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# Quantum Error Correcting Code Simulation

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Work with Andrew Landahl, Tzvetan Metodi

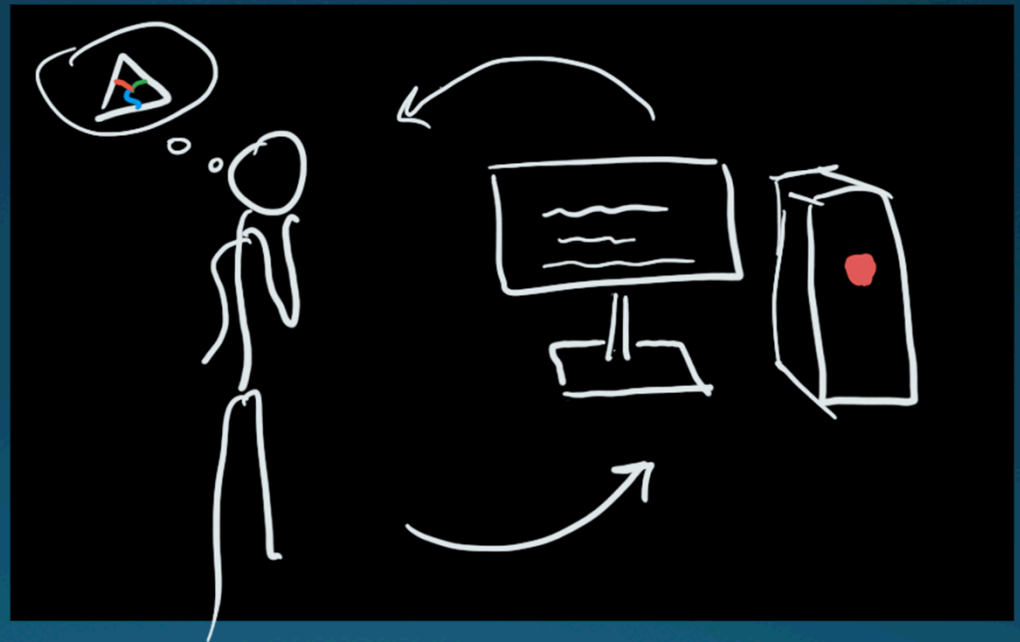
September 21, 2016



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# A Program for QECCs

Performance  
Estimator for  
Codes  
On  
Surfaces



# Focus

- Show the PECOS interface
- Present some concepts in QECC

# Modular Steps

choose QECCs



Define a sequence of Logical Ops



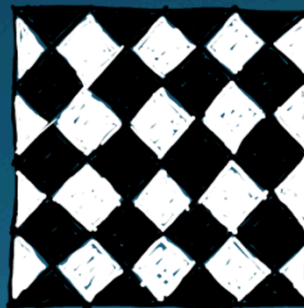
Select an error model



Run analysis on QECCs

# Choosing QECCs

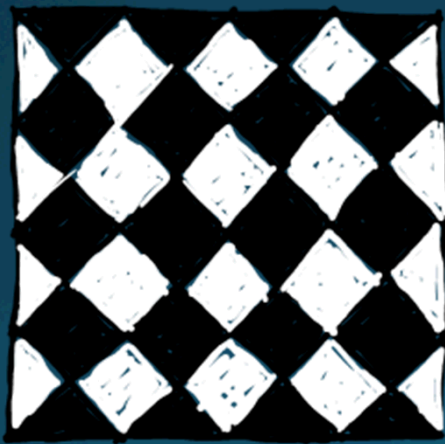
```
import pecos as pc  
Surface = pc.qecc.Surface4444(distance=5)
```



4.4.4.4 surface code

# Stabilizers

$$S_i |\psi\rangle = +1 |\psi\rangle$$



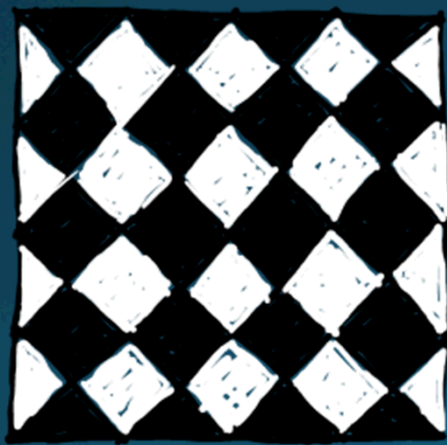
4.4.4.4

$$2^n \rightarrow 2^{n-\#S} \rightarrow 2^k$$

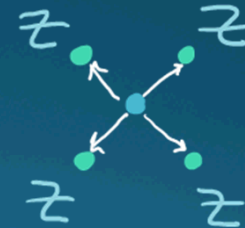
$$\prod_i S_i |\bar{\psi}\rangle = +1 |\bar{\psi}\rangle$$

# Stabilizers

$$S_i |\psi\rangle = +1 |\psi\rangle$$



4.4.4.4



# Stabilizers



$$\begin{cases} |++++\rangle \rightarrow 0 \\ |++-+\rangle \rightarrow 1 \end{cases}$$

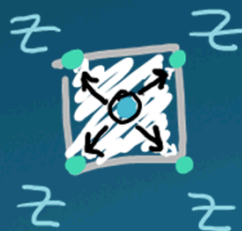
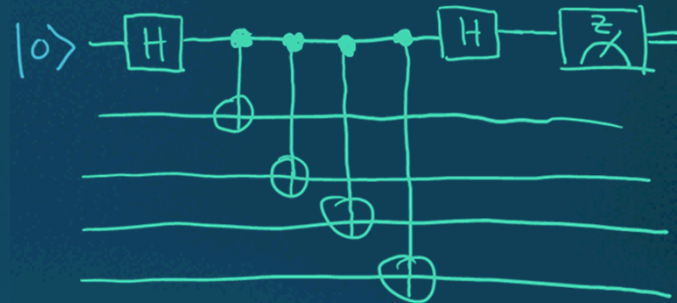


$$\begin{cases} |0101\rangle \rightarrow 0 \\ |0111\rangle \rightarrow 1 \end{cases}$$

