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INCLUSIVE PROCESSES AT HIGH TRANSVERSE MOMENTUM*

R. Blankenbecler, S. J. Brodsky, J. F. Gunion**
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

The interchange theory for inclusive scattering at large transverse momentum is extended into kinematic regions where Regge effects are important. Hadronic bremsstrahlung is shown to inevitably lead to Reggeization of the fundamental production process as s goes to infinity for fixed, large, p_T. The cross sections are shown to rise to their ultimate scaling limit. This transition zone connects smoothly with the Feynman scaling region at low p_T, and the deep scale-invariant region at large $p_T \sim 0(\sqrt{s})$. The inclusive results of the interchange theory have the form of the standard fragmentation, triple Regge, and pionization formulae in their respective regions, but in addition predict the behavior of the associated residues and trajectories at large t or p_T^2.

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**Present address: Physics Department, Massachusetts Institute of Technology.

The recent measurements at CERN ISR of single-particle inclusive scattering at 90° and large transverse momentum have revealed several dramatic features of pion inclusive reactions.[1,2] Of particular interest is the transition from steep exponential to power law behavior in p_T for $p_T > 4$ GeV/c. Other surprising features of the data are the rapid rise in the inclusive π° cross section at fixed p_T as a function of s, and the trend towards a large value for the total charged particle to a charged pion ratio indicated by the last two points of the British-Scandinavian collaboration at 3.7 and 4.5 GeV/c.

Large p_T reactions are theoretically extremely interesting because of the possibility that they may be related to simple and basic processes at small distances: For a sufficiently large energy and momentum transfer, one can assume that the interaction time becomes so brief that only a single elementary process can occur. In general, the elementary process can proceed via one of two possible mechanisms. The first type, consisting of elementary binding or gluon interactions, has been considered by Bjorken, Berman, and Kogut.[3] Whether or not elementary gluon exchange turns out to be experimentally important, a second and simpler type of interaction, namely, the interchange of elementary constituents, will certainly take place if hadrons are composite,[4,5] or at least effectively composite. In two recent papers, we have presented a theory of large momentum transfer scattering for both inclusive and exclusive processes based on the assumed dominance of the constituent interchange force. The resulting exclusive cross sections have the form[4]

$$\frac{d\sigma}{dt} \simeq s^{-N} P\left(\cos \theta_{cm}\right) \tag{1}$$

where the power N and the function P are determined by the asymptotic behavior of the electromagnetic form factors of the interacting hadrons. Comparison with the large-angle pp and πp elastic scattering data indicates that the interchange amplitude is, in fact, the dominant contribution in the deep scattering region (s, t, u large). The constituent interchange predictions for single particle inclusive cross sections in the "deep" scattering region, where t, u, and the missing mass $\mathcal{M}^2$ are finite fractions of s, are characterized by power law behavior in energy and transverse momentum. The inclusive cross section for $A + B \to C + X$ has the same form as that for deep inelastic electron scattering, $e + B \to e + X$, except that the electron-photon current interaction is replaced by an effective current depending in a simple fashion on the form factors of particles A and C.

Unfortunately the measured values of p_T^2 are so small in comparison to the huge s of the ISR that the data are not in the deep scattering region where the simplest processes could dominate. The physical complication is clear: because the interchange process falls rapidly with energy, real hadronic bremsstrahlung from the incident particles prior to the central event will play a major role; it enables the interchange interaction to take place at the smallest value of s consistent with the observed p_T of the final particle. As we shall show, this hadronic radiation process Reggeizes the energy dependence of the interchange contribution. The resulting expression rises to an asymptotic form independent of s, with a power law fall-off in p_T determined mainly by the hadronic electromagnetic form factors. The agreement with the p_T dependence of the ISR data is excellent if we assume a monopole pion form factor. We will also show how the resulting expression continues smoothly into the usual pionization, triple Regge, and Feynman scaling regions, yielding predictions for the "deep" limiting behavior of the standard Regge forms used to describe these regions.

