Computing nucleon EDM on a lattice

M. Abramczyk, T. Izubuchi

Physics Department

Brookhaven National Laboratory

DISCLAIMER

Computing nucleon EDM on a lattice

Michael Abramczyk[1], *Sinya* Aoki[2,3], *Tom* Blum[1,4], *Taku* Izubuchi[4,5], *Hiroshi* Ohki[4], and *Sergey* Syritsyn[4,6,*]

[1] *Physics Department, University of Connecticut, Storrs, Connecticut 06269, USA*
[2] *Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[3] *Center for Computational Sciences, University of Tsukuba, Tsukuba 305-8577, Japan*
[4] *RIKEN/BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[5] *Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[6] *Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*

Abstract. I will discuss briefly recent changes in the methodology of computing the baryon EDM on a lattice. The associated correction substantially reduces presently existing lattice values for the proton and neutron theta-induced EDMs, so that even the most precise previous lattice results become consistent with zero. On one hand, this change removes previous disagreements between these lattice results and the phenomenological estimates of the nucleon EDM. On the other hand, the nucleon EDM becomes much harder to compute on a lattice. In addition, I will review the progress in computing quark chromo-EDM-induced nucleon EDM using chiral quark action.

1 Introduction

Observing a non-zero nucleon electric dipole moment (EDM) will be evidence of violations of fundamental P,T-symmetries beyond the level of the Standard Model. Such violations are necessary to explain the excess of matter over antimatter in the Universe, and the $C\!P$ in the CKM matrix of the Standard model is insufficient for the observable amount of matter. Knowledge of nucleon structure and interactions is necessary to interpret constraints from precise experiments attempting to measure the EDM of nucleons and nuclei, the precision of which is projected to improve by two orders of magnitude in the next decade.

An EDM may be induced within the Standard Model by the QCD $\bar\theta$-term, the smallness of which is known as the strong CP problem. In order to constrain $\bar\theta_{\rm QCD}$ from nucleon EDM measurements, lattice calculations of the $\bar\theta_{\rm QCD}$-induced neutron EDMs have been performed earlier in Refs. [1–3] using the background electric field and in Refs. [4–10] from nucleon matrix elements in $C\!P$ vacuum. In our recent paper [11], we have found that the analysis of the nucleon EDM in Refs. [4–10] was flawed, and all previous results are largely consistent with zero.

In this report, we calculate the nucleon EDM induced by the CP-odd quark-gluon electric dipole interaction (so-called chromo-EDM)

$$\mathcal{L}_{cEDM} = i \sum_{\psi=u,d} \frac{\tilde{\delta}_\psi}{2} \bar{\psi} T^a (g_S G^a_{\mu\nu}) \sigma^{\mu\nu} \gamma_5 \psi = i \sum_{\psi=u,d} \tilde{\delta}_\psi O_{\psi\tilde{G}} , \qquad O_{\psi\tilde{G}} = \frac{1}{2} \bar{\psi} T^a (g_S G^a_{\mu\nu}) \sigma^{\mu\nu} \gamma_5 \psi , \qquad (1)$$

using $24^3 \times 64$ lattices generated with Iwasaki gauge action and $N_f = 2 + 1$ dynamical flavors of Domain Wall fermions by the RBC collaboration. These lattices have pion mass $m_\pi \approx 330$ MeV and lattice spacing $a^{-1} \approx 1.73$ GeV. We perform these calculations using two independent methods: (1) nucleon matrix elements of the vector current and (2) nucleon energy shifts in the background electric field.