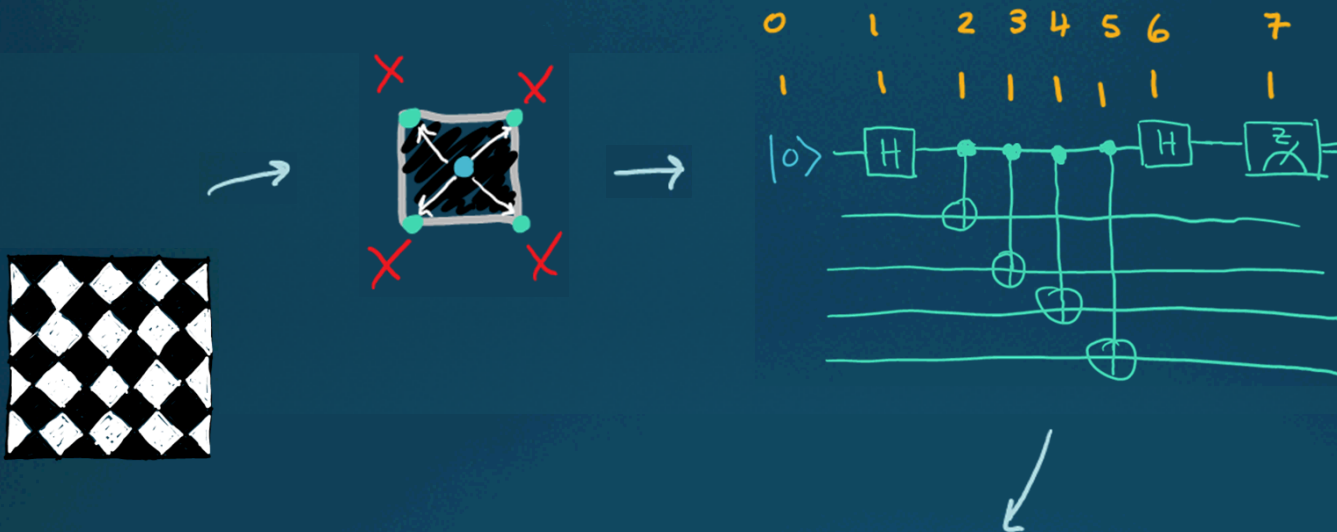
# Checks to Circuits



4.4.4.4



# Checks to Circuits



timing info

$\text{check}([(x,0), (x,1), \dots], \text{~~~~~})$

$\text{check}([x, (0,1,2,3)], \text{~~~~~})$

# Circuits

$check([X, (0,1,2,3)], \underline{\quad})$

$\downarrow \{ \}$

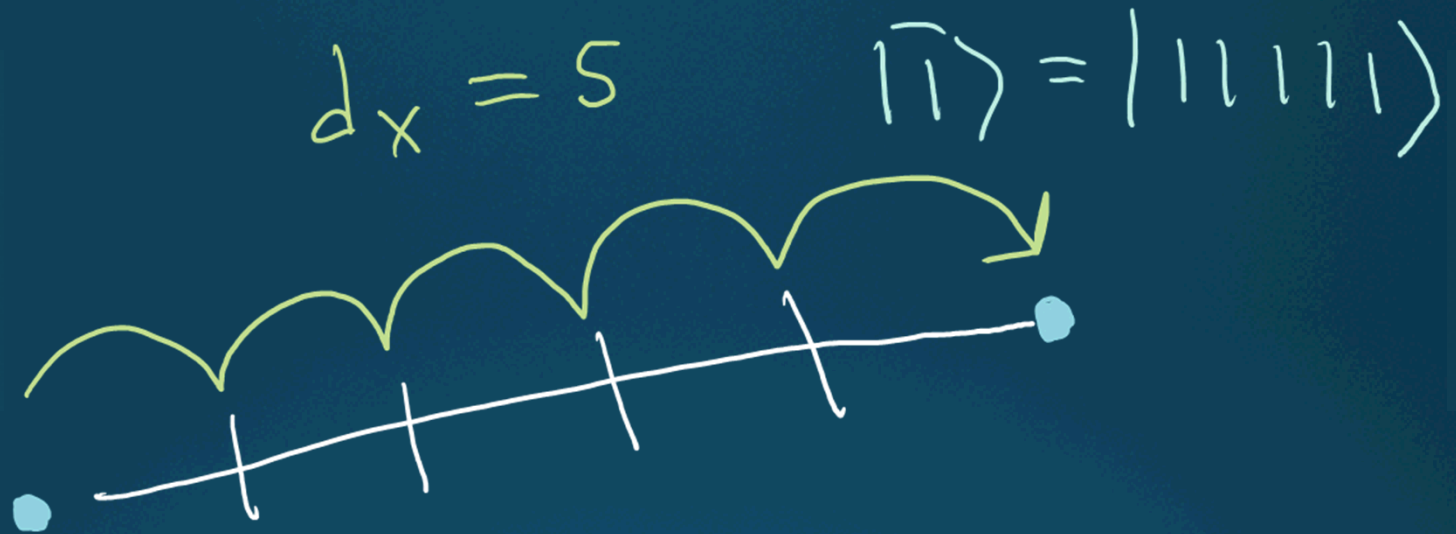
$[$        $\{ 'init\ 10>' : \{ 4, 7, 11, 52 \}, \}$        $\leftarrow 0$       *tick*  
           $\{ 'H' : \{ 4, 7, 11, 52 \}, \}$        $\leftarrow 1$   
           $\{ 'cNoT' : \{ (4, 0), \dots \}, \dots \}$        $\leftarrow 2$   
           $\dots ]$        $\vdots$