The basic elementary interchange process relevant to the large transverse momentum inclusive scattering $A + B \to C + X$ is illustrated in Fig. 1a.[5] The hadrons are assumed to be composite systems consisting of the elementary constituents (a or b) bound to a multiparticle core (represented as systems c or d in the figure). The basic mechanism is the rearrangement of the elementary constituents a and b. The kinematics are defined by

$$t = \left(p_A - p_C\right)^2 = q^2$$

$$u = \left(p_B - p_C\right)^2 = r^2$$

$$s = \left(p_A + p_B\right)^2 = 2M_A^2 + 2M_B^2 - (q + r)^2 \quad (2)$$

and

$$p_T^2 = tu/s, \mathcal{M}^2 = s + t + u - M_A^2 - M_B^2 - M_C^2$$

and we shall calculate the invariant cross section $d\sigma/\left(d3p_C/E_C\right)$. The calculations are performed in an infinite momentum frame using time-ordered perturbation theory, which greatly simplifies the covariant calculations of bound state scattering.

As we have shown previously, the asymptotic wave functions for hadrons A and C can be directly related to their electromagnetic form factors F_A and F_C. The two hadrons A and C in the interchange process act as an effective current which carries momentum q, in precise analogy with deep inelastic electron scattering (see Fig. 1b). We thus can relate the process "q"+ B→X directly to the measured scaling structure function $F_{2B}(x) = \nu W_{2B}\left(-q^2/\left(\mathcal{M}^2 - q^2\right)\right)$. We thus obtain for the interchange contribution[6]

$$sE_C d\sigma/d^3p_C = \left[R_0^{A+B\to C+X}\left(s, p_T^2, u\right) + R_0^{B+A\to C+X}\left(s, p_T^2, t\right)\right]$$

$$R_0^{A+B\to C+X}\left(s, p_T^2, u\right) = \sigma_{\rm eff}^{AC}\, F_{2B}\left(-t/\left(\mathcal{M}^2 - t\right)\right) \quad (3)$$

where

$$\sigma_{\rm eff}^{AC} \equiv N_0\,(s+u)\,(u/s)^2\left(s/\mathcal{M}^2\right)^2 F_A^2\left(s p_T^2/\left(\mathcal{M}^2 - t\right)\right)F_C^2\left(s p_T^2/\mathcal{M}^2\right)$$

and we have taken all particles to be spinless. In most cases, the results are unchanged when spin is included.[5]

Since R_0 falls as s^{-1} for fixed p_T^2 and u, hadronic bremsstrahlung plays an increasingly important role as $s \longrightarrow \infty$. If we define the distribution function $G_{H/A}(z)$ as the probability for the incident particle A to emit a hadron H with momentum $p_H = zp_A$ (in a frame in which $p_A \longrightarrow \infty$), then one obtains an additional contribution to the measured cross section of the form

$$\left[R_1^{A+B \to C+X}\left(s,\, p_T^2,\, u\right) + R_1^{B+A \to C+X}\left(s,\, p_T^2,\, t\right)\right]$$

where

$$R_1^{A+B \to C+X}\left(s,\, p_T^2,\, u\right) = \int_0^1 \frac{dz}{z} \sum_H G_{H/A}(z)\, R_0^{H+B \to C+X}\left(zs, p_T^2\,, u\right) . \qquad (4)$$

Note that the structure function $F_{2B}(x)$, in R_0, vanishes if its argument, x, is outside the range 0 to 1. This result is computed from graphs such as that represented in Fig. 1c. Due to the Regge behavior of the forward $A+\bar{H} \to A+\bar{H}$ elastic scattering amplitude represented in the upper half of the graph, one has

$$G_{H/A}(z) \sim (z)^{-\alpha_A} \beta_{AH} \qquad (z \sim 0) \qquad (5)$$

with $\alpha_A = 1$ for Pomeron behavior.[7] Note that the Regge behavior coming from hadronic bremsstrahlung of particle B is incorporated in the behavior of

$$F_{2B}(x) \sim (x)^{1-\alpha_B}$$

for $x \simeq 0$. It can be shown that additional bremsstrahlung from the internal particles or from the core d leads to results falling faster in p_T^2, as could be expected from the impulse approximation. Bremsstrahlung or resonance formation associated with the final particle C does not modify in an essential fashion the predictions for R_0 and R_1 given above. These latter processes affect mainly the normalization and not the dependence on the invariants.

