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# SCALING MODELS OF HIGH ENERGY MULTIPARTICLE PRODUCTION IN HADRON-HADRON COLLISIONS*

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ABSTRACT

Asymptotic high energy Koba-Nielsen-Olesen scaling of multiplicity distributions is shown to hold in a class of models. The shape of the scaling function is simply related to the shape of the topological cross sections. A statistical study of the models is made and the thermodynamic limit is investigated. An attempt to relate the impact parameter picture of high energy elastic hadron scattering and the scaling phenomenon is presented. Special realistic cases are explored in detail.

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## INTRODUCTION

Recent bubble chamber experiments at Serpukhov[1] and NAL[2] measuring topological cross sections for charged particle production in pp collisions have yielded surprising agreement with the scaling law of Koba, Nielsen and Olesen[3] (KNO).

KNO scaling states that

$$\frac{\sigma_N}{\sigma_{in}} = \frac{1}{<N>} \psi \left(\frac{N}{<N>}\right) \tag{1}$$

where $\sigma_N$ is the topological cross section for the production of N particles, $<N>$ the average multiplicity, and $\sigma_{in}$ the total inelastic cross section. $\psi$ is a universal function whose shape may depend on the projectile and target particles, and the only s dependence on the right hand side of the equation is the implicit one in $< N >$.

An alternative statement of KNO scaling can be given if one considers the multiplicity moments $<N^q>$, q = positive integer.

$$< N^q > = \sum_{N=1}^{\infty} \frac{N^q \sigma_N}{\sigma_{in}} = < N >^q \sum_{N=1}^{\infty} \left(\frac{N}{<N>}\right)^q \psi \left(\frac{N}{<N>}\right) . \tag{2}$$

If $< N >$ is large one can replace the discrete sum in Eq. (2) by an integral over the variable $z = N/< N >$ :

$$c_q \equiv \frac{< N^q >}{< N >^q} = \int_{1/<N>}^{\infty} dz\, z^q \psi(z) \simeq \int_0^{\infty} dz\, z^q \psi(z) . \tag{3}$$

If one considers charged particle production the sum in Eq. (2) runs only through even values of N, therefore Eq. (3) is changed to

$$c_q = \tfrac{1}{2} \int_0^\infty dz\, z^q\, \psi(z) . \tag{3a}$$

The experimental situation for the $c_q$ has been reported by Slattery[4] who finds that they indeed seem to be remarkably energy independent, as asserted by the right hand side of Eqs. (3) and (3a).

In this paper we consider a fairly general class of models which have the property of KNO scaling in the high energy limit. The presentation of the material is as follows: first we introduce the models and explicitly find the $\psi(z)$ scaling function. In the first section a statistical approach to multiparticle production in the context of the models is taken and a realistic case is considered in detail. The second section deals with a possible relationship between geometric models of elastic scattering and those considered here. A discussion of the results is made in the final paragraphs.

A model of multiparticle production belongs to the class we will be discussing if the N-particle cross section is of the form

$$\sigma_N = a(s)\,\Phi\big(b(s)\,N\big) \tag{4}$$

where $b(s)$ is small for large s, and provided that $\phi$ and $\kappa$ defined below exist.

We prove first that these models scale in the KNO sense. The proof is straightforward: the inelastic cross section is given by

$$\sigma_{in} = \sum_{N=1}^{\infty} a(s)\,\Phi\big(b(s)\,N\big) \underset{b\to 0}{=} \frac{a(s)}{b(s)}\,\phi \tag{5}$$

with

$$\phi = \int_0^{\infty} dx\,\Phi(x)\ . \tag{6}$$

The average number of particles is

$$\langle N \rangle = \frac{\sum\limits_{N=1}^{\infty} N\sigma_N}{\sigma_{in}} \underset{b\to 0}{=} \frac{\kappa}{\phi}\,\frac{1}{b(s)} \tag{7}$$

with

$$\kappa = \int_0^{\infty} dx\,x\Phi(x)\ . \tag{8}$$

We can define

$$r = \frac{\kappa}{\phi} \tag{9}$$

and write

$$\langle N \rangle \frac{\sigma_N}{\sigma_{in}} = \frac{r}{\phi}\,\Phi\left(r\frac{N}{\langle N \rangle}\right) = \psi\left(\frac{N}{\langle N \rangle}\right) \tag{10}$$

which proves our statement. An interesting fact is that the KNO scaling function turns out to be independent of both $a(s)$ and $b(s)$.

