off readily by setting $\vec{k}_\perp = -(1-x)\,\vec{q}_\perp$, taking the limit $\vec{q}_\perp^{\,2} \to \infty$ and looking at the degree of divergence of the remaining x-integral. A convergent x-integration yields a q-independent factor, a logarithmically divergent x-integration gives a $\log\left(\vec{q}_\perp^{\,2}/m_2^2\right)$, a linearly divergent one gives a factor $|\vec{q}_\perp|/m_2$. The particular combination of these separate terms in $\widetilde{S}_{1/2}$ and $\widetilde{S}_0$, however, gives rise to cancellations and therefore requires more careful calculation. Let first the spin-1/2 constituent be charged. With the above description the term $\propto \vec{k}_\perp^{\,2}$ of the spin factor gives an integral $\propto 1/\vec{q}_\perp^{\,2}$. So does the term $\vec{k}_\perp \cdot \vec{q}_\perp\,(1-x)$. But as the two terms appear with a different sign the $1/\vec{q}_\perp^{\,2}$-contributions cancel out. There are no contributions proportional to odd powers of $1/\vec{q}_\perp^{\,2}$. The constant $(m_1-m)^2$ gives rise to a term $\propto 1/\vec{q}_\perp^{\,4}\,\log\left(\vec{q}_\perp^{\,2}/m_2^2\right)$. Careful calculation shows that the whole integral (III. 16) goes like $1/\vec{q}_\perp^{\,4}\,\log\left(\vec{q}_\perp^{\,2}/m_2^2\right)$. In the case of a charged spin-0 constituent the $1/\vec{q}_\perp^{\,2}$-contributions cancel out again. But due to the additional factor $\frac{1}{1-x}$ the remaining contribution from $\frac{1}{1-x}\left(\vec{k}_\perp^{\,2} + \vec{k}_\perp \cdot \vec{q}_\perp\,(1-x)\right)$ is proportional to $1/\vec{q}_\perp^{\,3}$. The term $\propto m_2^2$ in the spin factor yields a contribution of the same behavior. Careful calculation shows that these two contributions to (III. 16) also cancel and that the whole integral goes like $1/\vec{q}_\perp^{\,4}\,\log^2\left(\vec{q}_\perp^{\,2}/m_2^2\right)$ for large $\vec{q}_\perp^{\,2}$.

For the evaluation of (III. 16) denominators are combined with the Feynman-trick:

$$\frac{1}{(AB)^n} = 2\int_0^1 dz\, z^{n-1}\,\frac{1}{(Az+B)^{2n}} \qquad \text{(III. 19)}$$

This allows us to shift the integration variable $\vec{k}_\perp$ and render the angular integration trivial.

Expressing νW_2 in the parametrization (III. 1) in the infinite momentum frame (II. 12) is straightforward. The θ-functions limit the x-integration to an interval $[0,1]$. One δ-function puts the uncharged particle X on mass-shell — which makes νW_2 comparable to F_1. Taking the Bjorken-limit

$$\begin{aligned} &\vec{q}_\perp^{\,2} \to \infty \\ &\nu \to \infty \\ &\frac{\nu}{\vec{q}_\perp^{\,2}} \text{ fixed} \end{aligned} \tag{III. 20}$$

in νW_2, the other δ-function becomes $\delta\left(2x\, m_2 \nu - \vec{q}_\perp^{\,2}\right)$, which makes the x-integration trivial.

$$\nu W_2(\omega) \propto \int d\vec{k}_\perp^{\,2} \frac{1}{\omega} \int_0^1 dx\, \delta\left(x - \frac{1}{\omega}\right) |g(\vec{k}_\perp)|^2 \frac{(1-x)^3}{\left(-\vec{k}_\perp^{\,2} - m_x^2 + m^2 x(1-x)\right)^2} \cdot \begin{Bmatrix} \overline{S_{1/2}} \\ \overline{S_0} \end{Bmatrix} \tag{III. 21}$$

with

$$\omega = \frac{2m_2 \nu}{\vec{q}_\perp^{\,2}}$$

$$\begin{aligned} \overline{S_{1/2}} &= -\frac{1}{x}\left((m_1 - xm)^2 + \vec{k}_\perp^{\,2}\right) \\ \overline{S_0} &= -\frac{1}{1-x}\left((m_2 - (1-x)m)^2 + \vec{k}_\perp^{\,2}\right) \end{aligned} \tag{III. 22}$$

Again the result coincides with the result in the model of Gunion, Brodsky, and Blankenbecler but for factors of x in the integrand of (III. 21). These do not affect the threshold behavior $\omega \to 1$. The results for the two cases are

$$\nu W_2(\omega) \propto \left(1 - \frac{1}{\omega}\right)^3 \tag{III. 23}$$

for a charged spin-1/2 constituent and

$$\nu W_2(\omega) \propto \left(1 - \frac{1}{\omega}\right)^2 \tag{III. 24}$$

for a charged spin-0 constituent.

