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What randomized benchmarking actually measures

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U.S. DEPARTMENT OF ENERGY

Office of the Director of National Intelligence
IARPA
BE THE FUTURE

NNSA
National Nuclear Security Administration

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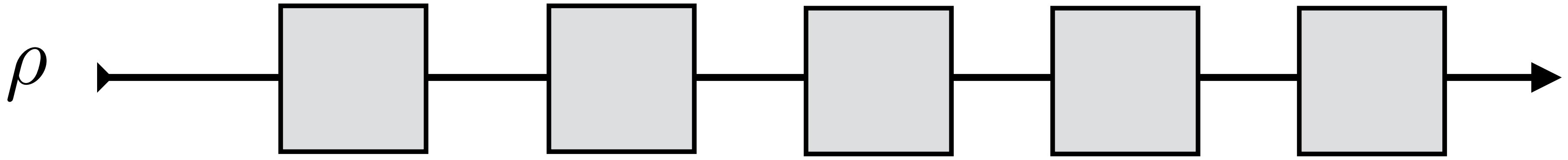
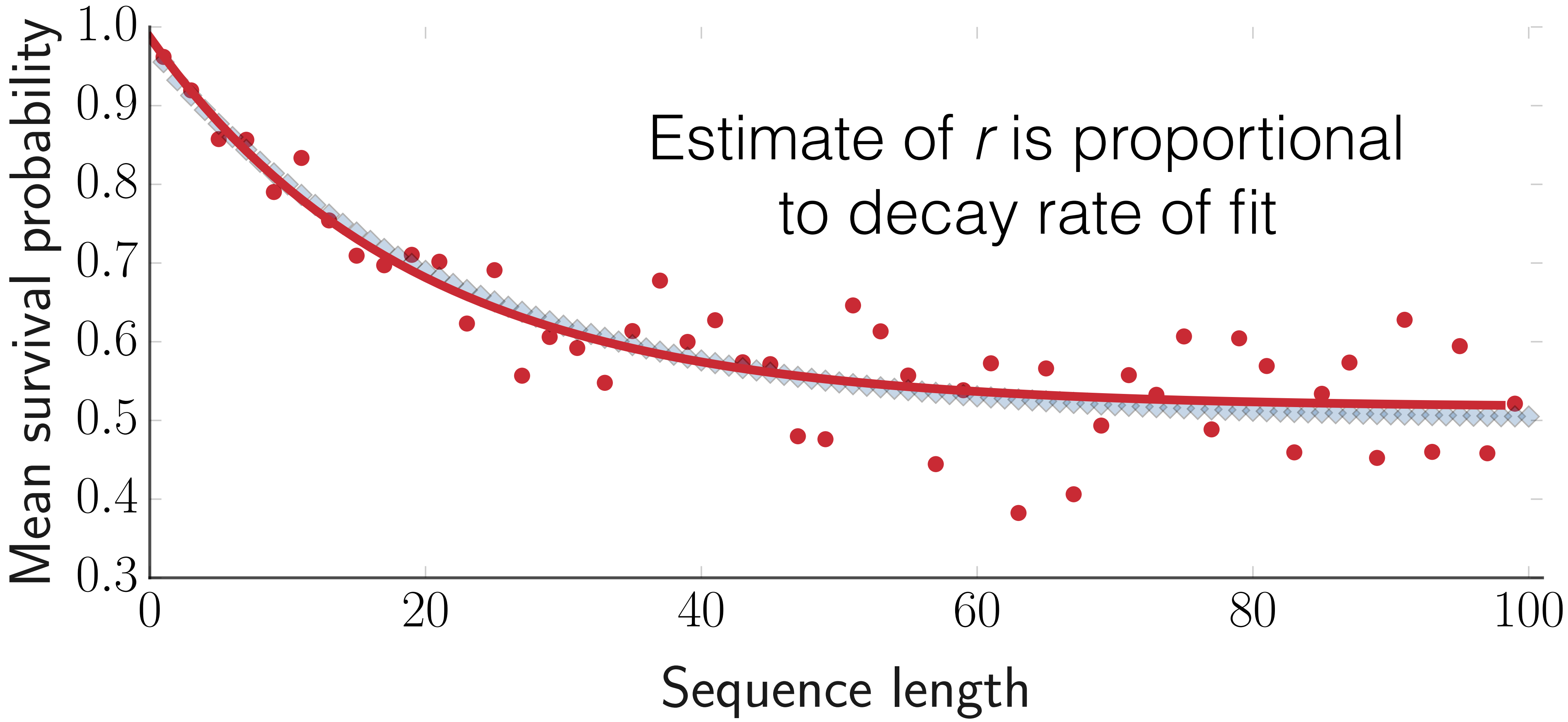
Randomized benchmarking

