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CURRENT AND CONSTITUENT QUARKS IN THE LIGHT CONE QUANTIZATION*

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ABSTRACT

Using the light cone quantization of the free quark model, a wide class of unitary transformations which relate the "current" to "constituent" quark models is constructed. The Melosh transformation is obtained as a special case. The construction clearly exhibits the kinematical aspects of the transformation. Phenomenological applications are discussed. A method for determining acceptable transformations in potential models is formulated.

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I. INTRODUCTION

Recently, valuable insights into a method of resolving certain long-standing difficulties of the "quark model" have been provided by Melosh.[1] These difficulties stem from the failure to distinguish between two separate "quark models".[2] In one model, dubbed the "current quark" model by Gell-Mann, operators with the algebra of $SU6_W$ are defined as integrals of certain vector and tensor densities formed from local quark fields.[3] The $SU2 \times SU2$ subalgebra of these operators leads to the highly successful Adler-Weisberger sum rules.[4] However, careful studies of the way these sum rules are saturated indicate the physical hadrons do not fall into multiplets of this algebra.[5] They transform as reducible representations.

The second kind of "quark model" treats the quarks as though they were constituents (in some abstract sense) of the hadrons.[6] Insofar as spectroscopic assignments are concerned, low lying hadron states seem to fall into pure multiplets of an $SU6_W$ group involving the unitary spins and Pauli spins of the quarks. The Johnson-Treiman relations,[7] as well as numerous predictions on transition rates,[8] indicate these classifications are also valid for collinear processes. The question arises whether this group is related to the "currents" group in any simple way.

It has been suggested from time to time that there may exist a transformation which takes the set of "current" $SU6_W$ generators into the "constituent" $SU6_W$ generators.[9] Melosh has given an example of the form such a transformation might take within the context of the free quark model. He arrives at the succinct expression[1]

$$V_M = \exp \frac{i}{2} \int d^3\vec{x}\, q^\dagger(x) \arctan \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{m} q(x) \quad . \qquad (1)$$

The similarity of this expression to the Foldy-Wouthuysen transformation suggests a simple physical interpretation. To the extent that in the "constituent" quark model hadrons are treated as nonrelativistic bound states of fermions, it is convenient to identify the spins of the fermions as those which have a correct nonrelativistic limit. The FW construction provides us with such an identification. Further, Bitar and Gursey[10] have established that the relation of these spin operators to operators relevant to interactions with currents, such as the magnetic momentum operator, is also given by the FW transformation. An essential restriction recognized by Melosh is that to maintain the collinear properties of the symmetry, the FW expression must be modified to have no dependence on p_z.

With this transformation it is possible[1] to resolve certain of the difficulties arising from the naive identification of the two "quark models." The algebraic structure of the transformation is such as to allow the anomalous magnetic moments of the $\underline{56}$ L=0 baryons to be nonvanishing. With further simplifying assumptions, Gilman and Kugler[11] have gone on to make a series of quantitative predictions.

Since the interesting applications of these transformations involve sum rules for matrix elements of charges between infinite momentum states, it is relevant to inquire whether the interesting qualitative features of the transformation become more transparent if one works directly with the so-called "light-like charges." The virtues of such charges in sum rule work have been discussed extensively by a number of authors.[12] For example, it is possible to rederive relations previously obtained only by using the $p \to \infty$ limit, such as fixed mass current algebra sum rules[13] and the algebraic realization of chiral symmetry,[14] without invoking that limiting procedure. The light-cone formalism is reviewed in Section II.

In Section III we will find that there is a large class of possible "light-like" transformations that lead from "current" charges to acceptable "constituent" charges. The general form of these transformations is

$$\widetilde{V} = \exp\left\{\frac{i}{\sqrt{2}}\int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x)\left[\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} F(|\vec{\partial}_\perp|)\right] q_+(x)\right\} . \qquad (2)$$

where F is an arbitrary function of its arguments with the property F(0)=0. In the remainding sections of the paper we will examine the interpretation and consequences of this nonuniqueness.

In Section IV we find that the large class of possible transformation in the light-cone formulation has a straightforward interpretation in terms of changes of possible spin bases. The light-cone formulation permits us to separate these purely kinematic aspects of the transformation from the obscuring complications due to pair states created by V_M in the equal-time formulation. This simplification is possible because $\widetilde{V}$, Eq. (2), is a "good" operator and does not create pairs in the light-cone formulation. We also discuss the manner in which the nonuniqueness of the transformation is present in the equal-time formulation.

The nonuniqueness of the constituent operators has bearing on the interpretation of the dynamical information supposedly revealed in certain SU2×SU2 calculations. We have in mind the notion[9] that the correction of $(Ga/Gv)_{nucleon}$ from its group theoretical value of (5/3) to its experimental value leads to the dynamical condition $\langle p^2\rangle \simeq m_q^2$. It is now clear that any transformation with the algebraic structure of V_M will give some correction to Ga/Gv; but the "dynamic" content of the correction depends on the particular representation in which the wave functions of the states and the relevant transition operators have been written. Not unexpectedly, one will arrive at a "right" $\tilde{V}$ if one exists when consistent interpretation of all matrix elements is possible. These matters are considered more extensively in Section IV.

So far an important fact has been neglected to allow for a clear presentation of the essential ideas of the investigation. It is that the light-like "current" charges (as defined in Section II) commute with the light-like Hamiltonian, whereas not all the equal-time "current" charges commute with the equal-time Hamiltonian. This shows that the light-like "current" charges and the equal-time "current" charges are essentially different operators. Such a possibility should be expected since there is no unitary transformation which relates equal-time and light-cone commutators. The light-cone commutators contain structures which are not limits of expressions away from the light-cone and which therefore cannot be obtained by taking the $p \to \infty$ limit of equal-time expressions. For example, it is known that the light-cone commutators determine the fixed mass sum rules;[12] the $p \to \infty$ method sometimes fails to convert an equal-time commutator into a light-cone commutator.[13]

The differences between the $p \to \infty$ limit of an equal-time theory and a light-cone formulation are irrelevant for understanding the kinematic aspects of $\tilde{V}$ discussed above. However, they are important in delineating the classes of $\tilde{V}$ which one may wish to consider. In Section III we have taken the simplest approach possible: Since the "current" charges are conserved, and the "consitutent" charges are to be conserved, a conserved $\tilde{V}$ has been chosen. There are other possible avenues of investigation. Section V represents an excursion into one of the possible new directions. The conditions the transformation must satisfy so there will be a conserved "spin" quantum number in potential theory models are discussed. The details are exhibited for the simple case of the two-dimensional transverse oscillator.

II. LIGHT-CONE ANALYSIS OF $SU(6)_W$ CURRENTS

In this section we introduce our notations for light-cone quantization,[15] and define the generators of $SU(6)_W$ currents in this formalism. We also give the form of the states that transform as representations of this group.

In addition to the usual quantization on the spacelike plane $t=0$, it is possible to quantize a field theory on a hyperplane tangent to the light cone, conventionally taken to be the hyperplane $x^+ = t+z/\sqrt{2} = 0$. The metric tensor is given by $g^{++} = g^{--} = 0$, $g^{+-} = g^{-+} = 1$, $g^{ij} = -\delta^{ij}$; where the standard notation for vectors, V^μ, is $V^\pm = V^0 \pm V^z/\sqrt{2}$, and V^i for i equal x or y; tensors are treated similarly.

For the free quark model the Lorentz generators may be written in the general form

$$P^\mu = i\int d^4x\ \delta\left(n_\sigma x^\sigma\right)\bar{q}(x)\ n_\sigma\gamma^\sigma \frac{\partial}{\partial x_\mu} q(x) \tag{3a}$$

$$M^{\mu\nu} = i\int d^4x\ \delta\left(n_\sigma x^\sigma\right)\bar{q}(x)\ n_\sigma\gamma^\sigma\left(x^\mu \frac{\partial}{\partial x_\nu} - x^\nu \frac{\partial}{\partial x_\mu} + \frac{1}{4}\left[\gamma^\mu, \gamma^\nu\right]\right)q(x) \tag{3b}$$

The Lorentz generators appropriate for the light-cone quantization are obtained by letting $n^2=0$ and $n^-=1$, $n^+=0=n^i$ in the above expressions. (We will distinguish light-cone operators from the usual equal-time operators by placing tildes on light-cone operators.) These expressions for the light-cone Lorentz generators can be simplified by removing the dependent components of the quark field; that is, those components that do not obey a dynamical equation of motion. An an example of how this is accomplished, consider the light-cone Hamiltonian

$\widetilde{H} \equiv \widetilde{P}^-$ given by

$$\widetilde{H} = \int dx^- \, d\vec{x}_\perp \, \bar{q}(x) \left(-i\gamma^- \frac{\partial}{\partial x^-} - i\gamma^j \frac{\partial}{\partial x^j} + m \right) q(x) \tag{4}$$

We may write $q(x) = q_-(x)+q_+(x)$, where $q_\pm(x) = P_\pm q(x)$, and $P_\pm$ are defined by

$$P_+ = \frac{\gamma^-\gamma^+}{2} \qquad \text{and} \qquad P_- = \frac{\gamma^+\gamma^-}{2} \; .$$

Using the Dirac equation it is possible to express $q_-(x)$ in terms of $q_+(x)$,

$$q_-(x) = \frac{-1}{2\eta} \, \gamma^+ \left(\gamma^j \frac{\partial}{\partial x^j} + im \right) q_+(x) \quad . \tag{5}$$

The operator $1/\eta$ is defined by $\frac{1}{\eta} f(x) = \frac{1}{2} \int dy^- \, \epsilon(x^- - y^-) \, f(x^+, x_i, y^-)$. The dependent components may now be eliminated from $\widetilde{H}$ to obtain

$$\widetilde{H} = \frac{-i}{\sqrt{2}} \int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x) \left[\frac{1}{\eta} \left(m^2 + \frac{\partial}{\partial x_i} \frac{\partial}{\partial x^i} \right) \right] q_+(x) \tag{6}$$

Explicit forms of the other generators are listed in Appendix A.