We will now consider a particular case of Eq. (4); namely,

$$\Phi(x) = x\,e^{-x^2}\ . \tag{11}$$

We can easily find that $\phi = \frac{1}{2}$ , $r = \sqrt{\pi}/2$ so that

$$\psi(z) = \frac{\pi z}{2}\,e^{-\frac{\pi}{4}z^2}\ . \tag{12}$$

For charged particle production, and according to Eq. (3a) we must multiply this result times 2:

$$\psi_{ch}(z) = \pi z\, e^{-\frac{\pi}{4} z^2} . \tag{13}$$

This is the form conjectured by Buras and Koba[5] who show that it fits the experimental data for charged particle production in pp and $\pi$p scattering very well.

One could now add some extra input to the problem. For example, if one knows the s dependence of $< N >$, one has determined b(s). If the $\sigma_{in}$ is also known as a function of s, one also determines a(s).

Hence, if $< N > = A \ln s/s_0$ and $\sigma_{in} = K =$ constant, then for $\Phi$ given by Eq. (11) one has

$$b(s) = \frac{\sqrt{\pi}}{2A \ln s/s_0} , \qquad a(s) = \frac{K\sqrt{\pi}}{A \ln s/s_0}$$

and the scaling model corresponds to

$$\sigma_N = \frac{K\pi}{2A^2 \ln^2 s/s_0} N \cdot \exp\left( - \frac{\pi N^2}{4A^2 \ln^2 s/s_0} \right) \tag{14}$$

## I. STATISTICAL STUDY

Multiparticle production in hadron-hadron scattering can be also approached from a statistical viewpoint. Feynman and Wilson[6] have introduced the notion of the Feynman fluid in the rapidity space which is a 3 - dimensional space with a rectangular coordinate system. Two of these coordinates are transverse momentum and the third one is the rapidity of the produced particles, each of which corresponds therefore to a point in the space. As it is an empirical fact that the transverse momentum is cutoff at about 500 MeV/c, the points can be thought of as molecules of a fluid contained in a cylinder of height $Y \sim \ln s$. This cylinder has the advantage that it only gets displaced under Lorentz boosts in the longitudinal direction.

The question arises if in the limit of high energy one can speak about a thermodynamic limit for the Feynman fluid. Bjorken[7] has studied the grand-canonical partition function

$$Q(z, Y) = \sum_N z^N \sigma(N, Y) \tag{15}$$

where $\sigma(N, Y)$ is the cross section for N-particle production at rapidity Y, which, of course, plays the role of the volume.

We turn now to study the scaling models of Eq. (4). The grand-canonical partition function is

$$Q(z, Y) = \sum_N a(Y) z^N \Phi\big(b(Y)N\big) \underset{b \to 0}{=} \frac{a(Y)}{b(Y)} \int_0^\infty dx\, e^{\frac{\ln z}{b(Y)} \cdot x} \Phi(x) = \frac{a(Y)}{b(Y)} R\left(\frac{\ln z}{b(Y)}\right) \tag{16}$$

where we have assumed that the integral exists for arbitrary values of $\frac{\ln z}{b(Y)}$.
We observe that R is an ordinary Laplace transform of $\Phi$ for $0 < z < 1$.

We can write now an equation of state for the Feynman fluid:

$$PY = \alpha \ln Q(z, Y) = \alpha \ln \frac{\sigma_{in}(Y)}{\phi} + \alpha \ln R\left(\frac{\ln z}{b(Y)}\right) \tag{17}$$

with $\alpha$ a constant, and z given through the average number of particles in the fluid by

$$< N > = \frac{\partial}{\partial(\ln z)} \ln Q(z, Y) = \frac{R'\left(\frac{\ln z}{b(Y)}\right)}{R\left(\frac{\ln z}{b(Y)}\right)} \frac{1}{b(Y)} . \tag{18}$$

We notice that for $z = 1$, Eq. (18) coincides with Eq. (7) as it should be.