Imperfect Cliffords gates: $\tilde{\mathcal{C}} = \{\tilde{C}_i\}$

Implement random identity Clifford circuits of length m:





Estimate of *r* is proportional to decay rate of fit

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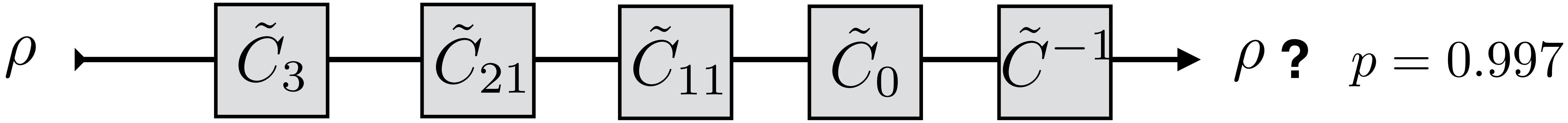
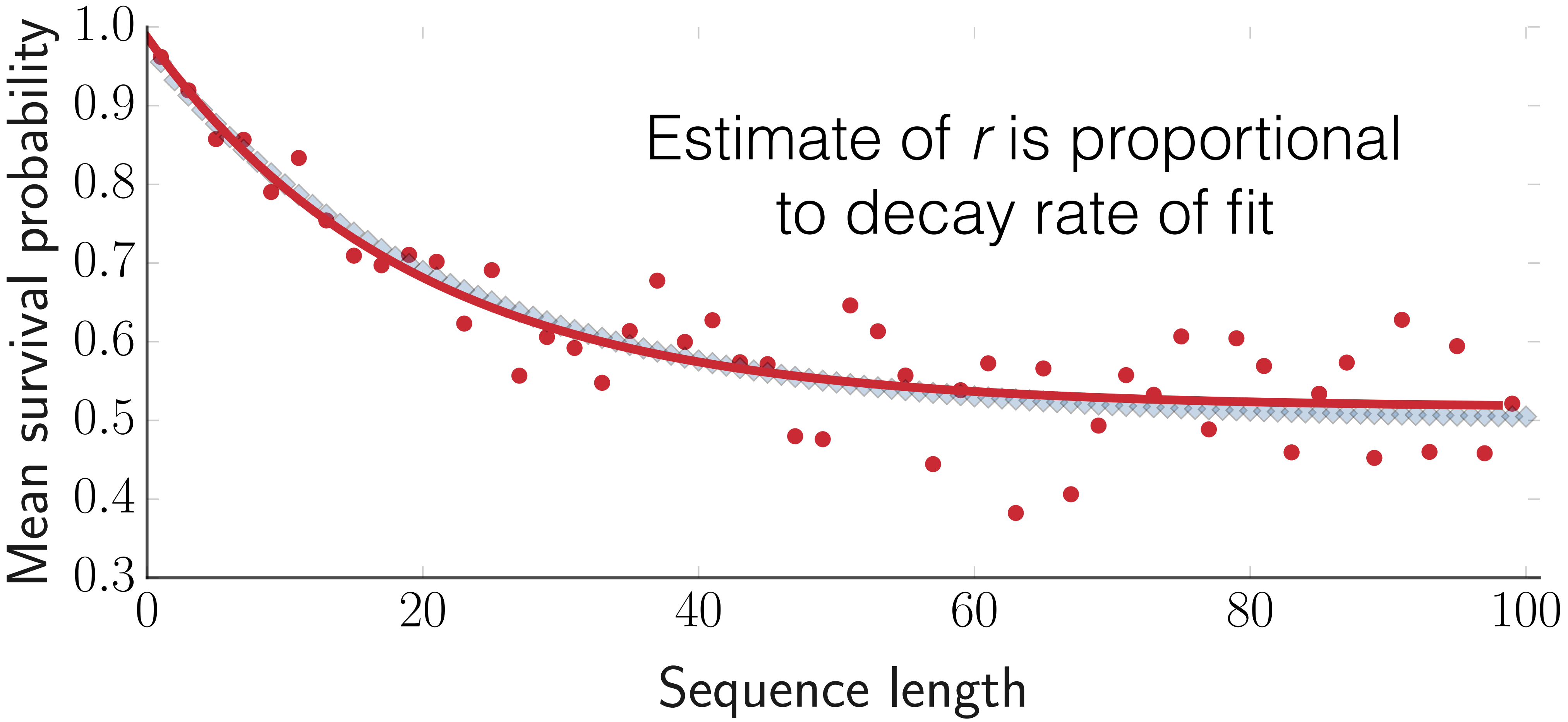


