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Title: Ir-Rational Number Institute Report 2017-2018

Author(s): Gonzales, Lorenzo

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Introduction

The 2017 – 2018 Los Alamos National Laboratory Math and Science Academy Ir-Rational Number Institute (IrRNI) six-all-day sessions were held on Northern New Mexico College campus (NNMC) between September 2017 and March 2018. The IrRNI included 7 Northern New Mexico school districts (Bloomfield Public Schools, Cuba Independent Schools, Española Public Schools, Los Alamos Public Schools, Pojoaque Valley School District, Pecos Independent School District and Santa Fe Public Schools) and one Bureau of Indian Education school district, Jemez Day School, located in Jemez Pueblo. Twenty-nine teachers of mathematics, kindergarten through eighth grade, 2 principals, 1 STEM Coordinator, and 1 New Mexico Public Education Math and Science Bureau staffer* attended regularly and were eligible for a Certificate of Completion or Certificate of Engagement on the last day of instruction (20 participants were new to the IrRNI and 13 were returnees from the previous year or the MSA Summer Institute).

The Los Alamos National Laboratory Math and Science Academy covered all budgetary costs for the IrRNI as part of the Laboratory's and Los Alamos National Security, LLC Community Commitment Plan for educational outreach, and Northern College provided the venue at no cost to CPO or MSA. The districts' committed teachers' time and travel to NNMC.

Ir-Rational Number Institute Big Learning Ideas

The IrRNI under the instruction of Richard Kitchen, PhD, University of Wyoming, Professor and Wyoming Excellence in Higher Education Endowed Chair in Mathematics Education, explored several big pedagogical and mathematical ideas over the six professional teacher development sessions:

1. Developing students' understanding of fractions is complex and requires significant time to develop. Students need lots of informal experiences with fractions before proceeding to formal fraction operations.
2. The role that mathematical modeling can play to support students making meaning of fractions and numbers.
3. The development of key mathematical ideas as students learn about fractions beginning in third grade and move through the Common Core State Standards for Expressions & Equations found in grades 6-8.
4. It is important that students have opportunities to engage in, solve, and develop meaning with regards to all five-rational number "subconstructs," not simply the part-whole interpretation.

Ir-Rational Number Institute End-of-Program Survey

On the final IrRNI session 22 teachers (1 teacher did not complete the survey, so it was not included in the results) responded to five open-ended questions about the quality, depth, and pedagogical value of the IrRNI and 2-word problems to ascertain their problem-solving skill, use of representational modeling, and ability to communicate their mathematical reasoning through writing. The reason behind the administration of the End-of-Program Survey is to answer the following question: "How can the program to be refined to better support and enhance teacher professional development and student learning and achievement?"

*IrRNI teacher participants: 13 elementary teachers, 6 math coaches, and 10 middle school teachers of math, 2 principals, 1 STEM Coordinator, and 1 NM-PED staffer.

(See Appendix A of this report for complete set of survey questions.)

Question 1: Has the Math and Science Academy's Ir-Rational Number influenced your knowledge and understanding of mathematics content? Yes ____ No _____ Please explain and feel free to extend your thoughts to the back side.
(This question attempts to elicit whether the content and pedagogical stance of the IrRNI influenced the teacher's pedagogical-content knowledge of mathematics, specifically her understanding of the five subconstructs of fractions.)

All responded in the affirmative, stating:

- "My knowledge and understanding of math has hit a new high. Not only do I understand math at a deeper level, but I totally enjoy teaching it!"

- "I feel much more confident with my math instruction. Before the Institute, I struggled with connecting models/manipulatives to the 'math' I was teaching. I now jump to the model before every new concept. . ."
- "The vertical connections between math concepts is clearer and how foundational understanding with one concept is necessary to support concepts in upper grades."
- "Not only has IrRNI deepened my understanding of and appreciation of fractions but it has profoundly influenced my teaching philosophy through a prolonged reflection on the pedagogy of mathematics instruction."

Question 2: What new mathematics ideas and/or teaching and learning strategies have you gained and how do you plan to implement these new ideas and strategies in your classroom?
(This question intents to have the teacher reflect on new knowledge gained, and how she is going to spread her new understanding of mathematics. In other words, did the IrRNI provide an opportunity for change in her mathematical thinking and pedagogy?)

