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Flexible Foam Model

Michael K. Neilsen, Wei-Yang Lu, Brian T. Werner, William M. Scherzinger, Chi S. Lo

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

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Michael K. Neilsen
Component Science and Mechanics, 1556
Sandia National Laboratories
P.O. Box 5800
Albuquerque, New Mexico 87185-0346

Wei-Yang Lu and Brian T. Werner
Mechanics of Materials, 8343
Sandia National Laboratories
P.O. Box 969
Livermore, CA 94551-0969

William M. Scherzinger, and Chi S. Lo
Solid Mechanics, 1554
Sandia National Laboratories
P.O. Box 5800
Albuquerque, New Mexico 87185-0840

Abstract

Experiments were performed to characterize the mechanical response of a 15 pcf flexible polyurethane foam to large deformation at different strain rates and temperatures. Results from these experiments indicated that at room temperature, flexible polyurethane foams exhibit significant nonlinear elastic deformation and nearly return to their original undeformed shape when unloaded. However, when these foams are cooled to temperatures below their glass transition temperature of approximately -35 ^{o}C, they behave like rigid polyurethane foams and exhibit significant permanent deformation when compressed. Thus, a new model which captures this dramatic change in behavior with temperature was developed and implemented into SIERRA with the name ***Flex_Foam*** to describe the mechanical response of both flexible and rigid foams to large deformation at a variety of temperatures and strain rates. This report includes a description of recent experiments. Next, development of the Flex Foam model for flexible polyurethane and other flexible foams is described. Selection of material parameters are discussed and finite element simulations with the new Flex Foam model are compared with experimental results to show behavior that can be captured with this new model.

Acknowledgments

The authors are thankful for early experimental studies of rigid polyurethane foam response to hydrostatic and tri-axial compression performed by Bill Olsson, Sandia retired, and Steve Bauer, 8864, which were very informative. Prior studies of rigid polyurethane foam by Helena Jin, 8343, with digital image correlation were also very helpful. Andy Kraynik, Sandia retired, studies of foam geometry and mechanics also contributed greatly to our understanding of foam behavior. Recent foam experiments performed by April Nissen, 8344, were also very informative. We would like to thank Kevin Long, 1554 and Brian Lester, 1554, for technical reviews of this report.

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Nomenclature

Abbreviation	Definition
DOE	Department of Energy
DMA	Dynamic Mechanical Analyses
FR	fire retardant
pcf	pounds per cubic foot
PMDI	polymethylene di-isocyanate
SEM	Scanning Electron Microscopy
SIERRA	A Code developed at Sandia National Laboratories for Finite Element Analyses
SNL	Sandia National Laboratories
UCPD	Unified Creep Plasticity Damage

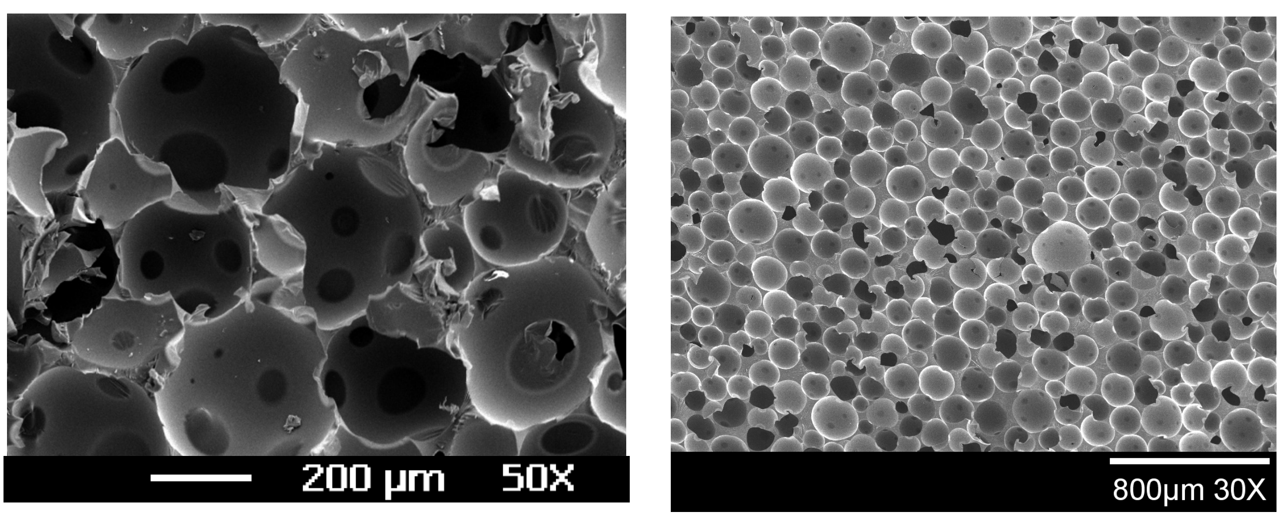
1. Introduction

Flexible and rigid foams are used in packaging to protect sensitive components from accidental impact events. These foams are typically designed to absorb energy during impact events by undergoing large inelastic deformation. Thus, constitutive models that describe foam response to large deformation at various rates and temperatures are needed for use in finite element analyses of impact events.

Blown polyurethane foams consist of nearly spherical voids with a typical diameter of 100 to 300 microns (Figure 1). The closed cells are separated by a polymer matrix that forms cells. Voids are less spherical and walls between neighboring cells are often very thin, fractured, or absent in polyurethane foams with densities of 192 kg/m^3 (12 pcf) or less. In higher density foams with densities of 320 kg/m^3 (20 pcf) or greater, cells are more spherical and walls between neighboring cells are typically intact. Solid polyurethane has a density of approximately 1200 kg/m^3 (75 pcf), so even the higher density 320 kg/m^3 (20 pcf) foam has an initial volume fraction of solid material of only 0.267.

Numerous experiments were performed to characterize the mechanical response of several different flexible and rigid polyurethane foams to large deformation. If a foam is glassy (below its glass transition temperature) at room temperature it is called a 'rigid' foam and if a foam is rubbery (above its glass transition temperature) at room temperature it is called a 'flexible' foam. This report includes a description of experiments on both rigid and flexible polyurethane foams. Recent experiments on a 15 pcf flexible polyurethane foam show that if you cool this 'flexible' foam to a temperature below its glass transition temperature it will exhibit behavior similar to a 'rigid' foam at room temperature.

A Foam Damage model [1] for rigid foams is discussed since a similar UCPD model is used as part of the new Flex Foam model. Development of a new Flex Foam model for flexible and rigid foams is then described. Selection of material parameters is discussed and finite element simulations with the Foam Damage and new Flex Foam model are compared with results from experiments on a flexible 15pcf polyurethane foam to show behavior that can be captured with these models. Finally, the new Flex Foam model is used to describe the mechanical behavior of a 40 pcf silicone foam.



(a) 176 kg/m^3 (11 pcf) foam (b) 320 kg/m^3 (20 pcf) PMDI foam

Figure 1. SEM images of polyurethane foam cell structure.

2. Experimental Observations

When rigid polyurethane foam is compressed, it exhibits an initial elastic regime followed by a plateau regime in which the load needed to compress the foam remains nearly constant (Figure 2). In the elastic regime, the foam sample is uniformly deformed. In the plateau regime, cell walls are plastically deformed and/or damaged and large permanent volume changes are generated. Uniform deformation is observed in foam samples that strain harden but localized deformation is often observed in foam samples that exhibit no hardening or strain softening in the plateau regime. When additional load is applied, cell walls are compressed against neighboring cell walls (Figure 3), the foam locks up, and the stiffness and strength of the foam increases. When rigid polyurethane foam is loaded in tension, it exhibits only a very small amount of plastic deformation before it fractures. Fracture surfaces generated by uniaxial tension are oriented such that the loading axis is normal to the fracture surface (Figure 4). In Figure 2, uniaxial stress and axial strain are plotted as positive for both compression and tension.

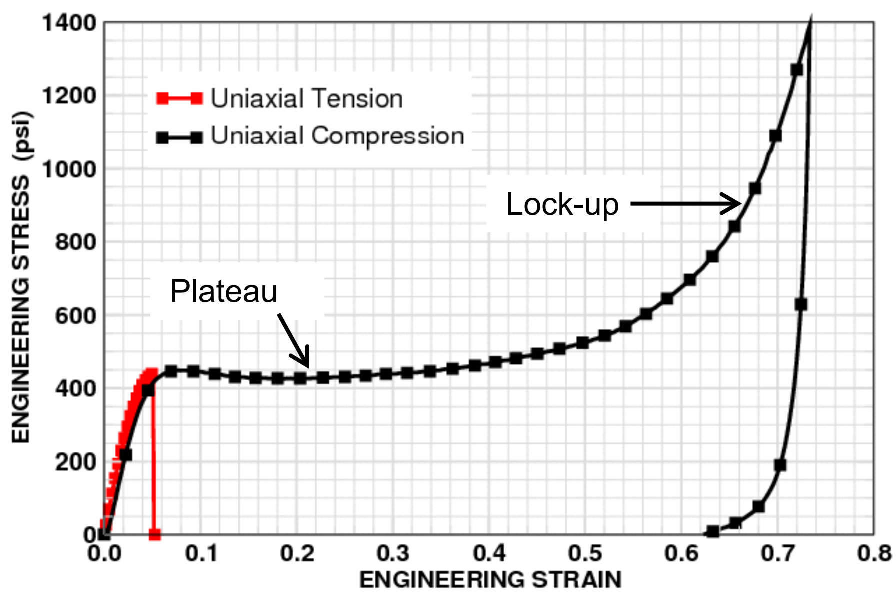
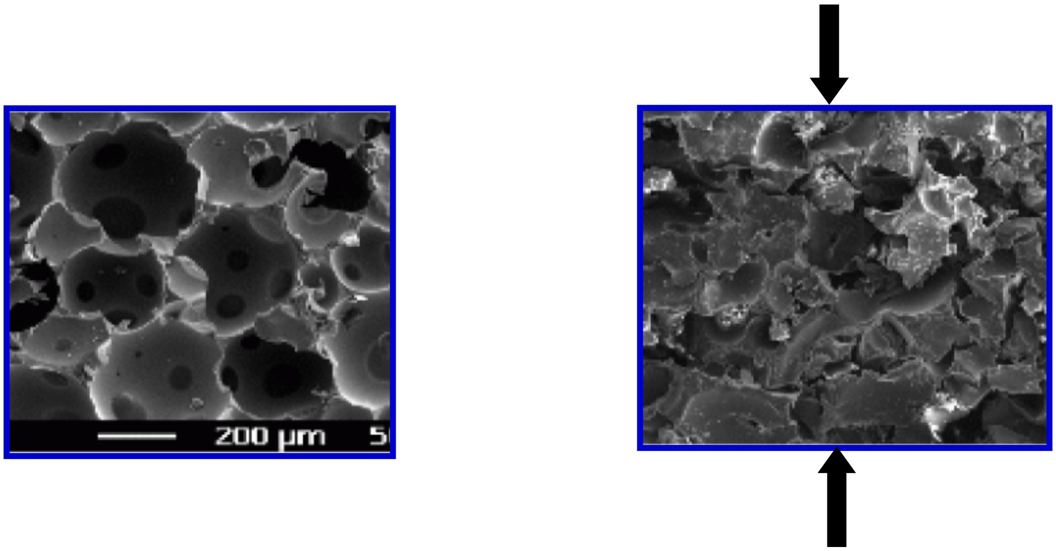


Figure 2. Typical stress-strain curves for 176 kg/m^3 (11 pcf) rigid polyurethane foam subjected to either uniaxial compression or uniaxial tension.



(a) undeformed (b) deformed shape after unloading

Figure 3. Cell walls compressed against neighboring cell walls when 176 kg/m^3 (11 pcf) rigid foam is compressed into the lockup regime and then unloaded.

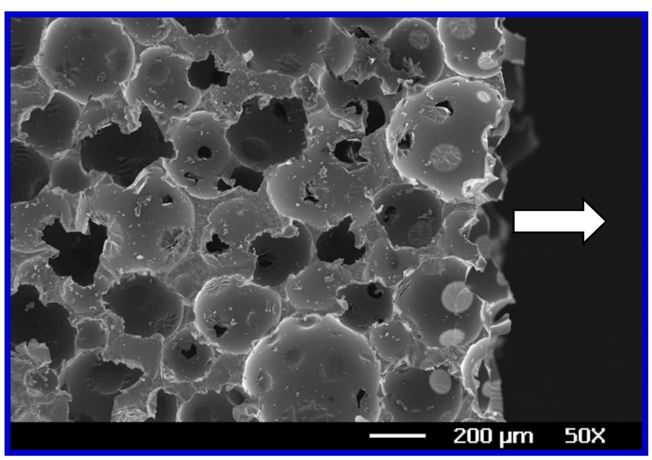


Figure 4. Fracture surface generated by uniaxial tension of 176 kg/m^3 (11 pcf) rigid polyurethane foam in the indicated direction.