$\rho \longrightarrow \boxed{\tilde{C}_3} - \boxed{\tilde{C}_{21}} - \boxed{\tilde{C}_{11}} - \boxed{\tilde{C}_0} - \boxed{\tilde{C}^{-1}} \longrightarrow \rho$ **?** $p = 0.997$



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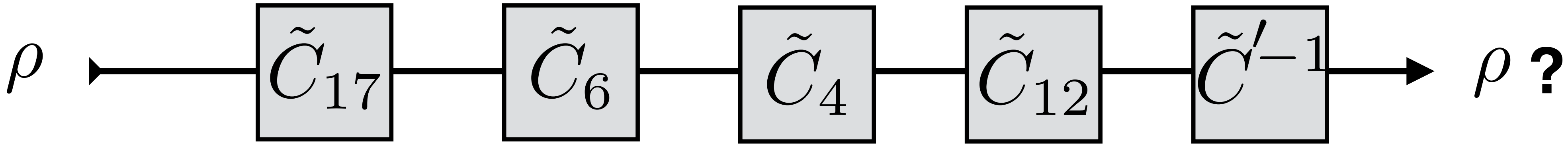

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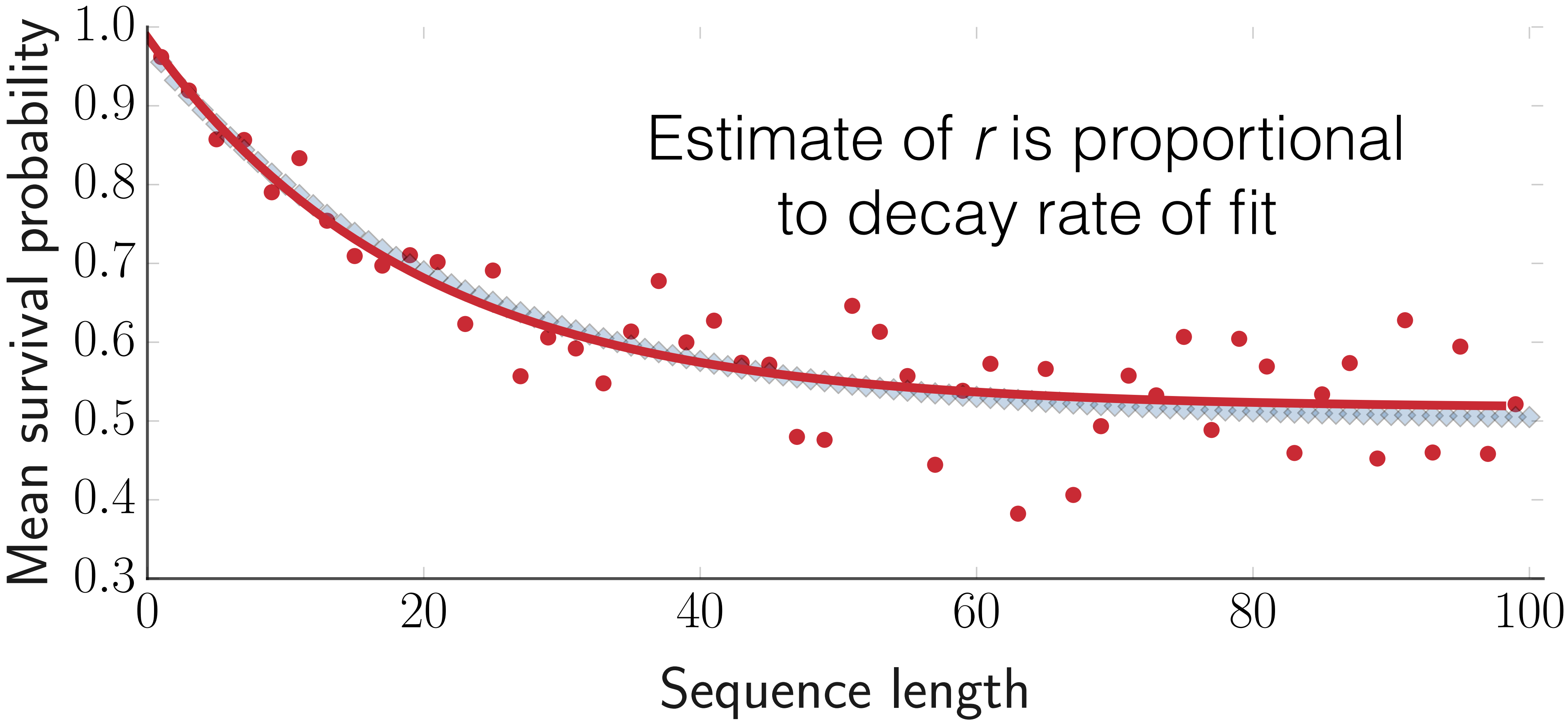

