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TOWARDS A DYNAMICAL INTERPRETATION OF DUAL AMPLITUDES WITH MANDELSTAM ANALYTICITY*

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ABSTRACT

It is argued that the infinite momentum frame loop variable x should be identified with the variable appearing in the integral representation of dual four point amplitudes with Mandelstam analyticity. Comparison with the amplitude of Gunion, Brodsky, and Blankenbecler for large angle scattering leads to flat trajectories at large negative arguments. The transition to Regge behavior can be discussed similar to Feynman's conjecture. The possible appearance of fixed pole effects and a modification of the model of Ademollo and Del Giudice is considered.

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The Veneziano model amplitude[1] beautifully implemented the idea of duality, but having real poles it stimulated the research for a similar dual form with Mandelstam analyticity. Cohen-Tannoudji et al. proposed the following amplitude[2,a]

$$A(u,t) = \int_0^1 dx\, x^{-\alpha[t(1-x)]-1}\,(1-x)^{-\alpha[ux]-1}\, f(t(1-x))\, f(ux) \qquad (1)$$

where $\alpha(t)$, $f(t)$ are real analytic functions in t with cuts starting at the physical threshold t_0. Their main idea is that t always appears multiplied by $(1-x)$, u by x in the dual formula or more general by functions $g(1-x)$, $g(x)$ with $g(0)=0$, $g(1)=1$, respectively. In the case of trajectories with a linear part — expected in a formalism which contains the Veneziano amplitude as a limiting case — the linear part should not be submitted to these prescriptions.[3,4,5,6] u-t crossing symmetry of ansatz (1) is obvious and one can show the following further properties:

i) Complex poles in u and t given by the complex trajectory α accompanied by multipoles on the daughter level. These multipoles, if sufficiently small, are not unwanted because they give a shift in the position and width of the daughter resonances.[2,3] No ancestors appear.

ii) A double singularity region contained in the Mandelstam region.

iii) Regge behavior for $u \to -\infty$, t fixed and for $u \to +\infty$, $|t| \le t_0$.

The further study[7,8,9,10] of the asymptotic behavior of dual amplitudes of type (1) in the right half u plane have led to the conclusion that Regge behavior for unlimited t is not compatible with a linear increasing trajectory. The same is true for the polynomial boundedness of the amplitude.[10] These troubles can be overcome by the introduction of nonlinear trajectories $\alpha(u)$ bounded by $\sqrt{u}$ [9,10,b,c] for $u \to \infty$ and amplitude forms more involved than (1) and still in discussion.

Unfortunately, nonlinear trajectories draw us far away from the simple original Veneziano amplitude. The function f now plays an important role[d] even in implying duality since it gives e.g., the leading t polynomial at a pole in u. We have a high degree of freedom in the choice of the trajectory function and of the function f. In this situation one should look for a better dynamical understanding of formula (1) and for further experimental information which might clarify the parameterization of (1).

A situation in which the old Veneziano amplitude fails because it contains a linear trajectory is the large angle (fixed θ, u, $t \to -\infty$) scattering. It gives an e^{-cs} dependence[e] which disagrees severely with the experimental s^{-n} behavior and the Cerulus-Martin lower bound $|A(s,\theta)| > c \exp(-c\sqrt{s})$. A very natural explanation of the fixed power behavior in s for large angle scattering is given by Gunion, Brodsky, and Blankenbecler[11] in a composite particle model. They use extensively the advantages of the infinite momentum frame method; in particular, that the composite system wave function can be assumed to have an especially simple limiting form in this frame. Their amplitude corresponding to the graph of Fig. 1 is given by

$$A(u,t) = \int_0^1 dx \frac{1}{x^2(1-x)^2} \int d^2k_\perp \, \Delta\psi_1(k_\perp)\, \psi_2(k_\perp + (1-x)q_\perp - xr_\perp) \times$$

$$\times \; \psi_3(k_\perp + (1-x)q_\perp)\, \psi_4(k_\perp - xr_\perp) \tag{2}$$

with transverse vectors $r_\perp$ and $q_\perp$ which fulfill $r_\perp \cdot q_\perp = 0$, $u = -r_\perp^2$, $t = -q_\perp^2$, in an infinite momentum frame given by

$$p_1 = \left(\mathbb{P} + \frac{m^2}{2\mathbb{P}},\ \vec{0},\ \mathbb{P}\right)$$

$$p_2 = \left(\mathbb{P} + \frac{m^2+q_\perp^2}{2\mathbb{P}},\ \vec{q}_\perp,\ \mathbb{P}\right) \text{ , etc. and } \mathbb{P} \to +\infty \ .$$