# Checks to Circuits

*Surface = pc.qecc.Surface4444(distance=5)*

```
# Intialize an instance of a QECC  
surface = pc.qecc.Surface4444(distance=5)
```

# Distance



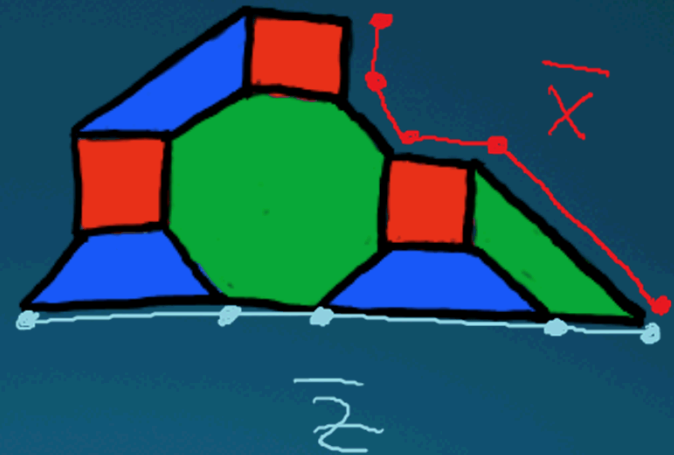
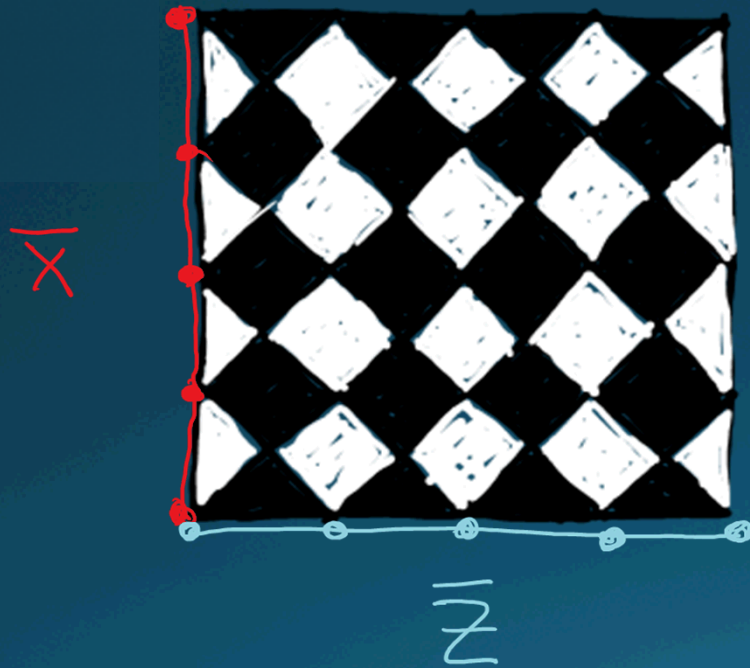
$$|\bar{0}\rangle = |00000\rangle$$