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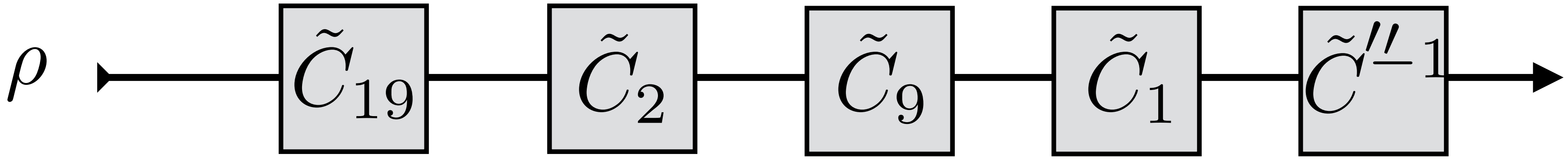
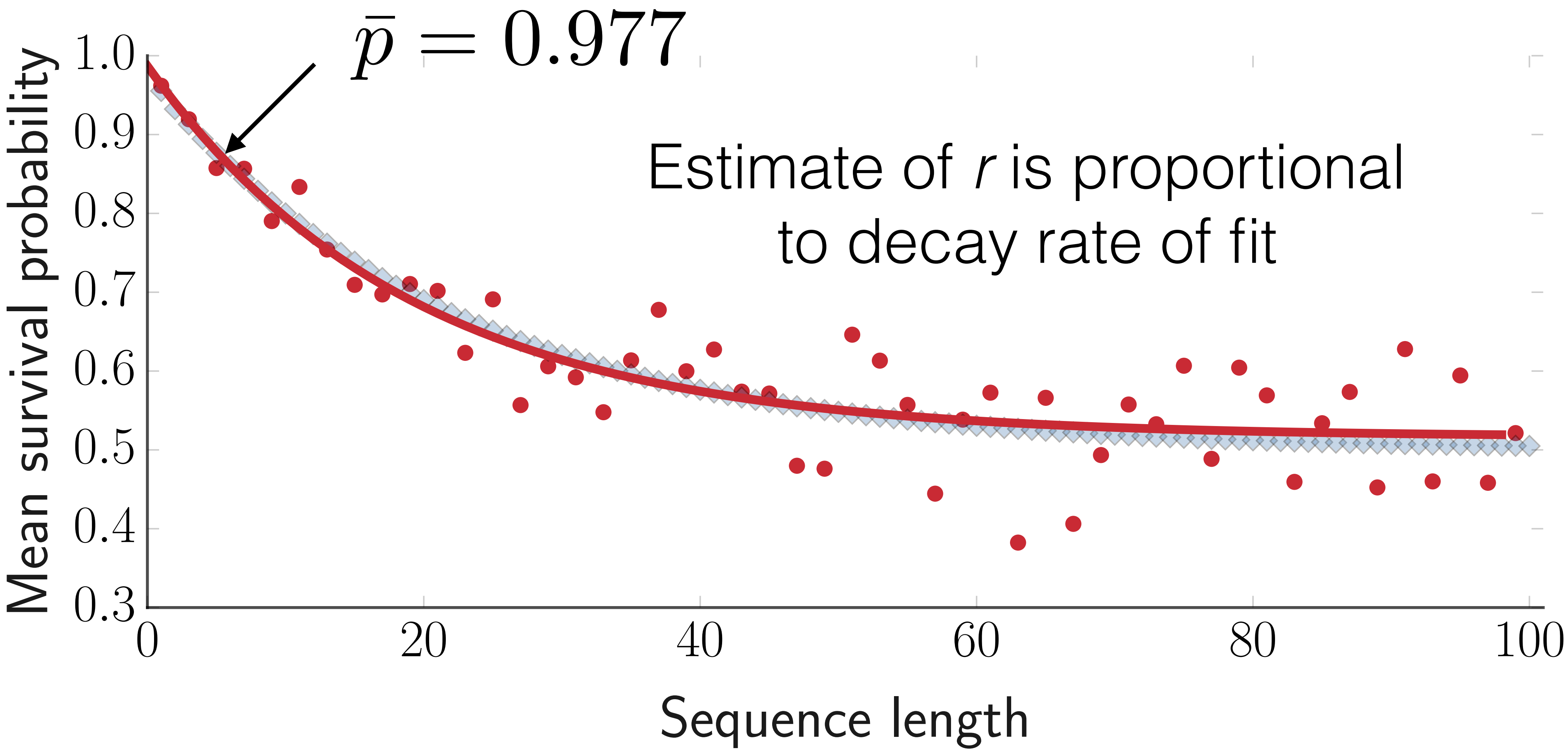