2 Nucleon EDM on a lattice

In order to compute the nucleon EDM on a lattice, one has to calculate nucleon matrix elements of the quark vector current in a $C\!\!\!/P$ vacuum $\langle N|J^\mu|N\rangle_{C\!\!\!/P}$. CP violation leads to "parity mixing" of the nucleon spinor $u \to \tilde{u} = e^{i\alpha_5\gamma_5}u$, which is equivalent to a chiral rotation of the nucleon mass $m_N \to e^{-2i\alpha_5\gamma_5}m_N$ in the Dirac equation. In order to extract the nucleon matrix elements, one has to analyze the nucleon-current correlators as

$$\langle N(p')J^\mu\bar{N}(p)\rangle \propto e^{i\alpha_5\gamma_5} \frac{-i\not{p}'_{\mathcal{E}uc} + m_N}{2E'_N} \{e^{i\alpha_5\gamma_5}\}^? \Gamma^\mu_{\mathcal{E}uc} \{e^{i\alpha_5\gamma_5}\}^? \frac{-i\not{p}_{\mathcal{E}uc} + m_N}{2E_N} e^{i\alpha_5\gamma_5} , \qquad (2)$$

where the phase factors in curly braces were deemed necessary as parts of the nucleon propagator in the $C\!\!\!/P$ vacuum in Refs. [4–10]. However, as we discuss in our recent paper [11], these factors lead to incorrect results and should be omitted because the correct decomposition of the vector current into CP-even and -odd form factors is

$$\langle p + q, \sigma'|J^\mu|p, \sigma\rangle = \bar{u}_{p+q,\sigma'}\Big[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\Big]u_{p,\sigma} \qquad (3)$$

with spinors satisfying the Dirac equation $(i\not{\partial} - m_N)u$ and having definite parity $\gamma_4 u_{\vec{p}} = +u_{(-\vec{p})}$. The improper chiral phase factors $\{e^{i\alpha_5\gamma_5}\}$ in Eq. 2 lead to mixing of form factors F_3 and F_2, resulting in a spurious contribution to the electric dipole form factor

$$F_3 \to \tilde{F}_3 = F_3 - 2\alpha_5 F_2 . \qquad (4)$$

(conventions are consistent with our definitions in Ref. [11]).

3 EDM from form factors

In this section we summarize briefly our methodology, with full details provided in Ref. [11]. The CP-odd ($\overline{CP}$) form factor F_3 is extracted from nucleon matrix elements in presence of $\overline{CP}$ interactions with the action $S_{CP-even} + i\delta^{\overline{CP}}S = S_{CP-even} + i\tilde{\delta}_\psi \sum_x [O_{\psi\tilde{G}}]_x$. To compute them, we evaluate corrections to the nucleon correlation functions $C^{C\!\!\!/P} = C^{CP-even} - i\tilde{\delta}_\psi \delta^{\overline{CP}}C$ linear in the CP-odd operator $O_{\psi\tilde{G}}$:

$$\delta^{\overline{CP}}C_{NJ\bar{N}}(\vec{p}', \vec{q}; t_{sep}, t_{op}) = \sum_{x,\vec{y},\vec{z}} e^{-i\vec{p}'\vec{y}+i\vec{q}\vec{z}} \langle N(\vec{y}, t_{sep}) [\bar{q}\gamma^\mu q]_{(\vec{z},t_{op})} \bar{N}(0) [O_{\psi\tilde{G}}]_x\rangle , \qquad (5)$$

$$\delta^{\overline{CP}}C_{N\bar{N}}(\vec{p}; t_{sep}) = \sum_{x,\vec{y}} e^{-i\vec{p}'\vec{y}} \langle N(\vec{y}, t_{sep})\bar{N}(0) [O_{\psi\tilde{G}}]_x\rangle , \qquad (6)$$

2
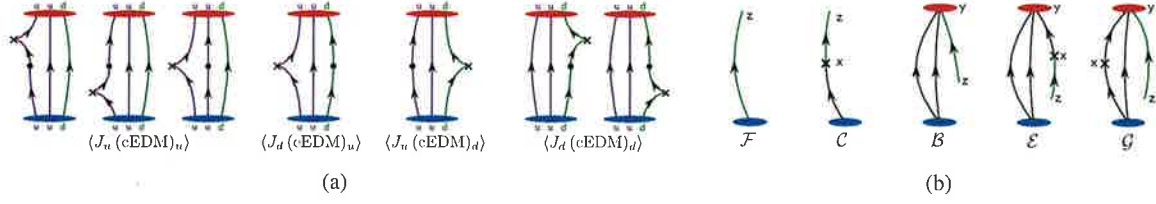
Figure 1. (a) Quark-connected contractions of the nucleon, quark current (dots), and cEDM operators (crosses). (b) Propagators required for computing quark-connected contractions of the nucleon, quark current, and cEDM operators.