# Distance



$$t = \left\lfloor \frac{d-1}{2} \right\rfloor$$

# Distance



# Logic

```
surface_layout = surface.layout
```

```
L = pc.Circuit.Logic(surface_layout)
```



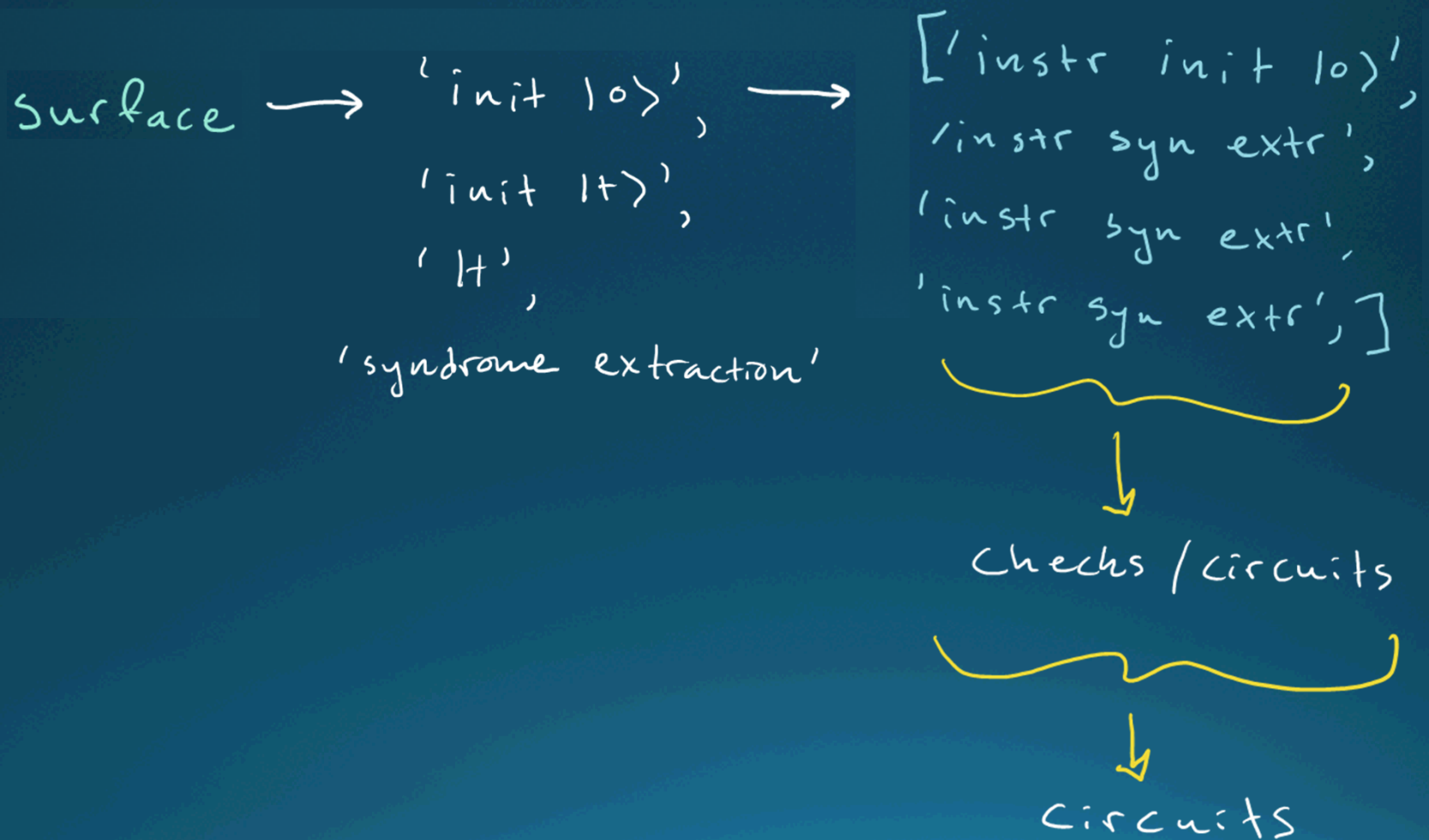
# Logical Gates

$L.\text{gate}(\text{surface}, \text{'ideal init } |\psi\rangle')$

$L.\text{gate}(\text{surface}, \text{'syndrome extraction'})$

$\underbrace{\quad\quad\quad}_{\begin{matrix} \langle S \\ I \end{matrix}}$

# Logical gates vs logical instructions



# Logic

```
surface = pc.qecc.Surface4444(distance=5)

L = pc.circuit.Logic(layout=surface.layout)
L.gate(surface_code, 'ideal init |psi>')
L.gate(surface_code, 'syndrome_extraction')
```

# Error generator/model

```
depolar = pc.error_gen.GateWise('symmetric-depolarizing')
```



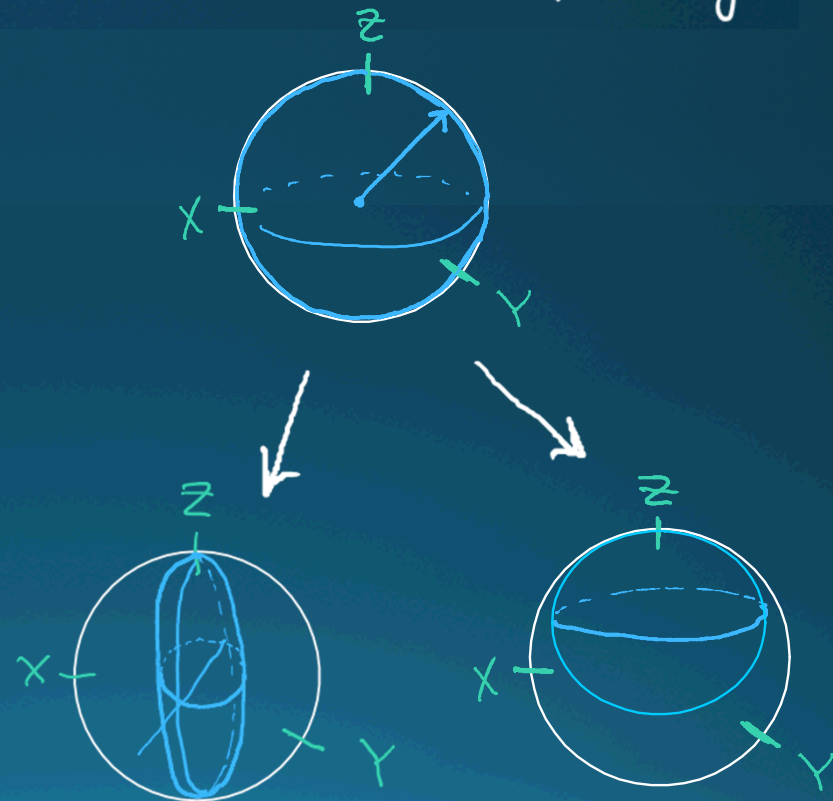
Per gate:

- insert error before
- insert error after
- replace the gate

# Error generator/model

```
depolar = pc.error_gen.GateWise('symmetric-depolarizing')
```

- Paulis
- Cliffords
- Measurements
- Leakage
- much more



# Error generator/model

```
surface = pc.qecc.Surface4444(distance=5)

L = pc.circuit.Logic(layout=surface.layout)
L.gate(surface_code, 'ideal init |psi>')
L.gate(surface_code, 'syndrome_extraction')

depolar = pc.error_gen.Gatewise(error_model='symmetric depolarizing')
```

# Simulation model

```
Sim_model = pc, Sim_model.StandardCodeCapacity( logic = L,  
                                                  error_gen = depolar)
```

- Initiates all qudits to  $|0\rangle$
- couples the circuits in logic and generates errors

# Stabilizer Simulation

- Realistic codes and error models  $\rightarrow$

$$\left[ \begin{array}{c|cccccc} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{array} \right]$$

- Normally much faster ( $n^{0.7}$  vs  $n^2$ )

# Levels of Modeling

- Code Capacity
  - Error on data qubits
- Phenomenological
  - Error on each data qubits and measurement
- Circuit
  - Error on each unitary and measurement