The mechanical response of polyurethane foam is also very sensitive to changes in either loading rate or temperature. For example, the crush strength of a General Plastic's FR3712 (12 pcf) rigid polyurethane foam with a glass transition temperature of 132 °C subjected to uniaxial compression decreases significantly with increases in temperature (Figure 5). The crush strength of polyurethane foam is also observed to increase significantly with increases in loading rate (Figure 6). This foam behavior is consistent with the underlying behavior of solid polyurethane which also exhibits temperature and strain-rate dependence. When FR3712 rigid polyurethane foam is subjected to hydrostatic compression, it exhibits a pressure versus volume strain curve (Figure 7) that is similar in shape to its uniaxial stress strain curve (Figure 6). There is again an initial elastic regime followed by a plateau regime and finally a lock-up regime.

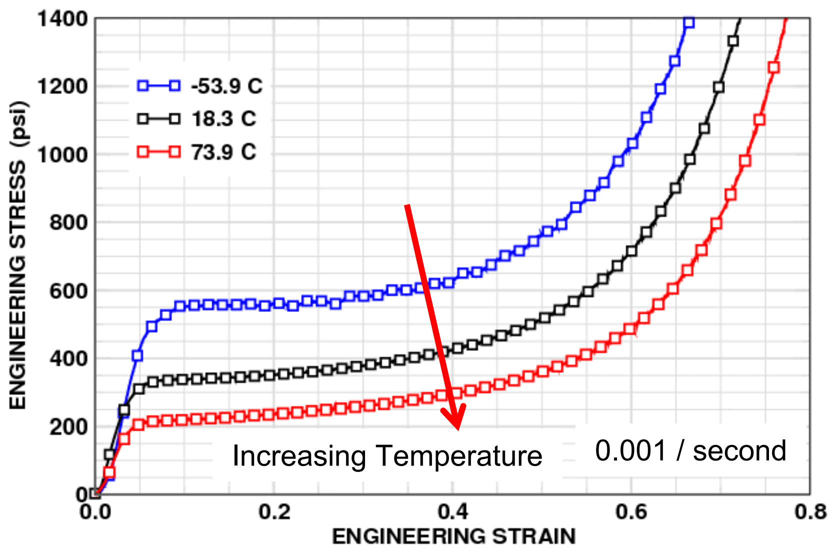


Figure 5. Stress-strain curves for 192 kg/m^3 (12 pcf) FR3712 foam subjected to uniaxial compression at engineering strain rate of 0.001 per second and at constant temperatures given by the legend.

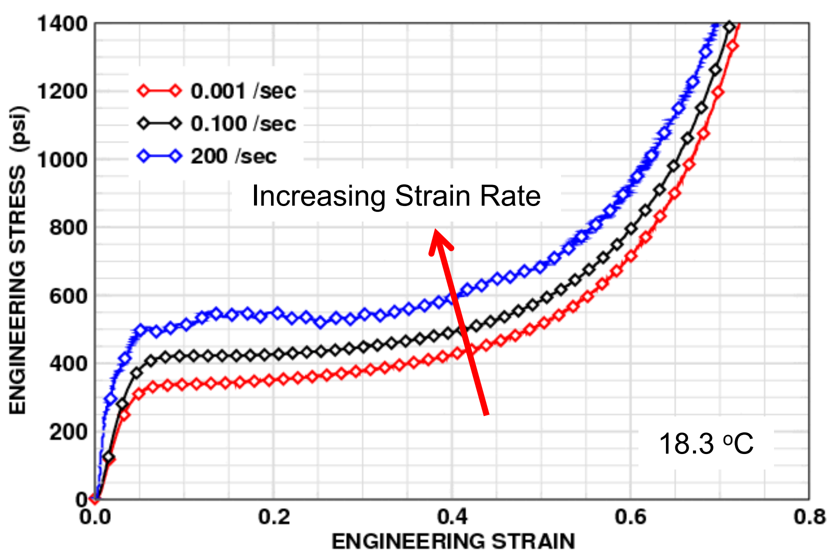


Figure 6. Stress-strain curves for 192 kg/m^3 (12 pcf) FR3712 foam subjected to uniaxial compression at an initial temperature of 18.3 ^{o}C and at constant engineering strain rates given by the legend.

In addition to hydrostatic compression, the FR3712 foam at 18.3 ^{o}C was also subjected to a variety of tri-axial compression load paths [2] in which the sample was initially subjected to hydrostatic compression at a pressurization rate of 0.1 MPa per second and then confining pressure was maintained while additional compressive strain was applied in the axial direction only. Results from this series of tri-axial compression experiments were then used to generate a plot of the initial yield surface for the foam in a von Mises effective stress versus mean stress space (Figure 8). The experimental results (blue symbols in Figure 8) indicate that the initial yield surface for FR3712 foam, in compression, could be described as an ellipse in this two dimensional space (solid black line in Figure 8) or as an ellipsoid about the hydrostat in three dimensional principal stress space.

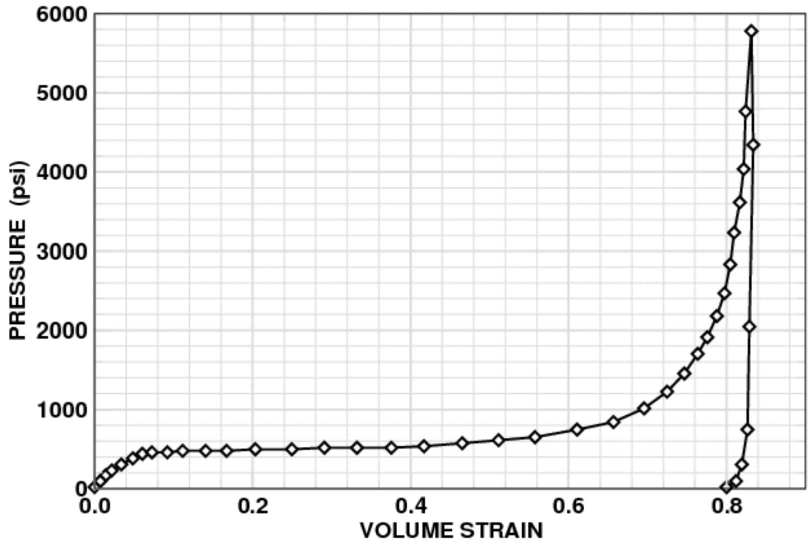


Figure 7. Pressure-volume strain curve for 192 kg/m^3 (12 pcf) FR3712 foam subjected to hydrostatic compression at 18.3 ^{o}C and a constant pressurization rate of 0.1 MPa/second.

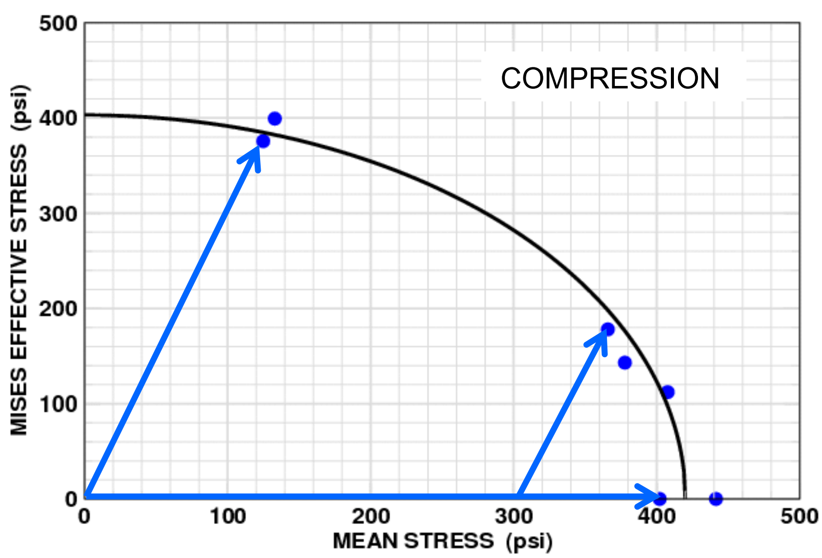


Figure 8. Yield surface obtained from uniaxial, hydrostatic and triaxial compression experiments (blue symbols) on 192 kg/m^3 (12 pcf) FR3712 foam at 18.3 ^{o}C [2].

A 15 pcf flexible urethane foam was recently studied. Dynamic mechanical analyses (DMA) showed that this foam has a glass transition temperature of approximately -35 ^{o}C (Figure 9). At temperatures below the glass transition temperature where the storage modulus plateaus, stress-strain curves exhibited by the flexible foam (Figure 10) are very similar to the stress-strain curves exhibited by a rigid polyurethane foam at room temperature (Figure 2). In other words, a flexible foam behaves as a rigid foam when it is cooled to temperatures below its glass transition temperature. Note that in Figure 10, the black curve is the first load-unload cycle, the red curve is the second cycle, and the green curve is the third cycle. The foam absorbs significantly more energy during the first cycle than during subsequent cycles.

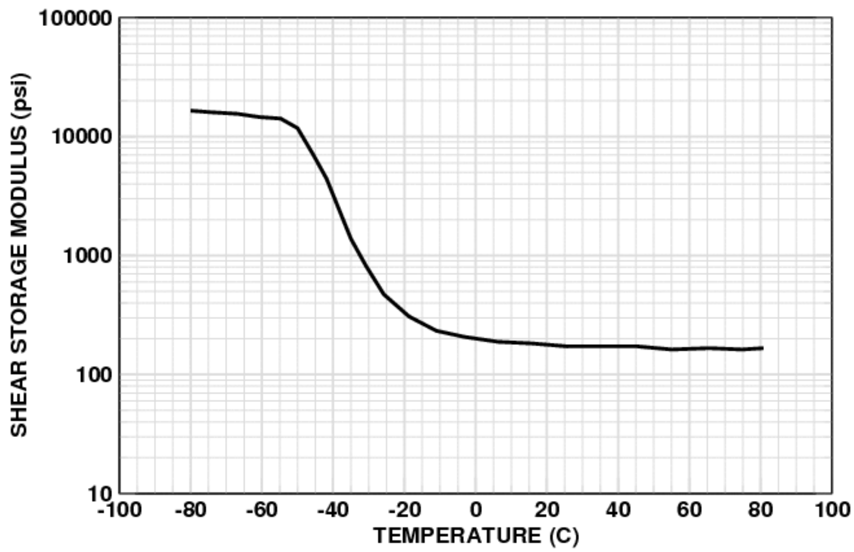


Figure 9. Shear storage modulus (psi) versus temperature from DMA experiment on 15 pcf flexible polyurethane foam.

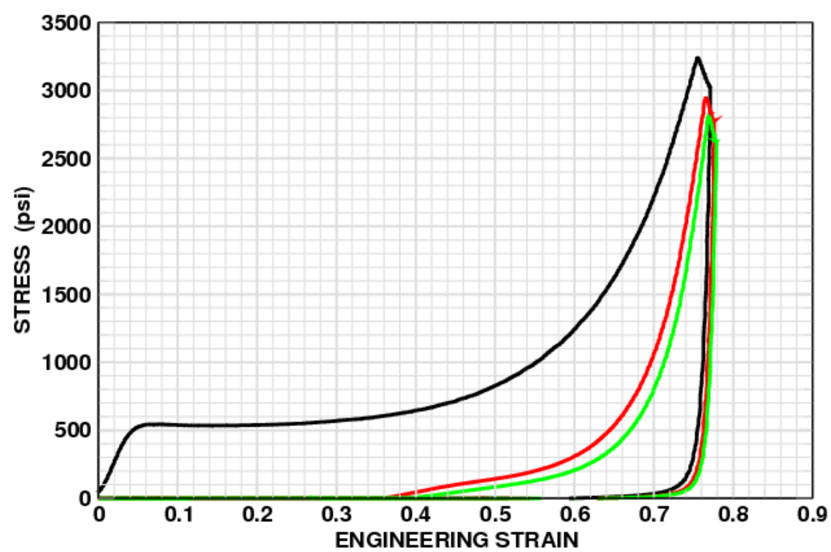


Figure 10. Stress-strain curves for 15 pcf flexible polyurethane foam subjected to uniaxial compression at -53.9 ^{o}C, engineering strain rate 0.05/second. Black curve is initial load-unload cycle, red

curve is second cycle, and green curve is third cycle (Brian Werner, 8343 and April Nissen, 8344).

At 21.1 °C, which is above the glass transition temperature of approximately -35 °C, the 15 pcf flexible polyurethane foam behavior is radically different than the foam behavior at -53.9 °C; the foam has much lower strength and nearly returns to its original undeformed shape when unloaded (Figure 11). Note that the stress axis for the rigid foam response (Figure 10) goes up to 3,500 psi and the stress axis for the flexible foam (Figure 11) only 1,500 psi. At room temperature, this flexible foam does show hysteresis and will dissipate a small amount of energy when it is compressed (Figure 11) but it dissipates significantly more energy when it is initially compressed at -53.9 °C (Figure 10).