in addition to the regular CP-even correlators $C_{N\bar{N}} = \langle N\bar{N} \rangle$ and $C_{NJ\bar{N}} = \langle N[\bar{q}\gamma^\mu q]\bar{N} \rangle$. Connected insertions of $O_{\psi\bar{G}}$ generate contractions (shown in Fig. 1(a) for nucleon-current correlators) that are computed using a set of sequential propagators (Fig. 1(b)). The gluon field strength $T^a[g_S G^a_{\mu\nu}]^{\text{cont}}_x$ in the lattice chromo-EDM operator is approximated using a "clover" plaquettes centered at site x,

$$(T^a[g_S G^a_{\mu\nu}]^{\text{cont}}_x) \rightarrow [G_{\mu\nu}]^{\text{clov}} = \frac{1}{8i}[(U^P_{x,+\hat{\mu},+\hat{\nu}} + U^P_{x,+\hat{\nu},-\hat{\mu}} + U^P_{x,-\hat{\mu},-\hat{\nu}} + U^P_{x,-\hat{\nu},+\hat{\mu}}) - \text{h.c.}] \tag{7}$$

Nucleon energies in CP-even vacuum are extracted from the $\langle N\bar{N} \rangle$ correlator with 2-state fits (Fig. 2(a)). As widely discussed in the literature, in a $\mathcal{CP}$ vacuum the lattice nucleon operator $(u^T C\gamma_5 d)u$ creates a parity-mixed state so that the nucleon propagator is

$$C^{\mathcal{CP}}_{N\bar{N}}(\vec{p}, t)\Big|_{g.s.} = \frac{-i\not{p}_{\mathcal{E}uc} + m_N e^{2i\alpha_5\gamma_5}}{2E_N}e^{-E_N t} = e^{i\alpha_5\gamma_5}\frac{-i\not{p}_{\mathcal{E}uc} + m_N}{2E_N}e^{i\alpha_5\gamma_5}e^{-E_N t}. \tag{8}$$

where $(-i\not{p}_{\mathcal{E}uc}) = E_N\gamma^4_{\mathcal{E}uc} - i\vec{p}\cdot\vec{\gamma}_{\mathcal{E}uc}$. In order to subtract the $F_{1,2}$ terms mixing with F_3, we calculate the parity-mixing angle α_5 from the ratio of the nucleon correlators,

$$\alpha_5^{eff}(t) = -\frac{\text{Tr}[T^+\gamma_5\delta^{\overline{CP}}C_{N\bar{N}}(t)]}{\text{Tr}[T^+ C_{N\bar{N}}(t)]} \overset{t\rightarrow\infty}{=} \alpha_5 \tag{9}$$

where $T^+ = \frac{1+\gamma_4}{2}$, and extract the value from its plateaus (see Fig. 2(b)).

To evaluate the ground state matrix elements, we use the three central plateau points of the ratios,

$$[\delta^{\overline{CP}}]\mathcal{R}_{NJ\bar{N}}(t_{sep}, t_{op}) = \frac{[\delta^{\overline{CP}}]C_{NJ\bar{N}}(t_{sep}, t_{op})}{c'_2(t_{sep})}\sqrt{\frac{c'_2(t_{sep})}{c_2(t_{sep})}\frac{c'_2(t_{op})}{c_2(t_{op})}\frac{c_2(t_{sep}-t_{op})}{c'_2(t_{sep}-t_{op})}}, \tag{10}$$

