

# Machine Learning + Physics

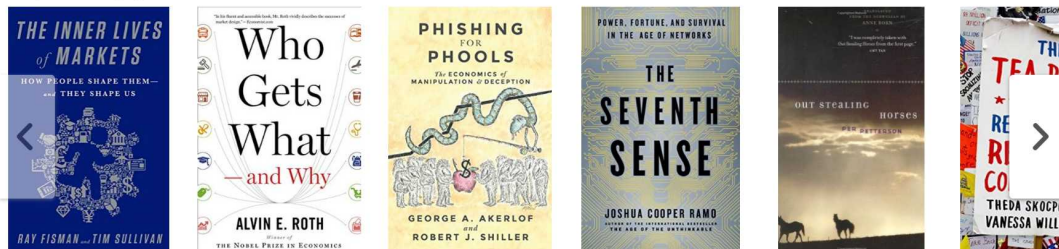
Julia Ling

May 2017

# What is Machine Learning?

- Data-driven algorithms to discern patterns and make predictions on big, high-dimensional data
- Linear regression, support vector machines, neural networks

Inspired by your Wish List [See more](#)



MOST EMAILED MOST

1. Trump Criticizes 'S. About His Son Barr

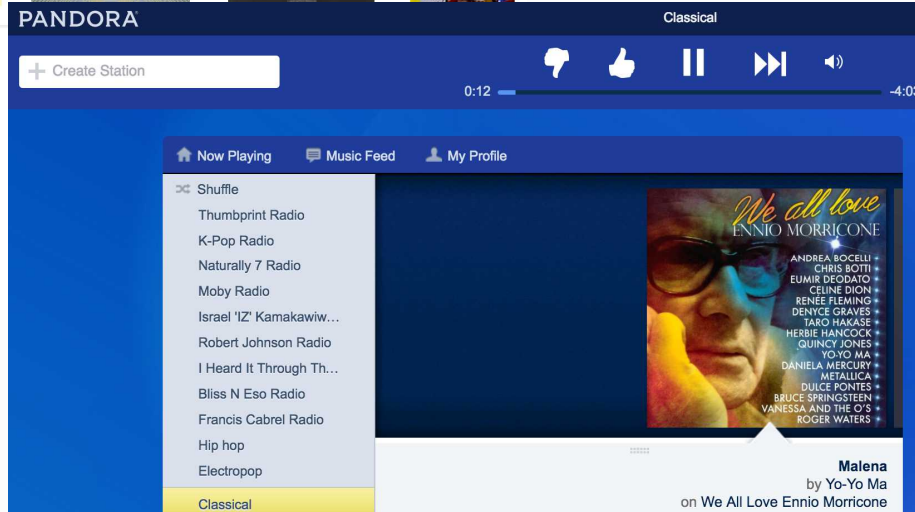
2. Top Russian Cyber on Charges of Treas

3. Pakistan Places Militant Tied to Mumbai Attacks Under House Arrest

4. OPINION A Crime in the Cancer Lab

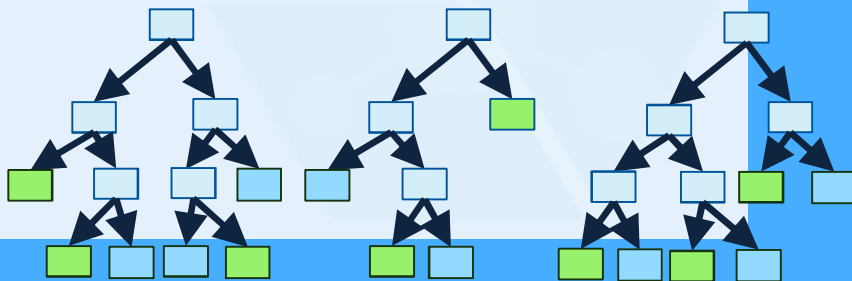
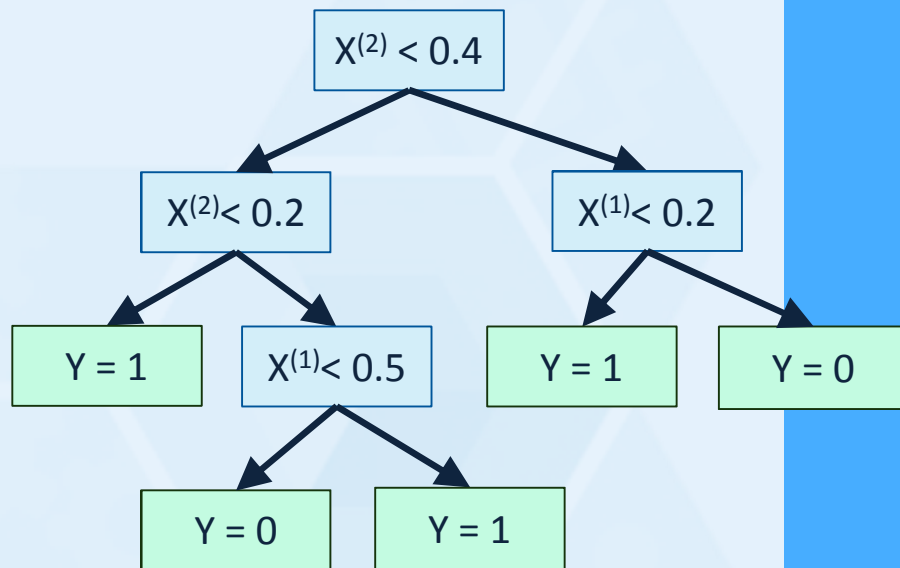