The following are quotes that exemplify the 22 teachers' responses:

- "The importance/value of concrete model for students to make sense of mathematics before moving onto the abstract processes or algorithms."
- "I learned a lot about inviting and engaging students in the discussion. I now regularly invite students to show and explain their work and allow other students to comment."
- "Since the mathematical focus was fractions, at first, I mumbled that it wasn't relevant to me as a kindergarten teacher. After much thought, I began to see how I can lay the foundation for future work on fractions: discussion about what is 'equal?', making groups and describing how many groups of size we've made, and practice dividing objects into equal parts."

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- "To teach well, I must be able to model problems pictorially, graphically, verbally, algebraically; to understand deeply, my students need to be able to do this too."
- "It is more beneficial for students to think deeply when solving 1- 4 problems, than working on 10 simple problems."
- "Organizing my math book… the notebook is huge!"

Question 3: Do you plan to share these strategies with your teacher colleagues or school PLC? Yes _____ No _____ Please explain below.

(This question is similar to Question 2 above, but the question is more direct, so the teacher has to make an expressed commitment to share what she has learned.) Below are several responses that typify the teachers' responses; all answered positively that they will share with colleagues:

All teacher responded in the affirmative:

- "I am definitely going to present this to the other teachers at my school."
- "In our PLCs… I want to share so that our whole school is on the same page."
- "Being a math coach, I will continue to try to find ways to share these problems and big ideas with colleagues."
- "I have been sharing with my colleagues and also with PLCs."

Question 4: What new mathematics ideas and/or teaching and learning strategies have you gained and how do you plan to implement these new ideas and strategies in your classroom?

(Again, this question is similar to questions 2 and 3 above, which were primers for this question. Question 4 was crafted and positioned after the aforementioned to give the IrRNI participants time and stimulus to think deeply about what she had learned and how was she going to use any new knowledge gain during the IrRNI experience in her classroom, instead of sharing it with her colleagues.)

Again, all teachers responded positively, and all give an example(s) of a big math idea gained:

- "Modeling was so hard to grasp at first. I feel much more confident in the variety of models … to multiple and divide."
- "Multiplicative thinking: The idea of introducing this to younger students, earlier on can make a phenomenal impact on their mathematical abilities."
- "Acknowledging student for way of solving problems; allowing students to make errors; and my teaching is more purposeful."
- Seeing the other teachers' solutions to word problems and being able to 'see' someone's thinking is extremely valuable."

Question 5: What specific suggestions do you have to improve the MSA Ir-Rational Number Institute?

(This is a straight-forward question that seeks to elicit from the 6-session participants recommendations that would improve the IrRNI's program and delivery of instruction.)

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Teachers expressed they wanted more of the same and that it the IrRNI be open to other teachers.

- "All teachers should have the opportunity to attend one of these Institutes, even a shortened version so as to expose all teachers to the extremely effective thinking and collaboration."
- "The Ir-Rational Number Institute was by far the best one [math PD] I've attended. . . . It has been awesome working with "math nerds" all day."
- "The sharing of student work from different schools more often would be awesome."
- "I really enjoyed this PD, it was unlike anything I have ever experienced.
- "Let it continue so we can keep learning from each other."
- "Have some way to connect online so when someone has a question many can contribute."

Using Numbers, Words, and Pictures to Represent and Communicate Mathematical Thinking

Two-word problems were included in the IrRNI Survey to gain an understanding of the teachers' use of representational problem solving using mathematical models, heuristics, and the ability to communicate their mathematical thinking through writing:

Question 6: Problem A. Using numbers, words, and a picture (i.e., model, drawing, etc.) represent and communicate your mathematical thinking of the following: How many times can I measure $\frac{5}{6} out\ of\ 2\frac{2}{3}$?

All survey respondents used a mathematical model (i.e., bar model, area model, number line, or double number line) to aid in solving the *measuring 5/6 out of 2 2/3 problem*; however, thirteen out of the 22 teacher participants correctly solved the problem, and seven communicated their mathematical reasoning through all three representational forms: numbers, models, and writing. Two errors typify the teachers' incomplete understanding for dividing a fraction by a fraction, 1) confusing the dividend as the divisor and 2) incorrectly identifying "what is the unit" for measuring out a fraction from another fraction or dividing a fraction by a fraction. For example, one teacher made a common error, she correctly interpreted the problem a division of a fraction by a fraction, but misidentified 5/6 as the dividend, so interpreted the question as "How many times does 2 2/3 go in 5/6?" Instead, of how many 5/6 can be measured out of 2 2/3? The teacher did leave a note stating, "Please be patient, you'll see more results next year. It was difficult for me" (See Figure 1, below).