The quark bound state wave functions ψ_i have the form[f]

$$\psi(k_\perp) = \left(m^2 - S(k_\perp) + i\epsilon\right)^{-1} \Phi\left(S(k_\perp)\right)$$

with (3)

$$S(k_\perp) = \left(k_\perp^2 + m_a^2(1-x) + m_c^2 x\right)/x(1-x)$$

and Δ is given by

$$\Delta = S(k_\perp - xr_\perp) + S(k_\perp + (1-x)q_\perp) - 2m^2 \quad .$$

The electromagnetic form factor $F(q^2)$ and the deep inelastic structure function $F_2(\omega) = \nu W_2(\omega)$ can be expressed in terms of the wave functions ψ appearing in Eq. (2).[g]

$$F(q_\perp^2) \sim \int_0^1 \frac{dx}{x(1-x)} \int d^2k_\perp \, \psi(k_\perp + (1-x)q_\perp) \, \psi(k_\perp) \tag{4}$$

and

$$F_2(\omega=1/x) = \int d^2k_\perp \frac{\psi^2(k_\perp)}{1-x} \tag{5}$$

with

$$\omega = -\frac{2\nu}{q^2} \quad ,$$

If we assume[11]

$$\Phi \sim S^{-n} \tag{6}$$

for large values of S (short distances) the behavior of ψ for $x \to 1$ and for $k^2 \to \infty$ is correlated and leads to the Drell-Yan relation[12] between the form factor[h] $F(q^2) \sim (q^2)^{-n-1}$ and the structure function $F_2(\omega) \sim (\omega-1)^{2n+1}$ for $q^2 \to -\infty$ and $\omega \to 1$. For high energy large angle scattering, $s \to +\infty$, $u, t \to -\infty$ with $t/u = (1-z)/(1+z)$ fixed, three of the wave functions ψ in Eq. (2) can be approximated by the asymptotic relation (6). With n=0 in (6) as proposed for the pion wave function (monopole form factor) in agreement with experimental consequences in Ref. 11, the amplitude (2) simplifies in this limit to

$$A(u,t) \sim -4 \int_{0(m_c^2/r_\perp^2)}^{1-0(m_a^2/q_\perp^2)} dx\, N_\psi(x) \left((1-x)\, q_\perp^2\right)^{-1} (x r_\perp^2)^{-1} \qquad (7)$$

Here we have canceled $\psi\left((1-x)q_\perp - xr_\perp\right)$ against Δ and $N_\psi = \int d^2k_\perp \frac{\psi(k_\perp)}{x(1-x)}$ is introduced as a smooth function (without factors x or (1-x)). The amplitude (7) has a power behavior[h] in $1/ut$ which in the realistic cases of pp and πp scattering leads to an agreement with experiments of the same quality as the dipole fit for the nucleon form factor.

A similarity in the structure of the dual amplitude (1) and the amplitudes (2) and (7) is remarkable. In both cases the u dependence is accompanied by a factor x and the t dependence by a factor (1-x) in the variable x integrated from 0 to 1.[i] We remember that this was the key idea in Eq. (1). Looking for details one detects differences; whereas Eq. (1) contains the u and t dependence in a strictly factored form, this is not the case for expression (2). In Eq. (7) we have a factored dependence only because of n=0. That allows the cancellation of Δ and the appropriate ψ.