where $c_2^{(\prime)}(t) = \text{Tr}[T^+ C_{N\bar{N}}(\vec{p}^{(\prime)}, t)]$ with a fixed sink momentum $\vec{p}' = \vec{0}$ and varying source momenta $\vec{p}$. Form factors $F_{1,2}$ are extracted with overdetermined analysis of matrix elements of the quark vector current averaged over directions of the current and the momentum transfer $\vec{q} = (\vec{p}' - p)$ that give equivalent combinations of $F_{1,2}$ as described, e.g., in Ref. [12]. The results for the Sachs form factors $G_E = F_1 - \tau F_2$ and $G_M = F_1 + F_2$ (where $\tau = \frac{Q^2}{4m_N^2} = \frac{E_N - m_N}{2m_N}$) are shown in Fig. 3(a) and demonstrate that there is no observable t_{sep} dependence indicating no substantial contamination from excited states. The electric dipole form factor F_3 can be extracted from both timelike and spacelike current. We find that the time component $\langle NJ^4\bar{N} \rangle$ yields much better precision than the spacelike component $\langle NJ^3\bar{N} \rangle$.
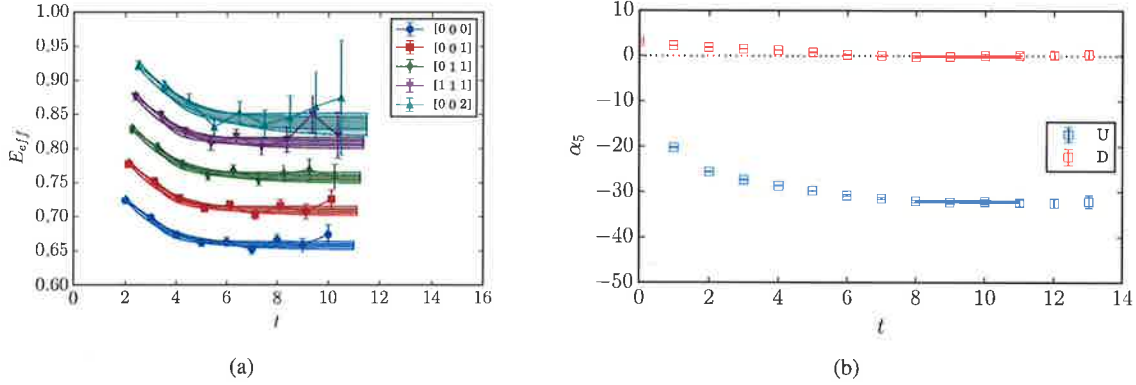
Figure 2. (a) Effective energy plots for $\vec{p}^2 \leq 4p_{\min}^2$, together with two-state fits. (b) Chiral rotation angle α_5 of the proton field induced by u- and d-quark cEDM interactions (no disconnected contractions, no renormalization);

Combining these components did not result in improved precision of the F_3 form factor. With only the $\langle NJ^4\bar{N} \rangle$ component, the form factor is given by the expression

$$(1 + \tau)F_3(Q^2) = \frac{m_N}{q_3 \mathcal{K}_\mathcal{R}} \text{Tr}[T_{S_z+}^+ \delta^{\overline{CP}} \mathcal{R}_{NJ^4\bar{N}}] - \alpha_5 G_E(Q^2), \tag{11}$$

where $T_{S_z+}^+ = T^+(1 + \Sigma_z) = \frac{1}{2}(1 + \gamma_4)(1 - i\gamma_1\gamma_2)$, $\tau = \frac{Q^2}{4m_N^2} = \frac{E_N - m_N}{2m_N}$ and $\mathcal{K}_\mathcal{R} = \frac{m_N}{\sqrt{2E_N(m_N + E_N)}}$. The second term in the rhs is the mixing subtraction. Following our recent paper [11], we stress that the correct subtraction of $F_{1,2}$ mixing contributions from F_3 is different from the expressions used in earlier Refs. [4–10] by a term $(-2\alpha_5 F_3)$ due to spurious mixing of F_2 and F_3 in the vector current vertex in these papers. The F_3 results for the four combinations of flavors in the cEDM and current operators are shown in Fig. 3(b), again demonstrating no visible t_{sep} dependence.