The question is now if the thermodynamic limit exists, i. e.

$$P(z) \stackrel{?}{=} \lim_{Y \to \infty} \frac{\alpha}{Y} \ln Q(z, Y) \tag{19}$$

with z given by the $Y \to \infty$ limit of Eq. (18). Barring any bizarre behavior of $\sigma_{in}(Y)$ (say, like $e^{-\epsilon Y}$) we can surely write

$$P(z) = \lim_{Y \to \infty} \frac{\alpha}{Y} \ln R\left(\frac{\ln z}{b(Y)}\right) . \tag{20}$$

To be specific, we can try the previously considered (Eq. (11)) absorbed gaussian form for $\Phi(x)$, and find:

$$R(w) = \frac{1}{2}\left[1 + \frac{\sqrt{\pi}}{2} w e^{w^2/4}\left\{1 + T\left(\frac{w}{2}\right)\right\}\right] \tag{21}$$

where

$$T(v) = \frac{2}{\sqrt{\pi}} \int_0^v e^{-t^2} dt . \tag{22}$$

It follows that

$$R'(w) = \frac{\sqrt{\pi}}{4}\left[e^{w^2/4}\left(1+\frac{w^2}{2}\right)\left\{1+T(\frac{w}{2})\right\} + \frac{w}{\sqrt{\pi}}\right]. \tag{23}$$

There are three separate cases to consider:

i) $0 < z < 1$, therefore $w < 0$. As $Y \to \infty$, $b \to 0$, we can use the asymptotic expansion[8] for $T(w/2)$ and write

$$R(w) \xrightarrow[w \to -\infty]{} \frac{1}{|w|^2} \tag{24}$$

$$R'(w) \xrightarrow[w \to -\infty]{} \frac{2}{|w|^3} . \tag{25}$$

So that

$$P(z) = 2\alpha \lim_{Y \to \infty} \frac{1}{Y} \ln b(Y) = \text{independent of } z . \tag{26}$$

Thus any behavior of $b(Y)$ of the type

$$b(Y) \sim \lim_{Y \to \infty} \frac{\ln^m Y}{Y^r} \tag{27}$$

with m and r consistent with $b(Y) \to 0$ as $Y \to \infty$ results in the existence of the thermodynamic limit in $0 < z < 1$ with $P(z) = 0$. On the other hand, if

$$b(Y) \underset{Y \to \infty}{\sim} Y^q e^{-\epsilon Y} \tag{28}$$

with $\epsilon > 0$ results in $P(z) = -2\alpha\epsilon$.

We can also calculate $< N >$ in this region:

$$< N > \;=\; \lim_{Y\to\infty} \frac{1}{b(Y)} \frac{2b(Y)}{|\ln z|} \;=\; \frac{2}{|\ln z|} \tag{29}$$

ii) $z = 1$. At this point

$$P(1) \;=\; \lim_{Y\to\infty} \frac{\alpha}{Y} R(0) \;=\; 0 \tag{30}$$

and

$$< N > \;=\; \lim_{Y\to\infty} \frac{\sqrt{\pi}}{2b(Y)} \;. \tag{31}$$

iii) $z > 1$. For this region we find

$$P(z) \;=\; \lim_{Y\to\infty} \frac{\alpha}{Y} \left\{ -\ln b(Y) + \frac{\ln^2 z}{4b^2(Y)} \right\} \tag{32}$$

so that, if Eq. (27) is valid

$$P(z) \;=\; \frac{\alpha}{4} \ln^2 z \left\{ \lim_{Y\to\infty} \frac{1}{Yb^2(Y)} \right\} \tag{33}$$

and the thermodynamic limit will exist only if $0 < r \leq \frac{1}{2}$, or $r = 0$, $m < 0$. One sees that in all these cases $P(z) = 0$, except if $b(Y) \underset{Y\to\infty}{\sim} 1/Y^{\frac{1}{2}}$ for which

$$P(z) \;=\; P \ln^2 z \tag{34}$$

with P a constant.