# Simulation Model

```
surface = pc.qecc.Surface4444(distance=5)

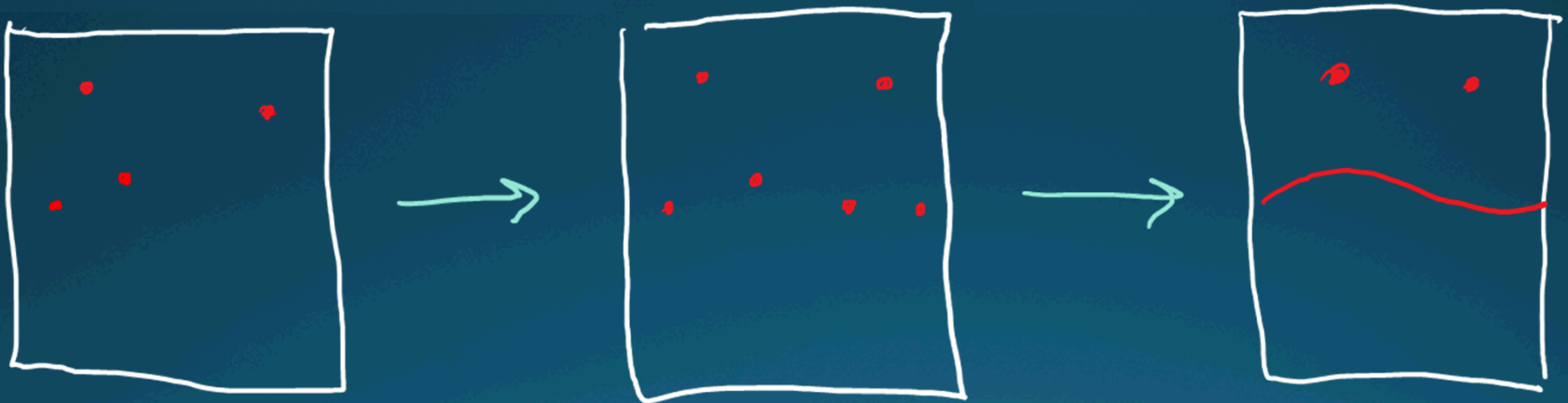
L = pc.circuit.Logic(layout=surface.layout)
L.gate(surface_code, 'ideal init |psi>')
L.gate(surface_code, 'syndrome_extraction')

depolar = pc.error_gen.Gatewise(error_model='symmetric depolarizing')

sim_model = pc.sim_model.StandardCodeCapacity(logic=L,
                                              error_generator=depolar)
```

# Decoder

```
diamonds = pc.decoder.ExpandingDiamonds(logic=L)
```



# Decoder

```
surface = pc.qecc.Surface4444(distance=5)

L = pc.circuit.Logic(layout=surface.layout)
L.gate(surface_code, 'ideal init |psi>')
L.gate(surface_code, 'syndrome_extraction')

depolar = pc.error_gen.Gatewise(error_model='symmetric depolarizing')

sim_model = pc.sim_model.StandardCodeCapacity(logic=L,
                                                error_generator=depolar)

diamonds = pc.decoder.ExpandingDiamonds(logic=L)
```

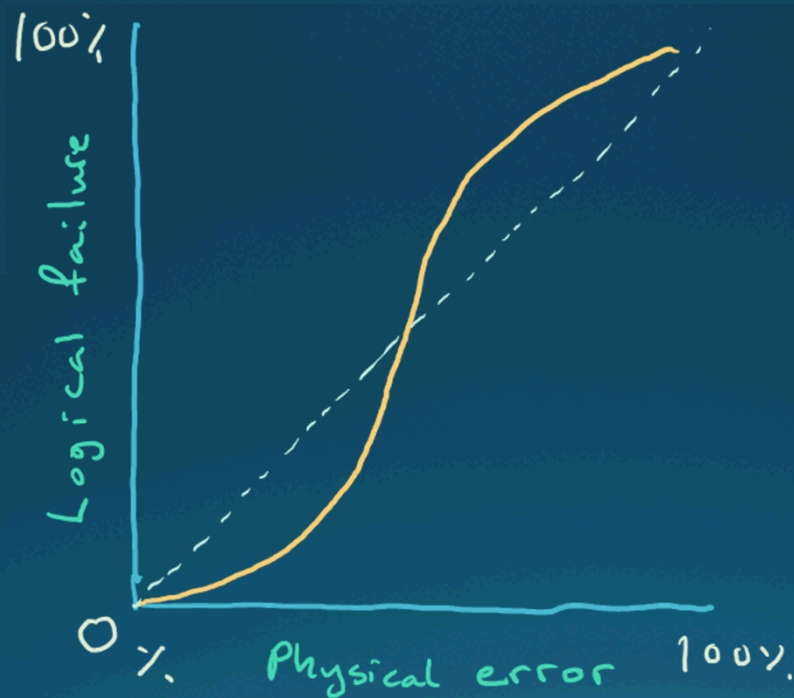
# Analysis Tool

```
pc.tool.monte_carlo.Defaultt(sim_model,  
                               error_params = um,  
                               decoder = diamonds)
```