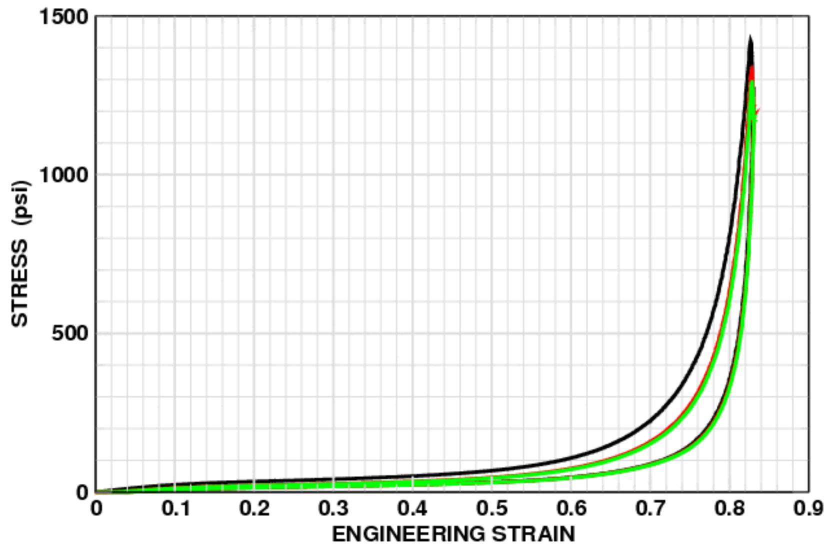


Figure 11. Stress-strain curves for 15 pcf flexible polyurethane foam subjected to uniaxial compression at 21.1 °C, engineering strain rate 0.05/second. Black curve is initial load-unload cycle, red curve is second cycle, and green curve is third cycle (Brian Werner, 8343 and April Nissen, 8344).

The existing Foam Damage model [1] could be used to describe the response of this flexible foam below the glass transition temperature but was not expected to capture the response of this flexible foam within and above the glass transition. Thus, a new material model which would capture essentially a rigid foam response at cold temperatures and a flexible foam response at temperatures above the glass transition was needed.

The next section includes a review of the Foam Damage model that was developed at Sandia to describe the behavior of rigid polyurethane foams [1]. Flexible foam behavior that can and cannot be captured with the Foam Damage model will then be shown. This is followed by the development of the new Flex Foam Model.

3. Foam Damage Model

The Foam Damage model in SIERRA [1] was developed to capture the inelastic deformation and cracking of rigid polyurethane foams. This model is similar to many existing foam models [e.g. 3-8]. Neilsen et al. [3] developed a plasticity model for polyurethane foams with a yield surface that has a cubic shape based on the use of a principal stress yield criterion. Deshpande and Fleck [4] developed a plasticity model for metal foams with a yield surface that is an ellipsoid about the hydrostat. Deshpande and Fleck [5] subsequently developed a yield surface for polymeric foams with a yield surface that is the inner envelope of the ellipsoidal surface previously developed for metal foams and a surface based on a minimum (compressive) principal stress criterion. The Foam Damage model has an ellipsoidal yield surface with damage surfaces based on a maximum (tensile) principal stress criterion.

The Foam Damage model was implemented into SIERRA using the unrotated Cauchy stress, σ, and unrotated deformation rate, $\dot{\varepsilon}$ [9, 10, 11]. For small elastic strains, the total strain rate, $\dot{\varepsilon}$, can be additively decomposed into elastic, $\dot{\varepsilon}^e$, and inelastic, $\dot{\varepsilon}^{in}$, parts as follows

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{in} \tag{1}$$

The elastic response is linear and isotropic such that stress rate for constant elastic moduli is given by the following equation

$$\dot{\sigma} = E : \dot{\varepsilon}^e = E : \left(\dot{\varepsilon} - \dot{\varepsilon}^{in}\right) \tag{2}$$

where E is the fourth-order, isotropic elasticity tensor. Based on the experimental results shown in Figure 8, the yield surface is an ellipsoid about the hydrostat described by the function

$$\varphi = \frac{\bar{\sigma}^2}{a^2} + \frac{p^2}{b^2} - 1.0 = 0 \tag{3}$$

where a and b are state variables that define the current deviatoric and volumetric strengths of the foam. State variables a and b are user-prescribed functions of ϕ_{max}, which is the maximum volume fraction of solid material obtained by the material during any prior loading. The current volume fraction of solid material is simply given by

$$\phi = \frac{\phi_0 V_0}{V} \tag{4}$$

where ϕ_0 is the initial volume fraction of solid material, V_0 the initial volume, and V the current volume. This relationship assumes that all of the volume change is accommodated through a change in porosity, and any volume change associated with the solid polymer matrix is ignored. $\bar{\sigma}$ is the von Mises effective stress, a scalar measure of the deviatoric stress and is given by

$$\bar{\sigma} = \sqrt{\frac{3}{2} s : s} \tag{5}$$

with p being the pressure or mean stress and is given by

$$p = \frac{1}{3} \sigma : i \tag{6}$$

where σ is the Cauchy stress and i is the second-order identity tensor. s is the second-order deviatoric stress tensor

$$s = \sigma - p\,i \tag{7}$$

Puso and Govindjee [7] and Zhang et al. [8] developed strain rate dependent models for foam that have the foam's inelastic rate given as a power-law function of stress. For the model developed here, we start with the yield function, Equation (3), rewritten as follows

$$\varphi = \sigma^{*} - a = 0 \tag{8}$$

where the effective stress, σ^{*}, is a function of the vonMises effective stress, $\bar{\sigma}$, and pressure, p, as follows

$$\sigma^{*} = \sqrt{\bar{\sigma}^2 + \frac{a^2}{b^2}p^2} \tag{9}$$

Next, to capture strain-rate effects a Perzyna-type formulation is used and the following expression for the inelastic rate, $\dot{\varepsilon}^{in}$, is developed

$$\dot{\varepsilon}^{in} = \begin{cases} \dot{\lambda}\, g = e^{h}\left|\frac{\sigma^{*}}{a} - 1\right|^{n} g, & \frac{\sigma^{*}}{a} - 1 > 0 \\ 0 & , \quad \frac{\sigma^{*}}{a} - 1 \leq 0 \end{cases} \tag{10}$$

where g is a symmetric, second-order tensor that defines the orientation of the inelastic flow. This type of model is sometimes referred to as an overstress model because the inelastic rate is a power-law function of the overstress (distance outside the yield surface). Note that this model has an ellipsoidal yield surface which bounds an elastic regime, a region of stress states that generate no inelastic deformation. For associated flow, flow direction g is simply normal to the yield surface and is given by

$$g_{associated} = \frac{\frac{\partial \varphi}{\partial \sigma}}{\left|\frac{\partial \varphi}{\partial \sigma}\right|} = \frac{\frac{3}{a^2} s + \frac{2}{3b^2} p\, i}{\left|\frac{3}{a^2} s + \frac{2}{3b^2} p\, i\right|} \tag{11}$$

with $|\cdot|$ denoting the L_2 norm of a tensor. When lower density foams are subjected to a simple load path like uniaxial compression, the inelastic flow direction at moderate strains appears nearly uniaxial. In other words, the flow direction is given by the normalized stress tensor as follows

$$g_{radial} = \frac{\sigma}{|\sigma|} = \frac{\sigma}{\sqrt{\sigma : \sigma}} \tag{12}$$

This type of flow is referred to as radial flow. The Foam Damage model has a parameter, β, which allows for the flow direction to be prescribed as a linear combination of associated and radial flow directions as follows

$$g = \frac{(1-\beta)\, g_{associated} + \beta\, g_{radial}}{\left|(1-\beta)\, g_{associated} + \beta\, g_{radial}\right|} \tag{13}$$

When β is equal to 0 the flow is associated, when β is equal to 1 the flow is radial, and β values between 0 and 1 give flow directions between radial and associated (Figure 12).

Rigid polyurethane foams have little ductility when they are subjected to tensile stress and behave more like elastic brittle materials for this load path. Even for uniaxial compression, these foams often exhibit cracking. The damage surfaces for the Foam Damage model are simply 3 orthogonal planes with normals given by the positive principal stress axes in principal stress space as shown in Figure 12 and are described by the following equation

$$\varphi_{damage}^{\ i} = \sigma^{**i} - c(1-w) = 0,\ i = 1,3 \tag{14}$$

where σ^{**i} is a principal stress, c is the initial tensile strength which is a material parameter, and w is a scalar measure of the damage. Damage has an initial value of 0.0 and is limited to a maximum value of 0.99 to prevent the tensile strength from going to zero or becoming negative due to numerical round-off. As damage occurs, the damage surface will collapse toward the origin and the foam will have very little tensile strength. The foam will, however, still have compressive strength. For most simulations, foam that is completely damaged should be removed using element death based on the damage variable reaching a value of 0.98, but removal of fully damaged elements from a simulation is not required.

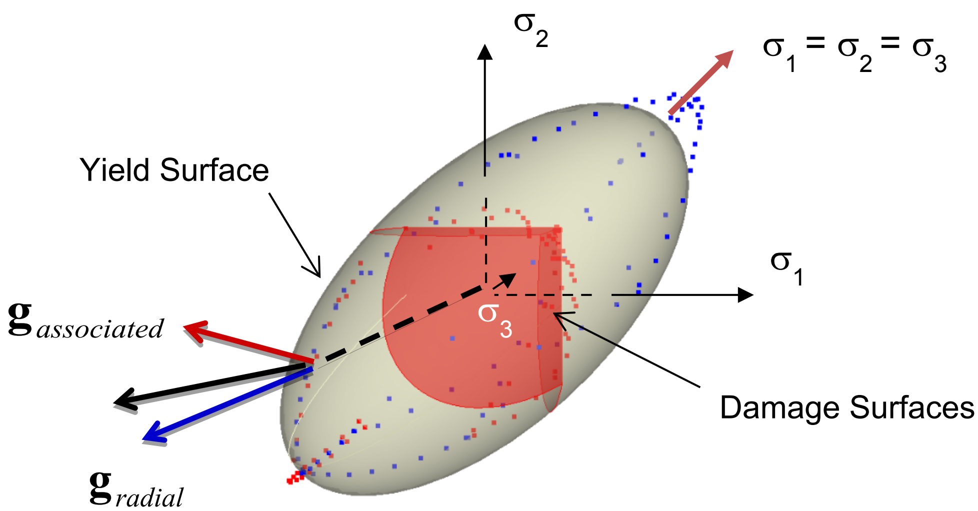


Figure 12. Yield (white) and damage (red) surfaces in principal stress space. Symbols represent results from either experiments or cell-level simulations on a representative volume of foam. Blue symbols are from cell-level simulations in which the matrix material was not allowed to crack.

Damage is given as a monotonically-increasing, user-prescribed function of damage **strain**, ε_{dam}, and damage strain is a function of the maximum tensile strain, ε_{max}, and the plastic volume strain, ε_{vol}^{p}, as follows

$$w = w(\varepsilon_{dam}) = w(a_{dam}\varepsilon_{max} + b_{dam}\varepsilon_{vol}^{p}) \tag{15}$$

where a_{dam} and b_{dam} are positive material parameters which allow the user to control the rate at which damage is generated in tension and compression. Note that in compression the plastic volume strain obtains a negative value, so the maximum tensile strain needed to generate damage is larger. Damage is never allowed to decrease even if the damage strain decreases which means that once foam is damaged, healing is not allowed.

To fully capture temperature, strain-rate, and lock-up effects several material parameters are not simply material constants but are instead functions of temperature, θ, and/or the maximum volume fraction of solid material obtained during any prior loading, ϕ_{max}, which depends on the volume strain. Material parameters defining the foams elastic response, Young's modulus and Poisson's ratio, are functions of temperature, θ, and ϕ_{max}. To be more specific, the current Young's modulus and Poisson's ratio used in a simulation are given by

$$\begin{aligned} E &= E_r \cdot E(\theta) \cdot E(\phi_{max}) \\ v &= v_r \cdot v(\theta) \cdot v(\phi_{max}) \end{aligned} \tag{16}$$

The natural log of the reference flow rate, h, and the power law exponent, n, in Equation 10 are also functions of temperature

$$\begin{aligned} h &= h_r \cdot h(\theta) \\ n &= n_r \cdot n(\theta) \end{aligned} \tag{17}$$

Also in the Foam Damage model, the parameter β which defines the fraction of associated and radial flow in Equation 13 is a user-prescribed function of ϕ_{max}.

Extensive use of user-prescribed functions in the Foam Damage model makes this model quite flexible for fitting data from a variety of foams; however, use of user-prescribed functions also makes the selection of material parameters more difficult because the user is no longer trying to find the best material constant but instead needs to define entire functions. When there is insufficient experimental data, the user is forced to select functions based on prior experience and good engineering judgment.

4. Foam Damage Parameters for 240 kg/m^3 (15 pcf) Polyurethane Foam

It was expected that the Foam Damage Model would be able to adequately describe the behavior of 15 pcf flexible polyurethane foam when it was at -53.9 ^oC and glassy. It was unknown how well this model would capture the measured behavior at and above room temperature.