The canonical commutation rules for the independent components of the field in the light-cone formulation are postulated to be

$$\left\{ q_+^\dagger(x), \quad q_+(y) \right\}_{x^+=y^+} = \frac{P_+}{\sqrt{2}} \, \delta(x^- - y^-) \, \delta(\vec{x}_\perp - \vec{y}_\perp) \tag{7}$$

The commutation relations involving redundant components of the field follow from the above commutator and the constraint equation (5).

In the free quark model the generators of the $SU(6)_W$ of "currents" may be written as follows:

$$Q_i = i \int d^4x \, \delta\left(n_\mu x^\mu\right) \, \bar{q}(x) \, n_\mu \gamma^\mu \, \frac{\lambda_i}{2} \, q(x) \tag{8a}$$

$$Q_{i\perp} = i \int d^4x \, \delta\left(n_\mu x^\mu\right) \, \bar{q}(x) \, n_\mu \gamma^\mu \gamma_\perp \gamma^5 \, \frac{\lambda_i}{2} \, q(x) \tag{8b}$$

$$Q_{iz} = i \int d^4x \, \delta\left(n_\mu x^\mu\right) \, \bar{q}(x) \, n_\mu \gamma^\mu \, \sigma_z \, \frac{\lambda_i}{2} \, q(x) \tag{8c}$$

The form appropriate when discussing the space-like quantized theory is obtained by setting $n^2=+1$ (conventionally $n^o=1$, $\vec{n}=0$). When $n^2=+1$ only the vector charges are conserved.

The light-like charges are constructed by using $n^-=+1$, $n^+=\vec{n}_\perp=0$ in Eqs. (8) above. It is easy to check that

$$\left[\tilde{Q}_i, \tilde{H}\right]=0, \qquad \left[\tilde{Q}_{i\perp}, \tilde{H}\right]=0, \qquad \text{and} \qquad \left[\tilde{Q}_{iz}, \tilde{H}\right]=0$$

the operator $\tilde{Q}_{iz}$ is equal to the light-like axial charge $\tilde{Q}_{i5}$. Therefore chiral $[SU(3)\times SU(3)]_{\text{currents}}$ is a conventional good symmetry in the light-cone formulation, as is $SU(6)_{W,\text{ currents}}$.

In the free quark model the operators $\tilde{Q}_i$ and $\tilde{Q}_{i\alpha}$ commute with the Lorentz generators $\tilde{K}_3$ and

$$\tilde{B}_i=\left(\tilde{K}_i+\epsilon_{ij}\tilde{J}_i\right)/\sqrt{2}\ ; \qquad i=1,2\ ; \qquad \epsilon_{12}=+1 \quad .$$

Thus it is possible to construct representations of $SU(6)_{W,\text{ currents}}$ for states of arbitrary momentum, provided these states are constructed in the "light-like helicity basis",[15] defined by

$$|p^+,\vec{p}_\perp,\lambda>_q \;=\; e^{+i\omega\tilde{K}_3}\,e^{+i\vec{\beta}_\perp\vec{B}_\perp}\left|\frac{m}{\sqrt{2}},\vec{0}_\perp,\lambda>\right.\equiv b^\dagger(p^+,\vec{p}_\perp,\lambda)|0> \tag{9}$$

This is another indication of the utility of working in the light-cone formulation.

We can go even further and construct direct product states of total momentum zero of the general form

$$\left|\frac{M}{\sqrt{2}},\vec{0}_\perp;\lambda,a>\right. = \prod_{i=1}^{n}\left(\int\frac{d\vec{p}_{\perp i}}{(2\pi)^3}\int_0^\infty\frac{d\eta_i}{2\eta_i}\right)\delta\left(\sum_{i=1}^{n}\vec{p}_{\perp i}\right)\delta\left(\sum_{i=1}^{n}\eta_i-\frac{M}{\sqrt{2}}\right)$$

$$f(\vec{p}_{\perp 1},\ldots\vec{p}_{\perp n};\eta_1,\ldots,\eta_n)\;C^{\lambda_1\ldots\lambda_n}_{a_1\ldots a_n}\prod_{i=1}^{\ell}\left(b^\dagger(\vec{p}_{\perp i};\eta_i;\lambda_i,a_i)\right)\prod_{i=\ell+1}^{n}\left(d^\dagger(p_{\perp i},\eta_i;\lambda_i,a_i)\right)|0> \tag{10}$$

In the free quark model the "wave function" $f(\vec{p}_\perp, \eta)$ is equal to unity, and the coefficients $C^{\lambda_1, \ldots \lambda_n}_{a_1, \ldots a_n}$ may be chosen in such a manner that the state transforms according to a given irreducible representation of the $SU(6)_{W,\,\text{currents}}$ group. The classification of the states will remain invariant if we give them nonzero momentum by means of $\widetilde{K}_3$ boosts and $\widetilde{B}_\perp$ "boosts" just as in Eq. (9) for the single particle state. How much of this can be extended to the case where there is field theoretic interaction is not clear; aspects of the situation in potential theory are discussed in Section V.

III. CONSTRUCTION OF THE GENERAL TRANSFORMATION

In this section we will consider the general form of a unitary transformation (in the free quark model) which acts on the $SU(6)_W$ of light-like "current" charges to give a set of charges $\widetilde{W}^i_\alpha = \widetilde{V}\widetilde{Q}^i_\alpha \widetilde{V}^{-1}$ that satisfy the following set of conditions:

$$\left.\begin{array}{ll}
\text{(a)} & \widetilde{W}^i_\alpha \text{ is a "good" operator. This is the analogue of requiring} \\
 & \text{the equal-time W's to be such that classification will be} \\
 & \text{meaningful between infinite momentum states;} \\
\text{(b)} & \left[\widetilde{W}^i_\alpha, \widetilde{K}_3\right] = 0, \text{ in order that the symmetry be collinear;} \\
\text{(c)} & \left[\widetilde{W}^i_3, \widetilde{J}_3\right] = 0, \text{ to preserve the spin projection classification;} \\
\text{(d)} & \text{CVC requires that } \widetilde{W}^i = \widetilde{Q}^i; \\
\text{(e)} & \widetilde{W}^i_\alpha \text{ has the same } \widetilde{C} \text{ and } \widetilde{P} \text{ properties as } \widetilde{Q}^i_\alpha; \\
\text{(f)} & \left[\widetilde{W}^i_\alpha, \widetilde{H}\right] = 0 \text{ so that } \widetilde{W}^i_\alpha \text{ classify single particle states.}
\end{array}\right\} \quad (11)$$

We write $\widetilde{V} = \exp i\widetilde{Y}$, where $\widetilde{Y}$ is a hermitian operator. The conditions that $\widetilde{Y}$ must satisfy are discussed below.

(a) Since $\widetilde{W}$ is to be "good", the operator $\widetilde{Y}$ should be "good". A "good" operator as the term is used in current algebra is one whose matrix elements do not vanish between states with infinite momentum. In the light-cone formulation such operators can be written in terms of densities local in x^-, involving only canonically independent fields.

We will restrict ourselves to consideration of a form bilinear in the quark field,

$$\tilde{Y} = i \int dx^- \, d\vec{x}_\perp \; \bar{q}(x) \; F_0(x^\mu, \gamma^\nu, \partial^\sigma, \lambda^i) \; q(x) \quad .$$

This assumption insures that $\tilde{W}^i_\alpha$ does not lead from nonexotic to exotic states.

The condition that $\tilde{V}$ is "good" restricts the possible Dirac structure of F_0 to the following form:

$$\begin{aligned} F_0 &= \gamma^+ \left[F_{(1)} + \gamma^5 \, F_{(2)} + \gamma_\perp \, F_{\perp(3)} + \gamma^5 \gamma_\perp \, F_{\perp(4)} + \frac{i}{2}\left[\gamma^x, \gamma^y\right] F_{(5)} \right] \\ &\equiv \gamma^+ \, E + \frac{i\gamma^+}{2} \left[\gamma^x, \gamma^y\right] F_{(5)} \end{aligned} \qquad (12)$$

Since the tensor $\frac{i}{2}\left[\gamma^x, \gamma^y\right]$ is equivalent to γ^5 between $q_+^\dagger(x)$ and $q_+(x)$, $F_{(5)}$ is not an independent function. Then $\tilde{Y}$ can be rewritten in terms of the independent fields as

$$\tilde{Y} = i\sqrt{2} \int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x) \; E \, q_+(x) \quad . \qquad (13)$$

The "goodness" assumption further restricts the functions above so they have no derivatives with respect to x^+, for these may be written

$$\frac{\partial}{\partial x^+} q_+(x) = \frac{i}{4} \int dy^- \, \epsilon(x^- - y^-) \left(-\vec{\nabla}_\perp^2 + m^2\right) q_+\left(x^+, \vec{x}_\perp, y^-\right) = \frac{1}{2\eta}\left(\vec{\nabla}_\perp^2 - m^2\right) q_+(x) \quad .$$

This expression is nonlocal in x^-.