The behavior of R_0 and R_1 must be discussed in several limiting regions. We consider first the fragmentation and triple Regge regions of projectile A. Both regions involve small t/s but large $|u| \sim s$. (t, however, is kept large compared to hadronic masses.) In this region, R_0 and R_1 for $A+B \longrightarrow C+X$ become

$$R_0 = N_0 s^{\alpha_B} x_F^{1+\alpha_B} \left(1-x_F\right)^{\alpha_B - 2(1-A-C)} \left(p_T^2\right)^{1-\alpha_B-2(A+C)} \tag{6}$$

$$R_1 = N_1 s^{\alpha_B} x_F^{1+\alpha_B} \int_{x_F}^{1} \frac{dz}{z} \sum_H G_{H/A}(z) \left(z-x_F\right)^{\alpha_B - 2(1-H-C)} \left(zp_T^2\right)^{1-\alpha_B-2(H+C)} \tag{7}$$

where we have defined the Feynman variable $x_F = t/s - u/s$. For small t, $\left(1-x_F\right) \sim \mathcal{M}^2/s$. In the fragmentation region of A, where $\mathcal{M}^2/s$ is not small, the p_T behavior is clearly controlled by the values of H, A, and C appearing in the expressions for R_0 and R_1, where

$$F_I(t) \sim (-t)^{-I} \qquad (I = A,\ B,\ C,\ H)$$

In Ref. 4, we showed that the pion-nucleon large-angle elastic scattering data require the form factor of the pion to decrease as a monopole for $-t < 10\ \mathrm{GeV/c}^2$. Such a weak fall-off ($H_\pi = 1$) for the pion wave function makes it clear that the large p_T dependence of the inclusive cross section is controlled by the virtual pions present in any hadronic beam.[8] However, as $x_F \longrightarrow 1$, i.e., $\mathcal{M}^2 \longrightarrow 0$, R_1 vanishes at least as fast as $\left(1-x_F\right)^{2H}$ relative to R_0. (This is due to the generalized Drell-Yan threshold dependence which says that $G_{H/A}$ vanishes at least as fast as $(1-z)^{2A-1}$ for $z \longrightarrow 1$.) This illustrates the general feature that hadronic bremsstrahlung of the projectile A vanishes in this threshold or "Triple Regge" region of small $\mathcal{M}^2/s$.

An interesting comparison can now be made with the usual triple Regge formula

$$R \sim s^{\alpha_B} \left(\mathcal{M}^2/s\right)^{\alpha_B - 2\alpha_{AC}(t)} \beta(t) \quad . \tag{8}$$

Using the vanishing of R_1 and the fact that $p_T^2 = -t$, we obtain

$$\alpha_{AC}(t \longrightarrow -\infty) = 1 - A - C$$

and the limiting behavior

$$\beta(t) \sim (-t)^{2\alpha_{AC}(-\infty) - \alpha_B - 1} \quad .$$

The above prediction for $\alpha_{AC}(-\infty)$ corresponds exactly to the value predicted by the interchange theory for the appropriate Reggeon-particle scattering process at fixed but large t.[9]

In contrast to the triple Regge and fragmentation regions, R_1 dominates the pionization region defined by $t = u \sim -\sqrt{s}$. In particular, if we examine the region appropriate to the ISR data, $x_F = 0$, i.e., $u = t = -p_T\sqrt{s}$, Eq. (4) yields

$$R = R_1 \sim s^{\frac{1}{2}\left(\alpha_A + \alpha_B\right)} \sum_H \left(p_T\right)^{2-\alpha_A-\alpha_B-4(H+C)} \beta_{AH}\, \Gamma_{HB}^{C}\left(p_T/\sqrt{s}\right) \quad . \tag{9}$$

In deriving this result, it is convenient to scale z in units of $p_T/\sqrt{s}$, which exposes the small z (Regge) behavior of $G_{H/A}$. Closer examination of the integrals involved shows that Γ_{HB}^{C} rises quite sharply to its ultimate limit as s increases at fixed p_T.

Discussion

To help clarify the behavior of the inclusive cross section over the full range of p_T, let us continue to examine the region $x_F = 0$. At small p_T, we expect the cross section to be independent of energy and to fall exponentially in p_T in accord with the standard predictions in this, the Feynman scaling region. For very large p_T, i.e., $2p_T/\sqrt{s}$ approaching the kinematical limit 1, only the R_0 term of our theory survives, yielding a cross section that falls as a power in s for fixed $p_T/\sqrt{s}$. This we term the deep scale-invariant region.[5] Between the Feynman scaling and deep regions, there is a transition zone where the cross section is dominated by R_1, in which pre-interchange hadronic bremsstrahlung partially Reggeizes the process. In this region, the cross section is dominated by a meson-initiated interchange process, and behaves as an inverse power of p_T times a decreasing function of the ratio $p_T/\sqrt{s}$. This form provides a smooth interpolation between the two more fundamental regions discussed above.