Material parameters for 240 kg/m^3 (15 pcf) flexible polyurethane foam at temperatures between -53.9 ^oC and 73.9 ^oC are given in Table 1 and Figure 13. The first step in the generation of these material parameters was to determine the initial volume fraction of solid material in the foam. Since the foam has a density of approximately 240 kg/m^3 and solid polyurethane a density of 1200 kg/m^3, the foam has an initial volume fraction of solid material, ϕ_0,equal to 0.20 (0.20 = 240/1200). Material parameters were obtained using an iterative fitting process in which parameters were selected, experiments simulated, parameters modified, and process repeated until a 'best' fit was obtained where goodness of fit is based on an L^2-norm of the stress difference between experimental data and model predictions. Tools to automate this fitting process and to generate an 'optimized' fit are currently being developed but are not yet available. For this fit, damage was excluded by simply setting the damage as a function of damage strain equal to zero for all values of damage strain.

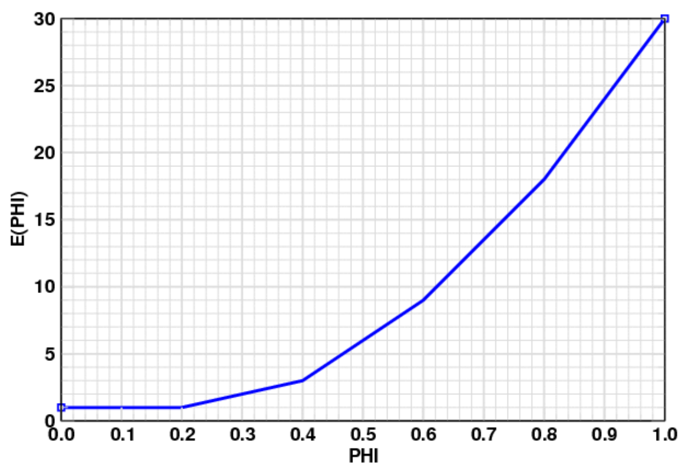
Table 1. Foam Damage Model Parameters for 240 kg/m^3 (15 pcf) flexible polyurethane foam

Parameter		Units	Value	Value	Value
Temperature		C	-53.9	21.1	73.9
Young's Modulus	$E_r \cdot E(\theta)$	MPa	99.31	5.96	1.99
Poisson's Ratio	$v_r \cdot v(\theta)$	-	0.250		
Initial Volume Fraction Solid	ϕ_0	-	0.200		
Flow Rate	$h_r \cdot h(\theta)$	-	-15.0	5.0	10.8
Power Exponent	$n_r \cdot n(\theta)$	-	8.5	6.0	3.0
Tensile Strength	c	MPa	6.90		
Adam	a_{dam}	-	1.00		
Bdam	b_{dam}	-	0.50		
Thermal Expansion Coefficient		1/C	60.0 x 10^{-6}		

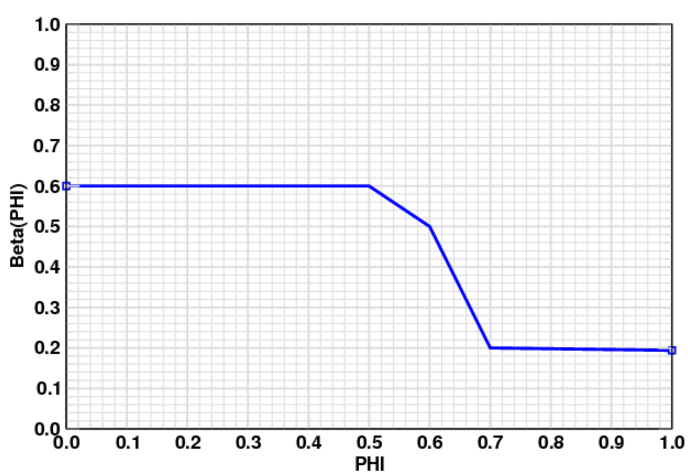
Plots in Figure 13 show dependence of Young's Modulus, $E(\phi_{max})$, flow direction parameter, $\beta(\phi_{max})$, shear strength, $a(\phi_{max})$, and hydrostatic strength, $b(\phi_{max})$, on maximum volume fraction of solid material obtained during any prior loading, ϕ_{max}. The Foam Damage material input block for 240 kg/m^3 (15 pcf) flexible polyurethane foam is listed in Appendix A.

Next, the uniaxial compression experiments were simulated using a simple 8-element model of a cube of material with a 25.4 mm (1.0 inch) edge length shown in Figure 14. In the first simulations, the unit block was subjected to uniaxial compression in the z-direction by preventing z-displacement of nodes on the back plane and displacing nodes on the front plane. The engineering stress-strain curves generated by these simulation are compared with the experimental data in Figures 15 and 16.

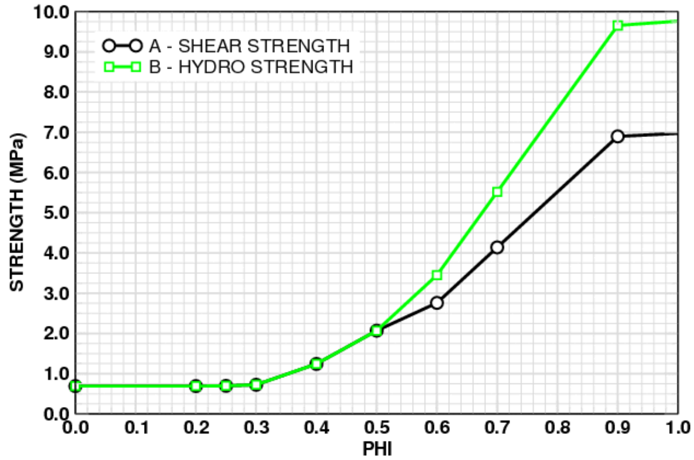
As expected, the Foam Damage model does a reasonably good job of capturing the mechanical response and permanent deformation exhibited by the flexible foam at -53.9 ^{o}C (Figure 15). Unfortunately, this model also predicts permanent deformation and is not able to capture the observed return to original undeformed shape on unloading at 21.1 ^{o}C (Figure 16). Also, the foam strength predicted by this model significantly exceeds the small measured strength at 21.1 ^{o}C for small and intermediate strains (Figure 16).



(a) $E(\phi_{max})$



(b) $\beta(\phi_{max})$



(c) shear strength, $a(\phi_{max})$, and hydrostatic strength, $b(\phi_{max})$

Figure 13. Foam Damage material paramet… ume fraction of solid material obtained during prior loading.

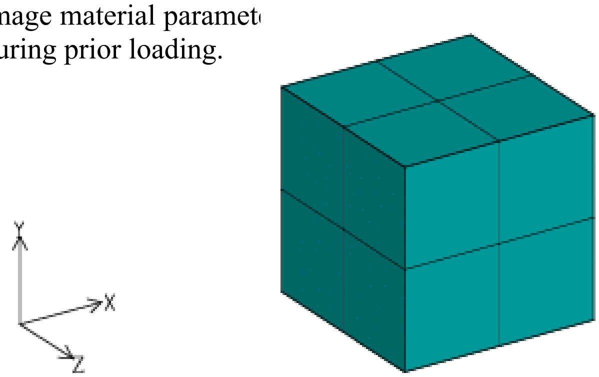


Figure 14. Eight element finite element model used for uniaxial compression simulations.

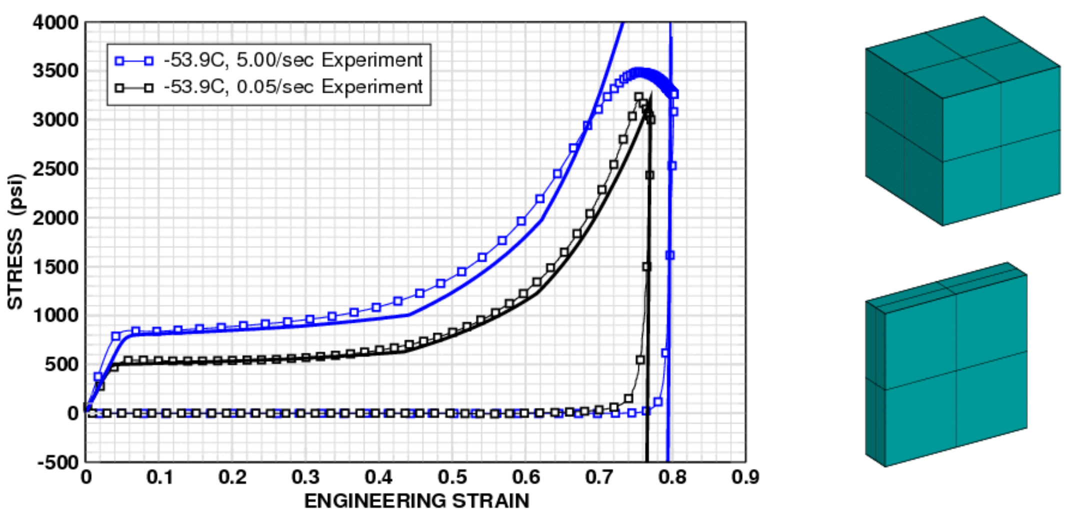


Figure 15. 15pcf flexible foam uniaxial compression experiments (symbols) and Foam Damage model predictions (solid lines) at -53.9 °C and an engineering strain rate of 0.05 or 5.0 per second.

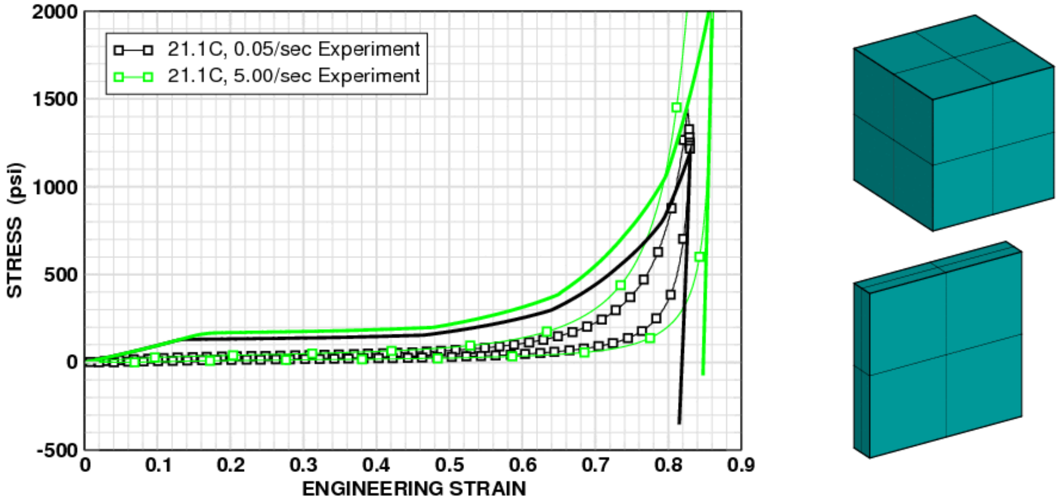


Figure 16. 15 pcf flexible foam uniaxial compression experiments (symbols) and Foam Damage model predictions (solid lines) at 21.1 °C and an engineering strain rate of 0.05 or 5.0 per second.

5. Flex Foam Model

The previous section clearly showed that the existing Foam Damage model could be used to describe the behavior of foam compressed at temperatures below the glass transition temperature; but at temperatures above the glass transition temperature the Foam Damage model was inadequate. Experiments on the 15pcf flexible polyurethane foam showed a dramatic change in foam response with temperature with this foam behaving as a flexible foam at room temperature and a rigid foam at cold temperatures.

The inelastic behaviors of flexible foam that need to be captured can be described simply in terms of idealized uniaxial (one-dimensional) mechanical units. Linear elasticity can be described with a linear spring that satisfies Hooke's law

$$\sigma = E \cdot \varepsilon_S \tag{18}$$

The ideal linear viscous unit used to describe creep is the dashpot for which the strain rate is proportional to the applied stress

$$\dot{\varepsilon}_D = \frac{\sigma}{\eta} \tag{19}$$

The Maxwell model which is simply a spring in series with a dashpot (Figure 17) has the strain rate given by

$$\dot{\varepsilon} = \dot{\varepsilon}_S + \dot{\varepsilon}_D = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \tag{20}$$

which can be rewritten as

$$\dot{\sigma} = E\left(\dot{\varepsilon} - \frac{\sigma}{\eta}\right) \tag{21}$$

Note the similarities between Equation 21 and Equation 2 which described response captured by the Foam Damage model. The Foam Damage model is essentially just a complex Maxwell element with the inelastic rate given as a power-law function of stress (Equation 10).

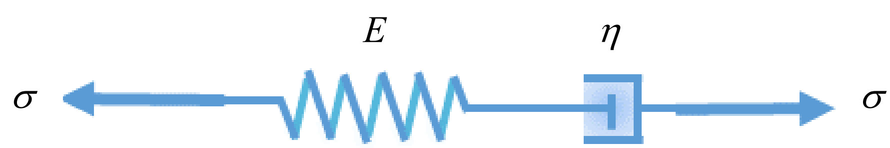


Figure 17. Maxwell model composed of spring in series with dashpot.