Comparing (7) with (1) one associates $f(t(1-x))$ with $x^{-1}\psi\left((1-x)q_\perp\right) = \left((1-x)q_\perp^2\right)^{-1}$ and similar $f(sx)$ with $(1-x)^{-1}\psi(xr_\perp)$. One further has to postulate $\alpha(t)$, $\alpha(u) \to -1$ for $t, u \to -\infty$.[j] Independent of this comparison a power behavior for large angle scattering leads to a $f(t) \sim t^{-c}$ and a constant or logarithmic trajectory[k] $\alpha(t)$ for large negative t. Logarithmic trajectories lead to a $c(\theta)$ dependence.[ℓ]

Thus we are led to trajectories which become constant or slowly (logarithmic) varying at large negative values of their argument.†

We can go further and ask how Regge behavior arises from ansatz (2). Of course, the amplitude (2) is not Regge behaved in itself. In the limit of large

$s \approx -u = r_\perp^2$ and fixed $t = -q_\perp^2$, we obtain from Eq. (2)

$$A(u,t) \sim \int_0^1 \frac{dx}{x(1-x)}\, g\left(r_\perp^2, x\right) \int d^2k_\perp\, \frac{\psi(k_\perp)\,\psi\left(k_\perp + (1-x)q_\perp\right)}{x(1-x)} \tag{8}$$

with

$$g\left(r_\perp^2, x\right) = \Delta\psi^2(xr_\perp) = \psi(xr_\perp)$$

in an x region to be discussed below. We recognize the last integral in expression (8) — referred to as function $\phi\left(1-x, q_\perp^2\right)$ later on — as the integrand in the expression for the form factor $F(-q_\perp^2)$ in Eq. (4). It is this function from which we can learn how the Reggeization comes in.[m] For $q_\perp^2 = 0$ according to Eq. (5) the function ϕ is related to the deep inelastic structure function as

$$\phi(1-x, 0) = x^{-1}\, F_2\left(\omega = \frac{1}{x}\right). \tag{9}$$

If we would evaluate (5) with $\psi \sim S^{-1}$ for $x \to 0$ we would get a result similar to that for $x \to 1$. $\left(F_2\left(\omega=\frac{1}{x}\right) \sim (1-x) \text{ for } x \to 1\right)$: $F_2 \sim x^2$. Actually Eq. (5) should be treated more carefully. If one introduces a parton-outer-particle amplitude a Reggeization[13] in the core mass m_c destroys the symmetry in x and (1-x) (respectively m_a, m_c) in (3) and gives the more general result

$$F_2\left(\omega - \frac{1}{x}\right) \sim x^{-\alpha(0)+1} \quad \text{for} \quad x \to 0\ , \tag{10}$$

but still $F_2\left(\omega = \frac{1}{x}\right) \sim (1-x)$ for $x \to 1$. Here α is the trajectory appearing in forward Compton scattering. It can be assumed to be exchange degenerate.[n] In the presence of Pomeron exchange we have to add a constant term in (10) corresponding to $\alpha = 1$.

If we go to large $q_\perp^2$ in Eq. (4) the function ϕ can be approximated by

$$\phi(1-x, q_\perp^2) = \psi\left((1-x)q_\perp\right) \int \frac{d^2k_\perp\, \psi(k_\perp)}{x(1-x)} = \psi((1-x)q_\perp)\, N_\psi(x) \tag{11}$$

with the smooth function $N_\psi(x)$ defined under Eq. (7). With constant N_ψ and the above identification of ψ this reads

$$\phi\left(1-x, q_\perp^2\right) = x\ f\ ((1-x)\ t) \sim \frac{x}{(1-x)t} \tag{12}$$

A representation of F(t) which embodies (9), (10), and (12) is

$$F(t) = \int_0^1 dx\ x^{-\alpha(t(1-x))}\ h\left(1-x,\ q_\perp^2\right) \tag{13}$$

with

$$h\left((1-x), q_\perp^2\right) = x^{-1}\ \psi((1-x)\ q_\perp) = f(t(1-x)) \tag{14}$$

in the range of validity of (12) and with $\alpha(-\infty) = -1$. Equation (9) and (10) are fulfilled if we take

$$h\left(1-x, q_\perp^2\right) \sim x^{-1}\left(\frac{\mu^2+(1-x)^2\ q_\perp^2}{x(1-x)}\right)^{-1}$$

for $x \to 1$ which is suggested by (3), which should be relevant in this limit, if we want h to join smoothly to the behavior (14). We are then led to[o]

$$F_2\left(\omega = \frac{1}{x}\right) \sim x^{-\alpha(0)+1}\ (1-x) = x^{-\alpha(0)+1}\ (1-x)^{-\alpha(-\infty)} \tag{15}$$

A similar parameterization in the realistic case of the proton deep inelastic structure function gives a good fit to the experimental data.[17] As one can see from (10) and (15), the Drell-Yan relation[12] — expected from the construction principle Eq. (6) — is fulfilled. For small t and $x \neq 1$ we can make the assumption that a smooth function[p] f(t(1-x)) conjuncts to the behavior given in Eq. (14). Equation (13) can be further justified by similar considerations as in the work of Ademollo and Del Guidice.[19] It can be derived from a heuristic "weak" amplitude[q] of two fictitious particles which constitute a current coupled to hadrons.