4 EDM from energy shift

Calculation of the dipole moment using a uniform background field is advantageous because it does not require momentum extrapolation. The calculation is straightforward for the neutron, but for the proton one has to analyze correlation functions that are complicated due to its constant acceleration [13]. However, since the uniform background field is quantized on a lattice, it cannot be made arbitrarily small. For small lattices, the minimal field strength is very large with the magnitude comparable to the QCD scale: $\mathcal{E}_0 = \frac{3 \cdot 2\pi}{a^2 L_x L_t}$, where the extra factor of 3 is due to the fractional quark charge. On the $24^3 \times 64$ lattice, the minimal electric field is $\mathcal{E}_0 \approx 0.04 \, \text{GeV}^2 \approx 200 \, \text{MV/fm}$. Such an electric field pulls the u quark in the neutron with tension $\approx (160 \, \text{MeV})^2$, or approximately 14% of the QCD string tension. Although it may deform the neutron away from the ground state, we believe that the correction is insignificant at our level of precision.

We introduce the uniform electric field along the z axis of the lattice as described in Ref. [11]. Using the modified QCD+$U(1)$ gauge links, we calculate the regular nucleon correlator $C_{N\bar{N},\mathcal{E}}$ and its linear perturbation in the CP-odd interaction $\sum_x[O_{\psi\tilde{G}}]_x$, similarly to the form factor F_3:

$$\delta^{\overline{CP}} C_{N\bar{N},\mathcal{E}}(\vec{p}, t) = \sum_{\vec{y}} e^{-i\vec{p}(\vec{y}-\vec{x})} \langle N(\vec{y}, t)\bar{N}(\vec{x}, 0) O_{\psi\tilde{G}}^{\overline{CP}} \rangle_\mathcal{E}. \tag{12}$$

4
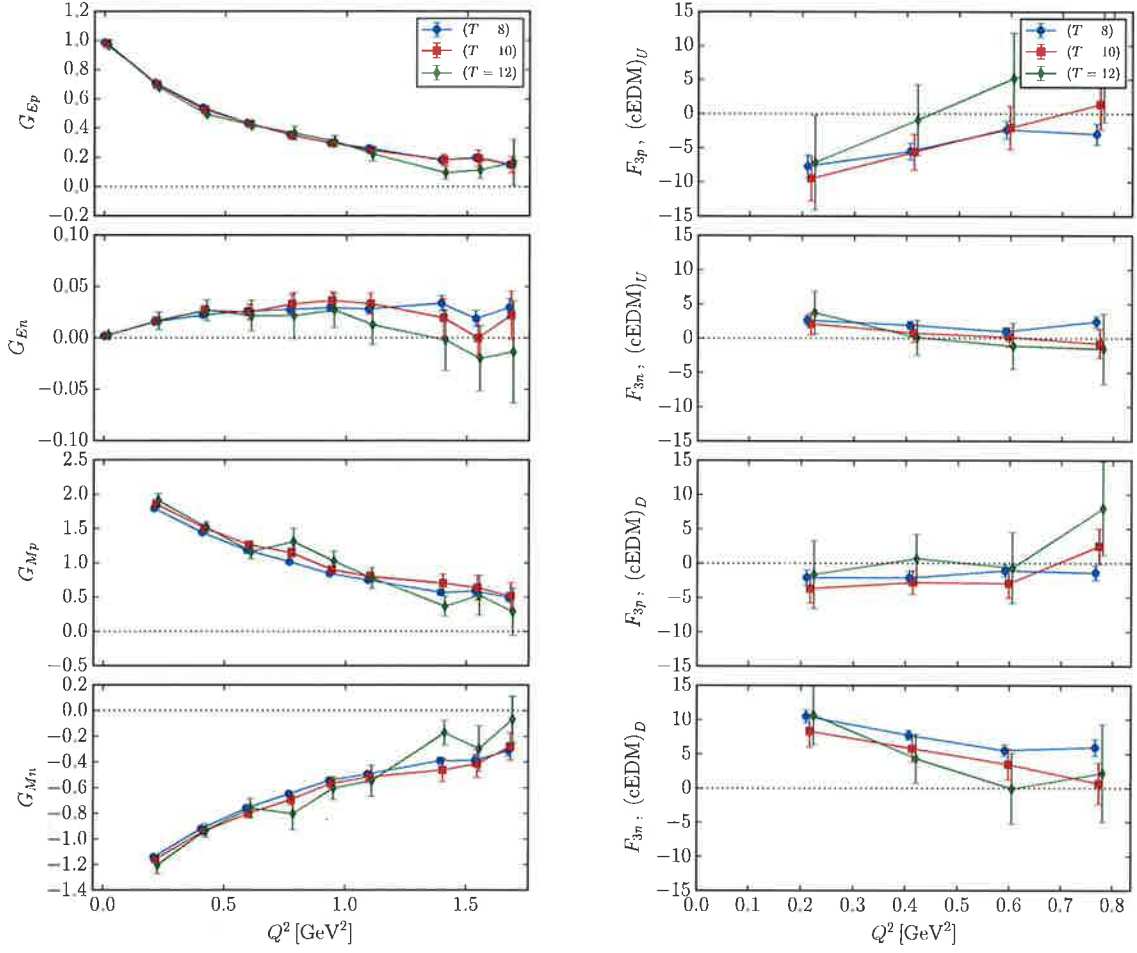
(a) P,T-even nucleon vector current (Sachs) form factors.