**amazonPrime**  
Original audio series

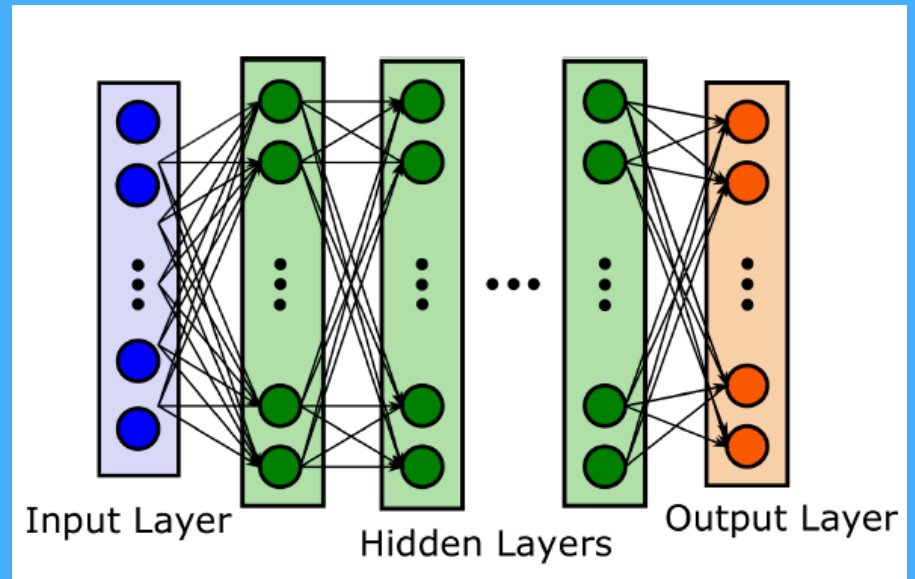




## Random Forest



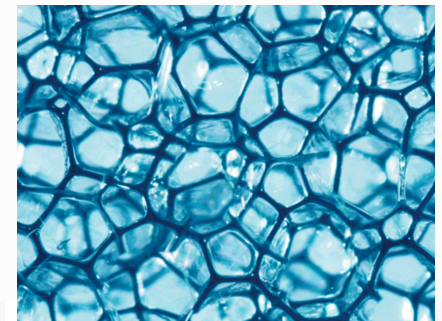
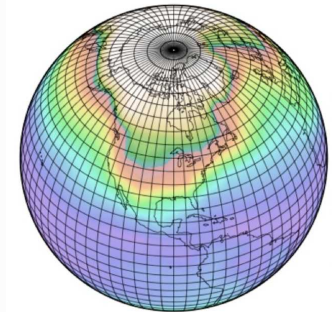
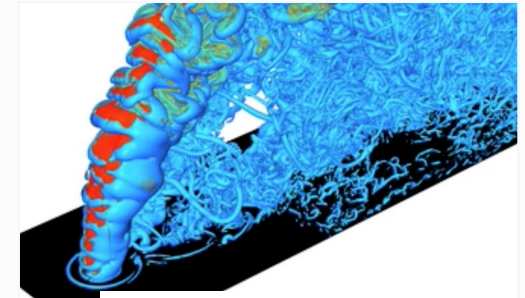
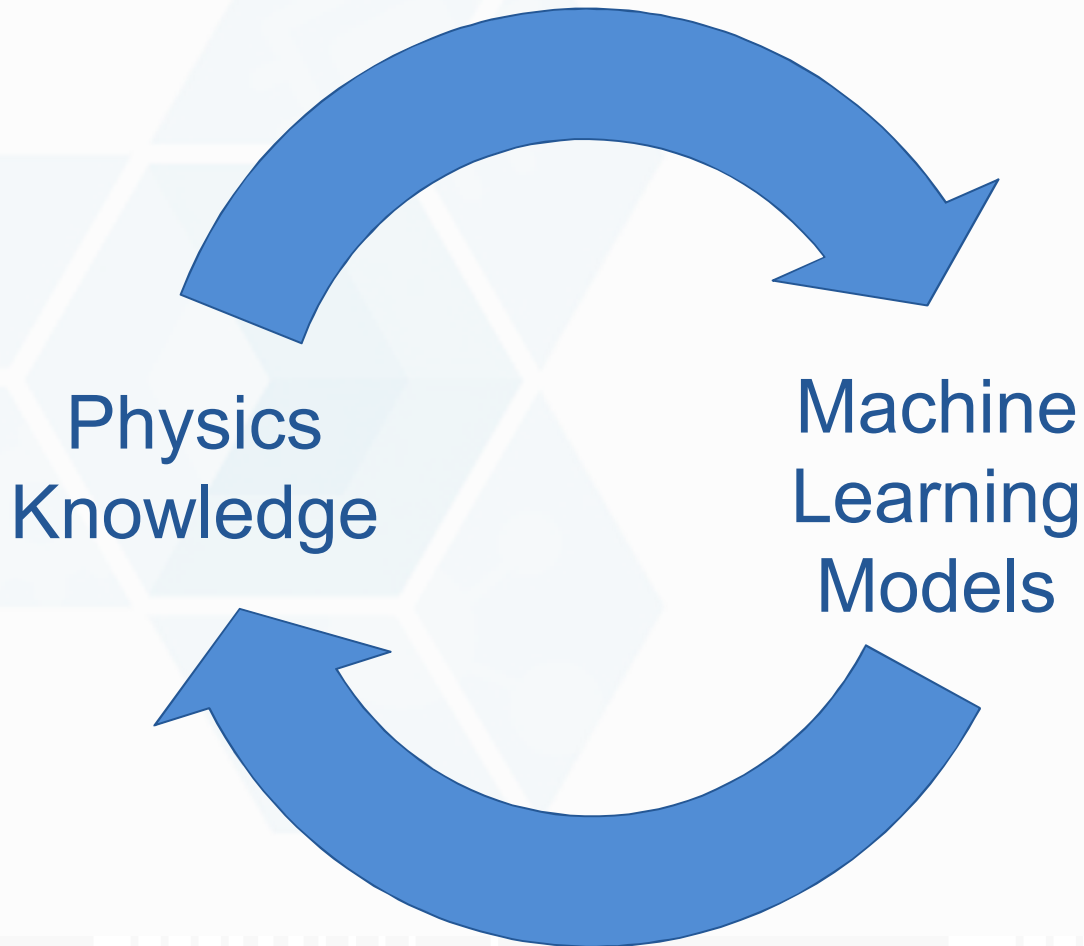
## Neural Network



$$y = f(w^T x)$$

# Machine Learning on Engineering Systems

- How should scientific domain knowledge interact with data-driven models?

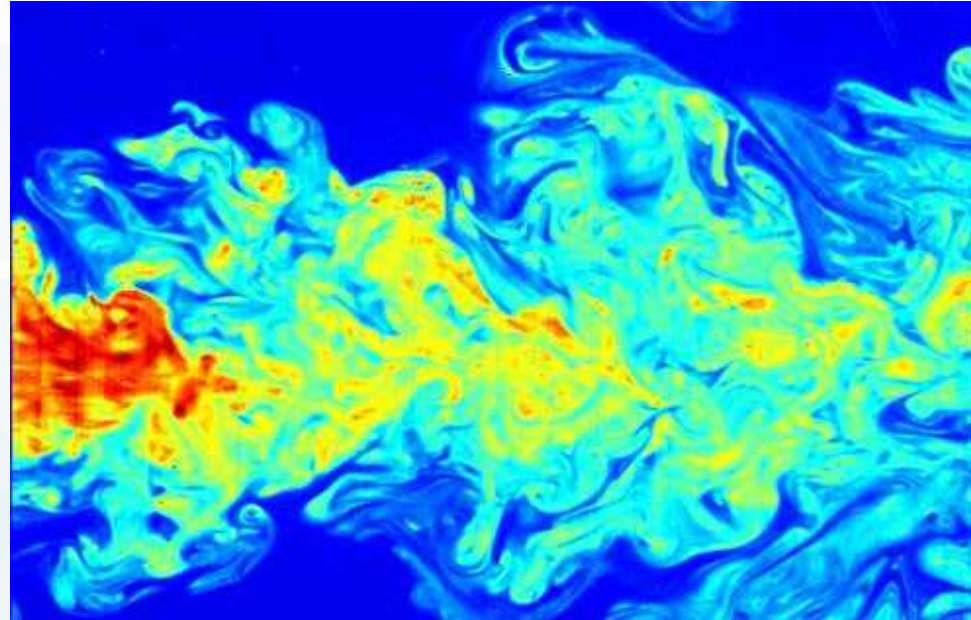


# Outline

- 1) Machine learning for turbulence uncertainty quantification
- 1) Rule extraction to gain physical intuition
- 1) Deep learning with embedded invariances

# Turbulence

Chaotic 3-D fluid motion at a continuum of scales



Fukushima et al.



Hokusai (c 1830)



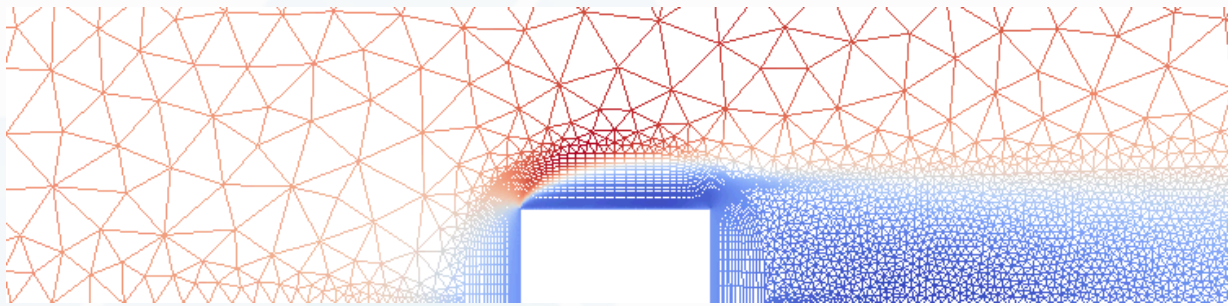
<http://www.windturbinesyndrome.com/2011/wind-turbine-turbulence-what-are-the-micro-climate-effects/>