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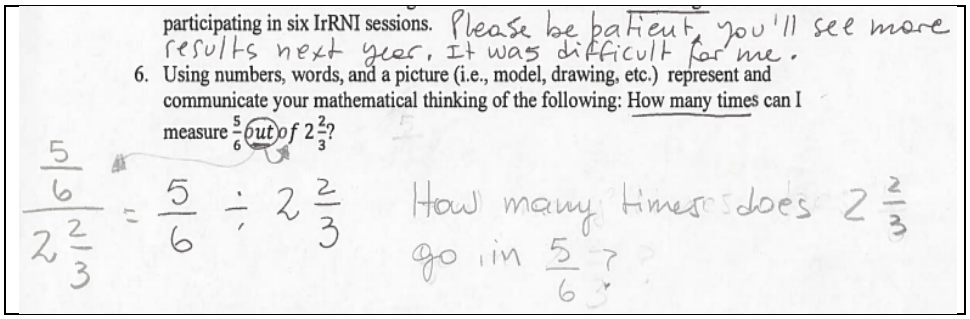

Figure 1. Teacher confused divisor for dividend.

Correctly identifying "What is the unit?" is a difficult idea to wrap one's mind around because "every fraction is a relative amount, it tells how much you have relative to the unit," that is, the whole unit. (Lamon, p. 97, 2012). Again, one example typifies this common error; the teacher correctly sees the problem as a division of a fraction by a fraction but misinterprets the unit to be measured out as sixths instead of fifths — See the teacher's model and explanation below. The question is asking, "How many 5/6 units can be measured out of a 2 and 2/3 whole? So, if the divisor, 5/6, is the unit that we are measuring out, we are, then, measuring out 5(1/6), so each 1/6 is 1/5 of the 5/6 unit. Therefore, 3 and 1/5 of units can be measured out of a 2 and 2/3 whole.

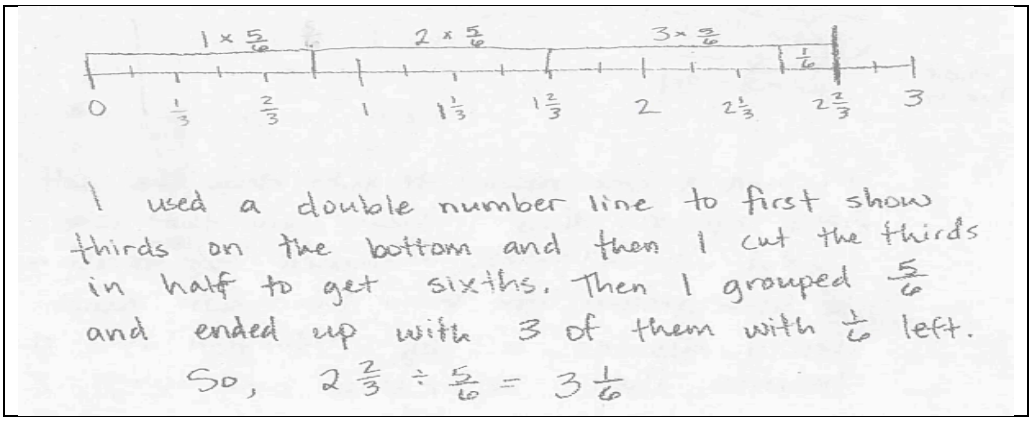

Figure 2. Teacher use of double number line, but incorrectly understands the unit of measure as sixths instead of fifths.

Question 7: Problem B. There are 64 fifth graders at Blanco Elementary. Three-eighths of them bring their own lunch to school. Using numbers, words, and a mathematical model determine how many fifth graders bring their own lunch.

One-hundred percent of the teachers correctly solved the *3/8 of 64 problem*; they used a variety of mathematical models (i.e., bar model, array model, open-array, number line, double number line, table, or discrete geometric figures) to assist in correctly solving Problem B. Also, 50% of the teachers used number, words, and pictures to communicating their mathematical thinking. Below are three samples of teacher solutions that exemplify the mathematical thinking and the problem-solving strategies teacher used.

