

HELICITY STRUCTURE OF NUCLEON RESONANCE  
ELECTROPRODUCTION AND THE SYMMETRIC QUARK MODEL\*

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ABSTRACT

The symmetric quark model with harmonic forces is shown to lead to a rapid change with  $q^2$  of the helicity structure for electroproduction of the second ( $D_{13}$ ) and third ( $F_{15}$ ) nucleon resonances. Preliminary results of coincidence measurements of pi-zero electroproduction in the resonance region are found to disagree with this prediction. Some further experimental tests and theoretical consequences of this failure are discussed.

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The symmetric quark model, with harmonic forces acting between pairs of quarks, has been rather successfully employed, both in classifying hadron states<sup>1</sup> and in calculating the electromagnetic transitions between different hadron states due to the emission or absorption of real photons.<sup>2</sup> Recently a relativistic quark model with harmonic forces has been developed by Feynman, Kislinger, and Ravndal<sup>3</sup> and used to calculate the matrix elements of both the vector and axial-vector currents, again with considerable quantitative success. With such a quark model, it becomes possible to treat the very relativistic processes involved in the electroproduction of nucleon resonances. Indeed, Ravndal<sup>4</sup> has pointed out that inelastic electron-proton scattering "will be the first serious test of the relativistic version" of the model.

One of the major successes of either the relativistic or non-relativistic versions of the symmetric quark model is the prediction of the remarkable helicity structure of the photoproduction amplitudes for the first three prominent nucleon resonances, the  $P_{33}$ (1236),  $D_{13}$ (1520), and  $F_{15}$ (1690). Let us work in the center of mass of the photon and nucleon (isobar rest frame) and consider the two independent amplitudes for formation of a given resonance to be  $A_{1/2}$  and  $A_{3/2}$  corresponding to net spin component  $\lambda$  equal to 1/2 and 3/2 along the photon's direction of motion. Then experimentally it is known<sup>5</sup> that the excitation of the  $P_{33}$ (1236) is dominantly of a magnetic dipole character ( $A_{3/2}/A_{1/2} = \sqrt{3}$ ), while the excitation of the  $D_{13}$  and  $F_{15}$  from protons proceeds almost entirely through the  $\lambda = 3/2$  state ( $A_{1/2}^p \approx 0$  for  $D_{13}$  and  $F_{15}$ ). This can be seen directly in the forward and backward differential cross sections for  $\gamma p \rightarrow \pi^+ n$  or  $\pi^0 p$  where only the  $\lambda = 1/2$  amplitude contributes by angular momentum conservation, and where there is no appropriate structure on passing

through the energy region of the  $D_{13}$  (1520) and  $F_{15}$  (1690). We note that the Drell-Hearn-Gerasimov sum rule<sup>6</sup>, which equates the square of the nucleon's anomalous moment (a manifestly positive quantity) to an integral over cross sections proportional to  $|A_{3/2}|^2 - |A_{1/2}|^2$ , has for protons the possibility of being saturated by low lying resonances precisely because  $|A_{3/2}| > |A_{1/2}|$  for all the prominent nucleon resonances.<sup>7</sup>

In the symmetric quark model with harmonic forces the dominance of the  $\lambda = 3/2$  excitation for the  $D_{13}$  and  $F_{15}$  comes about because of a cancellation between terms arising from the quarks orbital motion and terms arising from the magnetic moments of the quarks. Explicitly, in the non-relativistic model of Copley, Karl, and Obryk<sup>2</sup>, one has for the  $\lambda = 1/2$  amplitude with a proton target<sup>8</sup>:

$$\begin{aligned} A_{1/2}^p &\propto |\vec{q}_{\text{cm}}|^2 - \alpha^2/g \text{ for the } D_{13} \\ A_{1/2}^p &\propto |\vec{q}_{\text{cm}}|^2 - 2\alpha^2/g \text{ for the } F_{15}, \end{aligned} \tag{1}$$

where  $\vec{q}_{\text{cm}}$  is the three momentum in the isobar rest frame,  $g$  is the gyro-magnetic ratio for the quark, and  $\alpha$  is related to the harmonic oscillator strength. Since  $|\vec{q}_{\text{cm}}|^2$  is roughly twice as large for the  $F_{15}$  as the  $D_{13}$ , it is possible for the  $A_{1/2}^p$  amplitudes for both resonances to be very small. In fact, with quite reasonable choices<sup>2,3</sup> for  $g$  and  $\alpha$  both amplitudes in Eq. (1) are very small and are consistent in both sign and magnitude with photoproduction experiments.