(b) The condition that $\left[\tilde{K}_3; \tilde{W}^i_\alpha\right] = 0$ can be satisfied with $\left[\tilde{K}_3, \tilde{Y}\right] = 0$. The light-like generator $\tilde{K}_3$ is (see Appendix A)

$$\tilde{K}_3 = i\sqrt{2} \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x) \left(x^+ \frac{\partial}{\partial x^+} - x^- \frac{\partial}{\partial x^-} - \frac{1}{2} \right) q_+(x) \quad .$$

We thus require

$$\left[E\,(x^{\mu},\gamma^{\nu},\partial^{\sigma},\lambda^{i}),\ x^{-}\frac{\partial}{\partial x^{-}}\right]=0 \qquad (x^{+}=0) \quad .$$

Since E is a polynomial of finite degree in $(\partial/\partial x^{-})$ by condition (a) above,[16] the condition that $[E, x^{-}(\partial/\partial x^{-})]=0$ implies that E depends on (x^{-}) and $(\partial/\partial x^{-})$ only as a polynomial of finite degree in $x^{-}(\partial/\partial x^{-})$.

(c) The condition that $\left[\widetilde{J}_3, \widetilde{W}_3^i\right]=0$ can be satisfied with $\left[\widetilde{J}_3, \widetilde{Y}\right]=0$. In the light-cone formulation the rotation $\widetilde{J}_3$ is

$$\widetilde{J}_3 = i\sqrt{2}\int dx^{-}\, d\vec{x}_{\perp}\, q_{+}^{\dagger}(x)\left\{\epsilon^{ij}\, x_i\,\frac{\partial}{\partial x^{j}}+\frac{1}{2}\,\gamma^{x}\gamma^{y}\right\}q_{+}(x) \quad .$$

We thus require

$$\left[E,\ \epsilon^{ij}\, x_i\,\frac{\partial}{\partial x^{j}}+\frac{1}{2}\gamma^{x}\gamma^{y}\right]=0 \quad .$$

This implies that E may be written

$$E\left(x^{-}\frac{\partial}{\partial x^{-}},\ \vec{x}_{\perp}^{\,2},\vec{\nabla}_{\perp}^{\,2},\ \vec{x}_{\perp}\cdot\vec{\nabla}_{\perp},\ \epsilon^{ij}x_i\partial_j,\lambda^{9}\right)=F_{(1)}(\cdots)+\gamma^{5}F_{(2)}(\cdots)$$

$$+\gamma^{i}\,F_{i(3)}(\cdots)+\gamma^{5}\gamma^{i}\,F_{i(4)}(\cdots) \quad .$$

where

$$F_{i(3)}(\cdots)=x_i\,F'_{(3)}(\cdots)+\frac{\partial}{\partial x^{i}}\,F''_{(3)}(\cdots)$$

and

$$F_{i(4)}(\cdots)=x_i\,F'_{(4)}(\cdots)+\frac{\partial}{\partial x^{i}}\,F''_{(4)}(\cdots) \quad .$$

(d) CVC implies that $\widetilde{Y}$ is an SU(3) scalar. Thus E depends only on the unit matrix of SU(3).

(e) The discrete properties of $\tilde{Q}$ and $\tilde{W}$ imply that $\tilde{Y}$ has $\tilde{P}=+$, and $\tilde{C}=+$. This condition eliminates a number of possible terms. We now have

$$E = \gamma_i x^i F'_{(3)}\left(x^- \frac{\partial}{\partial x^-}, \vec{x}_\perp^2, \vec{\nabla}_\perp^2, \vec{x}_\perp \cdot \vec{\nabla}_\perp\right) + \gamma_i \partial^i F''_{(3)}\left(x^- \frac{\partial}{\partial x^-}, \vec{x}_\perp^2, \vec{\nabla}_\perp^2, \vec{x}_\perp \cdot \vec{\nabla}_\perp\right) \tag{14}$$

(f) The final property of $\tilde{W}^i_\alpha$ is that it commutes with $\tilde{H}$. Since in the free quark model $\left[\tilde{H}, \tilde{Q}_\alpha\right] = 0$, we have the possibility $[\tilde{Y}, \tilde{H}] = 0$.

Using Eq. (6) we find the restriction

$$\left[E, \frac{\vec{\nabla}_\perp^2 - m^2}{2\eta}\right] = 0 \quad .$$

This condition implies that E does not depend on $x^- (\partial/\partial x^-)$ or x^i. Therefore one may express E as

$$E = \gamma_i \partial^i F''_{(3)}(|\vec{\partial}_\perp|) \quad .$$

Thus the most general transformation satisfying the properties (a) through (f) stated above may be written as

$$\tilde{V} = \exp\left\{\frac{i}{\sqrt{2}} \int dx^- d\vec{x}_\perp q_+^\dagger(x) \left[\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} F(|\vec{\partial}_\perp|)\right] q_+(x)\right\} \quad , \tag{15}$$

where

$$F \equiv |\vec{\partial}_\perp| \; F_{(3)}(|\vec{\partial}_\perp|) \quad .$$

There is an additional physical constraint that one should impose on $\tilde{V}$.[17] It is that when $\tilde{W}_3$ acts on states at rest, it is equal to the "spin" part of $\tilde{J}_3$. In general this implies the restriction

$$e^{i\pi \vec{n}_\perp \cdot \vec{J}_\perp} \tilde{W}_3 \, e^{-i\pi \vec{n}_\perp \cdot \vec{J}_\perp} = - \tilde{W}_3$$

(where $n_\perp$ is a unit vector in any transverse direction) when the operators act on states at rest. In the free quark model, this implies $F(0) = 0$.

Two of the assumptions made in this derivation may have to be relaxed in field theory models with interactions:

1) The light-like charges no longer commute with $\tilde{P}^-$ unless the Fermion mass vanishes. Thus the transformation can no longer be constructed to commute with the Hamiltonian, if we require conserved $\tilde{W}$.

2) It may be necessary to include terms in $\tilde{V}$ that are not bilinear in the quark field. These terms may be required anyway in light of theorems regarding the necessity of exotics in the saturation of the current algebra.[18] It will be interesting to see if such terms are needed phenomenologically.

The explicit forms of the operators $\tilde{W}$ constructed using the general transformations $\tilde{V}$, Eq. (15), are given in Appendix A. The light-like Melosh transform corresponds to choosing $F = \arctan\,(|\vec{\partial}_\perp|/m)$. Another form which satisfies all the conditions above, except $F(0) = 0$, is $F = \text{constant}$; the resulting $\tilde{W}$ operators have many properties in common with the operators introduced by Gilman and Kugler in their phenomenological analysis. In particular, we may identify the operators, which satisfy in 0(5) algebra.[11] Defining Q_5^i and K^i from

$$W_\alpha^i = (\cos F)\, Q_5^i - (\sin F)\, K^i \quad , \tag{16a}$$

these operators are,

$$Q_5^i = \sqrt{2} \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x) \, \frac{\sigma_z \tau_i}{2} \, q_+(x) \tag{16b}$$

$$K^i = \sqrt{2} \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x) \, \frac{\vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} \, \frac{\tau_i}{2} \, q_+(x) \tag{16c}$$

$$S = \sqrt{2} \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x) \, \frac{i\sigma_z \vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} \, q_+(x) \tag{16d}$$

and

$$Q^i = \sqrt{2} \int dx^- \, d\vec{x} \; q_+^\dagger(x) \frac{\tau_i}{2} q_+(x) \tag{16e}$$

(Here τ_i is the isospin operators.) This algebra can be extended to SU(4) by appending the operators

$$Q_5 = \sqrt{2} \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x) \frac{\sigma_z}{2} q_+(x) \tag{17a}$$

$$K = \sqrt{2} \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x) \frac{\vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{2 \, |\vec{\partial}_\perp|} q_+(x) \tag{17b}$$

$$S^i = \sqrt{2} \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x) \frac{i\sigma_z \, \vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} \frac{\tau_i}{2} q_+(x) \tag{17c}$$

Finally, it can be extended to give another SU(6) by changing τ_i into λ_i. These generators are similar in form to the standard $SU(6)_{currents}$ generators, because after all there are only a limited number of possible structures in the free quark model. Nevertheless they are not exactly the same because the derivatives appearing imply they have different L properties.

IV. INTERPRETATION AND APPLICATIONS OF $\tilde{V}$

The transformation $\tilde{V}$ constructed in the preceding section has a very natural interpretation as a change of spin basis. As noted in Section II, the states in the light-like helicity basis, Eq. (9), transform as representations of $SU(6)_{W,\, currents}$, independent of the value of their momentum. States transforming according to the same representation of $SU(6)_{W,\, constituents}$, which is generated by $\tilde{W}^i$ and $\tilde{W}^i_\alpha$, may be constructed from the states of the LLHB using $\tilde{V}$: if

$$\tilde{Q}^i_\alpha \, |p^+, p_\perp; \lambda> = C^{i\alpha}_{\lambda\lambda'} \, |p^+, p_\perp; \lambda'>$$

then

$$\widetilde{W}^{i}_{\alpha}(\,\widetilde{V}\,|p^{+},\,\vec{p}_{\perp};\lambda\rangle) = C^{i\alpha}_{\lambda\lambda'}\,(\widetilde{V}\,|p^{+},\,\vec{p}_{\perp};\lambda'\rangle)$$

We will now show that the states $(\widetilde{V}\,|p^{+},\,p_{\perp};\lambda\rangle)$ are simply one particle states in a different spin basis.

First we compute the action of $\widetilde{V}$ on the covariantly normalized single particle states in LLHB.

$$\widetilde{V}\,|p^{+},\,\vec{p}_{\perp};\lambda'\rangle = \sum_{\lambda} \mathcal{V}_{\lambda\lambda'}(p)\,|p^{+},\,\vec{p}_{\perp};\lambda\rangle \tag{18}$$

where

$$\mathcal{V}_{\lambda\lambda'}(p) = \chi^{\dagger}_{\lambda}\left[\cos\,(f/2) - i\hat{p}_{\perp}\times\vec{\sigma}_{\perp}\sin\,(f/2)\right]\chi_{\lambda'}\ , \tag{19}$$

$$\chi_{+1/2} = \begin{pmatrix}1\\0\end{pmatrix}\ , \qquad \chi_{-1/2} = \begin{pmatrix}0\\1\end{pmatrix}\ ,$$

and

$$f \equiv \int d\,\vec{x}_{\perp}\; e^{i\,\vec{p}_{\perp}\cdot\vec{x}_{\perp}}\, F(|\vec{\partial}_{\perp}|) \tag{20}$$

Since $\widetilde{V}$ commutes with $\widetilde{H}$, we see it does nothing but rotate the spin of the state.