IV. THE DRELL-YAN RELATION

The Drell-Yan relation[1] states that if $\nu W_2(\omega)$ in the limit $\omega \to 1$ behaves like

$$\nu W_2(\omega) \propto \left(1 - \frac{1}{\omega}\right)^m \qquad m = 0, 1, 2 \ldots \tag{IV. 1}$$

then the behavior of $F_1\left(\vec{q}_\perp^{\,2}\right)$ for large $\vec{q}_\perp^{\,2}$ is

$$F_1\left(\vec{q}_\perp^{\,2}\right) \propto \frac{1}{\left(q_\perp^2\right)^p} \tag{IV. 2}$$

with

$$2p = m - 1 \tag{IV. 3}$$

The original derivation of the relation as proposed by Drell and Yan can now easily be reconstructed in the framework of the bound state model for spinless particles. Assuming an ansatz

$$\Phi_p(\kappa) = \frac{1}{(\kappa^2 - \lambda^2)^p} \tag{IV. 4}$$

for the bound state wavefunction one gets as in (III. 16) above

$$F_1\left(\vec{q}_\perp^{\,2}\right) = \int d^2\vec{k}_\perp \int \frac{dx}{(1-x)}\, f\left(x, \vec{k}_\perp^{\,2}\right)\; f^*\left(x, \left(\vec{k}_\perp + (1-x)\, \vec{q}_\perp\right)^2\right) \tag{IV. 5}$$

with

$$f\left(x, \vec{k}_\perp^{\,2}\right) = \frac{(1-x)^p}{\left(-\vec{k}_\perp^{\,2} - m_2^2\right)^p} \tag{IV. 6}$$

Further analogous to (III. 21)

$$\nu W_2(\omega) \propto \int d^2\vec{k} \int \frac{dx}{(1-x)}\, \delta\left(x - \frac{1}{\omega}\right) \; \left| f\left(x, \vec{k}_\perp^{\,2}\right) \right|^2 \qquad \text{(IV. 7)}$$

Drell and Yan argue that the q-dependence of $F_1\left(\vec{q}_\perp^{\,2}\right)$ in (IV. 5) stems from the x-integration, the transverse momentum overlap integral will be finite and q-independent — at least up to logarithms. That means

$$F_1\left(\vec{q}_\perp^{\,2}\right) \propto \int_0^{1-\frac{m}{|q_\perp|}} dx\,(1-x)^{2p-1} \qquad \text{(IV. 8)}$$

or

$$F_1\left(\vec{q}_\perp^{\,2}\right) \propto \left(\frac{1}{|q_\perp|}\right)^{2p} \qquad \text{(IV. 9)}$$

Clearly the $\left(1-\frac{1}{\omega}\right)$-behavior of $\nu W_2(\omega)$ is determined by the factors (1-x) in the integrand of (IV. 7). This time the integration over x will not add another factor of (1-x) because of the δ-function. Therefore one obtains

$$\nu\, W_2(\omega) \propto (1-\omega)^{2p-1} \qquad \text{(IV. 10)}$$

In the case of particles with spin the integral (IV. 5) is of the form

$$F_1\left(\vec{q}_\perp^{\,2}\right) = \int d^2\vec{k}_\perp \int \frac{dx}{(1-x)x}\, f\left(x, \vec{k}_\perp^{\,2}\right) f^*\left(x, \left(k_\perp + (1-x)\vec{q}_\perp\right)^2\right) h\left(x, \vec{k}_\perp \cdot \left(\vec{k}_\perp + (1-x)\vec{q}_\perp\right)\right) \qquad \text{(IV. 11)}$$

where h is the spin factor $S_{1/2}$ or S_0 respectively. The particular form of h can ruin the Drell-Yan argument. The overlap integral may no longer be q-independent. For n=2 the result of Chapter 3 is that in the case of a charged spin-0 constituent the relation is obviously violated. If there was no cancellation of $1/q_\perp^2$-terms in F_1, the relation would break down in both cases. In the case of a charged spin-1/2 constituent it is repaired by that cancellation and the fact that there are no $1/q_\perp^3$ -terms around. In the charged spin-0 constituent case there is a cancellation in nonleading terms $\propto 1/q_\perp^3$ too so that the relation is not repaired by the cancellation of the leading terms.