In the same fashion, we find

$$< N > \;=\; \lim_{Y\to\infty} \frac{\ln z}{2b^2(Y)} \;. \tag{35}$$

Assuming that one does have the thermodynamic limit for all $z > 0$ and the interesting case where $P(z) \not\equiv 0$, i.e. $b(Y) \underset{Y \to \infty}{\simeq} Y^{-\frac{1}{2}}$, we can write

$$P(z) = P \, \ell n^2 z \; \theta(z-1) \tag{36}$$

$$\rho(z) = \rho \, \ell n z \; \theta(z-1) \tag{37}$$

where $\rho(z) = \langle N \rangle / Y$, $P$ and $\rho$ constants.

These equations show that for this very particular case $z = 1$ is a phase transition point of the Feynman fluid.[9] More generally we see that for $b(Y) \sim 1/Y^p$, $p > \frac{1}{2}$, the point $z = 1$ is a critical point, but as there is no thermodynamic limit for $z > 1$ its nature is different from the limiting case $p = \frac{1}{2}$.

## II. POSSIBLE GEOMETRIC EXPLANATION

In this section we would like to motivate the shape of the KNO scaling function using some heuristic arguments.

The success of Eq. (13) as a description of the experimental data makes one conjecture a possible link between the absorption models of elastic hadron scattering and the scaling phenomenon in multiparticle production. Indeed, the shape $x e^{-x^2}$ is a widely used profile when describing two-body reactions in the framework of optical models[10] where x and the impact parameter are <u>directly</u> proportional.

Recently, Barshay[11] has pointed out that a simple geometrical model of the hadrons can account for the quantitative values and approximate constancy of the $c_q$ coefficients of Eqs. (3), (3a).

The geometrical model is the following: both target and projectile hadrons are extended objects with some matter density distribution. At high energy one can write the total inelastic cross section as an integral over the impact parameter b:

$$\sigma_{in} = 2\pi \int_0^\infty b\,db\left(1 - e^{-2A\rho(\mu b)}\right) \tag{38}$$

A is the absorption parameter, which is in principle energy dependent, and is fixed by either the total[12] or the inelastic cross section, and $\rho(\mu b)$ is the hadronic matter density overlap, given by

$$\rho(\mu b) = \frac{1}{2\pi} \int d^2k\, e^{i\vec{k}\cdot\vec{b}}\, G_p(k^2)\, G_t(k^2) \tag{39}$$

where $G_p$, $G_t$ are the projectile and target form factors. $\mu$ is a constant of dimensions of inverse length. The elastic scattering amplitude in this model can be written as[12,13]

$$f(t) = \frac{i}{2\pi} \int d^2b \, e^{-i\vec{q}\cdot\vec{b}} \left(1 - e^{-A\rho(\mu b)}\right) \tag{40}$$

where $-t = q^2$, and

$$\frac{d\sigma_{el}}{dt} = \pi \, | f(t) |^2 \tag{41}$$

$$\sigma_{total} = 4\pi \, \mathrm{Im} \, f(0) \; . \tag{42}$$

The model has been quantitatively successful[13] in describing the small angle elastic pp scattering data at ISR energies[14] if one uses as input in Eq. (39) the electromagnetically measured form factors.