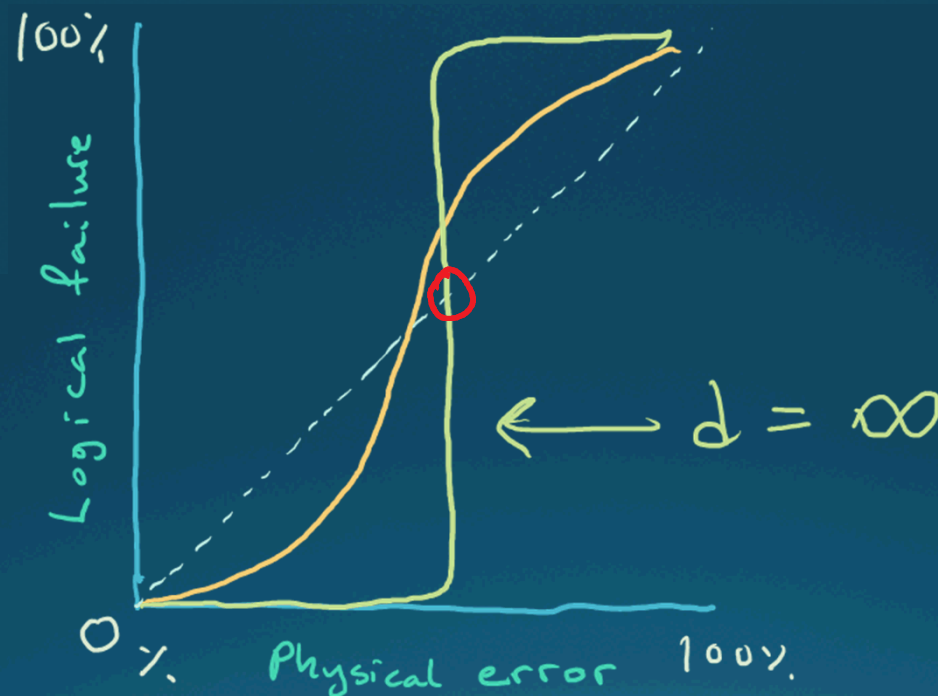


- Run the sim\_model many times.
- Determine the rate of logical error to physical error.

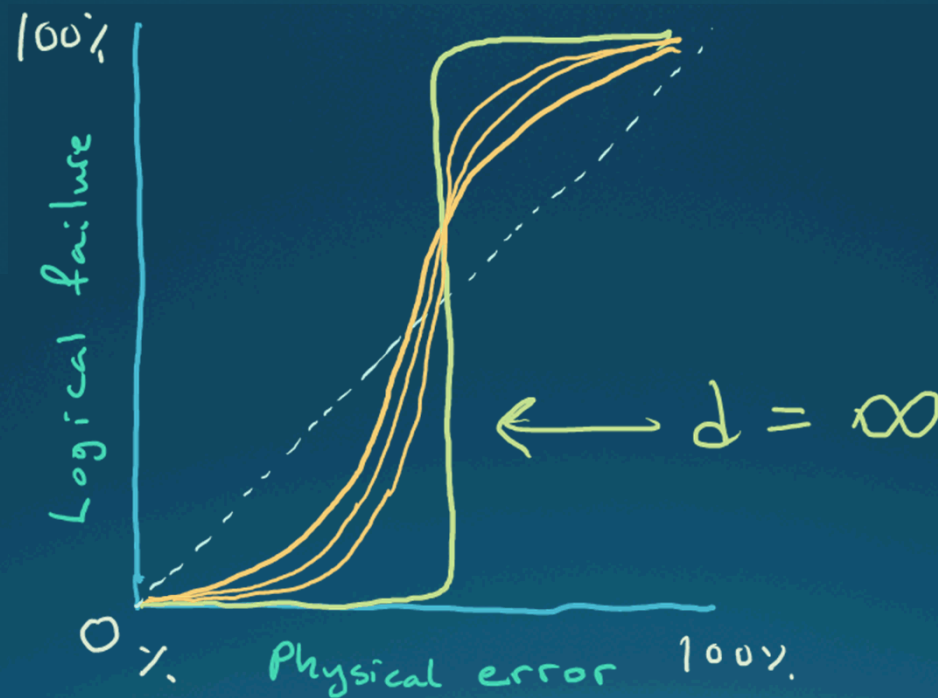
# Logical Error Rate



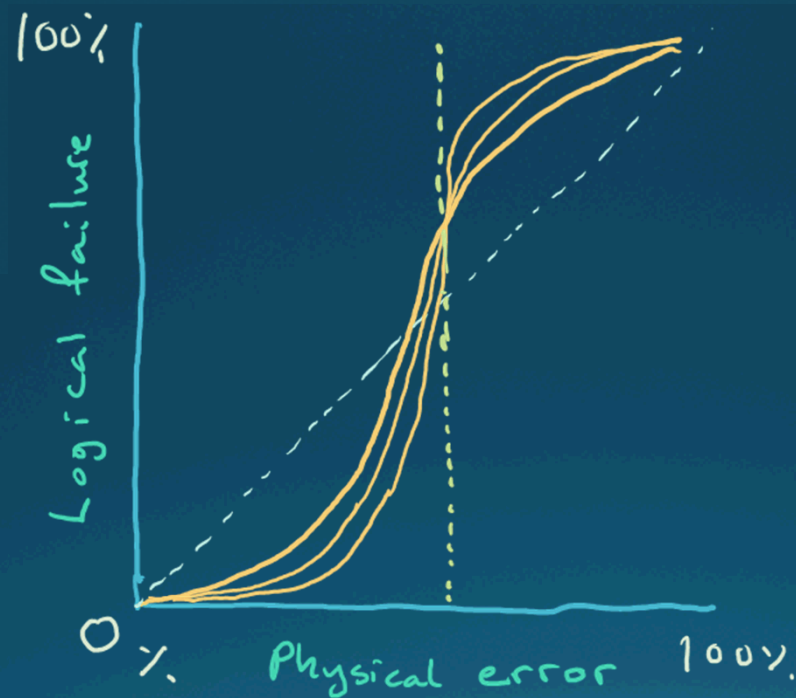
# Logical Error Rate



# Logical Error Rate



# Logical Error Rate



# Analysis Tool

```
surface = pc.qecc.Surface4444(distance=5)

L = pc.circuit.Logic(layout=surface.layout)
L.gate(surface_code, 'ideal init |psi>')
L.gate(surface_code, 'syndrome_extraction')

depolar = pc.error_gen.Gatewise(error_model='symmetric depolarizing')

sim_model = pc.sim_model.StandardCodeCapacity(logic=L,
                                              error_generator=depolar)

diamonds = pc.decoder.ExpandingDiamonds(logic=L)

monte = pc.tool.monte_carlo.Default(sim_model=sim_model,
                                   decoder=diamonds)
```