(b) Electric dipole form factors F_3 induced by u- and d-quark chromo-EDM interactions.

Figure 3. Nucleon CP-even and CP-odd electric dipole form factors. Disconnected contractions are not included for either currents or cEDM insertions.
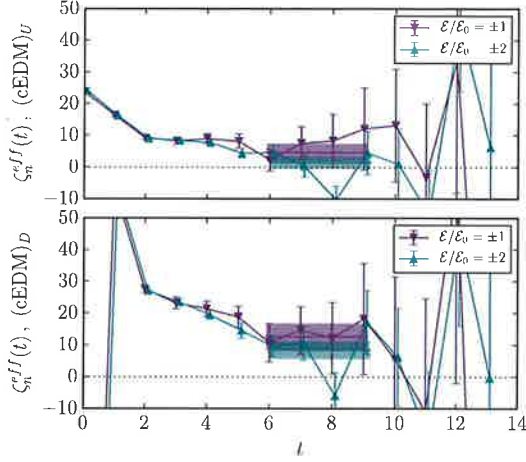
It is important to use the modified gauge links both in the computing of the propagators and in the construction of the smeared sources and sinks. Otherwise, since the individual quarks are charged, the results would depend on the gauge of the electromagnetic potential. For the purpose of source construction, the QCD gauge links are first APE-smeared, with the electromagnetic potential applied afterwards.

Following Ref.[11], the energy of a polarized ($\vec{S} \| \vec{E}$) particle on a Euclidean lattice is shifted by the imaginary value $\delta E = -(\zeta/2m)i\mathcal{E}$. The nucleon correlator at rest ($\vec{p} = 0$) is therefore
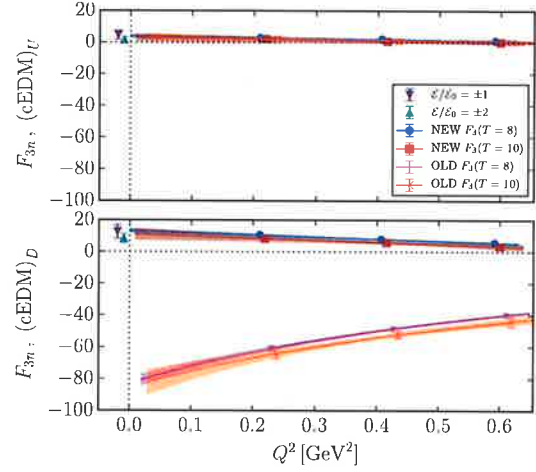
$$C^{\cancel{CP}}_{N\bar{N},\mathcal{E}}(\vec{p} = 0, t) = |Z_N|^2 e^{i\alpha_5\gamma_5} \frac{1+\gamma_4}{2} \left[\frac{1+\Sigma_z}{2} e^{-(m_N+\delta E)t} + \frac{1-\Sigma_z}{2} e^{-(m_N-\delta E)t} \right] e^{i\alpha_5\gamma_5} . \tag{13}$$