<https://brilliant.org/wiki/rocket-physics/>

# Turbulence Simulations

*“When I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.” –Horace Lamb*



## Direct Numerical Simulation (DNS)

- Hundreds of millions of core hours for even simple flows
- Exact

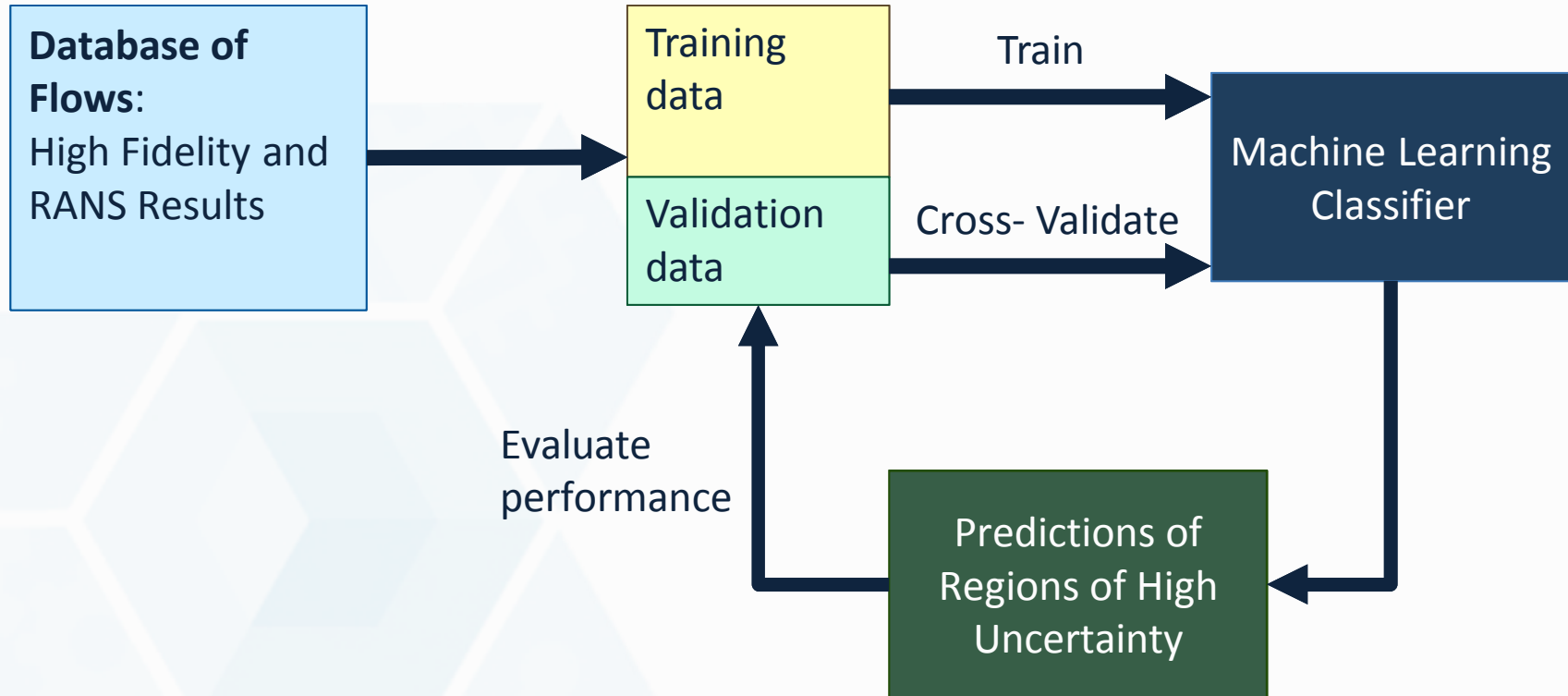
## Reynolds Averaged Navier Stokes (RANS)

- Orders of magnitude less computationally expensive
- Approximate

# Machine Learning Problem Set-Up

- Assemble a database of flows for which both DNS and RANS results are available
- Use the DNS as “truth” data for supervised machine learning
- Build a binary classifier to predict when RANS has high uncertainty based on when specific model assumptions are violated
- Inputs are local flow variables from RANS (velocity, pressure, density, ...)
  - Each point in the mesh represents a separate data point

# Classifier Development



# Classifier Development

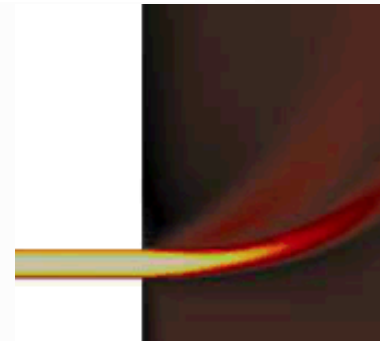
**Database of  
Flows:**  
High Fidelity and  
RANS Results