The cancellation between terms $\propto 1/\vec{q}_\perp^{\,2}$ is model-independent, i.e., it does not depend on the particular ansatz for the wavefunction.

The cancellation of $1/q_\perp^3$-terms in the second case can be traced back to the ansatz (III. 3) for the wavefunction. It appears not only for a wavefunction

$$\Phi_p(\kappa) \propto \frac{1}{\kappa^2 - \lambda^2} \tag{IV. 12}$$

but also for the general case

$$\Phi_p(\kappa) \propto \left(\frac{1}{\kappa^2 - \lambda^2}\right)^p \tag{IV. 13}$$

so that a relation similar to (IV. 3) holds:

$$2p = m-2 \tag{IV. 14}$$

V. CONCLUSIONS

A priori both of the two cases described above should contribute to elastic and inelastic scattering. The data for $\nu W_2(\omega)$,[10] however, suggest that νW_2 behaves like $\left(1 - \frac{1}{\omega}\right)^3$ for $\omega \to 1$. At least the admixture of scattering from a spin-0 parton must be very small. The same conclusion is independently drawn from the fact that $R = \sigma_L/\sigma_T$ is very small.

The pointlike coupling of the electromagnetic field to a spin-0 constituent may be ruled out if one views the spin-0 constituent as a core, that means an approximation for a number of further spin-1/2 constituents. Therefore the case of a charged spin-0 parton is only of theoretical interest, e.g., to study how the Drell-Yan relation can be violated.

The above results were also obtained in a softened field theory by Landshoff and Polkinghorne.[11]

In the Drell-Lee model G_E/G_M does not scale. Our result is valid for this kind of model. But it might well be that in a model with boson charged fields constructed to give G_E/G_M scaling, the Drell-Yan relation is retained.

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7. This was pointed out to me by M. G. Schmidt. M. G. Schmidt, "A simple connection between covariant Feynman formalism and time-ordered perturbation theory in the infinite momentum frame" (to be published).
8. It is interesting to note that κ^2 can be written as

$$\kappa^2 = (1-\beta)\, P^2 + \beta(p-P)^2 - \beta(1-\beta)\, m^2$$

That means that the wavefunction (III. 3) can be obtained from a DGS-representation[5,9]

$$\Phi_p(\kappa^2) = \int d\alpha \int d\sigma^2 \; \frac{\rho(\sigma^2, \alpha)}{(1-\alpha)\, \mathbb{P}^2 + \alpha\, (p-\mathbb{P})^2 - \sigma^2 + i\epsilon}$$

by choosing

$$\rho(\sigma^2, \alpha) = \delta\left(\sigma^2 - \lambda^2 + (1-\alpha)\, \alpha m^2\right) \delta(\alpha-\beta)\; \gamma_5\, u_p \quad .$$

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FIGURE CAPTIONS

1. Bound state wavefunction for the nucleon.
2. Electromagnetic form factor.
3. Feynman-diagram corresponding to $W_{\mu\nu}$.

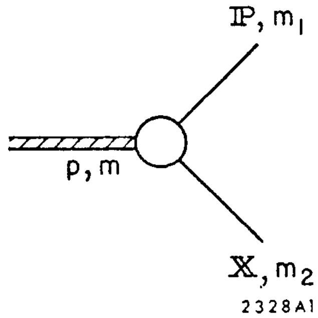

FIG. 1

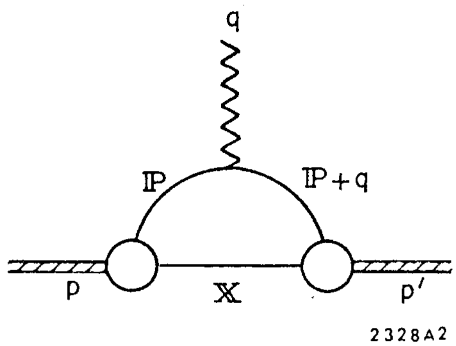

FIG. 2

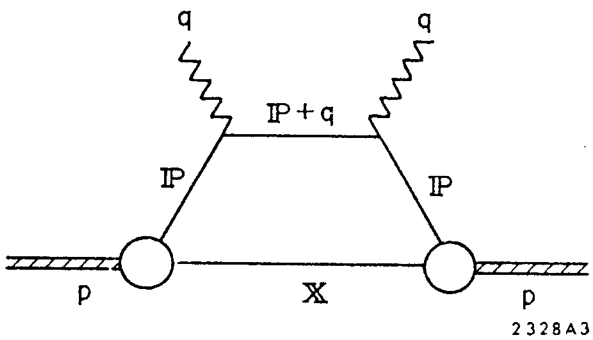

FIG. 3