Let us construct a state of arbitrary momentum by means of the prescription

$$|p^{+},\,\vec{p}_{\perp};\pm s_{W}\rangle \equiv e^{i\lambda\widetilde{K}_{3}}\, e^{i\vec{\beta}_{\perp}\cdot\widetilde{\vec{K}}_{\perp}}\, e^{i\theta\,\hat{n}_{\perp}\cdot\widetilde{\vec{J}}_{\perp}}\left|\frac{m}{\sqrt{2}},\,\vec{0}_{\perp};\pm 1/2\right\rangle \tag{21}$$

The parameters $\beta_{\perp}$ and λ have the values

$$\beta_{\perp} = \hat{p}_{\perp}\ \operatorname{arcsinh}\left(\frac{\sqrt{\omega^{2}-m^{2}}}{m}\right)\ ;$$

$$\lambda = \ln\left(\frac{\sqrt{2}\,p^{+}}{\omega}\right)\ ;$$

$$\omega = \sqrt{\vec{p}_{\perp}^{\,2} + m^{2}}$$

The remaining parameters θ and $\hat{n}_\perp$ are also functions of the momentum, and determine the specific spin basis. The prescription Eq. (21) has been adopted so the "spin" defined by the construction is invariant under boosts by $\tilde{K}_3$ if θ and $\hat{n}_\perp$ are functions of $p_\perp$ and m only, and not functions of p^+.

These states, Eq. (21), are related to the states in the LLHB in the following manner:

$$|p^+, \vec{p}_\perp; \pm s_W> = \sum_\lambda \mathscr{D}_{\lambda \pm s_W} |p^+, \vec{p}_\perp; \lambda> \tag{22}$$

where

$$\mathscr{D}_{\lambda\pm s_W}(p) = \chi^\dagger_\lambda \left\{ \sqrt{\frac{\omega+m}{2\omega}} \left[\cos(\theta/2) + i \sin(\theta/2)\, \hat{n}_\perp \cdot \vec{\sigma}_\perp\right] \right.$$
$$\left. + \sqrt{\frac{\omega-m}{2\omega}} \left[i\hat{p}_\perp \times \vec{\sigma}_\perp \cos(\theta/2) - i \sin(\theta/2) \left(\hat{n}_\perp \cdot \hat{p}_\perp \sigma_z + i\hat{p}_\perp \times \hat{n}_\perp\right)\right]\right\} \chi_{\pm 1/2} \tag{23}$$

Thus comparing Eq. (18) and Eq. (22), the states $|p^+, p_\perp, s_W>$ are just the states transforming according to $SU(6)_{W,\,\text{constituents}}$, $(\tilde{V}|p^+, p_\perp, \lambda>)$, provided we make the identifications:

$$\hat{n}_j = \epsilon_{jk}\, p^k / |\vec{p}_\perp| \tag{24}$$

$$\theta = f\left(\sqrt{\omega^2 - m^2}\right) - \arctan\left(\frac{\sqrt{\omega^2 - m^2}}{m}\right) \tag{25}$$

For the Melosh transformation, $f = \arctan\left(\frac{\sqrt{\omega^2 - m^2}}{m}\right)$, so by Eqs. (20) and (25), $\theta = 0$. Thus the corresponding spin basis is

$$|p^+, \vec{p}_\perp; \pm\omega> = e^{i \ln\left(\frac{\sqrt{2}p^+}{\omega}\right) \tilde{K}_3}\, e^{i\hat{p}_\perp \cdot \tilde{\tilde{K}}_\perp \operatorname{arcsinh}\left(\frac{\sqrt{\omega^2 - m^2}}{m}\right)} \left| \frac{m}{\sqrt{2}}, \vec{0}_\perp; \pm 1/2 \right> \tag{26}$$

A similar construction in the equal-time formulation leads to states classified by $W^{E.T.}_{i\alpha} = V_M F^{E.T.}_{i\alpha} V^{-1}_M$ in a momentum independent manner in the free quark model.

The "constituent" states corresponding to the "Gilman-Kugler" basis are obtained by taking $F=\alpha/2$, where α is a constant, so $\theta=\alpha/2-\arctan\left(\frac{\sqrt{\omega^2-m^2}}{m}\right)$. They are

$$
\begin{aligned}
|p^+, \vec{p}_\perp; \pm h> &= e^{i\ell n\left(\frac{\sqrt{2}p^+}{\omega}\right)\tilde{K}_3}\, e^{i\hat{p}_\perp\cdot\tilde{\vec{K}}_\perp \operatorname{arcsinh}\left(\frac{\sqrt{\omega^2-m^2}}{m}\right)} \\
&\quad e^{i\left[\frac{\alpha}{2}-\arctan\left(\frac{\sqrt{\omega^2-m^2}}{m}\right)\right]\hat{p}_\perp\times\tilde{\vec{J}}_\perp} \left|\frac{m}{\sqrt{2}}, \vec{0}_\perp; \pm 1/2>\right. \\
&= e^{i\ell n\left(\frac{\sqrt{2}p^+}{m}\right)\tilde{K}_3}\, e^{i\vec{p}_\perp\cdot\tilde{\vec{B}}_\perp\frac{\sqrt{2}}{m}}\, e^{i\frac{\alpha}{2}\hat{p}_\perp\times\tilde{\vec{J}}_\perp} \left|\frac{m}{\sqrt{2}}, \vec{0}_\perp; \pm 1/2>\right.
\end{aligned}
\tag{27}
$$

These are a complicated mixture of LLH states and "transverse helicity" states we will define below. The wavefunctions corresponding to the states (27) are

$$
<0|q_+(x)|p^+, \vec{p}_\perp; \pm h_\alpha> = e^{-ip\cdot x}\; u(p^+, \vec{p}_\perp, \pm h_\alpha)
$$

where

$$
u(p^+, \vec{p}_\perp, \pm h_\alpha) = \frac{1}{\sqrt{2}}\begin{bmatrix} \left(\frac{\sqrt{2}p^+}{m}\right)^{1/2}\left[\cos(\alpha/2) + i\hat{p}_\perp\times\vec{\sigma}_\perp \sin(\alpha/2)\right]\chi_{\pm 1/2} \\ \\ \left(\frac{m}{\sqrt{2}p^+}\right)^{1/2}\left[\cos(\alpha/2) + i\hat{p}_\perp\times\vec{\sigma}_\perp \sin(\alpha/2)\right]\sigma_z\chi_{\pm 1/2} \end{bmatrix}
$$

Now suppose that $p^+=m/\sqrt{2}$; and that we pick $\alpha=\pi/2$. Then these spinors take the simple form ("transverse helicity state")

$$
u(p^+, \vec{p}_\perp; +h_{\pi/2}) = \frac{1}{2}\begin{pmatrix} 1 \\ n_x+in_y \\ 1 \\ n_x+in_y \end{pmatrix}, \quad u(p^+, \vec{p}_\perp, -h_{\pi/2}) = \frac{1}{2}\begin{pmatrix} -n_x+in_y \\ 1 \\ -n_x+in_y \\ 1 \end{pmatrix}, \quad n_i = \frac{p_i}{|\vec{p}_\perp|}
$$

It is straightforward to verify that these are the eigenspinors of the "transverse helicity" operator K, Eq. (17b). We have, then, that the LLH spinors are eigenspinors of $\tilde{Q}_5$: the spinors $u(p^+, \vec{p}_\perp, \pm h_{\pi/2})$ are eigenspinors of K; but unfortunately the spinors of phenomenological interest,[11] $\alpha \simeq \pi/4$, are in some sense in between these cases. It is not clear whether this is of any physical significance.

As mentioned in the introduction, the nonuniqueness in the definition of $\tilde{V}$ which is so striking in the light-like formulation has an analogue in the equal-time formulation as well. We can see this analogue by constructing new equal-time W operators $W_{ET} = V\, F_{ET} V^{-1}$ using the equal-time transform

$$V = \exp\left\{\frac{i}{2}\int d^3\vec{x}\; q^\dagger(x)\; \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|}\; F(|\vec{\partial}_\perp|)\; q(x)\right\} \quad .$$

These W s will not commute with the Hamiltonian unless F coincides with F_M, but the matrix elements of $[H, W]$ vanish when taken between states at infinite z-momentum in the free quark model. Thus the theoretically interesting problem of classifying states at infinite momentum suffers from the same ambituity in the equal-time formulation that is immediately apparent in the light-cone quantization.

Let us now study the constraints placed on the transformation $\tilde{V}$ by phenomenological considerations. It is to be expected that matrix elements involving zero momentum transfer cannot fully determine a nonlocal function such as F. Consider for example the algebraic calculation of $(Ga/Gv)_{nucleon}$. It is determined by the matrix element of the third isospin component of the axial charge between collinear physical nucleon states. These states are complicated mixtures of states which transform as irreducible representations of the chiral algebra of charges, but fall in an irreducible representation of the "constituent" algebra. Thus one calculates the matrix element

$$
\begin{aligned}
\langle N'_{\text{physical}} | \tilde{Q}_5^3 | N_{\text{physical}} \rangle &= \langle N_{\text{current}} | \tilde{V}^{-1} Q_5^3 \tilde{V} | N'_{\text{current}} \rangle \\
&= \langle N'_{\text{current}} | \left[i\sqrt{2} \int dx^- d\vec{x}_\perp q_+^\dagger(x) \gamma^5 \frac{\tau^3}{a} \right. \\
&\qquad \left. \left\{ \cos F + i \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} \sin F \right\} q_+(x) \right] | N_{\text{current}} \rangle
\end{aligned}
$$

The state $|N_{\text{current}}\rangle$ is in the same representation under the "currents" group as the nucleon was under the "constituents" group. This gives a coefficient (5/3) as before, but it multiplies the integral $(\phi_{\text{currents}}, (\cos F)\phi_{\text{currents}})$, where ϕ_{currents} is the wavefunction of the nucleon <u>in the currents representation.</u> The wavefunction in the "constituents" representation is fixed to be the eigenfunction of the strong Hamiltonian, and is to be considered as unique.