Contours of velocity magnitude

Angled jet in crossflow



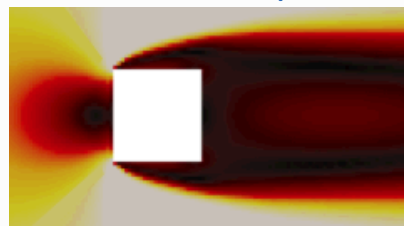
Jet in crossflow



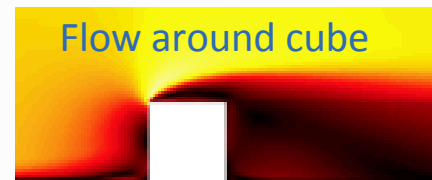
Flow over wavy wall



Flow around square

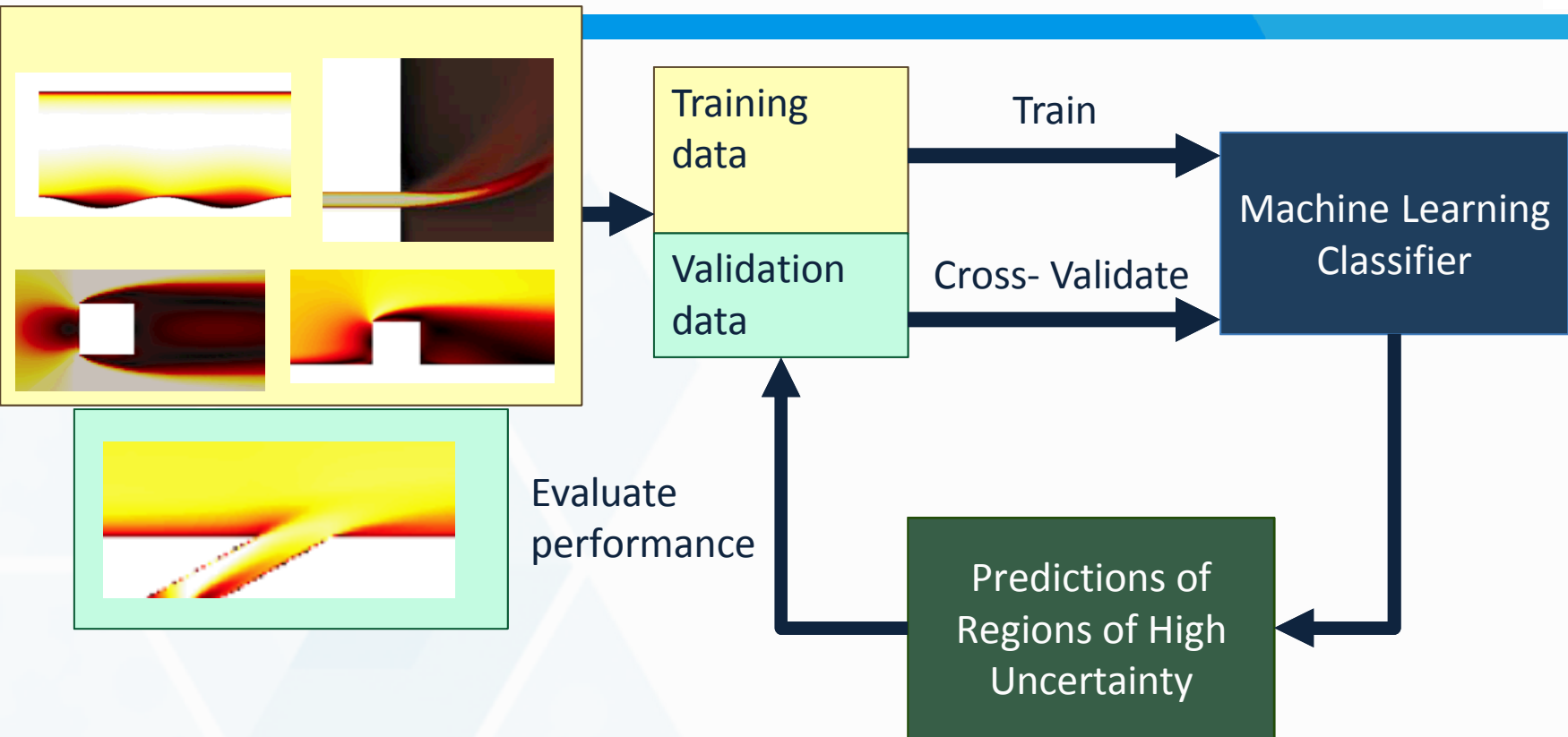


Flow around cube

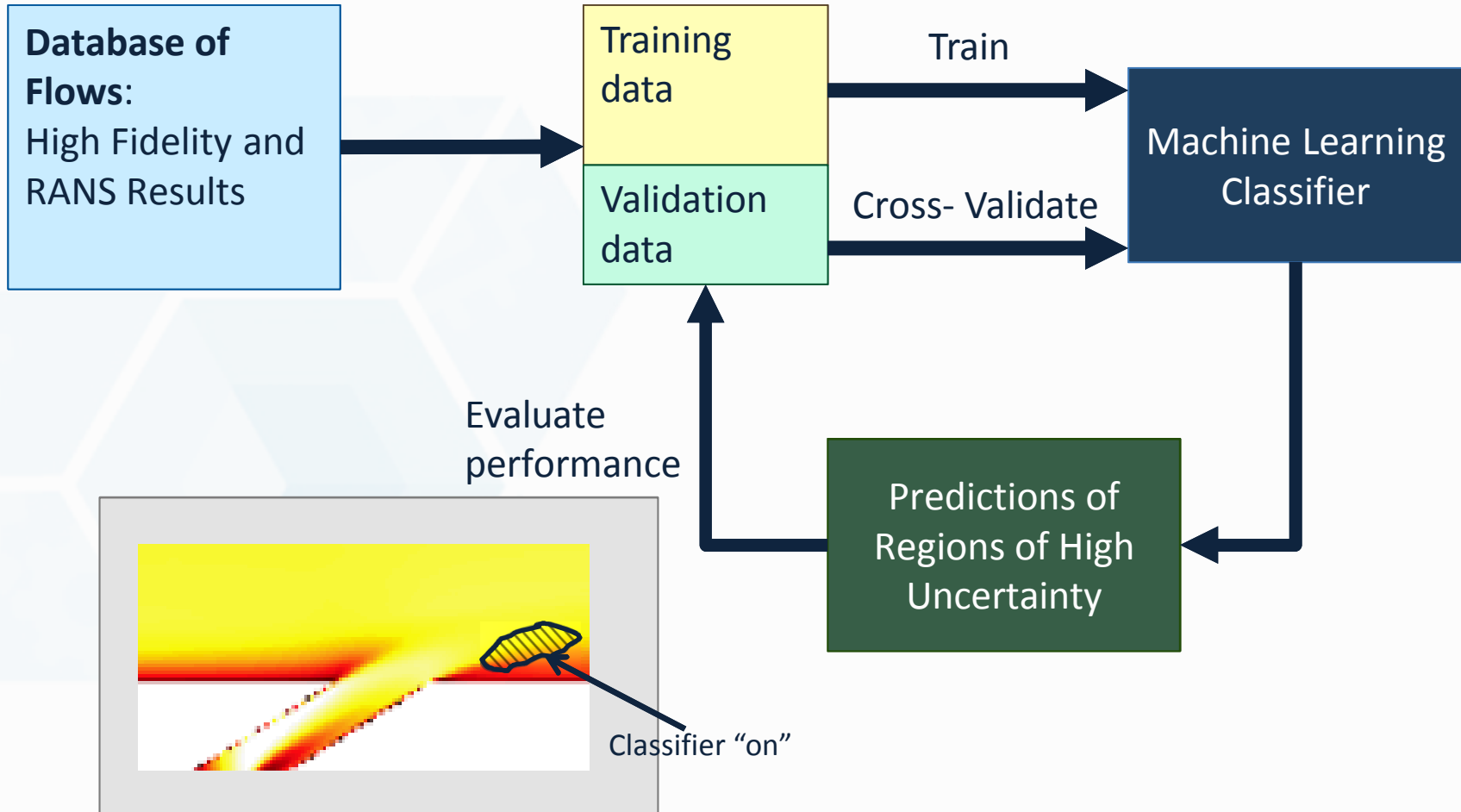


Machine Learning  
Classifier

# Classifier Development

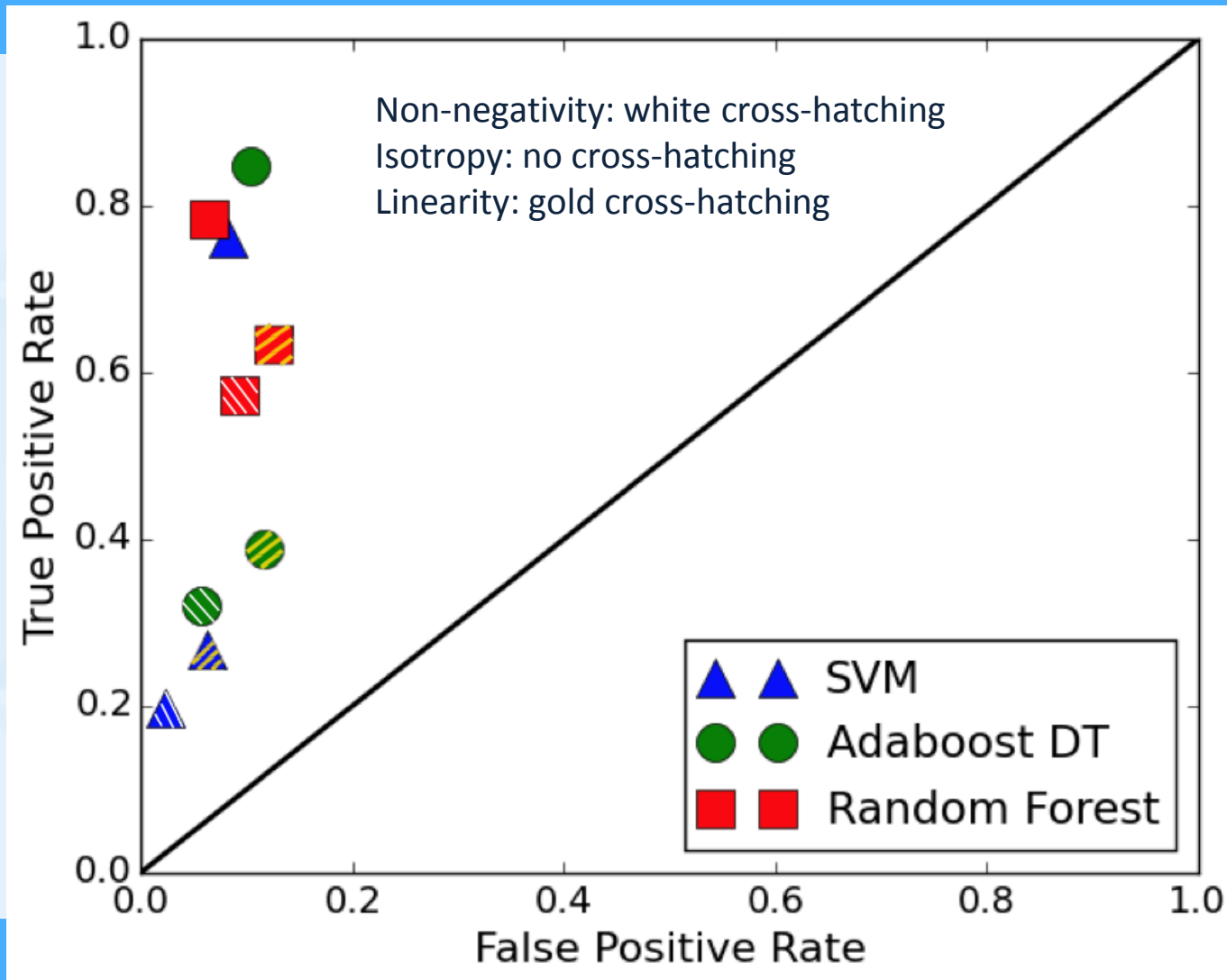


# Classifier Development

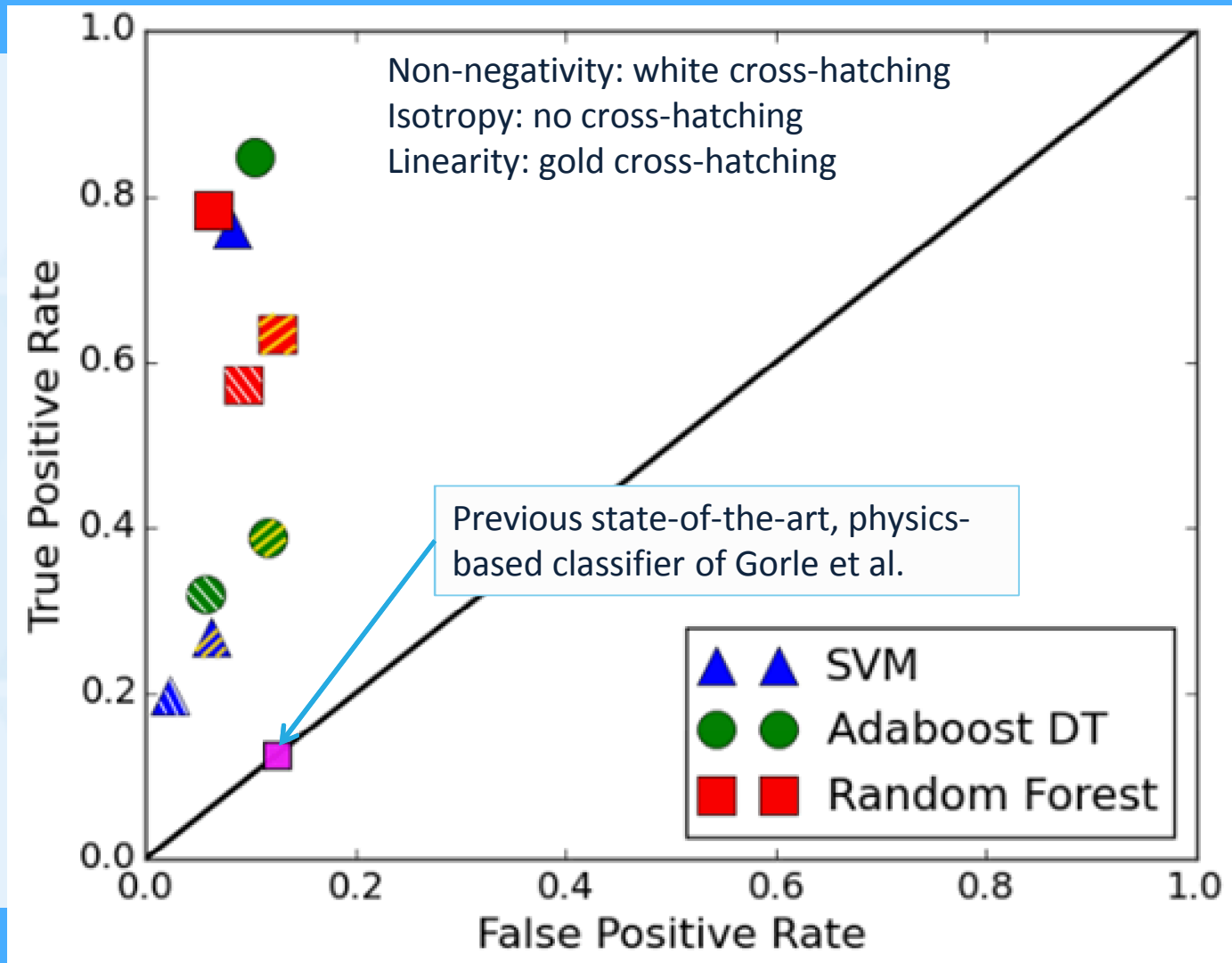