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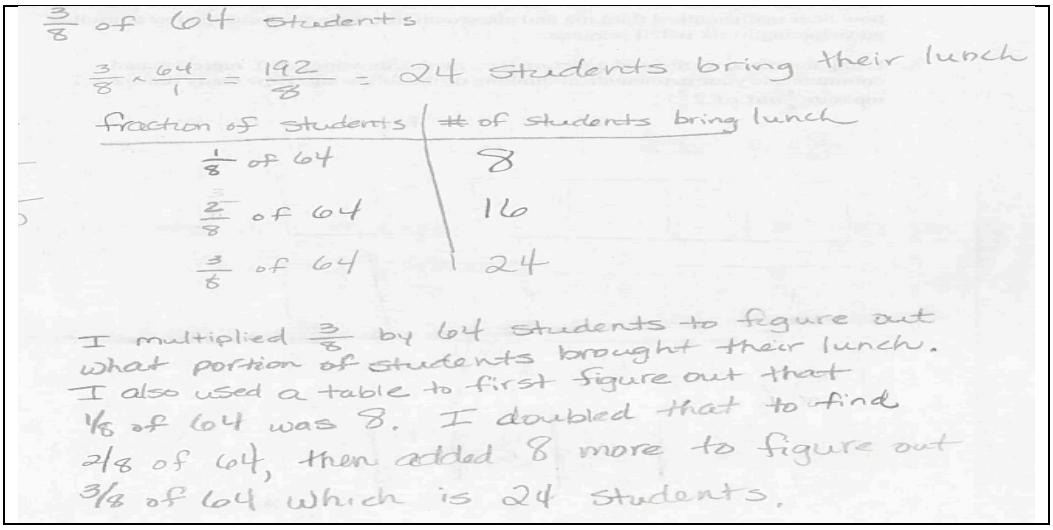

Figure 3. Teacher used a table to illustrate iteration of 1/8 of 64 to 3/8 of 64.

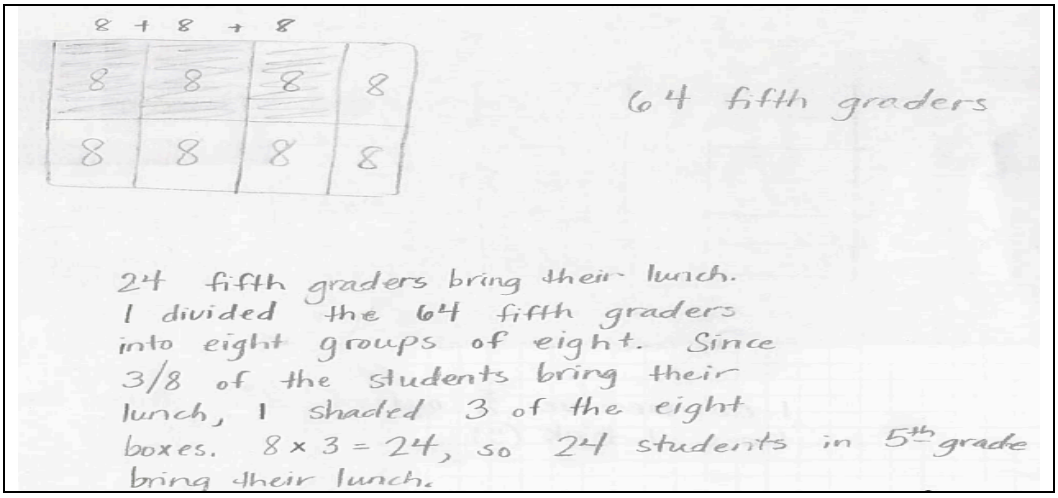

Figure 4. Teacher used a rectangle to illustrate 64 fifth graders as 8 groups of 8.