# The interface

Code →

layout →

Logical gates {

error →

How to model →

Find out something ↗

```
import pecos as pc

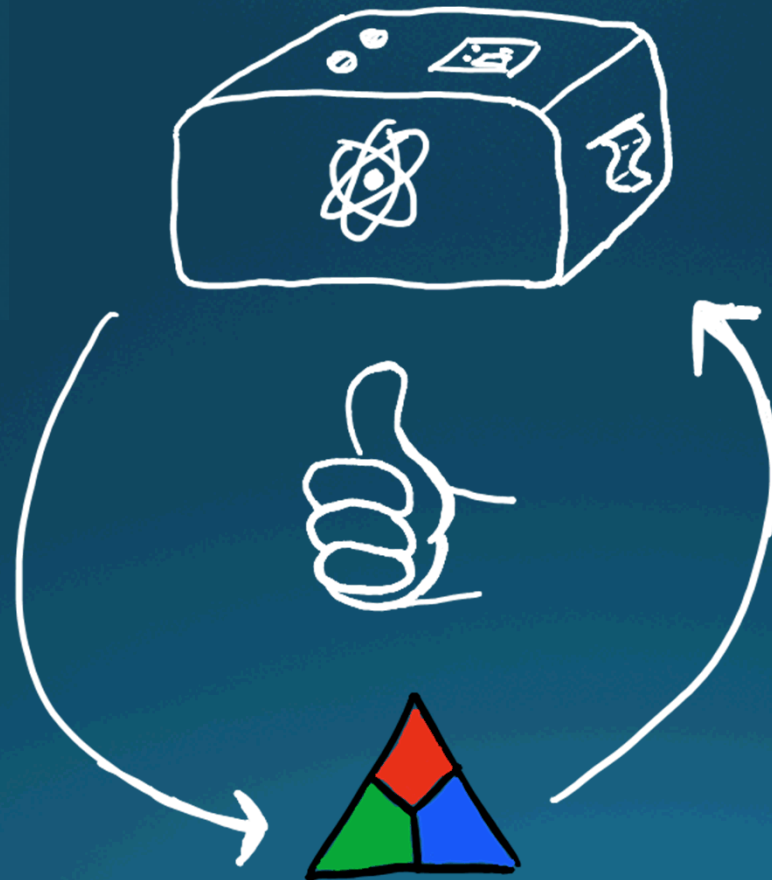
Surface = pc.qecc.Surface4444(distance=5)
surface_layout = Surface.layout

L = pc.Circuit.Logic(surface_layout)
L.gate(Surface, 'ideal init |psi>')
L.gate(Surface, 'Syndrome extraction')

depolar = pc.error_gen.GateWise('symmetric-depolarizing')

Sim_model = pc.Sim_model.StandardCodeCapacity(logic=L,
                                                error_gen=depolar)
pc.tool.monte_carlo.Default(Sim_model,
                             error_params=nm,
                             decoder=diamonds)
```

# Theory and experiment



# Future additions

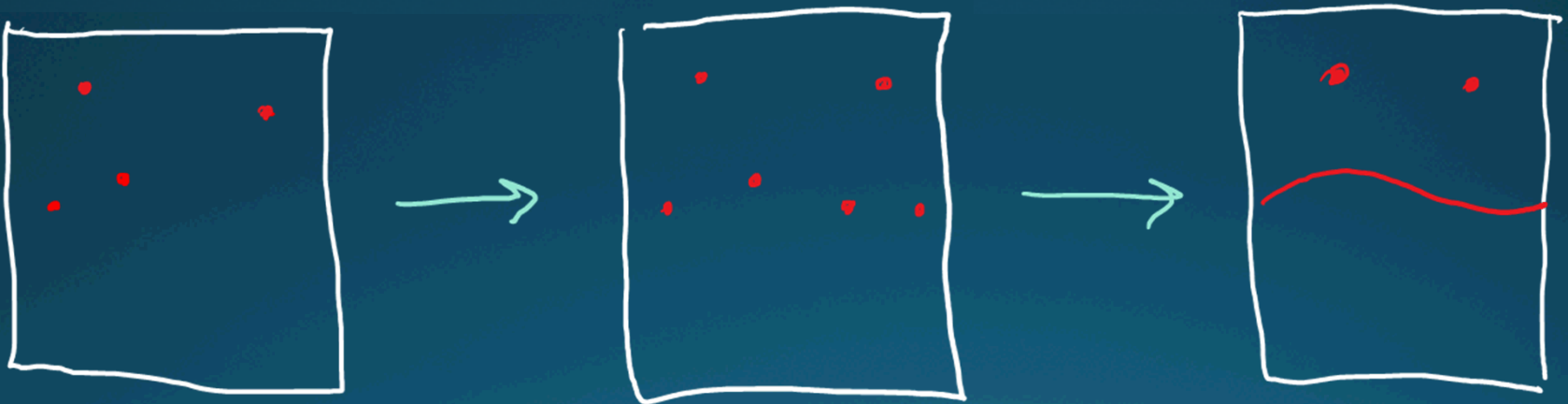
- Add gate duration
- A few more tweaks to efficiency
- Qudit stabilizer simulation
- Add the decoders being worked on
- Machine learning decoder