- Use classifier to make predictions on validation set
- Evaluate classifier by comparing to high fidelity results
- Leave-one-out cross-validation

# Classifier Performance



# Classifier Performance

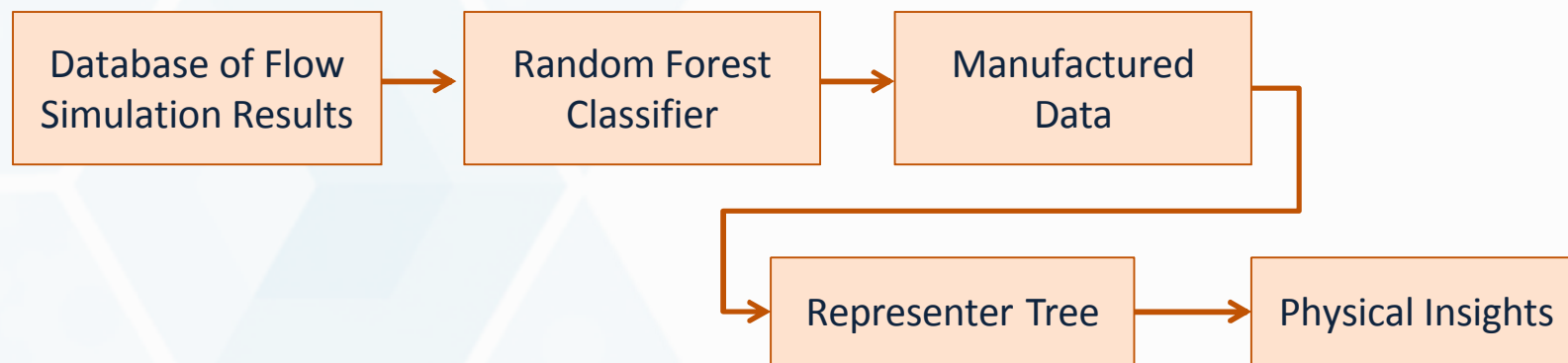


# Outline

- 1) Machine learning for turbulence model form uncertainty quantification
  - Achieved 3X more accurate error detection than previous state of the art
- 2) Rule extraction to gain physical intuition
  - 1) Deep learning with embedded invariances