To construct the $c_q$ coefficients of Eqs. (3) and (3a) one writes

$$< N^q > = \frac{1}{\sigma_{in}} \int d^2b \sum_{N=1}^{\infty} N^q \sigma_N(b, s) \; . \tag{43}$$

$\sigma_N(b, s)$ is the impact parameter representation of $\sigma_N$. With the definition

$$n(b, s) \equiv \frac{\sum_{N=1}^{\infty} N \sigma_N(b, s)}{\sigma_{in}(b, s)} \tag{44}$$

where

$$\sigma_{in}(b, s) = \sum_{N=1}^{\infty} \sigma_N(b, s) \tag{45}$$

is the impact parameter representation of $\sigma_{in}$; and accepting the Barshay[11] assumption

$$n^q(b,s) = \frac{\sum_{N=1}^{\infty} N^q \sigma_N(b,s)}{\sigma_{in}(b,s)} \tag{46}$$

we are enabled to write

$$c_q = \frac{1}{\sigma_{in}} \int d^2b \left(\frac{n(b,s)}{< N >}\right)^q \sigma_{in}(b,s) . \tag{47}$$

The change of variable

$$z = \frac{n(b,s)}{< N >} \tag{48}$$

defines b(z, s) if n(b, s) is monotonic in $0 \leq b < \infty$. Assuming that, Eq. (47) takes the form

$$c_q = \frac{2\pi < N >}{\sigma_{in}} \int_0^\infty dz\, z^q \left\{\frac{b\,\sigma_{in}(b,s)}{\left|\frac{dn}{db}\right|}\right\}_{b(z,s)} f(z,s) \tag{49}$$

with

$$f(z,s) = \begin{cases} \theta(z_\infty - z)\,\theta(z - z_0) & z_\infty > z_0 \\ \theta(z_0 - z)\,\theta(z - z_\infty) & z_0 > z_\infty \end{cases} \tag{50}$$

where $z_0 = \frac{n(0,s)}{< N >}$, $z_\infty = \frac{n(\infty,s)}{< N >}$ .

Comparison of Eq. (49) with Eq. (3a) suggests:

$$\psi_{ch}(z,s) = \frac{4\pi < N >}{\sigma_{in}} \left\{\frac{b\,\sigma_{in}(b,s)}{\left|\frac{dn}{db}\right|}\right\}_{b(z,s)} f(z,s) . \tag{51}$$

This equation can be derived rigorously by the Mellin transform technique provided one accepts Eq. (47) as valid for arbitrary complex values of q and provided further that

$$\int_0^\infty |\psi_{ch}(z, s)|^2 z\,dz \tag{52}$$

exists[15], with $\psi_{ch}$ given by Eq. (51). Eq. (38) shows that

$$\sigma_{in}(b, s) = \left(1 - e^{-2A\rho(\mu b)}\right) \tag{53}$$

for the geometrical model we are considering. The only possible dependence on s is in the absorption coefficient A.

If we insist on direct proportionality between the argument of the scaling function $\psi(z)$ and the impact parameter b, as suggested in the beginning of this section, we are led to try the simplest linear relationship[16]

$$n(b, s) = \nu(s)(\mu b) = \langle N \rangle z\,. \tag{54}$$

Substitution in Eqs. (51) and (53) yields

$$\psi_{ch}(z) = \frac{2r}{\phi}\Phi(rz) \tag{55}$$

with

$$\Phi(x) = x\left(1 - e^{-2A\rho(x)}\right) \tag{56}$$

which will be independent of s if A is a constant. The extra factor of 2 in Eq. (55) reflects the fact that we are talking about charged particle production. We can now study various different possibilities for $\rho(x)$; for example, those considered in reference 13.

The results for pp scattering are shown in Figure 1. Similar computations can be carried out for $\pi - p$ scattering with the result that the KNO scaling functions practically lie on top of each other, except for values of $z > 2$ where they are somewhat above the pp case. For this region there tends to be a larger difference if the pion radius is large, close to 1 fermi and better agreement as it is diminished to 0.5 fermi. The curves are practically insensitive to the choice of simple pole or dipole form factors for the pion, the dipole yielding a slightly more similar curve to the pp case, as is to be expected.

It should be emphasized that the values of A used in the computation were exactly those determined in reference 13, and as remarked before, they depend only on the total cross sections.