The integral which modifies (Ga/Gv) from (5/3) to its experimental value must be numerically $\approx 1/\sqrt{2}$. This is clearly a constraint on $\tilde{V}$. For instance, F=0 is ruled out. However, since ϕ_{currents} itself is an implicit function of F, any dynamical explanation of this numerical value of the integral is representation-dependent.

Next consider calculations in which the term involving (sin F) in $(\tilde{V}^{-1} \tilde{Q}_5^3 \tilde{V})$ gives a nonvanishing contribution. This would involve transitions with $\Delta L = \pm 1$.[1,11] In principle, comparison of the integral of this function to the integral of (cos F) between identical wavefunctions could give information about F. However, phenomenologically, it seems that the ratio of the integrals $(\phi_c, (\cos F)\phi_c)/(\phi_c, (\sin F)\phi_c) \approx \cot\theta$, where $\theta \approx (\phi_c, F\phi_c)$, when taken between low-lying states.[11] This is predictable if the relevant wavefunctions are sufficiently peaked in momentum space, and so does not restrict F in any clear manner.

Let us now examine matrix elements involving first order terms in transverse momentum transfer, such as the magnetic moments. These matrix elements give information on the average value of F' taken between wave functions. The moments may be calculated using the following expressions:

$$\left[M(2\pi)^3 \, \delta^3(0)\right] \frac{\mu_A}{2M} =$$

$$\frac{\partial}{\partial k_x} \left\langle A; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2} \right| \left\{ e^{\frac{-ik_x B_x}{\sqrt{2M}}} \int dx^- \, d\vec{x}_\perp \, e^{-ik_x \cdot x_1} J^+_{EM}(x) \, e^{\frac{-ik_x B_x}{\sqrt{2M}}} \right\} \left| B; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2} \right\rangle \Bigg|_{k_x=0} \tag{28}$$

$$\left[M(2\pi)^3 \, \delta^3(0)\right] \frac{\mu_T}{2M} =$$

$$\left\langle A; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2} \right| \left\{ \int dx^- \, d\vec{x}_\perp \, x_1 J^-_{EM}(x) \right\} \left| B; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2} \right\rangle \tag{29}$$

It is shown in Appendix B that these are kinematically correct light-cone expressions using the free quark model with an anomalous Pauli term. The calculation for the physical nucleons is straightforward, and the details are presented in Appendix B. Using

$$J^\mu_{EM}(x) = e \int dx^- \, d\vec{x}_\perp \, \bar{q}(x) \, \frac{\lambda^{EM} \gamma^\mu}{2} \, q(x)$$

we obtain for the anomalous magnetic moments

$$\frac{\mu^P_A}{\mu^N_A} = -\frac{3}{2}(1-\Delta)$$

where

$$\Delta = \frac{\left(\phi_c, \frac{\sqrt{2}}{M} \frac{\partial}{\partial x^-} \left[F' - \frac{\sin F}{|\vec{\partial}_\perp|}\right] \phi_c\right)}{\left(\phi_c, \left[F' - \frac{\sin F}{|\vec{\partial}_\perp|}\right] \phi_c\right)} \tag{A.3}$$

In the free quark model where the 3 quarks are at rest $\Delta = 1/3$. The mechanism responsible for giving a nonzero result is the same as that discussed by Melosh.[1] If F is zero, the anomalous moments vanish.

The ratio of the total magnetic moments of the proton and neutron is shown in Appendix B, Eq. (B.4) to be (-3/2), independent of the form of F. This is reassuring, as it is one of the principal results that drew attention to SU(6). Higher moments of the electromagnetic current give information on averages of higher derivatives of F.

To conclude this section, we stress that the notable qualitative changes in the predictions of the quark model effected by $\widetilde{V}$ are due to the algebraic structure of Eq. (15) under $SU(6)_{W,\text{ currents}}$.[1,11] However, to make quantitative predictions for processes involving nonzero momentum transfer we need to know the averages of the derivatives of the function F between wave functions.[19] For phenomenological analysis this means new undetermined constants must be introduced.

V. ASPECTS OF THE TRANSFORMATION IN MODELS WITH POTENTIALS

The discussion of the previous section indicates one has considerable lattitude to approach models with interaction. In the present section we will examine one such approach, allowing the quarks and antiquarks to interact by means of a potential. It makes sense to do this because models of this genre preserve particle number, a concept vital to the meaning of the "constituents" classification scheme of the hadrons. Furthermore, in spite of appearances, such theoretics are relativistic in a well-defined sense; namely, that the solutions span representations of the inhomogeneous Lorentz group.[20,21] In the light-cone quantization, the formalism under which relativistic invariance is possible has been discussed in detail by Bardakci and Halpern,[22] and we will utilize some of their results below. Our aim is to see how the conditions of relativistic invariance affect the "constituent" to "currents" mixing scheme effected by $\tilde{V}$.

Let us first cast the quark field into a form appropriate for examinging its properties in first-quantized notation in a simple manner. This involves eliminating the redundant components from the four-component Dirac spinors, and performing a unitary transformation so that in effect the Dirac matrices act like Pauli matrices. Bjorken, Kogut, and Soper[15] arrive at the simple expression

$$q_+(x) = \sum_{h=\pm\frac{1}{2}} \int \frac{d\vec{p}_\perp}{(2\pi)^3} \int_0^\infty \frac{dp^+}{2p^+} \Big[b(p^+, \vec{p}_\perp; h) \sqrt{2p^+}\, w(h)\, e^{-ip\cdot x}$$

$$+ d^\dagger(p^+, \vec{p}_\perp; h) \sqrt{2p^+}\, w(-h)\, e^{ip\cdot x} \Big] \tag{30}$$

where

$$w(\tfrac{1}{2}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad w(-\tfrac{1}{2}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

Note that the creation operators give states in the "light-like helicity basis," as in Eq. (9). The action of operations on the wave functions $<0\,|q_+(x)\, d^\dagger(p^+, \vec{p}_\perp, h)\,|0>$ and $<0\,|q_+^\dagger(x)\, b^\dagger(p^+, \vec{p}_\perp; h)\,|0>$ takes on a simple nonrelativistic form. For example,

$$Q_5^i = \int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\, \gamma^5\, \frac{\tau^i}{2}\, q_+(x) \longrightarrow \left(\frac{\sigma_3 \tau^i}{2}\right)_q - \left(\frac{\sigma_3 \tau^i}{2}\right)_{\bar{q}} \tag{31}$$

Consider next a two-particle system which we would want to correspond to a meson at rest:

$$|M\rangle = \sum_{\substack{\alpha, h \\ \beta, h'}} \int d\vec{p}_\perp \int_0^1 \frac{dx}{x(1-x)}\, f(\vec{p}_\perp, x)\, C^{\alpha\beta}_{hh'}\, b^\dagger_\alpha\!\left(\frac{M}{\sqrt{2}} x, \vec{p}_\perp, h\right) d^\dagger_\beta\!\left(\frac{M}{\sqrt{2}}(1-x), \vec{p}_\perp, h'\right) |0\rangle \tag{32}$$

To study the properties of the wave function of such a state under Lorentz transformations, we write the generators $\tilde{P}_\perp$, $\tilde{P}^+$, $\tilde{J}_3$, $\tilde{K}_3$ and $\tilde{B}_\perp$ as the sums of the generators for the two particles. A representation can be found in which these generators do not involve interaction terms.[20,22] All of the interaction dependence can be placed in just three generators, $\tilde{H}$ and $\tilde{S}_\perp$. Recall this is a virtue of the light-cone formulation of dynamics, for in the equal-time case there must be four generators which contain the effects of interaction. A field theoretic example is provided by QED.[15]

The "potential" is just the term added on to the sum of the two particles' free Hamiltonians,

$$\tilde{H} = \tilde{H}_q + \tilde{H}_{\bar{q}} + U \tag{33}$$

Introducing center-of-mass and relative coordinates and momenta in standard fashion, Bardacki and Halpern find that the theory will be invariant under Lorentz transformations if the potential is Galilei invariant, and rotationally invariant under a set of operators obeying the algebra SU(2) formed from relative variables exclusively. Labelling these "internal spin" operators j_i, the interaction-dependent Lorentz generators $\tilde{S}_k$ are

$$\tilde{S}_k = \left[\frac{1}{2}\left\{x_k, \frac{\mathcal{M}^2 + P_\perp^2}{2M}\right\} - \frac{1}{2}\left\{\frac{1}{M}, K_3\right\} P_k\right]_{c.m.} + j_3 \frac{\epsilon_{km}p_m}{2M} - \frac{\epsilon_{km}j_m}{2M}\sqrt{\mathcal{M}^2}$$

where

$$M = (x^-)_q + (x^-)_{\bar{q}}$$

$$\mathcal{M}^2 = 2HM - (P_\perp^2)_{c.m.}$$

We now focus attention on the action of the generator $\tilde{J}_3$ on the state Eq. (32), which is at rest. On such a state $\tilde{J}_3 = j_3$. Notice that the conventional partition into "orbital" and "spin" angular momentum

$$j_3 = \ell_3 + \sum_3 = (x_1\pi_2 - x_2\pi_1) + \left(\frac{\sigma_3}{2}\right)_q - \left(\frac{\sigma_3}{2}\right)_{\bar{q}} \tag{34}$$

is not unique. Since the transformation $\widetilde{V}$ commutes with $\widetilde{J}_3$, we may choose to write

$$j_3 = \widetilde{V}\left(\ell_3 + \sum_3\right)\widetilde{V}^{-1} \equiv L_3 + W_3$$

and not destroy the overall Lorentz properties of the system.