# Rule Extraction

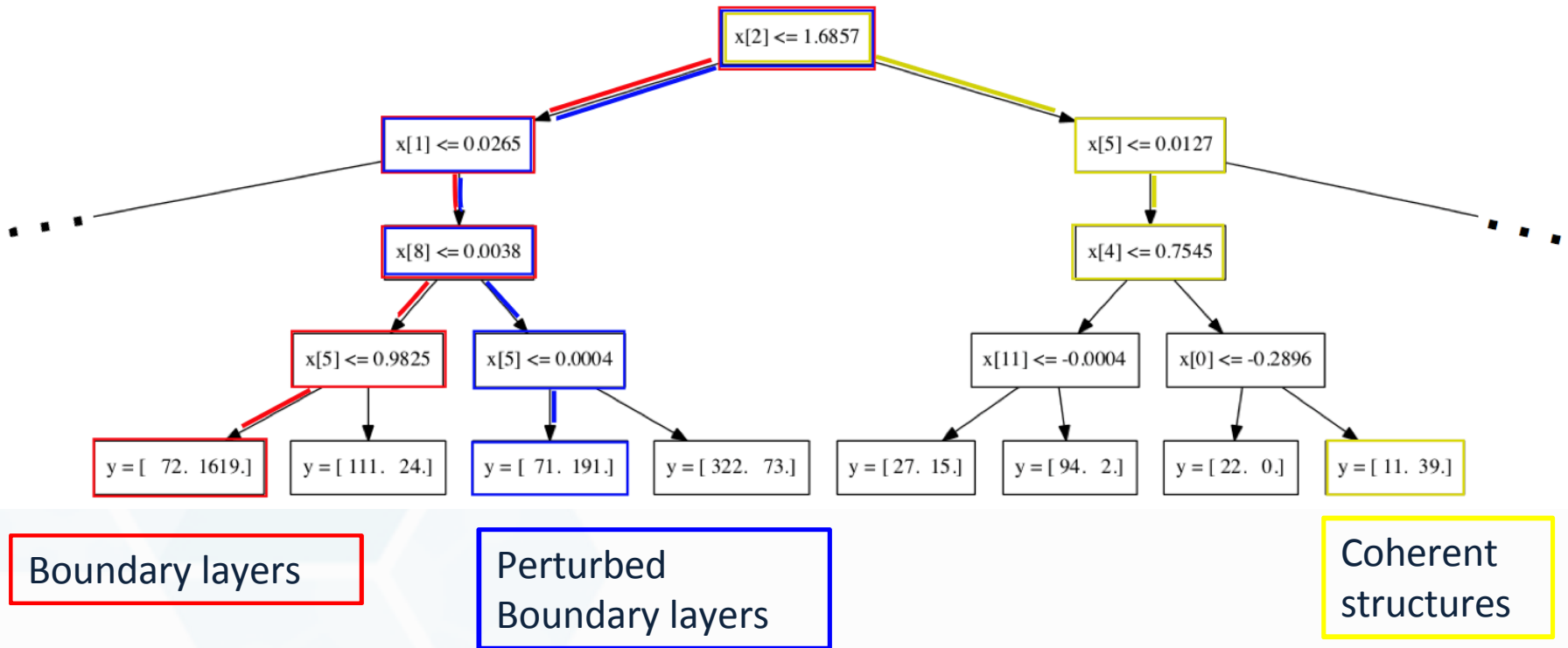
- Random Forests are much more robust and high-performance than single decision trees, but what have we lost?
  - Clarity—how can we understand these machine learned models?
- Representer Trees



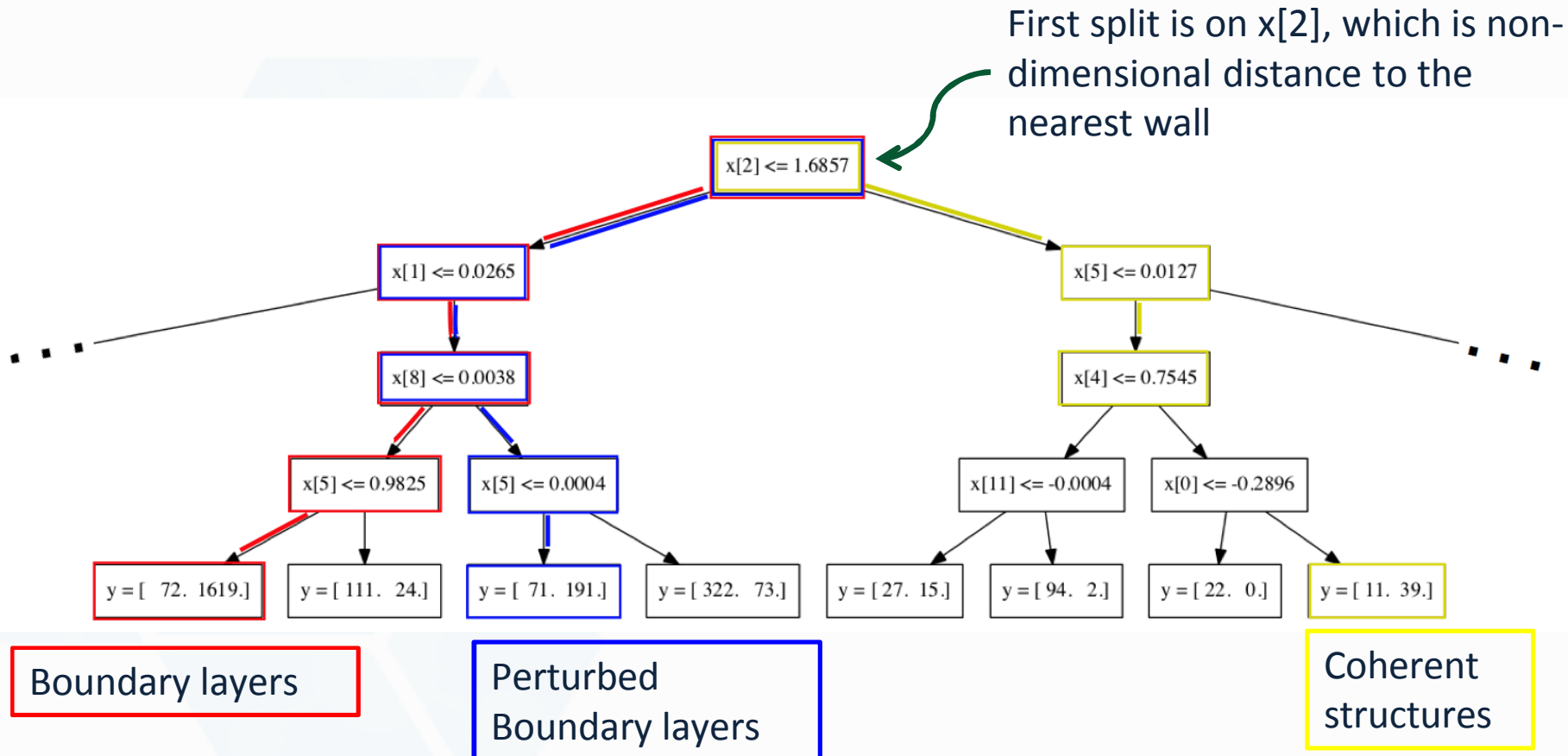
- Trained a representer decision tree based on Random Forest that predicted when the RANS isotropy assumption was invalid