## CONCLUSIONS AND DISCUSSION

We have studied the properties of the scaling models of Eq. (4) in the context of the statistical approach to multiparticle production and tried to motivate the shape of the scaling function using geometric arguments. We have seen that some of the models considered here can explain the experimental data fairly well, and that their statistical properties can be straightforwardly investigated. The value of such an investigation is that, granting the physical relevance of these models, it gives us an insight into the properties of the hadronic matter in the high energy limit. Thermodynamic properties are of such a general nature that we may reasonably expect that a model which correctly incorporates the physical ingredients that go into multiparticle production processes should at least describe them. Of course, the question is whether or not the presently available energies are already high enough as to tell us something about statistical properties of hadronic matter. Presumably properties such as phase transitions and critical phenomena should be among the most prominent, and looking for experimental evidence of them can indicate the onset of truly high energy physics.

The shape of the KNO scaling function led us to the consideration of the geometric-optical picture of hadron-hadron scattering, and of Barshay's assumption, Eq. (46). The obvious way to satisfy that equation is

$$\sigma_N(b,s) = \delta\Big(n(b,s) - N\Big)\,\sigma_{in}(b,s) \quad . \tag{57}$$

When this is combined with the linear form for $n(b,s) = \nu(s)(\mu b)$, Eq. (54), we see that it states that to produce more particles at a given energy the colliding hadrons should have a larger impact parameter, the production mechanism being dynamically cutoff by $\sigma_{in}(b, s)$ which will fall off with b since for large b there is no matter overlap. This behavior is to be contrasted with the intuitive idea that head-on collisions should result in more particles.

Finally, the problem arises to find basic dynamical schemes which will produce topological cross sections of the class considered here. It is hoped that this question will be answered in the near future.[17]

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16. This choice is different from the one of Barshay (loc. cit.), who takes a gaussian form. However, if one uses Barshay's form, or any bounded form in $0 < b < \infty$, one will have $\theta$-function discontinuities in $\psi(z)$ according to Eqs. (50) and (51). It is interesting that A. Chodos, M. Rubin and R. Sugar, University of Pennsylvania report UPR-0016T (1973), find $\theta$-function discontinuities in the context of a model with long range correlations in the rapidity variables of the produced particles.

17. See for example the models of H. D. I. Abarbanel and G. L. Kane, Phys. Rev. Letters 30, 67 (1973), and the one of S. Naito and T. Odaka, Osaka University preprint OCU 1 (73). The former authors' cross section is $\sigma_N \cong \frac{1}{N^2} \exp(-\frac{N^4}{s})$ which corresponds to $b(s) \sim 1/s^{\frac{1}{4}}$, $\Phi(x) = x^{-2} \exp(-x^4)$. This case, however, does not fall within our analysis because $\Phi$ and its first moment are not integrable from 0 to $\infty$ due to the $x = 0$ singularity. The latter authors' cross section takes the form $\sigma_N(s) \sim \frac{N^3}{s} B(N/s^{\frac{1}{4}})$, and they claim that the best fit to the pp data is (s in $GeV^2$):

$$\sigma_N = \frac{K}{s^{\frac{1}{4}}} \left(\frac{N}{s^{\frac{1}{4}}}\right)^3 \exp\left(-A \frac{N}{s^{\frac{1}{4}}}\right)$$

$K = 250$ mb, $A = 2.2$. Therefore such a form corresponds to $a(s) = \frac{K}{A^3} s^{-\frac{1}{4}}$, $b(s) = As^{-\frac{1}{4}}$, $\Phi(x) = x^3 \exp(-x)$. The KNO scaling function is, according to Eq. (10) $\psi_{ch}(z) = \frac{256}{3} z^3 e^{-4z}$ for this model.

## FIGURE CAPTION

Figure 1 Koba-Nielsen-Olesen scaling function for charged particle production in pp high energy collisions corresponding to $\Phi(x) = x\left(1 - \exp(-2A\rho(x))\right)$. $\rho(x)$ and A have been taken directly from reference 13: $\rho(x) = (\mu^2/48)\,x^3\,K_3(x)$, $\mu^2 = 0.71\ \mathrm{GeV}^2$, $A = 9.9\ \mathrm{GeV}^{-2}$. The experimental data are the points reported in reference 4.