$p = 0.853$



Estimate of *r* is proportional to decay rate of fit

Randomized benchmarking

Assume $\tilde{C}_i$ are fixed matrices.

(This assumption is often violated in real-world RB. In that case RB can fail at estimating *r.* We are *not* concerned with this in this talk.)

Fogarty et al, PRA 92, 022326 (2015); Epstein et al PRA A 89, 062321 (2014).

What is RB measuring?

or, equivalently

What is *r*?

What is RB measuring?

Average gate infidelity: $\epsilon_i = 1 - \int d\psi \ \mathrm{Tr}\left(\tilde{C}_i[|\psi\rangle\langle\psi|]C_i[|\psi\rangle\langle\psi|]\right)$

Imperfect Clifford

Perfect Clifford

This is the fidelity between perfect and imperfect output of the gate, averaged over all pure inputs.

Average gateset infidelity (AGsI): $\epsilon = \mathrm{avg}_i[\epsilon_i]$

What is RB measuring?

Average gate infidelity: $$\epsilon_i = 1 - \int d\psi \; \mathrm{Tr}\left(\underset{\substack{\uparrow \\ \text{Imperfect Clifford}}}{\tilde{C}_i}[|\psi\rangle\langle\psi|] \underset{\substack{\uparrow \\ \text{Perfect Clifford}}}{C_i}[|\psi\rangle\langle\psi|] \right)$$

The literature claims:

$$r \approx \epsilon$$

This is wrong!

Average gateset infidelity (AGsI): $\epsilon = \mathrm{avg}_i[\epsilon_i]$

E. Magesan, *et al.* PRL. 106, 180504 (2011); E. Magesan, *et al.* PRA 85, 042311 (2012); J. J. Wallman and S. T. Flammia, NJP 16, 103032 (2014); J. M. Epstein et al. PRA 89, 062321 (2014); E. Magesan et al. PRL 109, 080505 (2012); J. Wallman et al. NJP 17, 113020 (2015); J. M. Gambetta et al. PRL 109, 240504 (2012); J. Helsen et al. arXiv:1701.04299 (2017); A. Carignan-Dugas et al. PRA 92, 060302 (2015)

What is RB measuring?

Gauge

Initial state (density matrix): ρ

Gates (superoperators): $\tilde{C}_i$

Measurement (POVM element): E_ρ

See e.g., Merkel *et al.* PRA 87, 062119 (2013); Blume-Kohout *et al* Nat. Comm. 8, 14485 (2017)

The literature claims: $r \approx \epsilon$. *This is wrong…*

Proctor *et al.* arXiv:1702.01853

What is RB measuring?

Gauge

Initial state (density matrix): $\rho \longrightarrow M(\rho)$

Gates (superoperators): $\tilde{C}_i \longrightarrow M \circ \tilde{C}_i \circ M^{-1}$

Measurement (POVM element): $E_\rho \longrightarrow M^{-1}(E_\rho)$

All observable properties of a gateset are **invariant** under such a transformation.

See e.g., Merkel *et al.* PRA 87, 062119 (2013); Blume-Kohout *et al* Nat. Comm. 8, 14485 (2017)

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Gauge

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Gates (super $\tilde{C}_i \circ M^{-1}$

Measuremen (E_ρ)

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invariant under such a transformation.

