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Turbulence Compensation Using Micromirror Arrays: Comparing Tip-Tilt-Piston and Piston-Only Micromirrors

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Abstract: Micromirrors arrays can be used to correct wavefront aberrations due to atmospheric turbulence. In this note a simple method is presented for determining the number of micromirrors needed in an array of piston-only micromirrors to correct a given amount of atmospheric turbulence. We also compute the required number of piston-tip-tilt micromirrors.

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1. Introduction

In a telescope system, an array of micromirrors can be used to correct the wavefront errors introduced by atmospheric turbulence. The question is, should one use piston-only or piston-tip-tilt micromirrors? Furthermore, how many micromirrors are needed? Fortunately this problem can be cast to fit the body of work describing seeing through atmospheric turbulence developed by Fried[1], Roddier[2], Noll[3], and others[4,5]. We calculate the number of micromirrors required as a function of the Fried radius[1,2] r_0 for the two types of micromirror arrays. For the case where the Strehl ratio is 90%, one needs 63 piston-only micromirrors per Fried cell (πr_0^2) or 8.33 piston-tip-tilt micromirrors. There is a 12X ratio between these two numbers implying a 4X reduction in the total actuator count if piston-tip-tilt mirrors are used.

2. Turbulence Theory

It is assumed that the reader is familiar with the image degradation effects due to atmospheric turbulence. If a refresher is needed, we suggest a review of the very understandable web sites maintained by U. California at Berkeley[4] and by NOAO[5].

Noll[3], starting with the work of Fried[1], determined the probability functions for Zernike aberrations in an optical system that images through a turbulent atmosphere. He computed how much RMS wavefront error would remain after the lower order Zernike terms are removed. The two cases of interest here are: first, when the piston error is removed ($Z_0=0$), and second when the piston, tip, and tilt terms are removed ($Z_0 = Z_1 = Z_2 = 0$). For these two cases, the Strehl ratios associated with the residual wavefront errors (WFE) are[3,4,5]

$$S_{piston} = exp\{-1.03\ (D/r_0)^{5/3}\} \qquad (1)$$

and

$$S_{p\text{-}t\text{-}t} = exp\{-0.134\ (D/r_0)^{5/3}\}, \qquad (2)$$

where D is the diameter of the aperture of interest and r_0 is the Fried radius. The subscripts $_{piston}$ and $_{p\text{-}t\text{-}t}$ refer to the piston-corrected and the piston, tip, & tilt-corrected cases. Understand that Eqns (1) & (2) describe the time-averaged wavefront error and thus represent the most probable Strehl ratio.

This is all the turbulence theory needed to design a micromirror array for a telescope with atmospheric turbulence compensation.

3. The model

Our first inclination was to choose the aperture diameter D to be that of the whole telescope and then consider myriads of possible aberration configurations described by the turbulence statistics, but this is a very hard way to solve the problem. An alternate formulation applicable to round micromirrors is to choose D to be the diameter of the individual micromirrors. (Hexagonal micromirrors array are also well approximated by this analysis.) We know that each micromirror can be moved so as to locally minimize the WFE. After the micromirrors are fit to the wavefront, Eqns (1) and (2) will accurately describe the residual WFE for each individual piston-only or piston-tip-tilt micromirror.

Conveniently the WFE statistics are the same across the whole aperture of the telescope. Thus when each micromirror is moved to an optimum position to minimize the local wavefront error, the most probable RMS wavefront error for all of the individual micromirrors will be the same, and will be equal to the most probable RMS wavefront error of the whole array.

So in the design of a micromirror array to correct for turbulence, we first choose the system's Strehl ratio. A portion of this tolerance (determined elsewhere) will be allotted to the granularity of the micromirrors. This can be inserted into Eqns (1) and (2) as S_{piston} and $S_{p\text{-}t\text{-}t}$

Example I: How many piston-only micromirrors are needed per Fried cell if the Strehl ratio is $S_{piston} = 90\%$? Eqn. (1) can be used to calculate the number of micromirrors per Fried turbulence cell ($[r_0/D]^2$), which is $r_0/D = 7.9$, so one needs 62.5 micromirrors per Fried cell. If the diameter of the telescope were, for example, five Fried cells across [$D_{telescope}/(2r_0)=5$], then we would need ~$62.5*5^2$=1560 piston-only micromirrors for the whole telescope.

Example II How many piston-tip-tilt micromirrors would be needed for the same example? The answer is 8.33 micromirrors/Fried cell or 210 micromirrors in the whole telescope.

Number of micromirrors required to correct the WFE in a telescope

The total number of piston-only micromirrors required for a telescope with diameter $D_{telescope}$ is

$$N_{piston} \approx 1.04 * [-ln(Strehl)]^{-1.2} * [D_{telescope}/r_0]^2 \quad (3)$$

An identical development for piston-tip-tilt (p-t-t) micromirrors yields

$$N_{p\text{-}t\text{-}t} \approx 0.090 * [-ln(Strehl)]^{-1.2} * [D_{telescope}/r_0]^2 \quad (4)$$

Note that p-t-t micromirrors are ~12X better than piston-only micromirrors. This implies a 4X reduction in the number of actuators required—which is a significant reduction in complexity and cost.

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6. Bibliography

1. DL Fried, *Statistics of a geometric representation of wavefront distortion*, JOSA **55**, 1427 (1967)
2. F. Roddier, *The effects of atmospheric turbulence in optical astronomy*, Progress in Optics XIX, Amsterdam, North Holland, pp 281-376 (1981)
3. RJ Noll, *Zernike polynomials and atmospheric turbulence* JOSA **66** 207 (1976)
4. UC Berkeley, "Optical Effects of Atmospheric Turbulence", http://grus.berkeley.edu/~jrg/SEEING/node1.html
5. National Optical Astronomy Observatory, "Imaging through turbulence", http://www.ctio.noao.edu/~atokovin/tutorial/part1/turb.html