The behavior of the single pion inclusive cross section in the transition zone is predicted from Eq. (9) to have the form $\left(\alpha_A = \alpha_B = 1\right)$

$$p_T^N E_\pi \frac{d\sigma}{d^3 p_\pi} = \Gamma\left(p_T/\sqrt{s}\right) \tag{10}$$

Note that this form is highly reminiscent of the interchange predictions for exclusive scattering (Eq. 1).

For $C_\pi = 1$, i.e., a pion form factor that falls as t^{-1}, the value of N is 8. Assuming a simple model for the threshold behavior of the bremsstrahlung spectrum, $zG(z) \sim (1-z)^3$ and $F_{2p}(x) \sim (1-x)^3$, the function Γ has the behavior shown in Fig. 2a. The predicted transition zone cross sections for various energies are shown in Fig. 2b. The results for N = 8 are in excellent qualitative agreement[10] with the preliminary CERN-ISR data.[1,2]

Although the complete p_T dependence of R_1 in the pionization region is complicated by the question of the relative normalization of the contributions due to different hadrons H, it is true that for very large p_T, only H = meson (π, K, etc.) is likely to survive due to the slow decrease at large momenta of a typical meson's wave function. Thus in the context of this theory, the proton beam at the ISR behaves effectively like a meson beam of much lower energy. Pomeron dominance, $\alpha_A = \alpha_B = 1$, of course, implies that equal numbers of $\pi^{\pm 0}$ appear in the beam, and this, in turn, explains the continuing equality of their cross sections at large p_T. In addition, it implies that the number of produced particles and antiparticles will be the same at large p_T in the pionization region. The nucleon production cross sections should fall more rapidly with p_T than the meson cross sections, due to the more rapid fall-off of their form factors. Thus it would be very surprising if the pions (and kaons) turn out to be only a small fraction of the total charged particles at large p_T.

It is an inherent feature of the pionization region that Regge effects tend to mask the fundamental production mechanisms. Thus it is imperative that large p_T experiments be performed at lower energies where the contribution of R_1 relative to R_0 is suppressed by the threshold ($z \sim 1$) dependence of the spectrum $G_{H/A}(z)$. A comparison between pion and proton beam experiments at large $p_T/\sqrt{s}$ will be essential in unravelling the features of the basic interchange process, as it is only for $\pi p \to \pi X$ that R_0 is not suppressed by additional form-factor fall-off relative to R_1.

We wish to emphasize the fact that hadronic bremsstrahlung can, in general, be expected to Reggeize any fundamental interaction mechanism and will be particularly important for those that decrease with s. In the interchange case, it provides the key to establishing a remarkable and beautiful consistency between the fragmentation, triple Regge, pionization, and "deep" inclusive spectra.

Acknowledgments

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References

1. For the π° data, see R. Cool et al. (CCR Collaboration), XVI International Conference on High Energy Physics, September 1972.
2. For the charged data, see M. Banner et al. (Saclay-Strasbourg), ibid., and B. Alper et al. (British-Scandinavian Collaboration), ibid.
3. S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. 4, 3388 (1971). T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965).
4. J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Letters 39B, 649 (1972) and SLAC-PUB-1037. The general form of Eq. (1), without specific predictions for N and/or the function P, has been proposed by D. Horn and M. Moshe, Tel Aviv University Preprint TAUP-298/72, J. Kogut and L. Susskind, IAS preprint (1972), and W. R. Theis, DESY preprint DESY 72/35 (1972). A logarithmic scaling form has been suggested by S. D. Ellis and P. G. O. Freund, EFI/NAL preprint TH-82 (1972).
5. J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Rev. (to be published), SLAC-PUB-1053. J. Bjorken and J. Kogut have investigated the interpolation of the interchange predictions between inclusive and exclusive processes (private communication).
6. This result differs from that given in Ref. 5 by a factor of $\left(\mathcal{M}^2/(\mathcal{M}^2-t)\right)^{2(A-1)}$. This follows from a better choice for the wave function of particle A which includes the effect of time orderings that must be included when only one constituent is virtual. This will be discussed in detail in a later publication.
7. This is very much like the parton spectrum used by Feynman to get Regge behavior but here it describes the hadron spectrum. See R. Feynman, Phys. Rev. Letters 23, 1415 (1969).