Gauge transformations act on matrices ***representing*** physical gates

They are ***not*** physical transformations!

See e.g., Merkel *et al.* PRA 87, 062119 (2013); Blume-Kohout *et al* Nat. Comm. 8, 14485 (2017)

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Gauge transformations act on these.

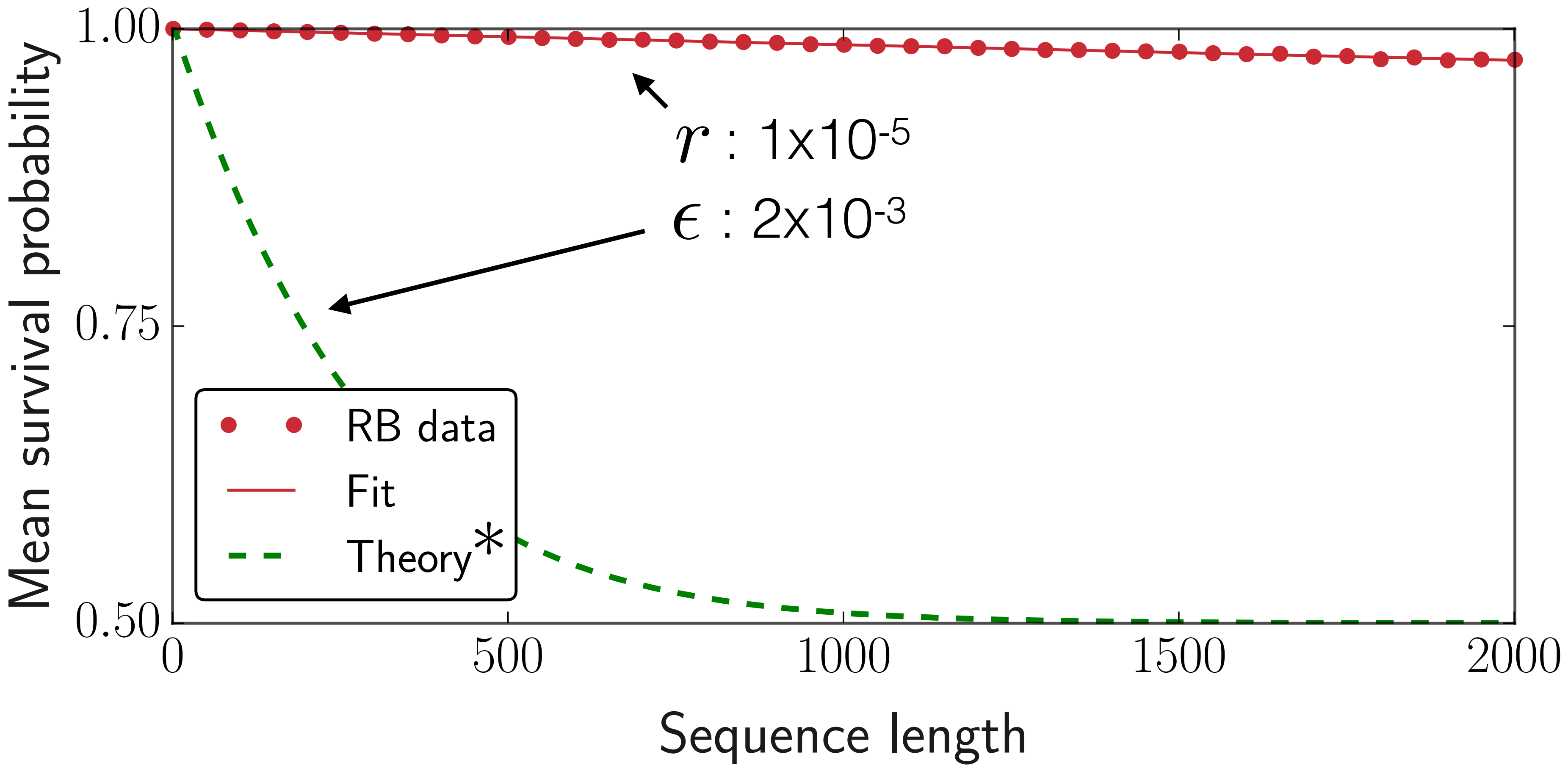
AGI and AGsI are not gauge-invariant!