# Thanks!

# Extras

# Decoder

```
diamonds = pc.decoder.ExpandingDiamonds(logic=L)
```



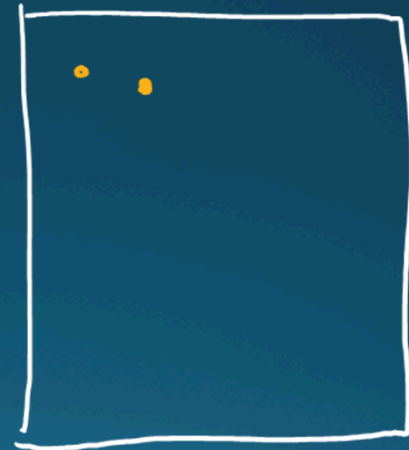
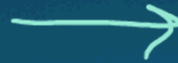
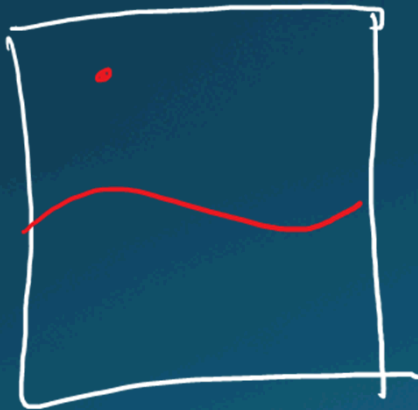
# Decoder

```
diamonds = pc.decoder.ExpandingDiamonds(logic=L)
```

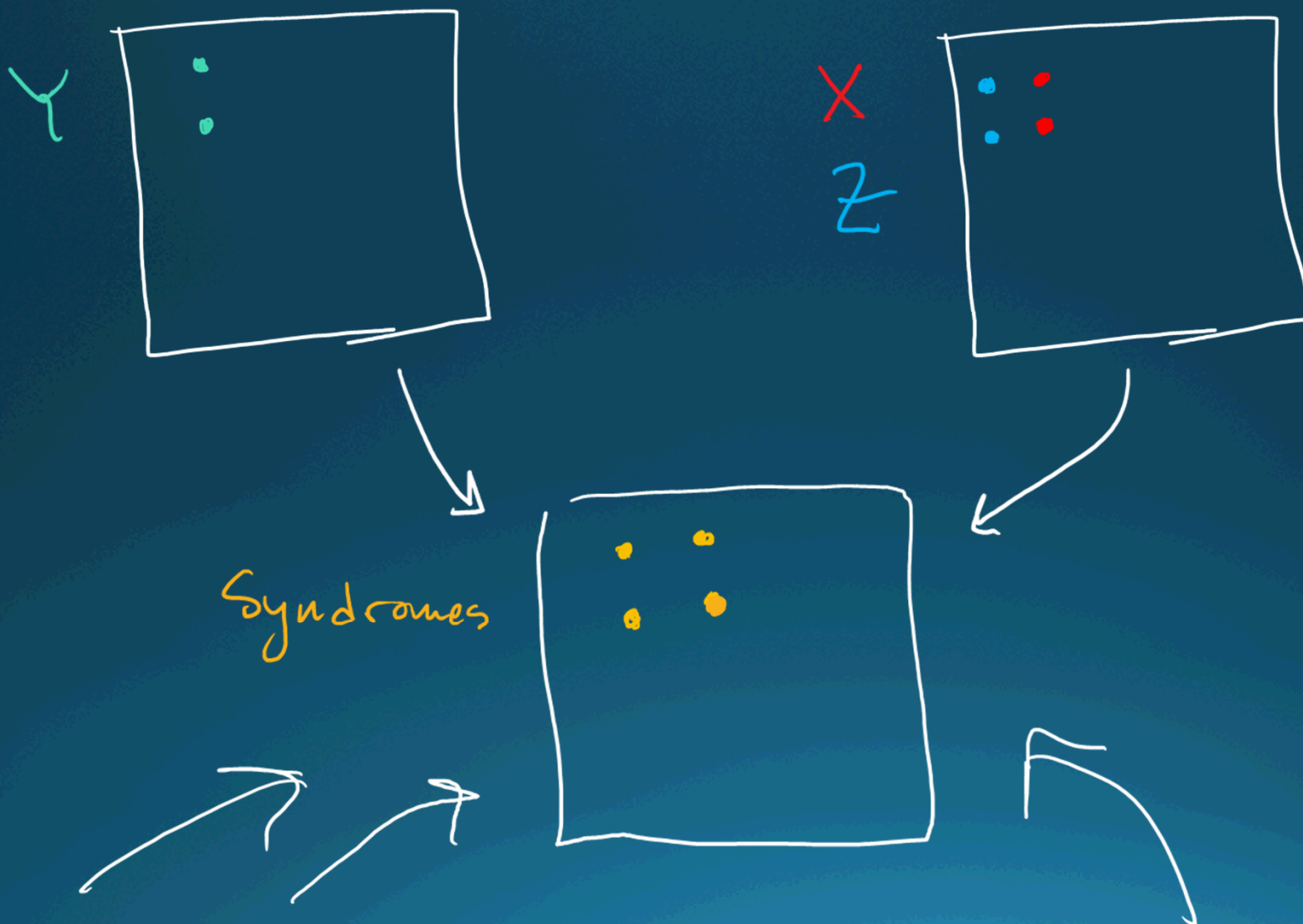


# Decoder

```
diamonds = pc.decoder.ExpandingDiamonds(logic=L)
```



# Decoder



# Error Model

↙ dictionary for replace, before, after

$\{ 'H' : \{ \text{set\_of\_errors}, 0.5, \dots \},$

$'CNOT' : \{ \dots \}, \dots \}$

$$\boxed{u} \in$$

$$\in \boxed{M_z}$$