8. As described in Ref. 5, the presence of antiparticles in the pion means that one must also include the (st) graphs with the (ut) graphs. These new terms in the transition region are of the same form as the ones explicitly discussed here but they are much smaller in magnitude.

9. A careful analysis of pp and π^-p scattering at high energies strongly supports this type of behavior of the leading effective α. See D. Coon, J. F. Gunion, J. Tran Thanh Van, and R. Blankenbecler (to be published).

10. The agreement is quite good quantitatively except for the lower p_T points of the 30.6-GeV data which lie well below our prediction.

Figure Captions

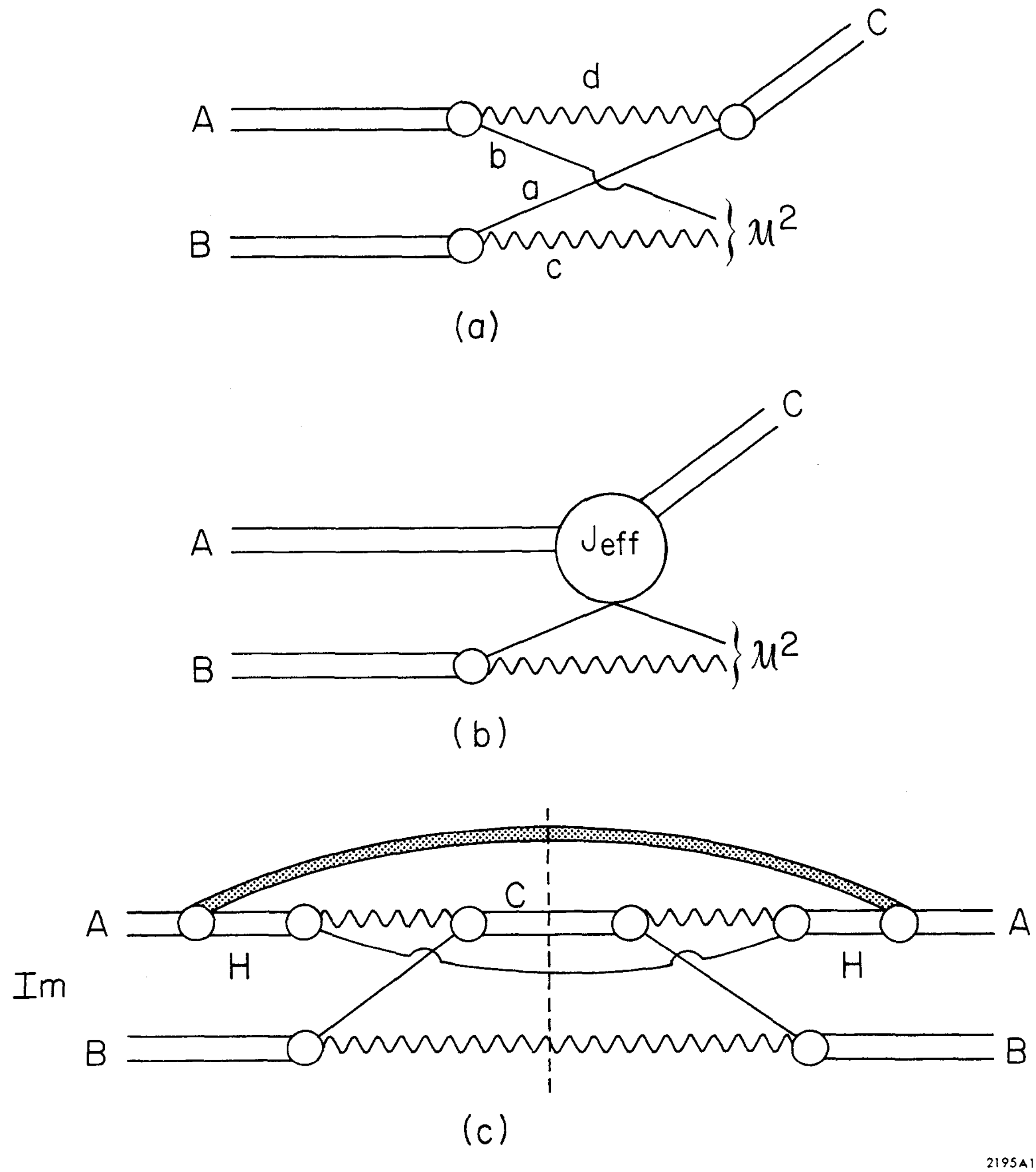
Fig. 1. (a) The basic interchange graph for the inclusive production of particle C.