RB does not measure AGsI.

The literature claims: $r \approx \epsilon$. *This is wrong…*

Proctor *et al.* arXiv:1702.01853

An example



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The error model

single qubit Cliffords compiled from

$$\tilde{X}_{\frac{\pi}{2}} = e^{-i\frac{\theta}{2}Z}e^{-i\frac{\pi}{4}X}$$

$$\tilde{Y}_{\frac{\pi}{2}} = e^{-i\frac{\theta}{2}Z}e^{-i\frac{\pi}{4}Y}$$

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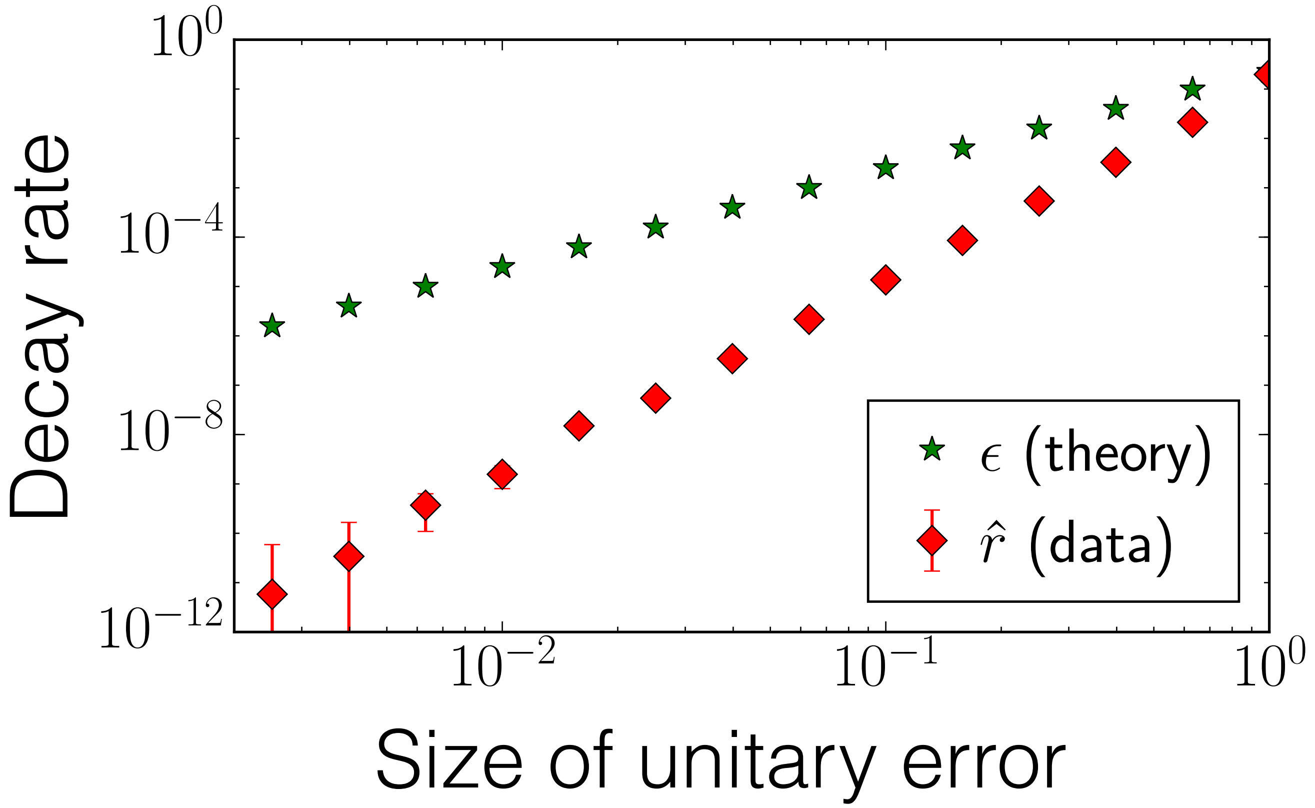

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Conclusions

RB does not measure AGsI ϵ (also called “average error rate”).

No protocol can estimate AGsI.

Why should you care?

The RB number as a metric relevant to e.g., fault tolerance, is justified by $r \approx \epsilon$ which is ***not*** true.

“Advanced” RB protocols have similar issues (e.g., interleaved RB, etc…)

Interested in further details? Including a new theory for the RB decay?
see: arXiv:1702.01853