The utility of a particular choice of this splitting is based on being able to find $\widetilde{L}_3$ and $\widetilde{W}_3$ such that they commute with $\widetilde{H}$ separately,

$$\left[\widetilde{L}_3, \widetilde{H}\right] = \left[\widetilde{W}_3, \widetilde{H}\right] = 0 \quad . \tag{35}$$

In this manner, the "$\widetilde{L}_3$ and $\widetilde{W}_3$" classification of the state will be meaningful. This imposes conditions on $\widetilde{V}$ once the Hamiltonian has been specified.

Consider first a potential which is a function of the internal coordinates and momenta only, and does not depend on the Pauli spins. Since the generators $\widetilde{J}_3$ and $\widetilde{K}_3$ do not involve interaction, the arguments presented in Section III still suffice to determine the general form of $\widetilde{V}$ given in Eq. (14). In order to be rotationally invariant under the "internal spins" j_i, the potential U must depend on both the relative momentum $\pi_\perp$ and relative position $x_\perp$. The constraint Eq. (35) will then be nontrivial to satisfy, and will give restrictions on $F_1(x_\perp^2, \pi_\perp^2, x_\perp \pi_\perp)$ and $F_2(x_\perp^2, \pi_\perp^2, x_\perp \pi_\perp)$ (which were denoted F_3' and F_3'' in Eq. (14)).

A simple example is provided by the two-dimensional harmonic oscillator. On states at rest we have

$$\widetilde{H} = \frac{1}{2}\left(\vec{\pi}_\perp^{\,2} + \vec{x}_\perp^{\,2}\right) \tag{36}$$

(in appropriate units for $\pi_\perp$ and $x_\perp$). Consider $\widetilde{V} = \exp(i\widetilde{Y})$, where $\widetilde{Y}$ is the Hermitian operator

$$\widetilde{Y} = \frac{i}{2}\left(\left\{\sigma_z \vec{\sigma}_\perp \cdot \vec{x}_\perp ,\ F_1\right\} + \left\{\sigma_z \vec{\sigma}_\perp \cdot \vec{\pi}_\perp ,\ F_2\right\}\right) \quad .$$

The condition Eq. (35) can be satisfied if

$$\left[F_1, H\right] = 2iF_2 \quad ; \tag{37a}$$

$$\left[F_2, H\right] = -2iF_1 \quad . \tag{37b}$$

A solution to this system is[23]

$$F_1 = -\frac{1}{4}\left\{x_\perp^i, \pi_{\perp i}\right\} \ ; \tag{38a}$$

$$F_2 = \frac{1}{4}\left[\vec{x}_\perp^{\,2} - \vec{\pi}_\perp^{\,2}\right] \quad . \tag{38b}$$

It is amusing that in addition to the sum

$$\widetilde{V}^{-1}\frac{\sigma_z}{2}\widetilde{V} = \frac{\sigma_z}{2} - \left[i\widetilde{Y}, \frac{\sigma_z}{2}\right] + \frac{1}{2i}\left[i\widetilde{Y}, \left[i\widetilde{Y}, \frac{\sigma_z}{2}\right]\right] + \ldots$$

each term in the sum by itself is a candidate for $(\widetilde{W}_3)_q$ up to factors of "i" required for hermiticity. Of course, the example, is not intended to be a realistic model, but only to show what a nontrivial solution can look like.

With this type of potential, it is still consistent to use the free form of the axial charge, Eq. (31). That is, this axial charge will have all the properties it is supposed to have under Lorentz transformations and under parity. It is also conserved. This is possible because the interaction conserves particle number, and is independent of Pauli matrices. In a field theoretic model, e.g., in the linear sigma model, one would have to append the contributions to the axial current of the other particles. The terms which break the conservation of axial charge will change particle number.

Using $\tilde{V}$ it is then possible to construct eigenstates of $\tilde{Q}_5$ from the eigenstates of the "spin" $\tilde{W}_3$. For instance, if $|M, s_z>$ is a state with a definite eigenvalue of $\tilde{W}_3$. $\tilde{V}^{-1}|M, s_z>$ is a state with the same eigenvalue of $\tilde{Q}_5$. However, it is not an eigenstate of $\tilde{L}_3$. Neither does it have the same energy as $|M, s_z>$. This is how the mixing scheme works in this type of model in all cases for which we can find solutions to Eq. (35) (which are essentially differential equations).

The conditions to be satisfied if the potential includes spin dependence are much more complicated. A simple way to construct a rotationally invariant U is to make it a function of j^2. Since the j_i include Pauli matrices, U will have what looks like "LS" coupling in it. Such a potential will no longer commute with $\tilde{Q}_5$, but there is still a possibility that a conserved $\tilde{W}_3$ operator can be found. The conditions to be satisfied will contain many terms, and it may be possible to contrive cancellations.

In any case, the point here is that one has a well-defined procedure for constructing the desired operators $\tilde{W}_3$, $\tilde{L}_3$ once a potential has been decided upon on some other physical grounds. It is an analogue of the FW construction for interacting field theories, but with the advantage that it focuses on the single condition one wants to maintain; namely, the conservation of the "spin" $\tilde{W}_3$. Our Lorentz generators are "fixed" in a given basis, and the bound states, as well as the vector and axial vector currents, must have the correct covariance properties with respect to this set of generators. We have seen these Lorentz conditions in no way conflict with the construction of $\tilde{W}_3$.

The problem of finding the representations of the chiral SU(3) × SU(3) charge algebra has thus been reduced to finding the operator $\tilde{V}$ such that $\tilde{V}\tilde{Q}_5\tilde{V}^{-1}$ commutes with the Hamiltonian (on top of the other conditions for $\tilde{V}$ given in

Section III). Thus, the problem is identical in form to the one solved by Melosh in the free quark model, but with the difference that the specific functional dependence of $\widetilde{V}$ depends on the interaction.

VI. SUMMARY AND CONCLUSIONS

In the attempt to find economical saturation schemes for charge algebra sum rules, one can use states at infinite momentum so that many-particle contributions to the intermediate states will give nondominant contributions. Alternatively, one can use states at finite momentum, state the sum rules in terms of light-like charges, and thus avoid a limiting procedure. The correctness of either procedure is tested by the phenomenological success of the derived sum rule.

In the light-cone formulation of the sum rules, as well as in the $p \to \infty$ formulation, the physical particle states lie in reducible representations of the charge algebra. By constructing the unitarily equivalent "constituent" algebra under which the particles transform as irreducible representations, the problem of finding the "mixture" of charge-algebra representations which make up a physical particle is elegantly solved.

We have found that in the light-cone quantization of the free quark model there is a large class of transformations which take us from the current algebra charges to such constituent charges. These transformations have a well-defined algebraic structure under $SU(6)_{W,\,\mathrm{currents}}$ as a simple consequence of essentially kinematic constraints. The same is true in potential models, since these are subject to an identical set of kinematic conditions.

In phenomenological applications, all of the allowed transformations give predictions for processes involving zero momentum transfer in terms of two reduced matrix elements involving the function F appearing in $\widetilde{V}$. In first order of momentum transfer, calculation of the nucleons' total magnetic moments gives the usual SU(6) result, $(\mu_p/\mu_N)_T = (-3/2)$, independent of the structure of V. The ratio of the anomalous moments involves a reduced matrix element in which F' enters. Continuing in this fashion, higher moments of the currents give information on the value of higher derivatives of F averaged between particle wave functions.

In addition to attempting to find more detailed information on V phenomenologically, it is of interest to try to construct V theoretically in models with interaction. Such transformations will undoubtedly have a higher algebraic structure than those permitted in the free quark model. Indeed, such structure may be necessary to do phenomenological analysis for higher-lying states.[11]

Within the framework of potential models, we have seen there are a small number of conditions to be satisfied in order to find conserved "constituent" z-component of spin once a potential has been specified. In principle, it is straightforward to solve the conditions for V, or else show such a V cannot exist for the given potential. Even the latter possibility could be very interesting, as it could indicate a relation between chiral symmetry breaking and "spin" symmetry breaking. We also note that even in potential models it may be possible to introduce more structure in the form of V by introducing more internal degrees of freedom, as in dual models. It will be interesting to see if two-dimensional models with spin admit a Foldy-Wouthuysen type transformation.

In interacting field theories, one may have to fall back on the FW procedure of constructing V in powers of (m^{-1}), though in general there is no assurance this will give us conserved charges. Such solutions will also lead to exotic configurations, and will have explicit dependence on the gluon fields. It is an open question whether these features are desirable even if they can be made tractable. Finally, once an infinite number of degrees of freedom are admitted, with either interacting field theory or dual models, one must allow for the possibility that the chiral symmetry is realized in the Nambu-Goldstone manner. Even in this case, there can be "algebraic consequences" of the chiral symmetry,[24] but all criteria based on commutativity with the Hamiltonian must be handled with great care.