# Analyzing the Representer Tree

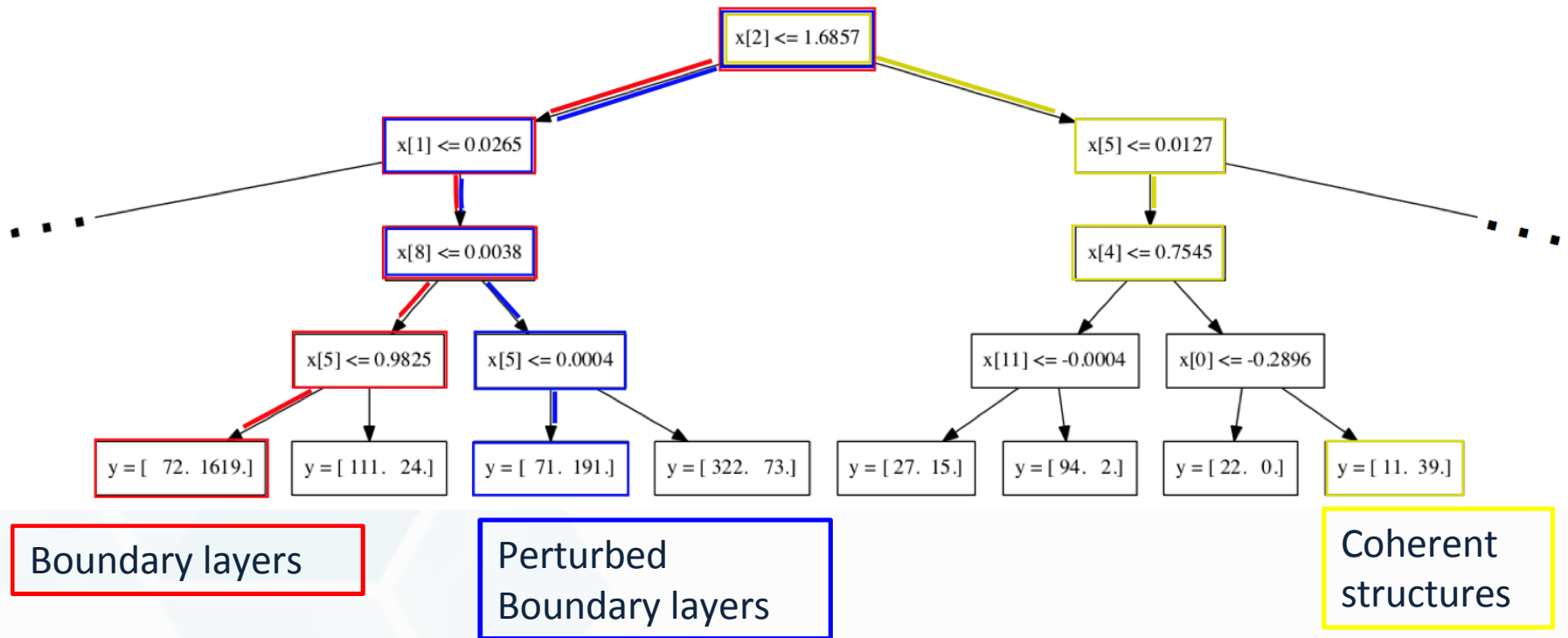
- Look for consistent branches



# Analyzing the Representer Tree



# Analyzing the Representer Tree



- Can determine physical regimes where assumptions are violated
- Can see that different mechanisms cause assumption to break down in near wall region than in free stream

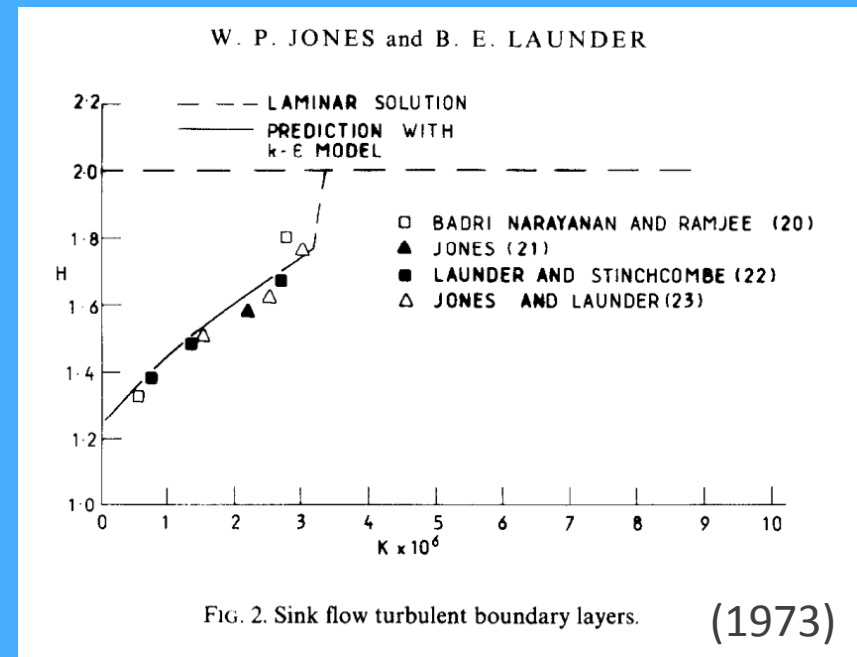
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  - Used rule extraction to understand how the complex machine learning algorithm was making its predictions
  - Increases confidence in ML predictions
  - Provides feedback to turbulence modeler
  
- 3) Deep learning with embedded invariances

# Deep Learning for Turbulence Modeling



- In RANS, use simplifying assumptions to get computational efficiency
  - Need model for unknown term: the Reynolds stress anisotropy tensor **A**
- Default model: Linear Eddy Viscosity Model
  - Based on theory + sparse experimental data
- Our approach: Deep neural network
- Inputs: Mean strain rate tensor **S**, mean rotation rate tensor **R**
- Outputs: Reynolds stress anisotropy **A**



# Deep Learning for Turbulence Modeling



- Inputs: Tensors  $\mathbf{S}$ ,  $\mathbf{R}$
- Output: Tensor  $\mathbf{A}$
- Would like to enforce Galilean invariance
  - Invariance to inertial coordinate frame transformations
  - Borrow some ideas from group theory, representation theory
  - All Galilean invariant tensors that are a function of  $\mathbf{S}$  and  $\mathbf{R}$  lie on a tensor basis: the *integrity basis* of  $\mathbf{S}$  and  $\mathbf{R}$  for the orthogonal group

$$\mathbf{A}(\mathbf{Q}\mathbf{S}\mathbf{Q}^T, \mathbf{Q}\mathbf{R}\mathbf{Q}^T) = \mathbf{Q}\mathbf{A}(\mathbf{S}, \mathbf{R})\mathbf{Q}^T$$

$$\mathbf{A} = \sum_{n=1}^{10} f^{(n)} \mathbf{B}^{(n)}$$

Unknown  
coefficients

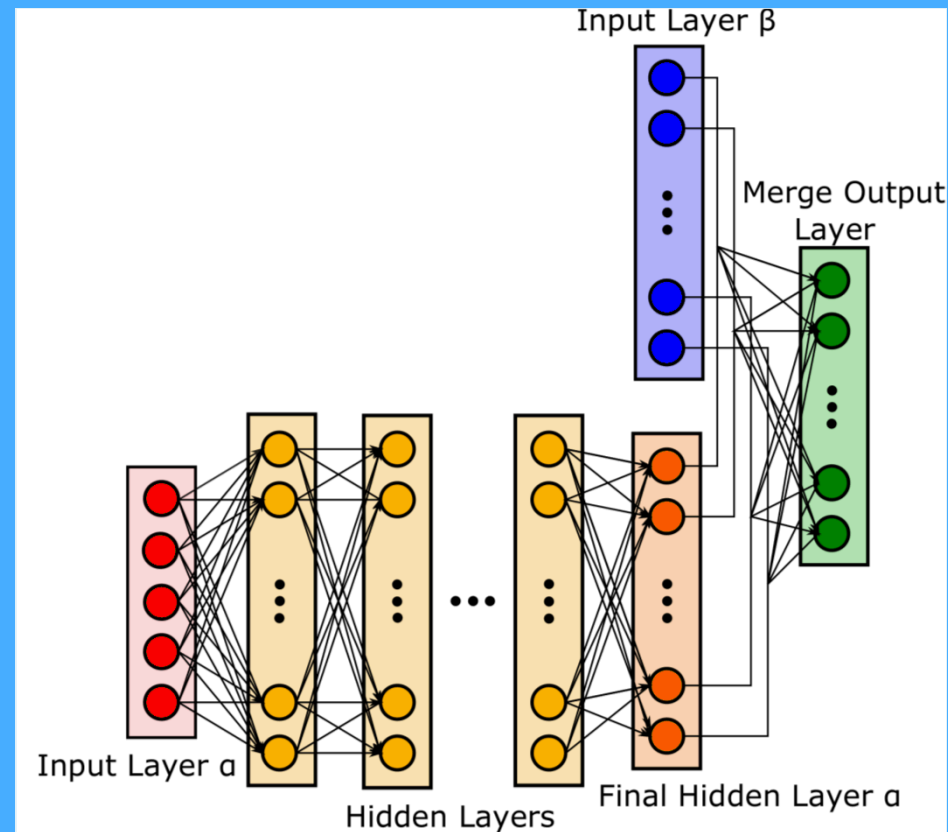
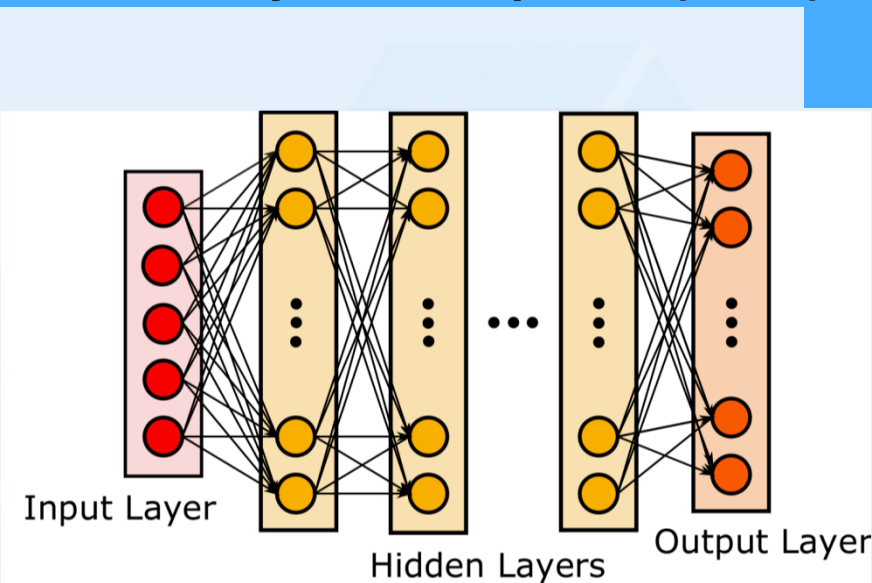
Known Tensor Basis