The simplest description of polymer behavior is often given by a Generalized Maxwell model (Figure 18) which is simply one or several Maxwell models in parallel with an equilibrium spring. For this model the stress rate is given by the combination of contributions from the equilibrium spring and the parallel Maxwell model

$$\dot{\sigma} = E_f \dot{\varepsilon} + E_r \left(\dot{\varepsilon} - \frac{\sigma}{\eta} \right) \tag{22}$$

The Maxwell model provides dissipation and the equilibrium spring is continuously trying to return the polymer to its original undeformed configuration

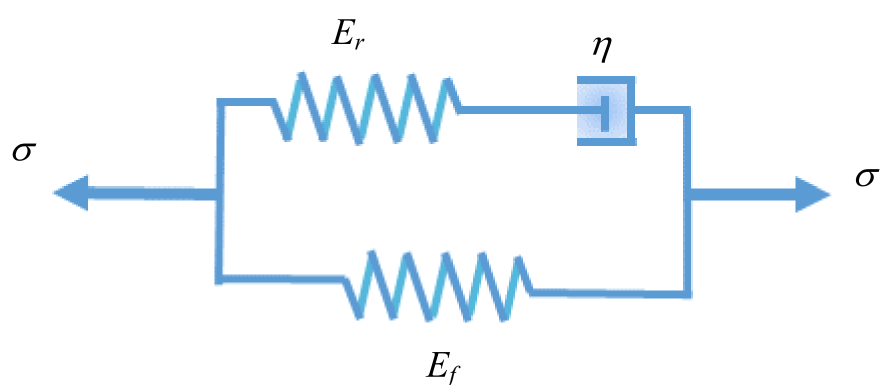


Figure 18. Generalized Maxwell model used to described polymer mechanical behavior.

This simple one-dimensional model is the basis for the new Flex Foam model. The Flex Foam model is simply a modified version of the Foam Damage model in parallel with an equilibrium non-linear elastic spring that is trying to return the foam to its original undeformed shape.

The total stress rate, $\dot{\sigma}$, is given as the sum of the stress rate from the rigid (Foam Damage) part of the model, $\dot{\sigma}_r$, and the stress rate from the flexible, non-linear elastic part of the model, $\dot{\sigma}_f$, as follows

$$\dot{\sigma} = \dot{\sigma}_r + \dot{\sigma}_f \tag{23}$$

Response of the flexible, non-linear elastic part is simply given by

$$\sigma_f = E_f : \varepsilon \tag{24}$$

where E_f, is a fourth-order, isotropic elasticity tensor.

The rigid part of the model is similar to the Foam Damage model presented in an earlier section. The stress rate for constant moduli is given by the following equation

$$\dot{\sigma}_r = E_r : \dot{\varepsilon}^e = E_r : \left(\dot{\varepsilon} - \dot{\varepsilon}^{in}\right) \tag{25}$$

where E_r is also a fourth-order, isotropic elasticity tensor. Unlike the Foam Damage model (Equation 10), the inelastic rate in the Flex Foam model is given by

$$\dot{\varepsilon}^{in} = \dot{\lambda}\, g = e^h \left(\frac{\sigma^*}{a}\right)^n g \tag{26}$$

Note that in the Flex Foam model (Equation 26) inelastic strain is generated whenever there is non-zero effective stress but the Foam Damage model (Equation 10) has an elastic regime of stress states that will not generate any inelastic strain. This change allows the equilibrium spring to eventually return the foam to its original undeformed shape. The effective stress σ^* depends only on stress, σ_r, in the rigid part of the model and not on stress in the parallel non-linear elastic flexible part as follows

$$\sigma^* = \sqrt{\bar{\sigma}_r^2 + \frac{a^2}{b^2} p_r^2} \tag{27}$$

where a and b are state variables that define the current deviatoric and volumetric strengths of the foam. State variables a and b are user-prescribed functions of ϕ, which is the **current** volume fraction of solid material given by

$$\phi = \frac{\phi_0 V_0}{V} \tag{28}$$

where $\phi_0$is the initial volume fraction of solid material, V_0 the initial volume, and V the current volume. $\bar{\sigma}_r$ is the von Mises effective stress, a scalar measure of the deviatoric stress and is given by

$$\bar{\sigma}_r = \sqrt{\frac{3}{2} s_r : s_r} \tag{29}$$

p_r is the pressure or mean stress and is given by

$$p_r = \frac{1}{3} \sigma_r : i \tag{30}$$

where σ_r is the Cauchy stress in the rigid part of the model and i is the second-order identity tensor. s_r is the second-order deviatoric stress tensor

$$s_r = \sigma_r - p_r\, i \tag{31}$$

g is a symmetric, second-order tensor that defines the orientation of the inelastic flow. For associated flow, flow direction g is simply normal to the yield surface and is given by

$$g_{associated} = \frac{\frac{\partial \varphi}{\partial \sigma_r}}{\left|\frac{\partial \varphi}{\partial \sigma_r}\right|} = \frac{\frac{3}{a^2} s_r + \frac{2}{3b^2} p_r i}{\left|\frac{3}{a^2} s_r + \frac{2}{3b^2} p_r i\right|} \quad (32)$$

and the radial flow direction is given by

$$g_{radial} = \frac{\sigma_r}{|\sigma_r|} = \frac{\sigma_r}{\sqrt{\sigma_r : \sigma_r}} \quad (33)$$

The Flex Foam model also has a parameter, β, which allows for the flow direction to be prescribed as a linear combination of associated and radial flow directions as follows

$$g = \frac{(1-\beta)\, g_{associated} + \beta\, g_{radial}}{|(1-\beta)\, g_{associated} + \beta\, g_{radial}|} \quad (34)$$

As in the Foam Damage model, damage surfaces for the Flex Foam model are simply 3 orthogonal planes with normals given by the positive principal stress axes in principal stress space as shown in Figure 12 and are described by the following equation

$$\varphi^{i}_{damage} = \sigma^{**i} - c(1-w) = 0, \ i = 1,3 \quad (35)$$

where σ^{**i} is a principal total stress, c is the initial tensile strength which is a material parameter, and w is a scalar measure of the damage. Damage has an initial value of 0.0 and is limited to a maximum value of 0.99 to prevent the tensile strength from going to zero or becoming negative due to numerical round-off. As damage occurs, the damage surface will collapse toward the origin and the foam will have very little tensile strength. The foam will, however, still have compressive strength. Damage is given as a monotonically-increasing, user-prescribed function of damage strain, ε_{dam}, and damage strain is a function of the maximum tensile strain, ε_{max}, and the plastic volume strain, ε^{p}_{vol}, as follows

$$w = w(\varepsilon_{dam}) = w(a_{dam}\varepsilon_{max} + b_{dam}\varepsilon^{p}_{vol}) \quad (36)$$

where a_{dam} and b_{dam} are positive material parameters which allow the user to control the rate at which damage is generated in tension and compression. Damage is never allowed to decrease even if the damage strain decreases which means that once foam is damaged, healing does not occur in the model.

To fully capture temperature, strain-rate, and lock-up effects several material parameters are functions of temperature, θ, and/or the **current** volume fraction of solid material, ϕ, which depends on the volume strain. Material parameters defining the foams elastic response, Young's modulus and Poisson's ratio, are functions of temperature, θ, and ϕ. To be more specific, the current Young's moduli and Poisson's ratios used in a simulation are given by

$$\begin{aligned} E_f &= E \cdot E_f(\theta) \cdot E_f(\phi) \cdot (1 + d_{mult}\varepsilon_{dev}) & v_f &= v \cdot v_f(\theta) \cdot v_f(\phi) \\ E_r &= E \cdot E_r(\theta) \cdot E_r(\phi) & v_r &= v \cdot v_r(\theta) \cdot v_r(\phi) \end{aligned} \quad (37)$$

where d_{mult} is a material parameter that can be used to increase the stiffness of the equilibrium spring when the foam is subjected to deviatoric (shearing) strain and ε_{dev} is a scalar measure of the deviatoric strain

$$\varepsilon_{dev} = \sqrt{\frac{2}{3} e : e} \qquad e = \varepsilon - \left(\frac{1}{3} \varepsilon : i\right) i \tag{38}$$

The natural log of the reference flow rate, h, and the power law exponent, n, in Equation 26 are also functions of temperature

$$\begin{aligned} h &= h_r \cdot h(\theta) \\ n &= n_r \cdot n(\theta) \end{aligned} \tag{39}$$

Also in the Flex Foam model, the parameter β which defines the fraction of associated and radial flow in Equation 34 is a user-prescribed function of ϕ.

Since, the elastic moduli are not constant, the contribution of the flexible, non-linear elastic part is computed using

$$\sigma_f = E_f(\theta, \phi, \varepsilon_{dev}) : \varepsilon \tag{40}$$

where θ is the current temperature, ϕ the current volume fraction of solid material, and ε_{dev} the current deviatoric strain. The contribution from the rigid part of the model is computed by writing the kinetic relation in rate form

$$\dot{\sigma}_r = \dot{E}_r : \varepsilon + E_r : \dot{\varepsilon} \tag{41}$$

or

$$\dot{\sigma}_r = \dot{E}_r : E_r^{-1} \sigma_r + E_r : \dot{\varepsilon} \tag{42}$$

Since the elasticity tensor is isotropic, it can expressed in terms of a fourth-order deviator projection and fourth-order spherical projection as follows

$$E_r = 2G\, P_d + 3K\, P_{sp} \tag{43}$$

$$P_{sp} = \frac{1}{3} i \otimes i \qquad P_d = I - P_{sp} \tag{44}$$

where i is the second-order identity and I the symmetric fourth-order identity. Since the deviatoric and spherical projections are orthonormal the inverse of the elasticity tensor is simply given by

$$E_r^{-1} = \frac{1}{2G} P_d + \frac{1}{3K} P_{sp} \tag{45}$$

Material parameters that a user must prescribe for the new Flex Foam model are listed in Table 2. State variables for this model are listed in Table 3. A Flex Foam material input block for 240 kg/m^3 (15 pcf) flexible polyurethane foam is listed in Appendix B.

Table 2. Parameter names and definitions.

Parameter	Definition
1 Youngs Modulus, E	Young's modulus reference value
2 Poissons Ratio, v	Poisson's ratio reference value
3 Flow Rate h_r	Reference flow rate that is multiplied by Rate Function

4 Power Exponent	n_r	Sinh exponent reference value multiplied by Exp. Function
5 Phi	ϕ_0	Initial volume fraction of solid material, Equation 28
6 Dev Multiplier	d_{mult}	Deviatoric strain effect on flexible spring, Equation 37
7 Tensile Strength	c	Initial tensile strength of material, Equation 35
8 Adam	a_{dam}	Contribution of maximum tensile strain to damage strain
9 Bdam	b_{dam}	Contribution of plastic volume strain to damage strain
1 Youngs Function	$E_f(\theta)$	Defines temperature dependence of Young's Modulus
2 Poissons Function	$v_f(\theta)$	Defines temperature dependence of Poisson's Ratio
3 Youngs Phi Function	$E_f(\phi)$	Defines Young's Modulus dependence on ϕ
4 Poissons Phi Function	$v_f(\phi)$	Defines Poisson's Ratio dependence on ϕ
5 Dmod Function	$E_r(\theta)$	Defines temperature dependence of Young's Modulus
6 Dpr Function	$v_r(\theta)$	Defines temperature dependence of Poisson's Ratio
7 Dmod Phi Function	$E_r(\phi)$	Defines Young's Modulus dependence on ϕ
8 Dpr Phi Function	$v_r(\phi)$	Defines Poisson's Ratio dependence on ϕ
9 Rate Function	$h(\theta)$	Defines temperature dependence of Flow Rate
10 Exponent Function	$n(\theta)$	Defines temperature dependence of Power Exponent
11 Shear Hardening Function	$a(\phi)$	Defines dependence of state variable *a* on ϕ
12 Hydro Hardening Function	$b(\phi)$	Defines dependence of state variable *b* on ϕ
13 Beta Function	$\beta(\phi)$	Defines dependence of flow direction parameter β on ϕ
14 Damage Function	$w(\varepsilon_{dam})$	Defines damage dependence on damage strain

Table 3. State variable names and definitions.

State Variable Name		Definition	Equation
1. ITER		Number of sub increments taken in subroutine	
2. EPVOL	ε_{vol}^{p}	Inelastic volume strain $\int \dot{\varepsilon}^{in}: i \ dt$	36
3. PHI	ϕ	Volume fraction of solid material	28
4. EQPS		Equivalent plastic strain $\int \dot{\lambda} \ dt$	26
5. FA	a	Deviatoric strength	26, 27
6. FB	b	Hydrostatic strength	26, 27
7. VSTRAIN		Total volume strain $\int \dot{\varepsilon} : i \ dt$ =ln(V/V_0)	
8. DSTRAIN	ε_{dev}	Total deviatoric strain	37, 38
9. EMAX	ε_{max}	Maximum tensile strain (total)	36
10. DAMAGE	w	Damage	36
11. PWORK		Plastic work rate $\sigma_r:\dot{\varepsilon}^{in}$	
12. DENERGY		Dissipated energy $\iint \sigma_r:\dot{\varepsilon}^{in} dV dt$	

The new Flex Foam model was given the name flex_foam in Sierra, so the input material block in SIERRA needed to use this model would be something like:

```
begin parameters for block block_1
   material foam
   solid mechanics use model flex_foam
end parameters for block block_1
```

Plastic work generated by the inelastic deformation is output as a stave variable PWORK and can be used as a volumetric heat source, $\dot{Q}$, in coupled thermal stress analyses. With English units, PWORK has units of in-lb/(in^3-sec) and would need to be scaled by the conversion $10.71x10^{-5}$ BTU/in-lb before it is used as the volumetric heating rate in ARIA. See [1] for more information on performance of coupled thermal stress simulations.