Analogously to the treatment of the CP-odd form factor F_3, the linear perturbation in $\sum_x[O_{\psi\bar{G}}]_x$ yields

$$C_{N\bar{N},\mathcal{E}} - i\tilde{\delta}_\psi \, \delta^{\overline{CP}} C_{N\bar{N},\mathcal{E}} \stackrel{t\to\infty}{=} |Z_N|^2 e^{-m_N t} \left[\frac{1+\gamma_4}{2} + i\alpha_5\gamma_5 - \Sigma_z \, \delta E \, t \right], \tag{14}$$

5

(a) The neutron EDM computed from energy shifts with two values of the electric field. The units are dimensionless and the scale is the same as for F_3.

(b) Comparison of the neutron EDFF $F_{3n}(Q^2)$ computed with the conventional [4–10] and the new (11) formulas to the energy shift results.

from which we obtain the following estimator for the electric dipole moment:

$$\zeta^{\text{eff}}(t) = 2m_N d_N^{\text{eff}}(t) = -\frac{2m_N}{\mathcal{E}_z}[R_z(t+1) - R_z(t)], \quad R_z(t) = \frac{\text{Tr}[T^+ \Sigma_z \delta^{\overline{CP}} C_{N\tilde{N},\mathcal{E}_z}(t)]}{\text{Tr}[T^+ C_{N\tilde{N},\mathcal{E}_z}(t)]}. \quad (15)$$

To estimate the effects of the ground state distortion by the background field, we have computed the neutron correlation functions with two values $\mathcal{E} = \mathcal{E}_0$ and $2\mathcal{E}_0$, with the results for $\zeta^{\text{eff}}(t)$ shown in Fig. 4(a). We choose $t = 6 \ldots 9$ as the common plateau to estimate the value of ζ for both flavors $\psi = u, d$. In the case of d-quark cEDM, we observe nonzero values of the energy shift. The EDM values computed with $\mathcal{E} = \mathcal{E}_0$ and $2\mathcal{E}_0$ agree well with each other, indicating that the energy shift is linear in $\mathcal{E}$ and our current EDM results are not sensitive to the nucleon ground state distortion by the electric field.

Our calculation of the neutron EDM from the energy shift does not depend on the parity mixing (chiral phase rotation) of the nucleon states in the $\mathcal{CP}$ vacuum. This mixing, if not taken into account correctly, may lead to the spurious mixing between F_3 and F_2. By comparing the energy shift and the form factor results, we demonstrate numerically that indeed our analysis of the parity mixing in Ref. [11] is correct. This comparison is shown in Fig. 4(b), together with form factor values $\tilde{F}_3$ computed using expressions from Refs. [4–10] given in Eq. 4. The $\tilde{F}_3$ values are different from F_3 only if the mixing angle α_5 is relatively large. Thus, in the case of u-quark cEDM, the value α_5 for the neutron is small, and there is no observable difference between F_3 and $\tilde{F}_3$. However, in the case of d-quark cEDM, the difference is remarkable. It is clear that even with the current uncertainties due to excited states and the $Q^2 \to 0$ extrapolation, the comparison favors agreement with F_3 but not $\tilde{F}_3$. This agreement between the energy shift and the form factor calculation of nEDM serves as a numerical cross-check of our analysis of the latter. We believe that this discrepancy has not been detected in earlier lattice studies of the $\bar{\theta}_{QCD}$-induced nucleon EDM because of the small corresponding values of the mixing angle as well as the relatively large statistical errors, which made the comparison between Refs. [2] and [4] inconclusive.

5 Summary

We have performed calculations of the nucleon electric dipole moments induced by CP-odd quark-gluon electric dipole interactions using two different methods. In the first method we calculate nucleon matrix elements in the $\mathcal{CP}$ vacuum by evaluating up to four-point connected nucleon correlation functions. The CP-odd interaction is treated as perturbation of the CP-even vacuum and is straightforward to analyze using sequential propagators. We have performed calculations with three different source-sink separations and we do not observe any significant excited states in either the CP-even form factors $F_{1,2}$ or the CP-odd electric dipole form factor F_3.