As can be seen directly from Eq. (13) it contains the right pole structure in q^2. Whereas in the model of Ref. 19 the behavior for $q^2 \to -\infty$ originates from a linear trajectory structure it is given by the bound state function in ansatz (13). If we now insert $\phi\left(1-x, q_\perp^2\right) = x^{-\alpha(q_\perp^2(1-x))} f(t(1-x))$ into (8) we obtain

$$A(u,t) \sim \int_0^1 \frac{dx}{x(1-x)}\, g\left(r_\perp^2, x\right)\, x^{-\alpha(t(1-x))}\, f(t(1-x)) \tag{16}$$

for large u and fixed t. Here we had to take the limit of small t and $x \neq 1$ for the function h.[r] For $x \gtrsim \frac{\mu}{|r_\perp|}$ we are allowed to put $g\left(r_\perp^2, x\right) = \psi(r_\perp x) =$ $= (1-x)\, f(ux)$ with the identification above. With $\alpha(ux) \to -1$ for $u \to -\infty$ we end up with exactly the dual amplitude form (1). For $x < \frac{\mu}{|r_\perp|}$ this procedure is not allowed anymore but we see what has to happen if we want to reproduce (1): $g\left(r_\perp^2, x\right)/1-x$ has to become a smooth function $f'\left(-r_\perp^2 x\right)$ regular at zero. Evaluating this part of the integral in the Regge limit as

$$\int_0^{\mu/|r_\perp|} dx\, x^{-\alpha(t)-1}\, f\left(x\, r_\perp^2\right) = s^{\alpha(t)} \int_0^{\mu|r_\perp|} d\lambda\, f(\lambda)\, \lambda^{-\alpha(t)-1} \tag{17}$$

we get Regge behavior. We see that the "wee" region $x < \frac{\mu}{|r_\perp|}$ gives the Regge behavior,[s] in agreement with arguments of Feynman.[16] Actually a different approach to get Regge behavior in a smooth way out of ansatz (2) is the introduction of hadronic bremsstrahlung.[15,t]

As a side remark we mention that we could have equally well introduced the Pomeron with $\alpha = 1$ in Eq. (17) to get a diffractive component $A \sim s$. The role of the Pomeron in dual amplitudes of type (1) has to be investigated.

The integral (17) for $x \gtrsim \frac{\mu}{|r_\perp|}$ with the choice of $f\left(r_\perp^2, x\right)$ discussed above leads to fixed poles. They could be canceled by (nonleading) contributions in the wee region though this is not necessary.[u] Actually they may play an important role in understanding the behavior of scattering amplitudes in cases where the known Regge trajectories do not contribute or are suppressed.[v] Normally these effects show up only at large t values and the smooth relation between large angle ($x \neq 0$) and fixed t ($x \sim 0$) high energy scattering realizes in a concrete way the connection between light cone and Regge behavior.

A well-known way to introduce fixed poles into Veneziano amplitudes is to keep constant trajectories in some channels. This heuristic procedure has found some theoretical justification[19] for photon amplitudes and was successful phenomenologically[20, 21] in fitting polynomial form factors and fixed pole effects in photoproduction and Compton scattering. We now have constant or slowly varying trajectories for large negative arguments. A modified version of this model may survive which could be applied to purely hadronic processes too. The generalization of Eq. (13) to more complicated photon amplitudes (photoproduction, Compton scattering) is suggestive.

As our last remark we mention that property (ii) of the dual amplitude (1) is easy to understand if we think in terms of graphs like Fig. 1 which should contribute only in the Mandelstam region.