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APPENDIX A

We list the Poincare generators in the light cone quantization of the free quark model. In the notation of Kogut and Soper[15]

$$P^i = i\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\, \frac{\partial}{\partial x_i}\, q_+(x) \tag{A.1}$$

$$P^+ = i\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\, \frac{\partial}{\partial x^-}\, q_+(x) \tag{A.2}$$

$$H = i\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x) \left[\frac{-1}{2\eta}\left(m^2 + \frac{\partial}{\partial x^i}\frac{\partial}{\partial x_i}\right)\right] q_+(x) \tag{A.3}$$

$$M^{i+} \equiv B^i = i\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\left(x^i\frac{\partial}{\partial x^-} - x^+\frac{\partial}{\partial x_i}\right) q_+(x) \tag{A.4}$$

$$\begin{aligned} M^{i} \equiv S^i = i\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x) \Bigg[& x^i\left(\frac{-1}{2\eta}\right)\left(m^2 + \frac{\partial}{\partial x^i}\frac{\partial}{\partial x_i}\right) \\ & - x^-\frac{\partial}{\partial x_i} - \frac{\gamma^i}{2\eta}\left(\gamma^i\frac{\partial}{\partial x^j} + im\right)\Bigg] q_+(x) \end{aligned} \tag{A.5}$$

$$M^{i} \equiv J^3 = i\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\left(x^1\frac{\partial}{\partial x_2} - x^2\frac{\partial}{\partial x_1} + \frac{1}{2}\gamma^1\gamma^2\right) q_+(x) \tag{A.6}$$

$$M^{+-} \equiv K^3 = i\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\left[x^+\left(\frac{-1}{2\eta}\right)\left(m^2 + \frac{\partial}{\partial x_i}\frac{\partial}{\partial x^i}\right) - x^-\frac{\partial}{\partial x^-} - \frac{1}{2}\right] q_+(x) \tag{A.7}$$

The Dirac equation has been used to eliminate the dependent components of the quark field and to eliminate derivatives with respect to x^+.

We also list in this appendix the general form of the "constituent" charges $\widetilde{W}_{I\alpha} = \widetilde{V}\widetilde{Q}_{i\alpha}\widetilde{V}^{-1}$ where $\widetilde{V}$ is given in Eq. (2). We have

$$\widetilde{W}_i = \widetilde{Q}_i = \sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\, \frac{\lambda_i}{2}\, q_+(x) \tag{A.8}$$

$$\widetilde{W}_{iz} = \widetilde{V}\left(\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\, \gamma_x\gamma_y\, \frac{\lambda_i}{2}\, q_+(x)\right)\widetilde{V}^{-1}$$

$$= \sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\left\{\frac{\gamma_x\gamma_y\lambda_i}{2}\left(\cos F + i\,\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{|\vec{\partial}_\perp|}\,\sin F\right)\right\} q_+(x)$$

(A.9)

$$\widetilde{W}_{i\perp} = \widetilde{V}\left(\sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\, \gamma_\perp\gamma^5\, \frac{\lambda_i}{2}\, q_+(x)\right)\widetilde{V}^{-1}$$

$$= \sqrt{2}\int dx^-\, d\vec{x}_\perp\, q_+^\dagger(x)\left\{\gamma_\perp\gamma^5 + i\,\frac{\partial_\perp\gamma^5}{|\vec{\partial}_\perp|}\,\sin F + \frac{\partial_\perp}{|\vec{\partial}_\perp|}\,\frac{\vec{\gamma}_\perp\cdot\vec{\partial}_\perp}{|\vec{\partial}_\perp|}\,\gamma^5\,(1-\cos F)\right\}\frac{\lambda^i}{2}\, q_+(x) \quad .$$

(A.10)

APPENDIX B

We will first show that in the free quark model the expressions in Eq. (28) and Eq. (29) for the total and the anomalous magnetic moments are correct by considering an electromagnetic current with an anomalous Pauli term:

$$J^{\mu}_{EM}(x) = e\,\bar{q}(x)\gamma^{\mu}\,q(x) + \mu_A\left(i\,\frac{\partial}{\partial x^{\nu}}\right)\bar{q}(x)\quad \Sigma^{\mu\nu}q(x)$$

where

$$\Sigma^{\mu\nu} = \frac{1}{2}\left[\gamma^{\mu},\gamma^{\nu}\right]$$

According to Eq. (28), the anomalous magnetic moment is given by

$$(N)\frac{\mu_A}{2M} = \frac{\partial}{\partial k_x}\left\langle \frac{M}{\sqrt{2}}, \vec{0}_{\perp}; -\frac{1}{2}\right| e^{-i\frac{k_xB_x}{\sqrt{2M}}}\int dx^-\,d\vec{x}_{\perp}\,e^{ik_x\cdot x_1}\,J^{+}_{EM}(x)$$

$$e^{-i\frac{k_xB_x}{\sqrt{2M}}}\left|\frac{M}{\sqrt{2}}, \vec{0}_{\perp}; +\frac{1}{2}\right\rangle\Bigg|_{k_x=0}$$

where

$$(N) = \sqrt{2M}\,(2\pi)^3\,\delta^3(0)$$

Now for the anomalous magnetic moment we have to first order in k_x

$$(N)\frac{\mu_A}{2M} = \frac{\partial}{\partial k_x}\left\langle \frac{M}{\sqrt{2}}, \vec{0}_{\perp}; -\frac{1}{2}\right|\left(1-\frac{ik_xB_x}{\sqrt{2M}}\right)\left[Q+\sqrt{2}\,ie\int dx^-\,d\vec{x}_{\perp}\,k_xx_1\right.$$

$$\left. q^{\dagger}_{+}(x)\,q_{+}(x) + \sqrt{2}\,\mu_A(-ik_x)\int dx^-\,d\vec{x}_{\perp}\,q^{\dagger}_{+}(x)\,i\gamma_xq_{+}(x)\right]\left(1-\frac{ik_xB_x}{\sqrt{2M}}\right)$$

$$\left|\frac{M}{\sqrt{2}}, \vec{0}_{\perp}; +\frac{1}{2}\right\rangle\Bigg|_{k_x=0}$$

$$\therefore (N)\frac{\mu_A}{2M} = \left\langle \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2} \right| \sqrt{2}\, e\, i \int dx^- \, d\vec{x}_\perp \; x_1 q_+^\dagger(x)\, q_+(x)$$

$$+ \sqrt{2}\,\mu_A \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x)\, \gamma_x q_+(x) - i\sqrt{2}\, \frac{QB_x}{M} \left| \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2} \right\rangle$$

The generator B_x may be written (at $x^+=0$) as

$$B_x = \sqrt{2}\, i \int dx^- \, d\vec{x}_\perp \; q_+^\dagger(x)\, x_1 \frac{\partial}{\partial x^-} q_+(x)$$

and the expression for the anomalous magnetic moment becomes (terms with x_1 give zero contribution)

$$\frac{\mu_A}{2M} = \sqrt{2}\, i\, \mu_A \frac{1}{\sqrt{2M}} U^\dagger(0, -1/2)\, \gamma_x\, P_+\, U(0, +1/2)$$

$$= \frac{i\mu_A}{2M}\, \chi^\dagger_{-1/2}\, \sigma_y\, \chi_{+1/2} = \frac{\mu_A}{2M}$$

The calculation for the total magnetic moment is also straightforward. According to Eq. (29), it is given by

$$(N)\frac{\mu_T}{2M} = \left\langle \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2} \right| \int dx^- \, d\vec{x}_\perp \; x_1\, J^-_{EM}(x) \left| \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2} \right\rangle$$

First we note

$$\int dx^- \, d\vec{x}_\perp \; x_1\, J^-_{EM}(x) = \sqrt{2}\, e \int dx^- \, d\vec{x}_\perp \left[x_1\, q_-^\dagger(x)\, q_-(x) - \frac{\mu_A}{\sqrt{2}}\, q_-^\dagger(x)\, \gamma_x\, q_-(x) \right] .$$

Since

$$q_-^\dagger(x) = \frac{-\gamma^+}{2} \left(im + \gamma_j \frac{\partial}{\partial x_j} \right) q_+(x)$$

we may reexpress J^-_{EM} in terms of canonically independent variables

$$\int dx^- \, d\vec{x}_\perp \, x_1 \, J^-_{EM}(x) = \sqrt{2}\, e \int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x) \left[\gamma_x \frac{\left(m - i\gamma^j \frac{\partial}{\partial x^j}\right)}{2\eta^2} \right.$$

$$\left. + x_1 \frac{\left(m^2 - \frac{\partial}{\partial x_j}\frac{\partial}{\partial x^j}\right)}{2\eta^2} \right] q_+(x) + \sqrt{2}\, \mu_A \int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x)$$

$$\left[\gamma_x \frac{\left(m^2 - \frac{\partial}{\partial x_j}\frac{\partial}{\partial x^j}\right)}{2\eta^2} - i \frac{\gamma_x \gamma_y \frac{\partial}{\partial x_2}}{2\eta^2} \left(m - i\gamma_j \frac{\partial}{\partial x_j}\right) \right] q_+(x)$$

Since between the rest states the x' term and transverse derivatives give zero we have

$$\text{(N)} \; \frac{\mu_T}{2M} = \left(\frac{\sqrt{2}\, e}{M} + \sqrt{2}\, \mu_A\right) \left\langle \frac{M}{\sqrt{2}}, \vec{0}_\perp ; -\tfrac{1}{2} \right| \left[\int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x) \gamma_x \frac{m^2}{2\eta^2} q_+(x) \right] \left| \frac{M}{\sqrt{2}}, \vec{0}_\perp ; +\tfrac{1}{2} \right\rangle$$

$$\therefore \; \frac{\mu_T}{2M} = \left(\frac{\sqrt{2}\, e}{M} + \sqrt{2}\, \mu_A\right) U^\dagger(0, -1/2)\, \gamma_x P_+ U(0, +1/2) \frac{1}{\sqrt{2M}}$$

$$= \frac{e}{2M^2} + \frac{\mu_A}{2M} \quad .$$

These expressions agree with the expected results in the free quark model. Consider the calculations of the anomalous and total magnetic moment for physical states. We may write

$$\text{(N)} \; \frac{\mu_A}{2M} = \frac{\partial}{\partial k_x} \left\langle A; \frac{M}{\sqrt{2}}, \vec{0}_\perp ; -\tfrac{1}{2} \right| e^{-iB_x \frac{k_x}{\sqrt{2M}}} \int dx^- \, d\vec{x}_\perp \left(e^{ik_x x_1} J^+_{EM}(x) \right)$$

$$e^{-iB_x \frac{k_x}{\sqrt{2M}}} \left| B; \frac{M}{\sqrt{2}}, \vec{0}_\perp ; +\tfrac{1}{2} \right\rangle$$

and

$$(N)\frac{\mu_T}{2M} = \left\langle A; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2}\right| \int dx^- \, d\vec{x}_\perp \, x_1 \, J^-_{EM}(x) \left| B; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2}\right\rangle$$