(b) The effective current operator for hadronic processes.

(c) The bremsstrahlung process which will ultimately produce Regge behavior. Similar processes occur along the bottom of the graph and are included in F_{2B}.

Fig. 2. (a) The function $\Gamma\left(p_T/\sqrt{s}\right)$ for the parameters described in the text.

(b) The predicted inclusive cross section, $pp \to \pi X$, using the above Γ, plotted against p_T for several energies. The curves were normalized to the value 1.45×10^{-32} cm^2/GeV^2 at $p_T = 4$ GeV and $\sqrt{s} = 53.4$ GeV.


A
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$\mathcal{M}^2$
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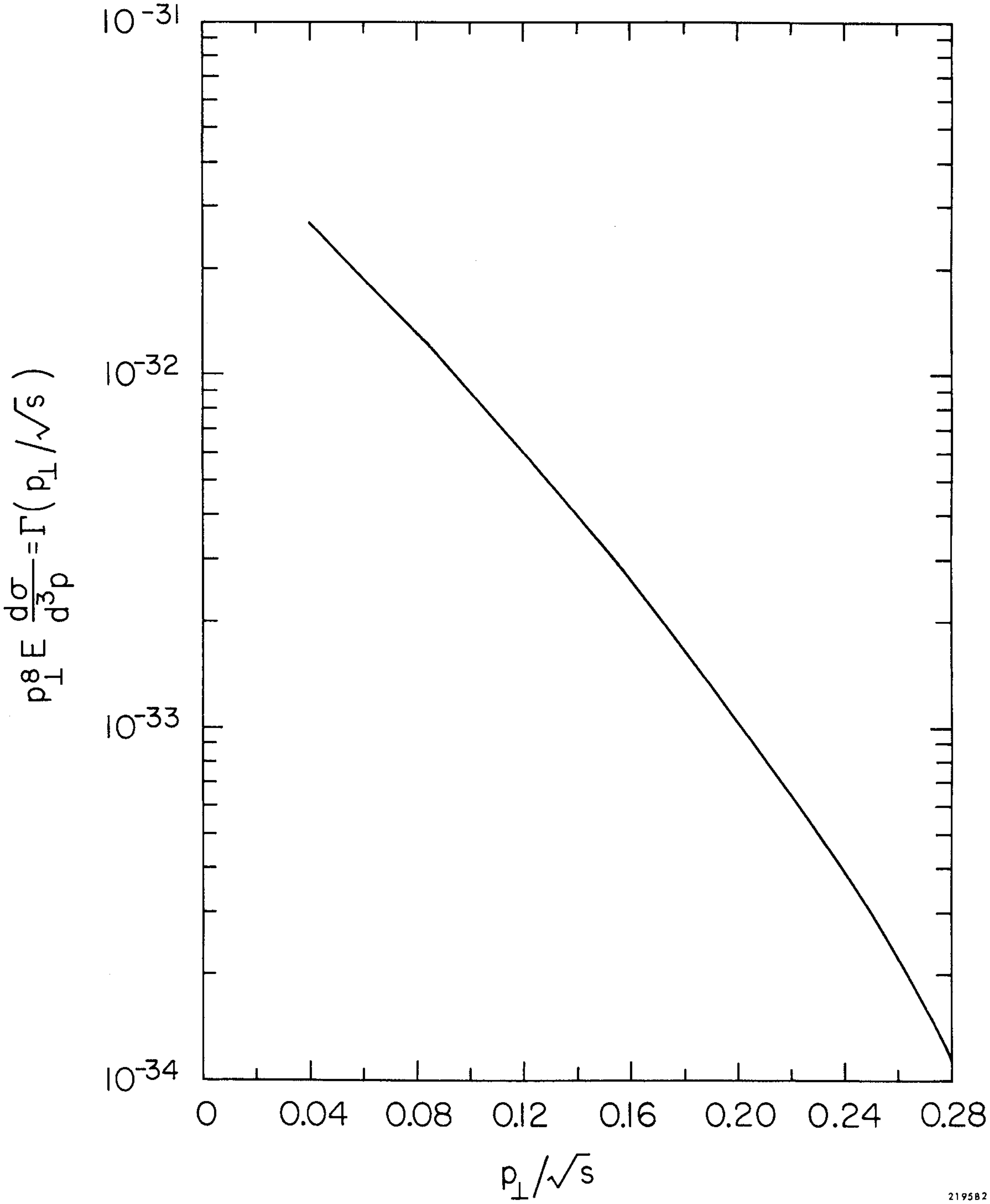
Fig. 1