Concluding, we propose to identify the infinite momentum frame variable x with the variable appearing in the integral representation of dual amplitudes and to interprete the dual amplitude in terms of wave functions modified in a definite way by a Reggeization procedure as indicated above. The introduction of the infinite momentum frame in more formal approaches to a dual theory should be very useful. The generalization to n point functions[2, 3] is still unexplored. From a practical point of view one should try to modify (1)[w] and expressions like (2) with the proposed identification in mind. Amplitude (1) can now be calculated[3, 9] and compared with experiments. A modification of the

Ademollo-Del Giudice model[19] along the lines proposed above should be of considerable phenomenological interest. A dynamical interpretation of dual amplitudes might help to overcome difficulties related to the introduction of spin; in particular, the unwanted parity doubling. The predicted convergence of trajectories for $t \to -\infty$ has severe and interesting consequences. The power behavior at high energy large angle scattering may turn out to be of similar fundamental importance as the scaling behavior in deep inelastic electron scattering.

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LIST OF FOOTNOTES

a. We restrict our discussion to a scalar four particle amplitude with one exotic channel.

b. Eventually modified by a log u factor.

c. We mention that a $\sqrt{t}$ type effective trajectory for positive large t arises naturally in a fixed impact parameter picture for resonances.[22,23]

d. In the Veneziano amplitude f gives the satellite terms.

e. With a finite number of satellite terms corresponding to polynomials f.

f. We take all outer masses and the wave functions 1 to 4 equal and distinguish only between quark and core mass for convenience of a later discussion. We have spinless partons. In a more careful investigation spin should be taken into account.

g. We assume one scalar parton which carries the total charge of the outer particle for simplicity here.

h. Up to logarithmic factors.

i. Actually in the nonasymptotic form (2) $r_\perp$, not $r_\perp^2$ is accompanied by x and analog for $q_\perp$. In a more refined treatment one could try to compare (2) to an amplitude (1) with g(1-x), g(x) instead of (1-x), x.

j. We should mention here similar conclusions in Ref. 15 for inclusive processes.

k. It would be interesting to see a connection between (1) and the model of D. D. Coon, Phys. Letters 29B (1969) 559 which can give an excellent fit to high energy NN data (J. Tran Thanh Van, private communication).

ℓ. Not included in the model calculations of Ref. 11.

m. This procedure has some similarity to that of Ref. 14 in the case of inclusive scattering.

n. Consistent with the assumption of an exotic channel in the process considered.

o. To be multiplied by the smooth function $N_\psi(x)$ more generally.

p. Thus the limits $t \to 0$ and $x \to 1$ cannot be interchanged.

q. We plan to report about these questions in an extended version with a more careful discussion.

r. Otherwise we do not get a (u, t) symmetric form easily compared to (1) for $x \sim 1$. In the limit $t \to -\infty$ and u fixed the region $x \sim 1$ has to give Reggeization and the role of the partons in Fig. 1 is interchanged.

s. If we insist on a function f dependent on $xr_\perp^2$ only the change from a regular behavior at zero to a $1/xr_\perp^2$ dependence for $x > \frac{u}{|r_\perp|}$ has to appear at $x \sim \frac{c}{|r_\perp|^2}$, i.e., $\lambda \sim c$ in Eq. (11). This could be changed in more involved versions of amplitude (1).

t. Feynman argues in the cms system, therefore it is hard to compare in detail. There is no contradiction in arguing in different systems with different kinds of bremsstrahlung.

u. The possibility of fixed poles in strong processes is discussed in a recent paper by A. Capella, B. Diu and J. M. Kaplan (LPTPE 72/8, Orsay 1972).

v. E.g., in near forward meson photoproduction, vector meson production or pn CEX. The appearance of fixed pole effects from diagrams of the type of Fig. 1 in the case of meson photoproduction has been discussed by Dosch.[18]

w. As we learned from A. Capella after completing this work, L. Gonzalez Mestres and R. Hong Tuan were able to construct a dual amplitude similar to (1) which contains the Veneziano amplitude as a limiting case, factorizes, and has Regge behavior. It should be interesting to formulate the ansatz in terms of "dual wave functions". (E. Del Guidice, R. Musto, P. Di Vecchia, and S. Fubini, CERN TH. 1553 (1972).) It is interesting to note that the new amplitude form has the connection ux^2, $t(1-x)^2$ mentioned in footnote i.