DENERGY is a state variable that prescribes the energy dissipated in each finite element so to compute the total energy dissipated by an entire foam block you would need to sum contributions from each finite element using User Output in Sierra [9] or post-processing the results with ALGEBRA [12] and a command like:

```
diss_energy = SUM(DENERGY)
```

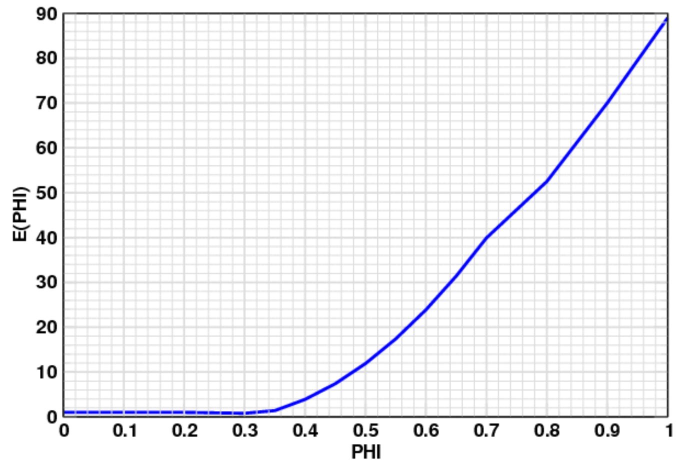
6. Flex Foam Parameters for 240 kg/m³ (15 pcf) Polyurethane Foam

New Flex Foam model parameters for 240 kg/m³ (15 pcf) flexible polyurethane foam at temperatures between -53.9 °C and 73.9 °C are given in Table 4 and Figure 19. Material parameters were obtained using an iterative fitting process in which parameters were selected, experiments simulated, parameters modified, and process repeated until a 'best' fit was obtained. Tools to automate this fitting process and to generate an 'optimized' fit are currently being developed but are not yet available.

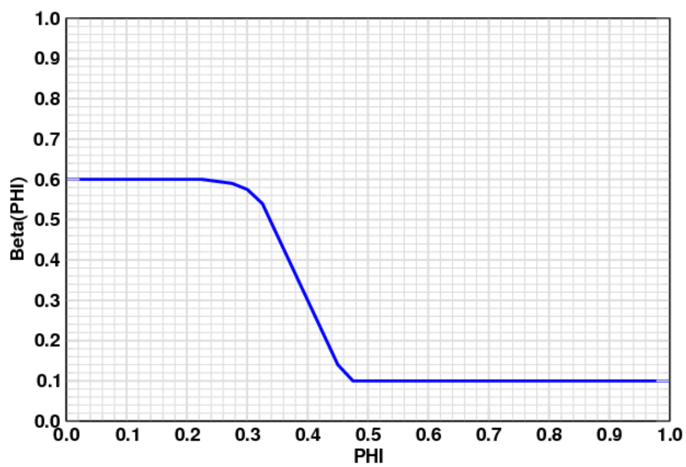
Plots in Figure 19 show dependence of Young's Moduli, $E_r(\phi)$ and $E_f(\phi)$, flow direction parameter, $\beta(\phi)$, shear strength, $a(\phi)$, and hydrostatic strength, $b(\phi)$, on current volume fraction of solid material, ϕ. For this fit, the Poisson's ratios were assumed constant and did not change with changes in temperature or volume strain. Also, damage was excluded by simply setting the damage as a function of damage strain equal to zero for all values of damage strain. An input block for this foam is listed in Appendix B.

Table 4. Flex Foam Model Parameters for 240 kg/m³ (15 pcf) flexible polyurethane foam

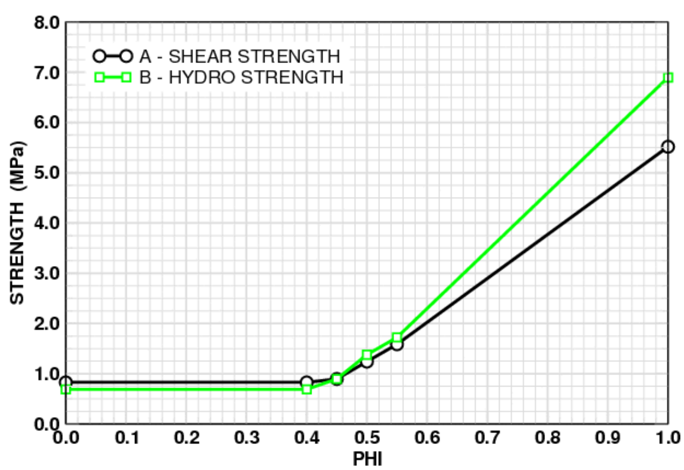
Parameter		Units	Value	Value	Value
Temperature		C	-53.9	21.1	73.9
Young's Modulus	$E \cdot E_r(\theta)$	MPa	99.3	0.596	0.546
Poisson's Ratio	$\upsilon \cdot \upsilon_r(\theta)$	-	0.250		
Young's Modulus	$E \cdot E_f(\theta)$	MPa	0.993	0.298	0.248
Poisson's Ratio	$\upsilon \cdot \upsilon_f(\theta)$	-	0.250		
Initial Volume Fraction Solid	ϕ_0	-	0.200		
Flow Rate	$h_r \cdot h(\theta)$	-	-15.0	4.8	8.8
Power Exponent	$n_r \cdot n(\theta)$	-	8.5	5.0	3.0
Dev. Multiplier	d_{mult}	-	0.20		
Tensile Strength	c	MPa	6.90		
Adam	a_{dam}	-	1.00		
Bdam	b_{dam}	-	0.50		
Thermal Expansion Coefficient		1/C	60.0 x 10^{-6}		



(a) $E_r(\phi)$ and $E_f(\phi)$ are both given by above curve



(b) $\beta(\phi)$



(c) shear strength, $a(\phi)$, and hydrostatic strength, $b(\phi)$

Figure 19. Flex Foam model parameter dependence on volume fraction of solid material for 15 pcf Flexible Polyurethane Foam.

Next, the uniaxial compression experiments were simulated using a simple 8-element model of a cube of material with a 25.4 mm (1.0 inch) edge length shown in Figure 14. In the first simulations, the unit block was subjected to uniaxial compression in the z-direction by preventing z-displacement of nodes on the back plane and displacing nodes on the front plane. The engineering stress-strain curves generated by these simulation are compared with the experimental data in Figures 20 and 21. As expected, the model predicts permanent deformation of the foam at -53.9 ^{o}C and a return to original undeformed shape at 21.1 ^{o}C.

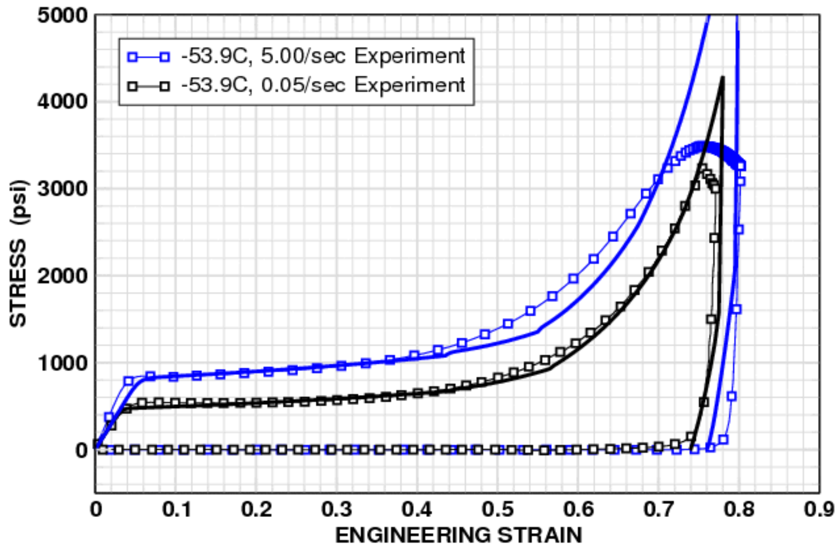


Figure 20. Flexible shipping container foam uniaxial compression experiments (symbols) and Flex Foam simulations (solid lines) at -53.9 ^{o}C and an engineering strain rate of 0.05 or 5.0 per second.

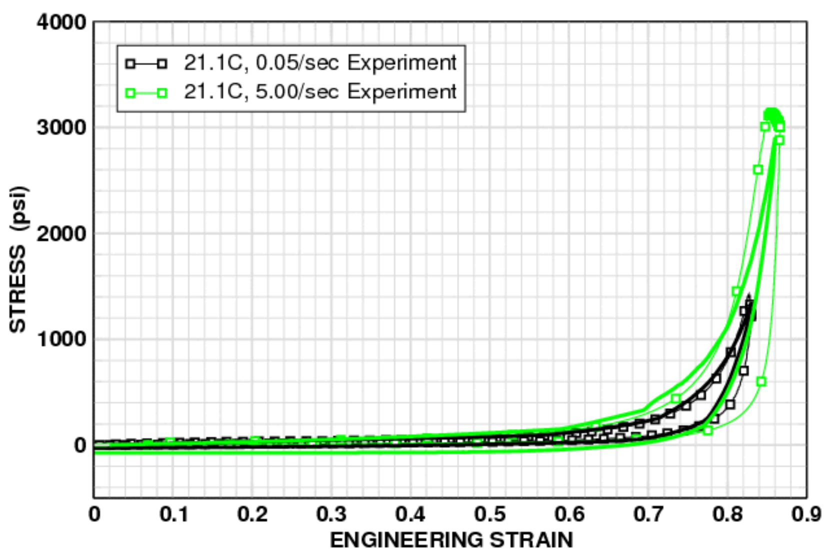


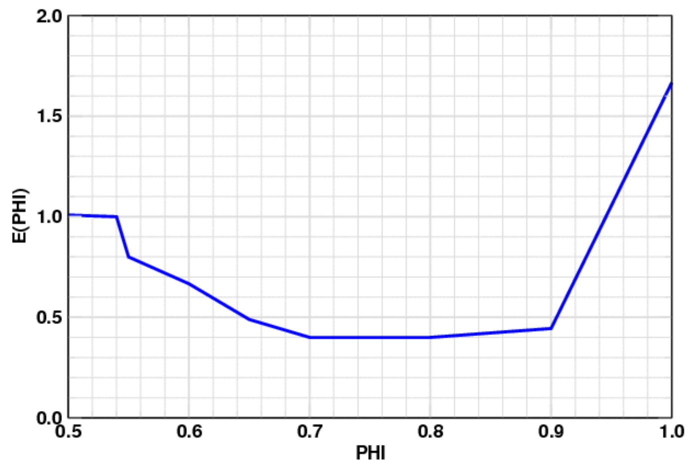
Figure 21. Flexible shipping container foam uniaxial compression experiments (symbols) and Flex Foam simulations (solid lines) at 21.1 ^{o}C and an engineering strain rate of 0.05 or 5.0 per second.

7. Flex Foam Parameters for 640 kg/m^3 (40 pcf) Cellular Silicone

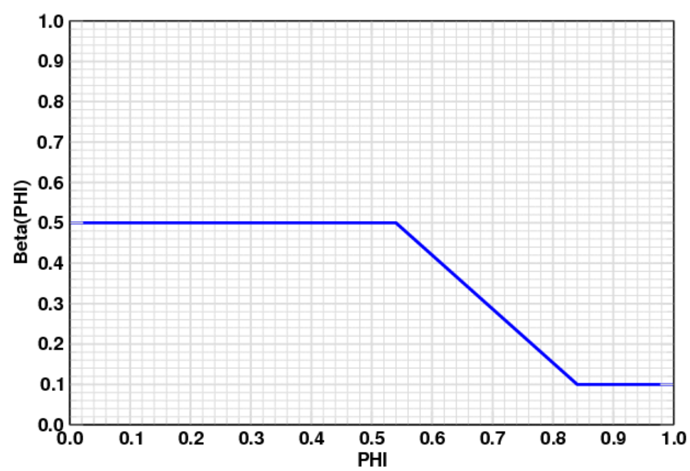
Flex Foam model parameters for 640 kg/m^3 (40 pcf) Cellular Silicone Foam were obtained next and are given in Table 5, Figure 22, and Appendix C. Since solid silicone has a density of approximately 74 pcf; this foam has an initial volume fraction of solid material of 0.54 = 40/74. Material parameters were again obtained using a manual iterative fitting process in which parameters were selected, experiments simulated, parameters modified, and process repeated until a 'best' fit was obtained. Damage was excluded by simply setting the damage as a function of damage strain equal to zero for all values of damage strain.