10^{-31}
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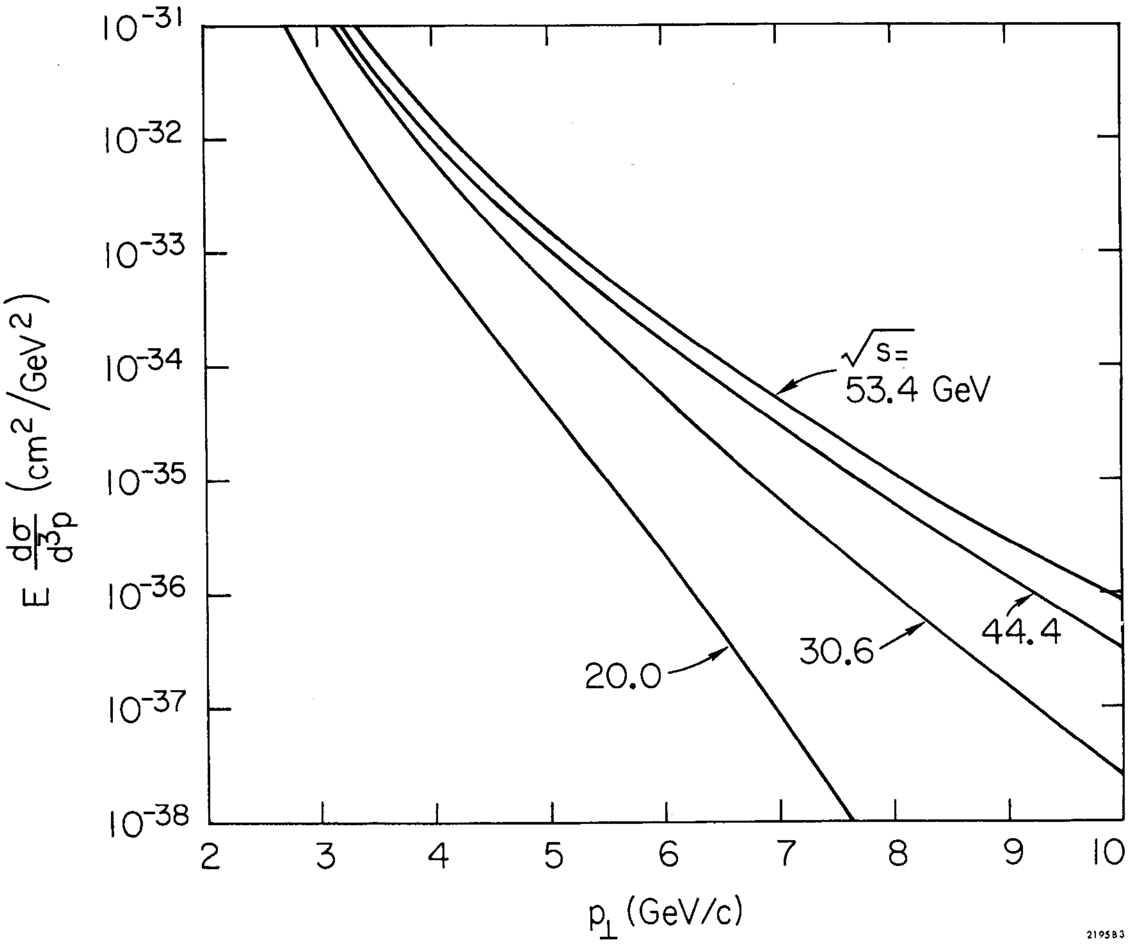
$p_\perp^8 E \frac{d\sigma}{d^3p} = \Gamma(p_\perp/\sqrt{s})$
0
0.04
0.08
0.12
0.16
0.20
0.24
0.28
$p_\perp/\sqrt{s}$
219582

Fig. 2A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10^{-33}
10^{-34}
10^{-35}
10^{-36}
10^{-37}
10^{-38}
$E \frac{d\sigma}{d^3p}$ (cm^2/GeV^2)
2
3
4
5
6
7
8
9
10
$p_\perp$ (GeV/c)
$\sqrt{s}=$ 53.4 GeV
44.4
30.6
20.0
219583

Fig. 2B