The electromagnetic current is taken to be that of the free quark model (with no anomalous Pauli term)

$$J^\mu_{EM}(x) = e \, \bar{q}(x) \, \gamma^\mu \, \frac{\lambda^{EM}}{2} \, q(x)$$

Using the expression for B_x given above and using the expression for $\int dx^- \, d\vec{x}_\perp \, x_1 \, J^-_{EM}$ with no anomalous Pauli term we have

$$(N)\frac{\mu_A}{2M} = \left\langle A; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2}\right| i \int dx^- \, d\vec{x}_\perp \, x_1 \sqrt{2} \, e \, q_+^\dagger(x) \, \frac{\lambda^{EM}}{2} \, q_+(x)$$

$$- \frac{\sqrt{2}\, iQ}{M} \left(\sqrt{2} \, i \int dx^- \, dx_1 \, q_+^\dagger(x) \left(x_1 \frac{\partial}{\partial x^-} \right) q_+(x) \right) \left| B; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2}\right\rangle$$

and

$$(N)\frac{\mu_T}{2M} = \left\langle A; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2}\right| \int dx^- \, d\vec{x}_\perp \sqrt{2} \, e \, q_+^\dagger(x) \left[\gamma_x \frac{m - i\gamma^j \partial_j}{2\eta^2} \right.$$

$$\left. + \, x_1 \frac{m^2 - \partial_j \partial^j}{2\eta^2} \right] q_+(x) \left| B; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2}\right\rangle$$

The physical states A and B which are constructed as simple irreducible representations of the strong charges can be related to the states classified simply under $SU(6)_W$ currents by

$$\left\langle A_{strong}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2}\right| = \left\langle A_{current}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2}\right| \tilde{V}^{-1}$$

$$\left| B_{strong}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2}\right\rangle = \tilde{V} \left| B_{current}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2}\right\rangle$$

where

$$\widetilde{V} = \exp\left\{ \frac{i}{\sqrt{2}} \int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x) \left(\frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} F |\vec{\partial}_\perp| \right) q_+(x) \right\}$$

Thus, in terms of states that transform simply under $SU(6)_W$ currents one may write

$$(N)\frac{\mu_A}{2M} = i \left\langle A_{current}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2} \right| \left(\left(\int dx^- \, d\vec{x}_\perp \, x_1 \, J^+_{EM}(x) - \frac{\sqrt{2}\, Q}{M} B_x \right) \right.$$
$$+ \sqrt{2} \int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x) \left\{ \left(\frac{e\lambda^{EM}}{2} - \frac{\sqrt{2}\, Q}{M} \, i \frac{\partial}{\partial x^-} \right) \left[\left(\frac{\cos F - 1}{2} + \frac{i}{2} \sin F \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} \right) \right. \right.$$
$$\left. \left. \left(\frac{\partial_x}{|\vec{\partial}_\perp|^2} - \gamma_x \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\partial_\perp|^2} \right) - \frac{i}{2} \left(\frac{\partial F}{\partial |\vec{\partial}_\perp|} \right) \frac{\partial_x}{|\vec{\partial}_\perp|^2} \, \gamma_\perp \cdot \vec{\partial}_\perp \right] \right\} q_+(x)$$
$$\left| B_{current}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; +\frac{1}{2} \right\rangle \qquad \text{(B. 1)}$$

and

$$(N)\frac{\mu_T}{2M} = i \left\langle A_{current}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2} \right| \sqrt{2}\, e \int dx^- \, d\vec{x}_\perp \, q_+^\dagger(x) \frac{\lambda_{EM}}{2}$$
$$\left\{ \frac{m}{2\eta^2} \left[\gamma_x - \epsilon_{xi} \vec{\partial}_i \epsilon_{jk} \gamma_j \vec{\partial}_k \frac{1}{|\vec{\partial}_\perp|^2} (\cos F - 1 + i \sin F) \right] - i \frac{\gamma_x \gamma_y \vec{\partial}_y}{2\eta^2} \right.$$
$$\left[\cos F + i \sin F \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|} \right] - \frac{i\vec{\partial}_x}{2\eta^2} + \frac{\left(m^2 - \vec{\partial}_j \vec{\partial}^j \right)}{2\eta^2}$$
$$\left[x_1 + \frac{1}{2} \left(\cos F - 1 + i \sin F \frac{\vec{\gamma}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|^2} \right) \left(\frac{\partial_x}{|\vec{\partial}_\perp|^2} - \frac{\gamma_x \vec{\partial}_\perp \cdot \vec{\partial}_\perp}{|\vec{\partial}_\perp|^2} \right) \right.$$
$$\left. \left. - \frac{i}{2} \left(\frac{\partial F}{\partial |\vec{\partial}_\perp|} \right) \vec{\gamma}_\perp \cdot \vec{\partial}_\perp \frac{\partial_x}{|\vec{\partial}|^2} \right] \right\} q_+(x) \left| B_{currents}; \frac{M}{\sqrt{2}}, \vec{0}_\perp; -\frac{1}{2} \right\rangle \qquad \text{(B. 2)}$$

Now consider the neutron and proton magnetic moments. The baryon SU(3) octet belong to the representations

$$(6,3)\ L_z=0 \quad \text{for spin up} \qquad \text{and} \qquad (3,6)\ L_z=0 \quad \text{for spin down}\ .$$

Therefore the only terms that may connect $(3,6)\ L_z=0$ and $(6,3)\ L_z=0$ have $(3,\bar{3})+(\bar{3},3)\ L_z=0$; the contribution of such terms is given below

$$(N)\ \frac{\mu_A^a}{2M} = i\left\langle (3,6)\ L_z=0 \text{ current};\ \frac{M}{\sqrt{2}},\ \vec{0}_\perp;\ a \right| \sqrt{2}\int dx^-\, d\vec{x}_\perp\ q_+^\dagger(x) \left\{ \frac{e\lambda^{EM}}{2} \right.$$

$$\left. - \left(\frac{\sqrt{2}\,Q}{M}\, i\,\frac{\partial}{\partial x^-}\right)\left(\frac{-i\gamma_x}{4}\right)\left(\frac{\partial F}{\partial\,|\vec{\partial}_\perp|} - \frac{\sin F}{|\vec{\partial}_\perp|}\right)\right\} q_+(x) \left| (6,3)\ L_z=0 \text{ current};\right.$$

$$\left.\frac{M}{\sqrt{2}},\ \vec{0}_\perp;\ a\right\rangle$$

and

$$(N)\ \frac{\mu_T^a}{2M} = \left\langle (3,6)\ L_z=0 \text{ current};\ \frac{M}{\sqrt{2}},\ \vec{0}_\perp;\ a \right| \sqrt{2}\, e\int dx^-\, d\vec{x}_\perp\ q_+^\dagger(x)\ \frac{\lambda^{EM}}{2}$$

$$\gamma_x \left\{ \frac{m}{2\eta^2}\,\frac{(\cos F + 1)}{2} + \frac{|\vec{\partial}_\perp|}{2\eta^2}\sin F + \frac{\left(m^2 - |\vec{\partial}_\perp|^2\right)}{2\eta^2}\left(\frac{-i}{4}\right)\right.$$

$$\left.\left[\left(\frac{\partial F}{\partial|\vec{\partial}_\perp|}\right) - \frac{\sin F}{|\vec{\partial}_\perp|}\right]\right\} q_+(x) \left| (6,3)\ L_z=0 \text{ current};\ \frac{M}{\sqrt{2}},\ \vec{0}_\perp;\ a\right\rangle$$

From these expressions we see that the ratio of the proton to neutron anomalous magnetic moments is given by

$$\frac{\mu_A^p}{\mu_A^n} = -\frac{3}{2}(1-\Delta)$$

where

$$\Delta = \frac{\left\langle (3,6)\ L_z{=}0\ \text{current};\ \frac{M}{\sqrt{2}},\ \vec{0}_\perp \middle| \frac{\sqrt{2}}{M}\, i \frac{\partial}{\partial x^-} \left(F' - \frac{\sin F}{|\vec{\partial}_\perp|}\right) \middle| (6,3)\ L_z{=}0\ \text{current};\ \frac{M}{\sqrt{2}},\ \vec{0}_\perp \right\rangle}{\left\langle (3,6)\ L_z{=}0\ \text{current};\ \frac{M}{\sqrt{2}},\ \vec{0}_\perp \middle| \left(F' - \frac{\sin F}{|\vec{\partial}_\perp|}\right) \middle| (6,3)\ L_z{=}0\ \text{current};\ \frac{M}{\sqrt{2}},\ \vec{0}_\perp \right\rangle}$$

$$F' \equiv \frac{\partial F}{\partial |\vec{\partial}|} \tag{B.3}$$

In the free quark limit

$$\Delta = \frac{1}{3} \qquad \text{so} \qquad \frac{\mu_A^p}{\mu_A^n} = -1 \quad .$$

The ratio of proton and neutron total magnetic moments is given by

$$\frac{\mu_T^p}{\mu_T^n} = -\frac{3}{2} \qquad \text{independent of } F \quad . \tag{B.4}$$

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