Table 5. Flex Foam Model Parameters for 640 kg/m^3 (40 pcf) Cellular Silicone

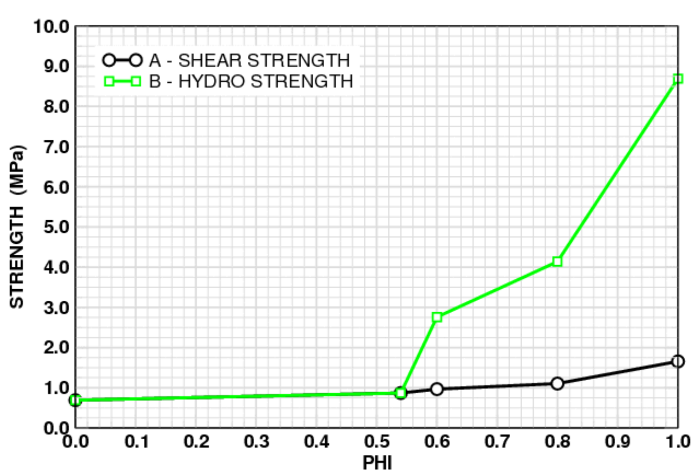
Parameter		Units	Value	Value	Value
Temperature		C	-53.9	18.3	73.9
Young's Modulus	$E \cdot E_r(\theta)$	MPa	0.931	0.745	0.559
Poisson's Ratio	$v \cdot v_r(\theta)$	-	0.050 - 0.499		
Young's Modulus	$E \cdot E_f(\theta)$	MPa	0.462	0.372	0.276
Poisson's Ratio	$v \cdot v_f(\theta)$	-	0.055		
Initial Volume Fraction Solid	ϕ_0	-	0.540		
Flow Rate	$h_r \cdot h(\theta)$	-	-20.0	-4.5	4.5
Power Exponent	$n_r \cdot n(\theta)$	-	8.0	8.0	8.0
Dev. Multiplier	d_{mult}	-	0.0		
Tensile Strength	c	MPa	1.38		
Adam	a_{dam}	-	1.00		
Bdam	b_{dam}	-	0.50		
Thermal Expansion Coefficient		1/C	210.0×10^{-6}		



(a) $E_r(\phi)$ and $E_f(\phi)$ are both given by above curve



(b) $\beta(\phi)$



(c) shear strength, $a(\phi)$, and hydrostatic strength, $b(\phi)$

Figure 22. Flex Foam material parameter dependence on volume fraction of solid material for 40 pcf Cellular Silicone.

Next, uniaxial and confined compression experiments were simulated using a simple 8-element model of a cube of material with a 25.4 mm (1.0 inch) edge length shown in Figure 14. In the first simulation, the unit block was subjected to uniaxial compression in the z-direction by preventing z-displacement of nodes on the back plane and displacing nodes on the front plane. The engineering stress-strain curve generated by this simulation (solid black line in Figure 23) captures the load and unload behavior of the foam reasonably well. The confined compression simulation was identical to the uniaxial compression simulation except that displacement in the x and y directions was constrained as it was in the confined compression experiment. The model predicts (solid red line) lock up at a much smaller strain and much less energy dissipation which was consistent with the experimental measurements (red symbols in Figure 23).

Next, to better understand the effects of loading platen friction on model predictions, a series of uniaxial compression simulation were performed using an axisymmetric model of the actual 1.10 diameter and 0.275 inch thick sample (cyan material in Figure 24) with loading platens and friction between the loading platens (blue material in Figure 24) and the sample.

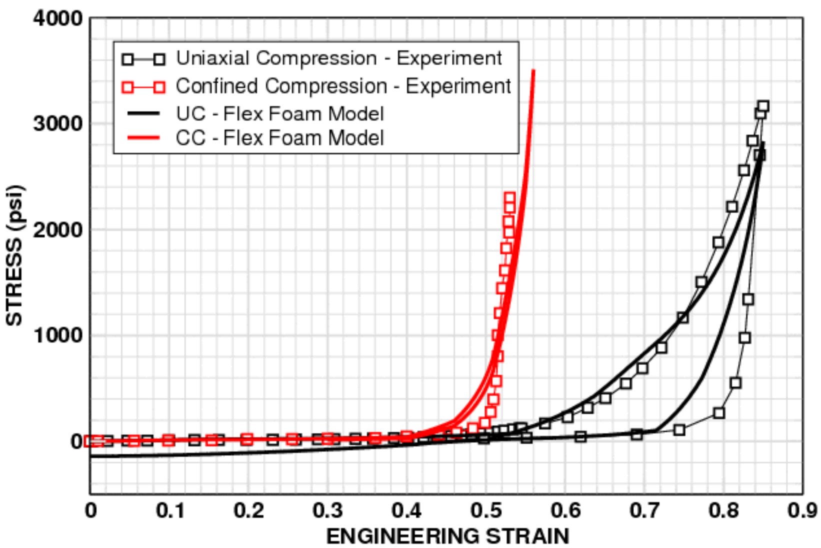


Figure 23. Cellular silicone uniaxial compression experiments (symbols) and simulations (solid lines) at three different temperatures and four different engineering strain rates.

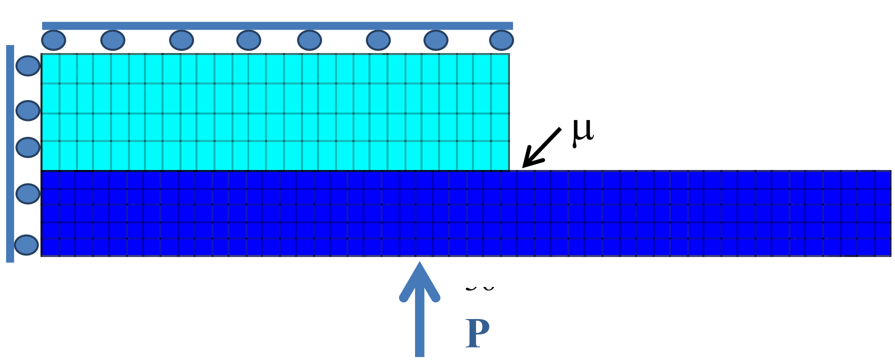


Figure 24. Axisymmetric model of 1.10 inch diameter, 0.275 inch tall foam sample subjected to uniaxial compression. Only half of foam disc thickness modeled due to symmetry.

Uniaxial compression with loading platen simulations were performed using coefficient of friction values of 0.000001, 0.01, and 0.50. A comparison of the predicted engineering stress strain curves generated by these simulations is shown in Figure 25. The simulation with nearly zero friction generated predictions that were close to the previous unconfined compression simulations with the unit block (compare black solid curve with black dashed curve in Figure 25). As friction is increased the predicted stress-strain curves tend to lock up sooner but do not lock up as soon as the confined compression simulation. Plots of the deformed shape of the model at maximum compression (Figure 26) show that the foam sample is predicted to squeeze out much less with increases in foam sample to platen friction. These results show it will be extremely important to not only measure the load vs displacement when characterizing these foams but also to measure the amount of lateral deformation of the foam since the coefficient of friction between the loading platen and sample is generally poorly characterized.

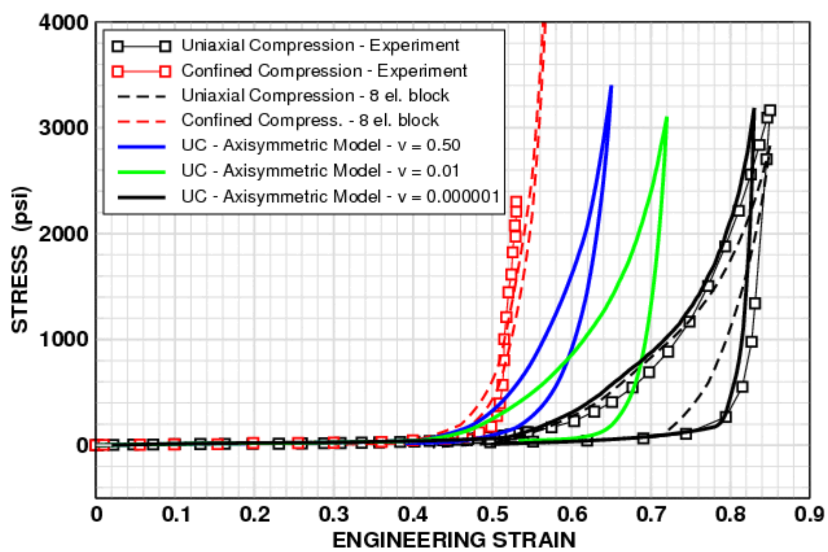
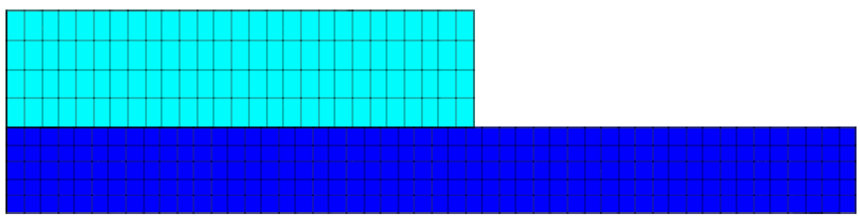
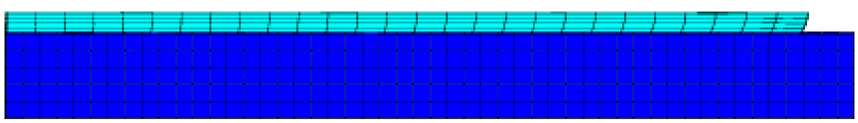


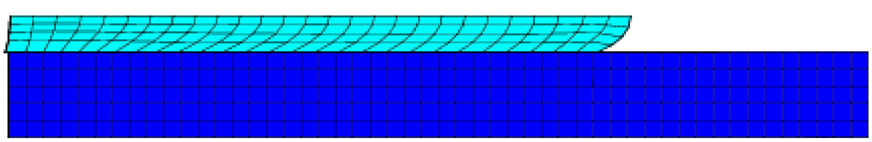
Figure 25. Cellular silicone uniaxial compression experiments (symbols) and simulations (solid lines) at three different temperatures and four different engineering strain rates.



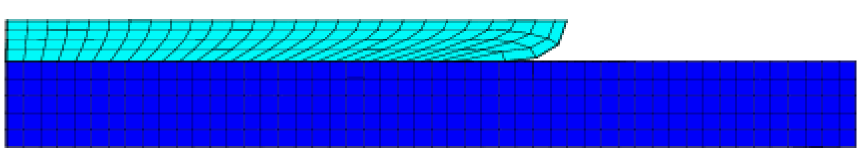
(a) undeformed model



(b) coefficient of friction = 0.000001



(c) coefficient of friction = 0.10



(d) coefficient of friction = 0.50

Figure 26. Predicted deformed shapes of foam samples at maximum compression.

8. Summary

Experiments were performed to characterize the mechanical response of both rigid and flexible polyurethane foams to large deformation. In these experiments the effects of load path, loading rate, and temperature were investigated. Results from these experiments indicated that flexible polyurethane foam will nearly return to its original undeformed shape after being compressed to large strain at room temperature. However, flexible polyurethane foam that is flexible at room temperature can become rigid and behave like a rigid polyurethane foam at cold temperatures and not return to its original undeformed shape after being compressed to large strains. Both flexible and rigid foams exhibit damage, as evidenced by a reduction in stiffness and strength, when compressed to large strain.

A new ***Flex_Foam*** model was developed to describe the mechanical response of both rigid and flexible polyurethane foams to loading experienced during accidental impact events. This model is based on a simple Generalized Maxwell model and has a Unified Creep Plasticity model to capture foam dissipation in parallel with an non-linear elastic element that is continuously trying to return the foam to its original undeformed shape. Various parts of the model are temperature dependent so the model can capture the transition from rigid foam behavior at cold temperatures to flexible foam behavior at room temperature. Essentially, contributions from the Unified Creep Plasticity part of the model decrease as the foam is heated through its glass transition temperature, the amount of energy dissipated by the foam decreases, and the contribution of the non-linear elastic spring becomes much more significant.

The new Flex Foam model captures most of the mechanical behavior exhibited by flexible and rigid polyurethane foams and other flexible foams like cellular silicone. Experiments indicated that both flexible and rigid polyurethane foams are damaged when they are first compressed to large deformation and will not be able to absorb as much energy during subsequent cycles. The Flex Foam model currently does not capture the reduction in foam stiffness due to damage and only reduces the tensile strength of the foam due to damage. Future work will be to include the effects of foam damage on foam moduli. Work is also in progress to create optimization tools based on a Levenberg-Marquardt non-linear least squares fitting algorithm to make selection of material parameters for the new Flex Foam model easier.

The current Flex Foam model should provide accurate predictions for the deformation and energy dissipation of both flexible and rigid polyurethane foams during impact events. The Flex Foam model outputs a state variable, PWORK, which can be used in fully-coupled thermal stress analyses to compute temperature increases in the foam due to plastic work which will, in turn, affect the mechanical response of the foam.