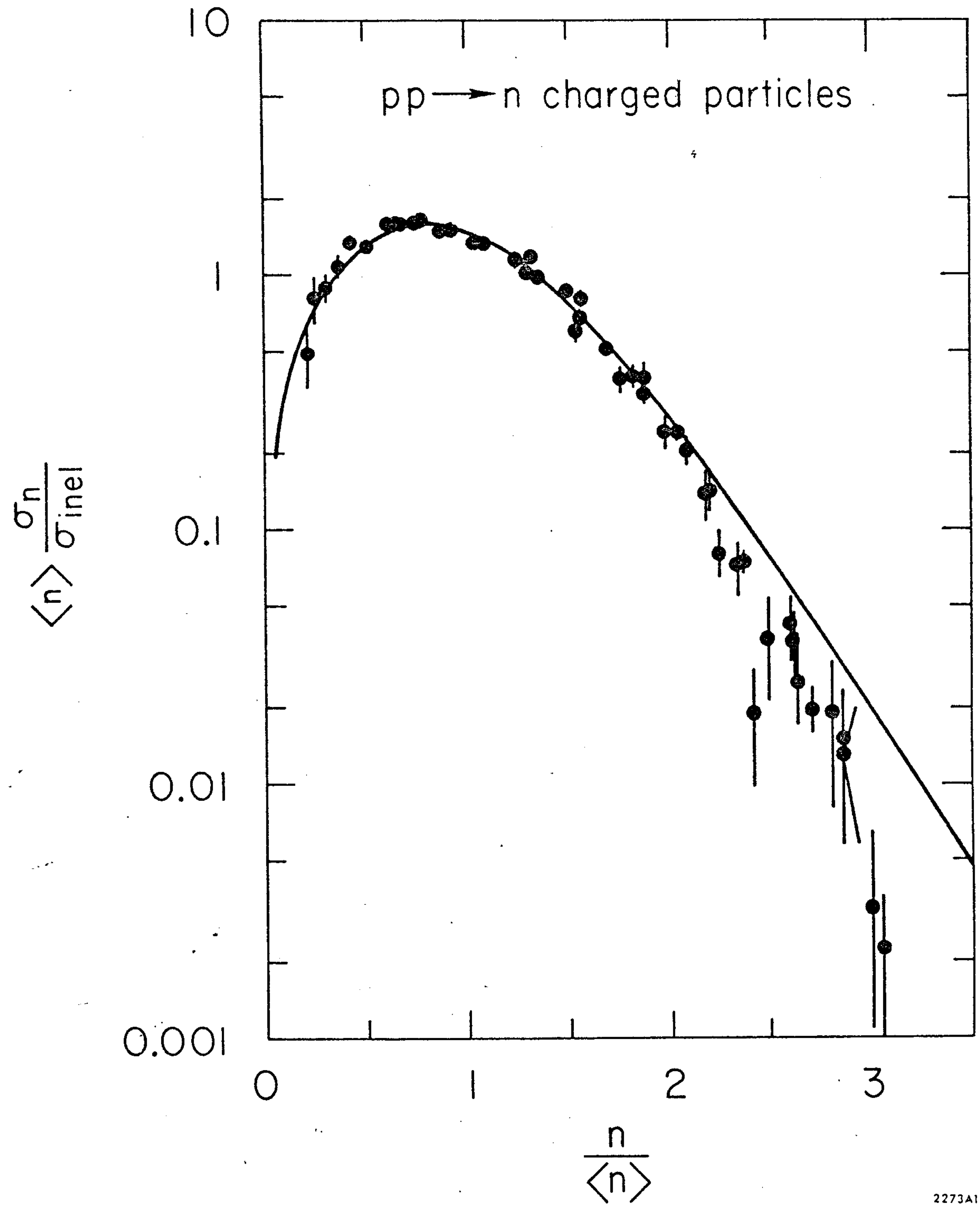
pp ⟶ n charged particles
10
1
0.1
0.01
0.001
0
1
2
3
$\langle n \rangle \frac{\sigma_n}{\sigma_{inel}}$
$\frac{n}{\langle n \rangle}$
2273A1

Fig. 1