However, given values of  $g$  and  $\alpha$ , this cancellation for real photon ( $q^2 = 0$ ) induced transitions will no longer hold for  $q^2 \neq 0$ . This is because  $|\vec{q}_{\text{cm}}|^2$  for a given resonance increases monotonically with increasing  $q^2$ , destroying the

balance between the two terms. For example, while the ratio of cross sections  $\sigma_{3/2}/\sigma_{1/2} = |A_{3/2}|^2/|A_{1/2}|^2$  is predicted by the relativistic quark model<sup>3</sup> to be more than 10 at  $q^2 = 0$  for the  $D_{13}(1520)$  resonance excited from protons, by  $q^2$  of  $0.3 \text{ GeV}^2$  (space-like) this ratio is predicted to be less than one. By  $q^2 \approx 1 \text{ GeV}^2$  the ratio is predicted to be  $\sim 1/10$ . We find that for both the  $D_{13}$  and  $F_{15}$  the  $A_{1/2}$  amplitude rapidly overtakes the  $A_{3/2}$  amplitude in magnitude as  $q^2$  changes from zero to a few tenths of a  $\text{GeV}^2$ , and that the  $A_{1/2}$  amplitude becomes more and more dominant with further increases in  $q^2$ .

Experiment on the other hand gives no indication for such a change.

While the excitation of the first resonance is known to maintain its magnetic dipole character (and therefore  $\sigma_{3/2}/\sigma_{1/2} = 3/1$ ) out to a least<sup>9</sup>  $q^2 = 1.0 \text{ GeV}^2$  (in agreement with the quark model and dispersion theory calculations<sup>10</sup> as well), recent experiments at Daresbury<sup>11</sup> on  $ep \rightarrow e\pi^0 p$  indicate that the  $D_{13}(1520)$  maintains a strongly  $\lambda = 3/2$  excitation from  $q^2 = 0$  out to  $q^2 = 0.6 \text{ GeV}^2$ . An experiment<sup>12</sup> on backward  $\pi^0$  electroproduction at DESY indicates the same dominance of the  $\lambda = 3/2$  amplitude for the third resonance (the  $F_{15}$ ) region out to at least  $q^2 \approx 0.5 \text{ GeV}^2$ . Thus, at values of  $q^2$  where such a change should already be clearly visible, there is no indication of the change in helicity structure of the  $D_{13}$  and  $F_{15}$  resonance excitations predicted by the symmetric quark model.

The small value of the  $\lambda = 1/2$  amplitude for photoproduction of the  $D_{13}$  and  $F_{15}$  "predicted" by the quark model with harmonic forces thus appears to be an accident, which evaporates as  $q^2$  changes. In light of this it would be interesting to check whether relations hold which depend only on the  $SU(6) \times O(3)$  symmetry of the symmetric quark model, and not on the specific dynamics

of the quarks' interaction (like the  $\vec{q}_{\text{cm}}^2$  term in Eq. (1)). For example, since the four transition amplitudes ( $\lambda = 1/2$  and  $3/2$  on proton and neutron) arise from only two terms, the quarks orbital motion and their magnetic moments, there are two linear relations between amplitudes for the excitation of each resonance. In the case of the  $D_{13}$  these are, in an obvious notation, (for photoproduction and the transverse multipoles in electroproduction)

$$\begin{aligned} A_{3/2}^n &= - A_{3/2}^p \\ A_{1/2}^n &= - \left( \frac{2}{3\sqrt{3}} \right) A_{3/2}^p - \left( \frac{1}{3} \right) A_{1/2}^p : \end{aligned} \tag{2}$$

Thus if the excitation of the  $D_{13}$  remains almost purely  $\lambda = 3/2$  on protons, there must be a sizeable  $\lambda = 1/2$  excitation on neutrons. Similarly, for the  $F_{15}$ :

$$\begin{aligned} A_{3/2}^n &= 0 \\ A_{1/2}^n &= \left( \frac{2}{3\sqrt{2}} \right) A_{3/2}^p - \left( \frac{2}{3} \right) A_{1/2}^p . \end{aligned} \tag{3}$$

Decisive experimental information with neutron targets to test the relations for  $A_{1/2}^n$  is lacking at present.

Finally, we note that the quark parton model for deep inelastic electron-proton scattering predicts<sup>13</sup> that  $\sigma_{1/2}$  is considerably larger than  $\sigma_{3/2}$  at large hadron invariant masses and large values of  $q^2$ . A similar conclusion results from analyzing certain sum rules.<sup>14</sup> Aside from the elastic peak (which always has  $\sigma_{3/2} = 0$ ), we have just seen that experiment indicates that the excitation of the prominent nucleon resonances from protons has just the opposite behavior ( $\sigma_{3/2} > \sigma_{1/2}$ ) out to at least  $q^2 = 0.5 \text{ GeV}^2$ . Whether this

behavior persists beyond  $q^2 \simeq 1 \text{ GeV}^2$  where scaling begins for the spin averaged cross sections, and whether the many less prominent nucleon resonances do the same is unknown at present.<sup>15</sup> Further coincidence measurements clearly are needed to separate the helicity amplitudes for each resonance. It will also be very interesting in forthcoming single arm experiments with polarized leptons and a polarized target to measure the asymmetry  $(\sigma_{1/2} - \sigma_{3/2}) / (\sigma_{1/2} + \sigma_{3/2})$  summed over all final hadronic states in both the deep inelastic region and the resonance region, and to study the transition between them as a function of  $q^2$ .

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