† If the trajectory function $\alpha(t)$ is a real analytic function which fulfills a dispersion relation and has no oscillations for $t \to +\infty$ constancy for $t \to -\infty$ implies the same constant limit for Re $\alpha(t)$ for $t \to +\infty$. This deviates from the idea of duality with infinitely rising trajectories but may well be in agreement with the usual FESR analysis in favor of duality if the trajectory rises over a finite range. A special case of a limited spectrum in the context of an ansatz of type (1) was discussed in Ref. 10. The model with a logarithmic trajectory mentioned in footnote k has similar features.

FIGURE CAPTIONS

1. (u, t) scattering graph.

2. Example of hadronic bremsstrahlung.

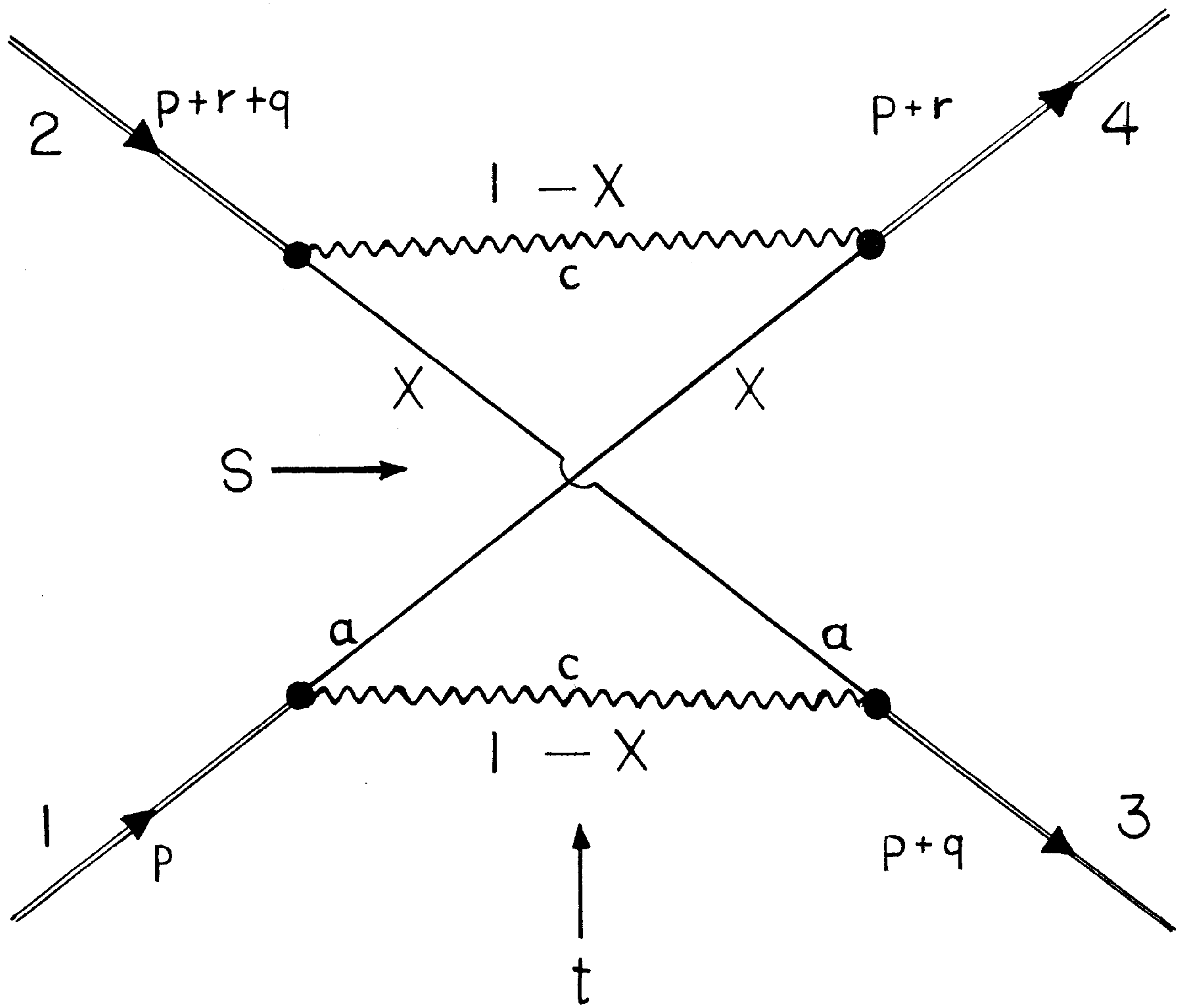

FIG. 1

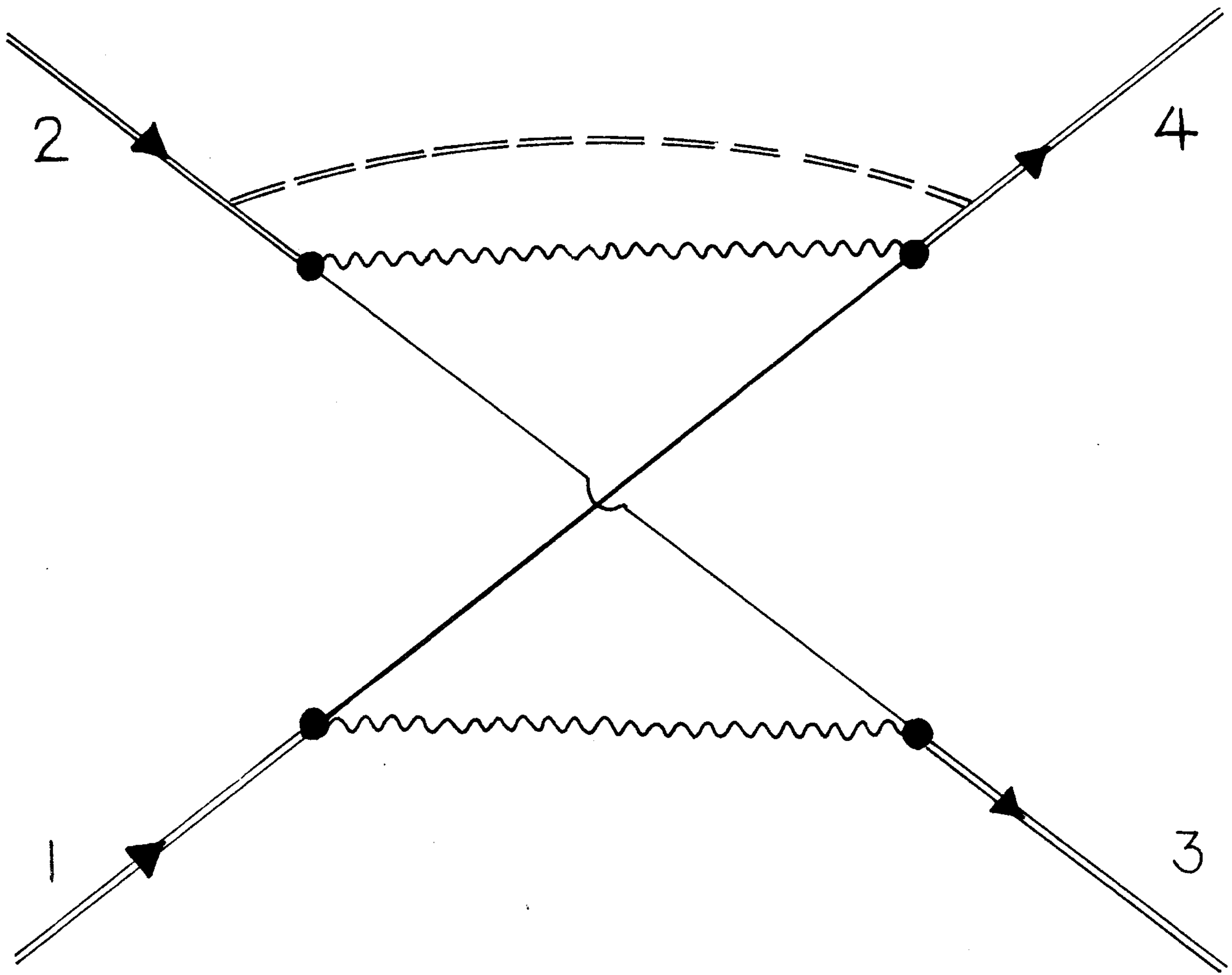

FIG. 2