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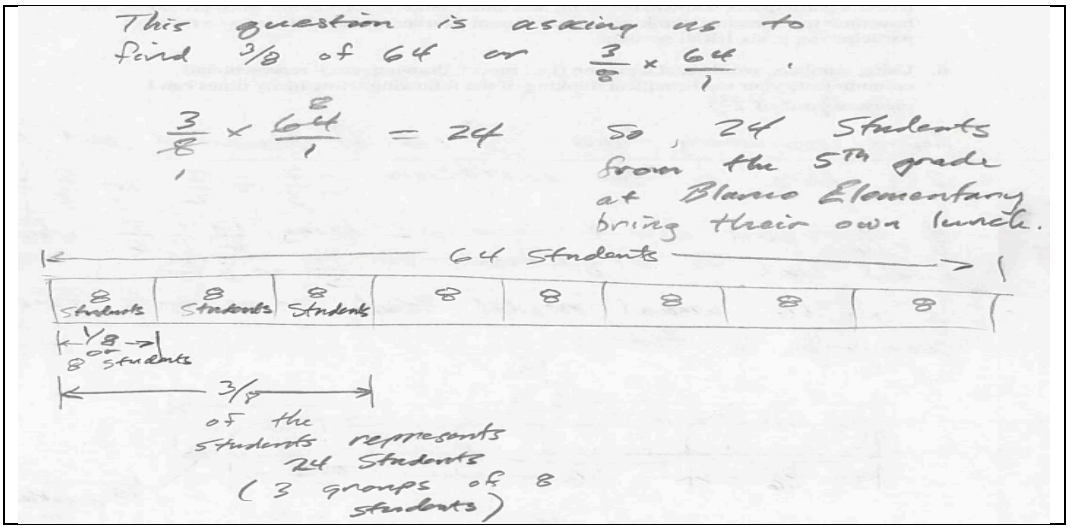

Figure 5. Teacher use of numbers, words, and bar model to illustrate and communicate mathematical thinking

Concluding Remarks: How can the program to be refined to better support and enhance teacher professional development and student learning and achievement?

Since its inception in the spring of 2009, the idea behind The MSA Ir-Rational Number Institute is to create mathematics learning opportunities beyond the established mathematics curriculum for K-8 teachers of mathematics. As teachers come to understand the reform elementary mathematics programs as concept-based curricula (i.e., Common Core State Standards for Mathematics), the subsequent professional development sessions must inherently carry relevant and substantive mathematics. Professional development must add to the teacher's profound understanding of mathematical concepts and pedagogy; otherwise, the teacher becomes a rhetorical figure teaching to a passive classroom of students who struggle to understand the images, concepts, facts, language, and procedures that govern mathematics (Gonzales, 2009).

Although the IrRNI teacher participants were a mixture of elementary and middle school teachers of mathematics with a wide-range of educational backgrounds and teaching experiences in mathematics, the IrRNI curriculum and pedagogical stance of successfully exploring big pedagogical and mathematical ideas was achieved. One teacher's End-of-Program Survey response clearly underscores that her thinking of mathematics instruction has changed as well as reflect the thoughts of the other 21 IrRNI participants; she writes, "The Math and Science Academy has opened my eyes to how 'traditional' teaching when I went to school actually caused self-doubt in my mathematical abilities."

The other indicator of teachers' changing view of mathematics reasoning and teaching is illustrated in the teachers' responses to the word problem instruction, "Using numbers, words, and a picture (i.e., model, drawing, etc.) represent and communicate your mathematical thinking." For example, thirteen out of the 22 teacher participants correctly solved Problem A, and seven communicated their mathematical reasoning through all three representational forms: numbers, models, and writing. This outcome was anticipated because of the mixed mathematical background of the K-8 teacher participants; elementary teachers'— as part of the traditional pre-

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service teacher programs and inservice professional development — learning opportunities do not delve deeply into understanding mathematics at a profound, foundational level; they are educated more as subject area generalist. A teacher's mathematical thinking, reasoning, and use of mathematical strategies can easily be seen in the computational strategies, operations, and mathematical models they used, but with the addition of directing the teacher to represent and communicate their mathematical thinking through the three mediums of numbers, models, and words, it becomes much more challenging for them because of the added cognitive demand of modeling and scripting their mathematical thinking.

The other indicator of the IrRNI teachers' evolving view of mathematics as a concept-based curriculum that can be visualized is the teachers' new appreciation of the use of modeling as a strategy for representing and solving word problems. All teachers used mathematical models to communicate their problem-solving strategies and mathematical reasoning of the two problems presented in the survey. Below is a teacher's quote that reflects the IrRNI participants' new perspective on modeling as a problem solving and mathematical thinking strategy:

> MODELING: There has always been modeling in my mathematics history but the new emphasis on it to gain better understanding has really given myself and my students strength to build a better understanding of the 'Why' in mathematics.