In the second method, we also treat the CP-odd interaction as a perturbation but compute the neutron EDM from the energy shift in a uniform background electric field. The energy-shift method for computing the nucleon EDM has been used before [1–3], but our calculation is the first one that uses the uniform background electric field respecting periodic boundary conditions [13]. The statistics is identical in both methods, and the energy shift results agree with the form factors F_3 if they are computed with the new formula [11]. Also, both methods yield comparable uncertainty. The energy-shift method may be preferable in the future because it does not require forward-limit extrapolation, and excited states may be easier to control [14].

In both methods, we observe statistically significant signal for the cEDM-induced nucleon EDM, although we do not include disconnected diagrams in our calculation, and our results are not yet renormalized. Comparison of the results from the two methods serve as a numerical cross-check of our recent finding that the extraction of the CP-odd form factor F_3 and the nucleon EDM in previous works was incorrect. Specifically, the reported values contain spurious contributions $(-2\alpha_5\kappa)$ due to mixing with the anomalous magnetic moment κ of the nucleon.

We plan to calculate disconnected contributions to the F_3 form factors as an extension to the presented work, combining the existing nucleon correlators and quark-disconnected loops with insertions of the quark current, chromo-EDM, and both. Although the disconnected contractions are simpler to construct (they do not require four-point correlators and associated sequential propagators), we expect that the stochastic noise will be a much bigger problem compared to the connected contractions.

ACKNOWLEDGEMENTS

References

[1] S. Aoki, A. Gocksch, Phys. Rev. Lett. **63**, 1125 (1989), [Erratum: Phys. Rev. Lett.65,1172(1990)]

[2] E. Shintani, S. Aoki, N. Ishizuka, K. Kanaya, Y. Kikukawa et al., Phys.Rev. **D75**, 034507 (2007), hep-lat/0611032

[3] E. Shintani, S. Aoki, Y. Kuramashi, Phys.Rev. **D78**, 014503 (2008), 0803.0797

[4] E. Shintani, S. Aoki, N. Ishizuka, K. Kanaya, Y. Kikukawa et al., Phys.Rev. **D72**, 014504 (2005), hep-lat/0505022

[5] F. Berruto, T. Blum, K. Orginos, A. Soni, Phys.Rev. **D73**, 054509 (2006), hep-lat/0512004

[6] R. Horsley, Y. Nakamura, D. Pleiter et al., *The Electric dipole moment of the nucleon from simulations at imaginary vacuum angle theta* (2008), 0808.1428

[7] F.K. Guo, R. Horsley, U.G. Meissner, Y. Nakamura, H. Perlt, P.E.L. Rakow, G. Schierholz, A. Schiller, J.M. Zanotti, Phys. Rev. Lett. **115**, 062001 (2015), 1502.02295

[8] A. Shindler, T. Luu, J. de Vries, Phys. Rev. **D92**, 094518 (2015), 1507.02343

[9] C. Alexandrou, A. Athenodorou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, G. Koutsou, K. Ottnad, M. Petschlies, Phys. Rev. **D93**, 074503 (2016), 1510.05823

[10] E. Shintani, T. Blum, T. Izubuchi, A. Soni, Phys. Rev. **D93**, 094503 (2016), 1512.00566

[11] M. Abramczyk, S. Aoki, T. Blum, T. Izubuchi, H. Ohki, S. Syritsyn, Phys. Rev. **D96**, 014501 (2017), 1701.07792

[12] S. Syritsyn, J. Bratt, M. Lin, H. Meyer, J. Negele et al., Phys.Rev. **D81**, 034507 (2010), 0907.4194

[13] W. Detmold, B.C. Tiburzi, A. Walker-Loud, Phys. Rev. **D79**, 094505 (2009), 0904.1586

[14] C. Bouchard, C.C. Chang, T. Kurth, K. Orginos, A. Walker-Loud (2016), 1612.06963