Element death based on foam damage reaching a critical level can be used to simulate complex foam cracking in either tension or compression. However, the newly created contact surfaces from foam cracking could prove to be challenging for the contact algorithm. Foam density depends on the foaming process and will vary and unfortunately, foam properties are sensitive to changes in initial foam density so product performance will also be sensitive to variations in foam density. This is an area where uncertainty quantification (UQ) analysis could play a key role in understanding expected variability in product performance. It will be interesting to see how far we can push this new modeling capability.

9. References

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Appendix A: Foam Damage input for 15pcf Flexible PU Foam

```
##
## 15 pcf flexible urethane foam
##
## Units: lb, second, inch, psi, temperature C
  begin property specification for material foam
      density = 2.2488e-5   # lb - s2/in4
      thermal strain function = foam_Thermal

      begin parameters for model foam_damage
           youngs modulus    =   720000.0
           poissons ratio    =      0.250
           phi               =      0.200
           flow rate         =      1.000
           power exponent    =      1.000
           tensile strength  =     1000.0  ## psi
           adam              =        1.0
           bdam              =        0.5
           youngs function          = foam_Modulus
           poissons function        = foam_Constant
           youngs phi function      = foam_E
           poissons phi function    = foam_Constant
           rate function            = foam_Rate
           exponent function        = foam_Expo
           shear hardening function = foam_Shearx
           hydro hardening function = foam_Hydrox
           beta function            = foam_Beta
           damage function          = foam_Damage
      end parameters for model foam_damage
  end property specification for material foam

##
## currently no damage = no failure of foam in tension
begin definition for function foam_Damage
  type is piecewise linear
   begin values
     0.00000     0.00000
     0.04000     0.00000
     0.30000     0.00000
   100.00000     0.00000
```

```
   end values
end definition for function foam_Damage

begin definition for function foam_Beta
  type is piecewise linear
   begin values
          0.000        0.600
          0.200        0.600
          0.500        0.600
          0.600        0.500
          0.700        0.200
         10.000        0.000
   end values
end definition for function foam_Beta

begin definition for function foam_E
   type is piecewise linear
    begin values
     0.00     0.020
     0.20     0.020
     0.40     0.060
     0.60     0.180
     0.80     0.360
     1.00     0.600
     1.50     1.000
     2.00     4.000
    10.00    10.000
    end values
 end
##
## currently just using 60 ppm/C based on measured rigid urethane foam
## flexible should probably be higher.
##
 begin definition for function foam_Thermal
      type is piecewise linear
      ordinate is strain
      abscissa is temperature
      begin values
      -500.0    -0.0300
         0.0     0.0000
       500.0     0.0300
      end values
 end

 begin definition for function foam_Modulus
   type is piecewise linear
    begin values
        -53.90      1.000
        -40.00      0.300
        -20.00      0.150
          0.00      0.085
         21.10      0.060
```

```
        73.90       0.020
   end values
end

begin definition for function foam_Constant
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -53.90          1.0
      21.10          1.0
      73.90          1.0
     end values
end definition for function foam_Constant

begin definition for function foam_Rate
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -53.90       -15.00
      21.10         5.00  # 1.0
      73.90        10.80
     end values
end definition for function foam_Rate

begin definition for function foam_Expo
     type is piecewise linear
     ordinate is temperature
     abscissa is time
     begin values
     -53.90          8.5
      21.10          6.0  ## 5.0
      73.90          3.0
     end values
end definition for function foam_Expo

begin function foam_Shear1
     type is piecewise linear
     begin values
          0.000         120.0
          0.200         120.0
          0.900         800.0
         10.000        1000.0
     end values
end function

begin function foam_Shearx
     type is piecewise linear
     begin values
          0.000         100.0
          0.200         100.0
```

```
          0.250          101.0
          0.300          105.0
          0.400          180.0
          0.500          300.0
          0.600          400.0
          0.700          600.0
          0.900         1000.0
         10.000         2000.0
    end values
end function

begin function foam_Hydrox
    type is piecewise linear
    begin values
          0.000          100.0
          0.200          100.0
          0.250          101.0
          0.300          105.0
          0.400          180.0
          0.500          300.0
          0.600          500.0
          0.700          800.0
          0.900         1400.0
         10.000         2800.0
    end values
end function
```

Appendix B: Flex Foam input for 15 pcf Flexible PU Foam

```
##
## 15 pcf flexible urethane foam
##
## Units: lb, second, inch, psi, temperature C
  begin property specification for material foam
     density = 2.2488e-5   # lb - s2/in4
     thermal strain function = foam_Thermal

     begin parameters for model flex_foam
          youngs modulus    =   120000.0
          poissons ratio    =      0.250
          phi               =      0.200
          flow rate         =      1.000
          power exponent    =      1.000
          dev multiplier    =        0.2
          tensile strength  =     1000.0
          adam              =        1.0
          bdam              =        0.5
          youngs function   =        foam_yModulus
          poissons function =        foam_Constant
          youngs phi function =      foam_E
          poissons phi function =    foam_Constant
          rate function     =        foam_Rate
          exponent function =        foam_Expo
          shear hardening function = foam_Shear
          hydro hardening function = foam_Hydro
          beta function     =        foam_Beta
          dmod function     =        foam_Modulus
          dpr function      =        foam_Constant
          dmod phi function        = foam_E
          dpr phi function         = foam_Constant
          damage function          = foam_Damage
     end parameters for model flex_foam
  end property specification for material foam

##
## currently no damage = no failure of foam in tension
##
begin definition for function foam_Damage
  type is piecewise linear
```

```
   begin values
     0.00000     0.00000
     0.14000     0.00000
     0.40000     0.00000
   100.00000     0.00000
   end values
end definition for function foam_Damage

begin definition for function foam_Beta
  type is piecewise linear
   begin values
          0.000         0.600
          0.200         0.600
          0.225         0.600
          0.250         0.595
          0.275         0.590
          0.300         0.575
          0.325         0.540
          0.350         0.460
          0.375         0.380
          0.400         0.300
          0.425         0.220
          0.450         0.140
          0.475         0.100
          1.000         0.100
         10.000         0.100
   end values
end definition for function foam_Beta

 begin definition for function foam_E
   type is piecewise linear
    begin values
     0.00     0.020
     0.20     0.020
     0.25     0.018
     0.30     0.016
     0.35     0.028
     0.40     0.078
     0.45     0.148
     0.50     0.238
     0.55     0.348
     0.60     0.478
     0.65     0.628
     0.70     0.798
     0.80     1.050
     0.90     1.400
     1.00     1.780
     1.50     3.000
     2.00     6.000
    10.00    10.000
    end values
 end
```

```
##
## currently just using 60 ppm/C based on measured rigid urethane foam
## flexible should probably be higher.
##
 begin definition for function foam_Thermal
      type is piecewise linear
      ordinate is strain
      abscissa is temperature
      begin values
      -500.0    -0.0300
         0.0     0.0000
       500.0     0.0300
      end values
 end

 begin definition for function foam_yModulus
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
        -53.90       0.060
        -40.00       0.050
        -20.00       0.030
          0.00       0.021
         21.10       0.018
         73.90       0.015
      end values
 end

 begin definition for function foam_Modulus
   type is piecewise linear
    begin values
        -53.90       6.000
        -40.00       1.000
        -20.00       0.080
          0.00       0.048
         21.10       0.036
         73.90       0.033
    end values
 end

 begin definition for function foam_Constant
      type is piecewise linear
      ordinate is temperature
      abscissa is time
      begin values
      -53.90          1.0
       21.10          1.0
       73.90          1.0
      end values
 end definition for function foam_Constant
```

```
begin definition for function foam_Rate
    type is piecewise linear
    ordinate is temperature
    abscissa is time
    begin values
    -53.90       -15.00
     21.10         4.80
     73.90         8.80
    end values
end definition for function foam_Rate

begin definition for function foam_Expo
    type is piecewise linear
    ordinate is temperature
    abscissa is time
    begin values
    -53.90          8.5
     21.10          5.0
     73.90          3.0
    end values
end definition for function foam_Expo

begin function foam_Shear
    type is piecewise linear
    begin values
         0.000        120.0
         0.400        120.0
         0.450        130.0
         0.500        180.0
         0.550        230.0
         1.000        800.0
        10.000       1000.0
    end values
end function

begin function foam_Hydro
    type is piecewise linear
    begin values
         0.000        100.0
         0.400        100.0
         0.450        130.0
         0.500        200.0
         0.550        250.0
         1.000       1000.0
        10.000       2000.0
    end values
end function
```

Appendix C: Flex Foam input for Cellular Silicone

```
##
## 0.54 = 40 pcf cellular silicone
##
## Units: lb, second, inch, psi, temperature C
  begin property specification for material foam
     density = 5.997e-5   # lb - s2/in4
     thermal strain function = foam_Thermal

     begin parameters for model flex_foam
          youngs modulus    =    30000.0   # psi
          poissons ratio    =      0.499
          phi               =      0.540
          flow rate         =      1.000
          power exponent    =      1.000
          dev multiplier    =        0.0
          tensile strength  =      200.0
          adam              =        1.0
          bdam              =        0.5
          youngs function   =        spring_Modulus
          poissons function =        spring_PR
          youngs phi function =      foam_E
          poissons phi function =    foam_Constant
          rate function     =        foam_Rate
          exponent function =        foam_Expo
          shear hardening function = foam_Shear
          hydro hardening function = foam_Hydro
          beta function     =        foam_Beta
          dmod function     =        foam_Modulus
          dpr function      =        foam_Constant
          dmod phi function =        foam_E
          dpr phi function  =        foam_PR
          damage function   =        foam_Damage
     end parameters for model flex_foam
  end property specification for material foam

begin definition for function foam_Damage
  type is piecewise linear
   begin values
     0.00000     0.00000
```

```
     0.60000     0.00000
   100.00000     0.00000
   end values
end definition for function foam_Damage

 begin definition for function foam_Beta
   type is piecewise linear
    begin values
          0.000         0.500
          0.540         0.500
          0.840         0.100
          0.900         0.100
          0.950         0.100
         10.000         0.100
    end values
 end definition for function foam_Beta

 begin definition for function foam_E
   type is piecewise linear
    begin values
      0.00000  0.005000000
      0.54000  0.004500000
      0.55000  0.003600000
      0.60000  0.003000000
      0.65000  0.002200000
      0.70000  0.001800000
      0.75000  0.001800000
      0.80000  0.001800000
      0.90000  0.002000000
      1.00000  0.007500000
      1.10000  0.020000000
      1.20000  0.050000000
      1.50000  0.200000000
      1.80000  0.800000000
     10.00000  2.400000000
    end values
 end definition for function foam_E

 begin definition for function foam_PR
   type is piecewise linear
    begin values
      0.00000  0.1000000
      0.54000  0.1000000
      0.55000  0.1000000
      0.60000  0.1600000
      0.65000  0.1800000
      0.85000  0.2000000
      1.00000  0.4000000
      1.10000  0.8000000
      1.30000  1.0000000
      1.50000  1.0000000
      1.70000  1.0000000
```

```
     1.90000  1.0000000
    10.00000  1.0000000
   end values
end definition for function foam_PR

begin definition for function foam_Thermal
    type is piecewise linear
    ordinate is strain
    abscissa is temperature
    begin values
    -500.0    -0.1050
       0.0     0.0000
     500.0     0.1050
    end values
end definition for function foam_Thermal

begin definition for function foam_Modulus
    type is piecewise linear
    ordinate is temperature
    abscissa is time
    begin values
    -53.90          1.0
     18.30          0.8
     73.90          0.6
    end values
end definition for function foam_Modulus

begin definition for function spring_Modulus
    type is piecewise linear
    ordinate is temperature
    abscissa is time
    begin values
    -53.90          0.50
     18.30          0.40
     73.90          0.30
    end values
end

begin definition for function spring_PR
    type is piecewise linear
    ordinate is temperature
    abscissa is time
    begin values
    -53.90          0.1111
     18.30          0.1111
     73.90          0.1111
    end values
end

begin definition for function foam_Constant
    type is piecewise linear
    ordinate is temperature
```

```
    abscissa is time
    begin values
    -53.90         1.0
     18.30         1.0
     73.90         1.0
    end values
end definition for function foam_Constant

begin definition for function foam_Rate
    type is piecewise linear
    ordinate is temperature
    abscissa is time
    begin values
    -53.90       -20.00
     18.30        -4.50
     73.90         4.50
    end values
end definition for function foam_Rate

begin definition for function foam_Expo
    type is piecewise linear
    ordinate is temperature
    abscissa is time
    begin values
    -53.90         8.0
     18.30         8.0
     73.90         8.0
    end values
end definition for function foam_Expo

begin function foam_Shear
    type is piecewise linear
    begin values
         0.000        100.0
         0.540        126.0
         0.600        140.0
         0.800        160.0
         1.000        240.0
         1.200        420.0
         1.400        900.0
         2.000       2000.0
        10.000       5000.0
    end values
end function

begin function foam_Hydro
    type is piecewise linear
    begin values
         0.000        100.0
         0.540        126.0
         0.600        400.0
         0.800        600.0
```

```
          1.000        1260.0
          1.200        5000.0
          1.450       20000.0
          2.000       48000.0
         10.000      100000.0
     end values
end function
```

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