An implication of the IrRNI and the End-of-The Year Survey is that teachers' pedagogical-content knowledge and skill (i.e., prior knowledge) play a critical role in assisting students to understand and communicate their mathematical thinking and reasoning. Specifically, the teachers' lack of profound understanding of fractions and their subconstructs (i.e., part-whole, ratios and rates, operator, measure, and as a quotient) will directly limit her students' understanding and use of fractions; because, and this brings us back to big ideas number 1 and 4 (See Page 1), fractions are complex and require significant time to develop. Both students and teachers need lots of informal and formal experiences with fraction operations; and it is important that teachers have opportunities to work with, solve, and develop meaning of all five-rational number 'flavors' not simply the part-whole explanation.

So how does the MSA refine the IrRNI to better support and enhance teacher professional development and student learning and achievement? Question 5 of the survey collected teachers' specific suggestions as to how to better support and enhance teacher professional development in mathematics:

- Go deeper into 3rd and 4th grade fractions, and the sharing of student work more often.
- All teachers should have the opportunity to attend one of these institutes to expose all teachers to extremely effective thinking and collaboration, offer it more often.
- More exposure and practice using a greater variety of models to solve problem, such as discrete models, double number lines, to help build my familiarity and confidence in using these models.

Teachers did not have specific suggestions for refining the structure, format, instructional approach, or content of the IrRNI; it seems they want more of the same content but at a deeper, more profound level. For example, one elementary teacher stated, "…. I knew I was not ready yet for this kind of math PD because in some ways, I'm still learning the math I am teaching…. This

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was just the beginning for me." Another elementary teacher wrote, "I have not studied math since high school, when I had terrible teachers. I now have a much deeper understanding of fractions, and a greater confidence in my ability to solve problems involving fractions." In contrast, a middle school teacher wrote, "The IrRNI has influenced my understanding of math. I always took for granted that I understood [fractions] already. Forcing myself to create models has led to a MUCH deeper understanding of the conceptual knowledge of math — especially of operations with fractions.

The above, I think, speaks to the mixed nature of the teachers' experiential backgrounds; that is, both middle school teachers (although by training and teaching background have more education and experience in mathematics, it is still limited) and elementary teachers came away with a deeper and more robust understanding of fraction regardless of their prior mathematical understanding, pedagogical-content knowledge, and skills level as they entered the IrRNI. One teacher wrote: "I am more confident and relaxed when approaching math in the classroom" after having attended the Los Alamos National Laboratory Math and Science Academy Ir-Rational Number Institute.

Recommendations

- The Ir-Rational Number Institute (IrRNI) is a focused mathematical content/pedagogy professional development series for K-12 teachers. The six IrRNI sessions' content are usually tied to the MSA Summer Math Week math content with content-pedagogical support for teachers during the school-year. To this end, I purpose that MSA redirect the IrRNI to the professional development of the Regional Partnership School 4^{th} – 8^{th} teachers.
- In addition to the IrRNI, Richard Kitchen has proposed working directly with RPS teachers to develop mini-lessons aligned with NM Common Core State Standards for Mathematics standards in Number & Operations – Fractions (upper elementary grades) as well as Expressions & Equations (middle school). The mini-lessons could be collaboratively implemented with MSA staff, Dr. Kitchen, other participating RPS teachers, and/or NMHU pre-service teachers. The execution of the mini-lessons would be systematically researched using design research methodologies to examine student learning, teaching methodologies, and how to use assessment as a means to improve learning. A particular focus of the mini-lessons will be on how to use mathematical models to support student understanding of fractions, decimals and percentages.

Appendix A

1. Has the Math and Science Academy's Ir-Rational Number influenced your knowledge and understanding of mathematics content? Yes ____ No _____ Please explain and feel free to extend your thoughts to the back side.

2. What new mathematics ideas and/or teaching and learning strategies have you gained and how do you plan to implement these new ideas and strategies in your classroom? (Feel free to extend your thoughts to the back side.)

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3. Do you plan to share these strategies with your teacher colleagues or school PLC? Yes _____ No _____ Please explain below.

4. What topic or area was most valuable to you? Please explain how it added value to your teaching and/or professional development.

5. What specific suggestions do you have to improve the MSA Ir-Rational Number Institute?

6. Using numbers, words, and a picture (i.e., model, drawing, etc.) represent and communicate your mathematical thinking of the following: How many times can I measure $\frac{5}{6} out\ of\ 2\frac{2}{3}$?

7. There are 64 fifth graders at Blanco Elementary. Three-eighths of them bring their own lunch to school. Using numbers, words, and a mathematical model determine how many fifth graders bring their own lunch.

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