# Embedding Galilean Invariance

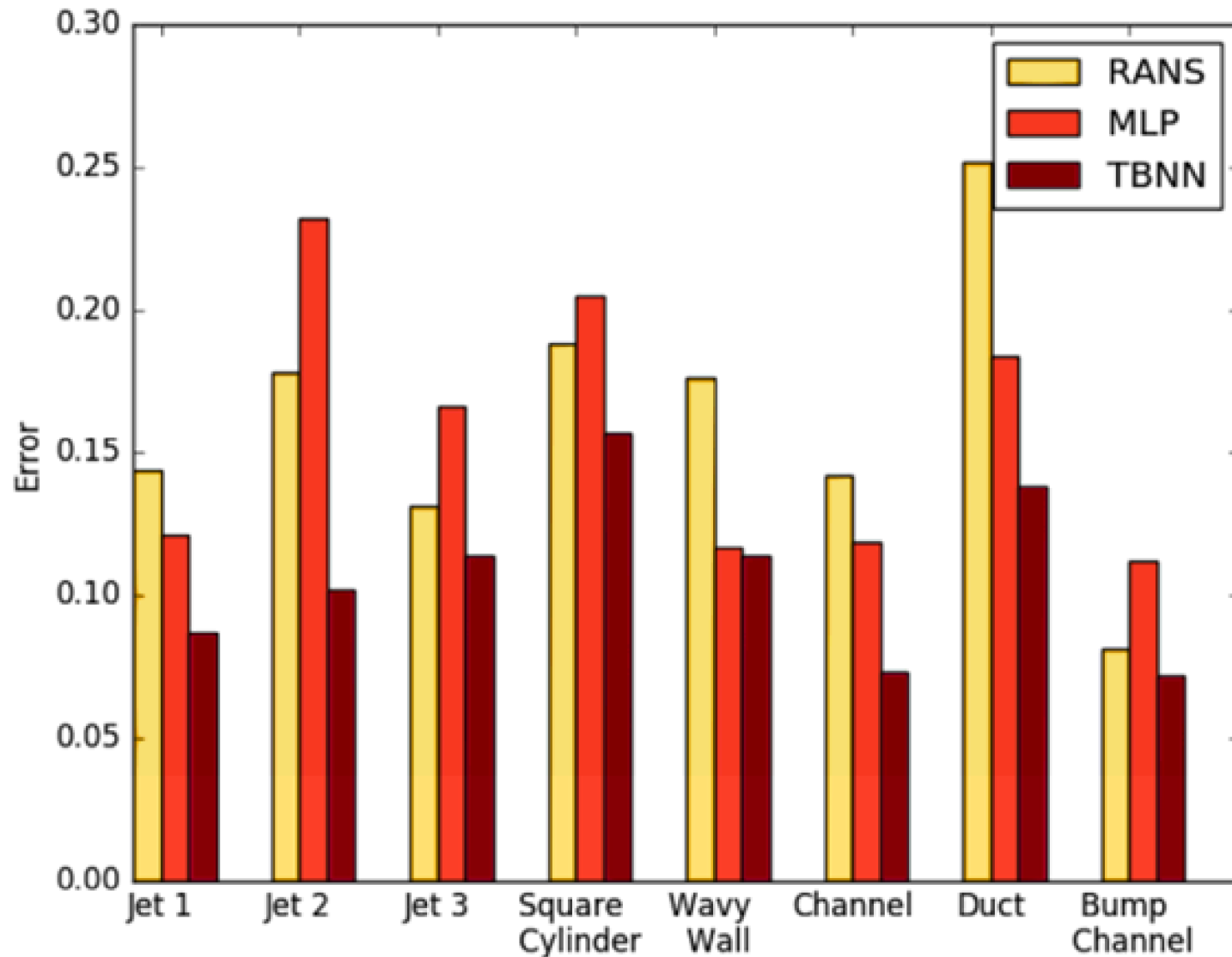


## Multi-Layer Perceptron (MLP)

## Tensor Basis Neural Network (TBNN)



# Deep Learning for Turbulence Modeling

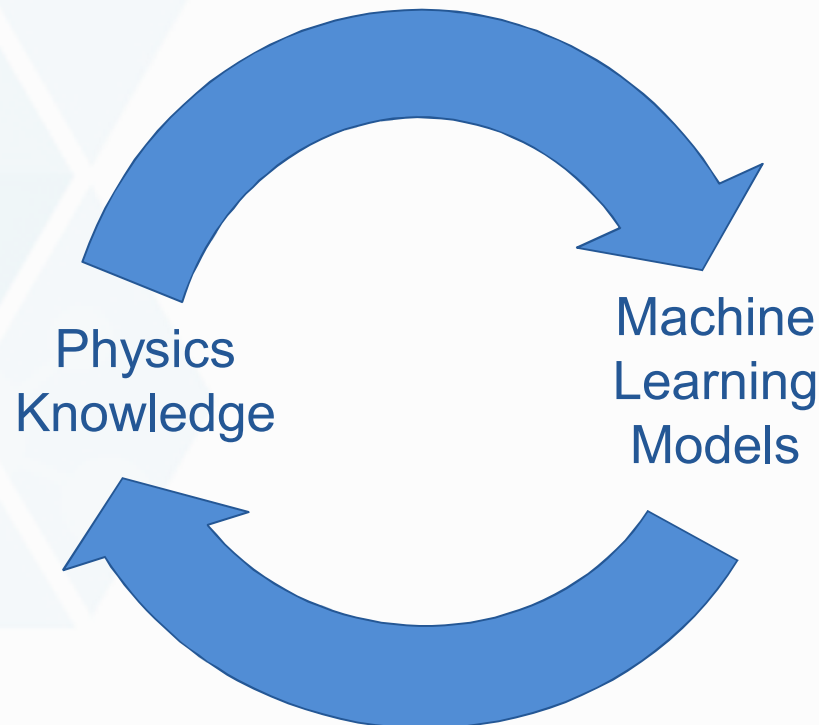


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- 3) Deep learning with embedded invariances
  - Embedded known invariance property into network architecture
  - Achieved significantly improved Reynolds stress anisotropy predictions
  - First application of deep learning to turbulence modeling

# Big Picture

1. Directly embedding scientific domain knowledge into machine learning models can give improved performance, especially in data-limited scenarios
2. Rule extraction on machine learning models can give new physical understanding



# References

- **J. Ling**, A. Kurzawski, and J. Templeton, “Reynolds Averaged Turbulence Modeling using Deep Neural Networks with Embedded Invariance,” *Journal of Fluid Mechanics*, (2016).
- **J. Ling** and J. Templeton, “Evaluation of machine learning algorithms for prediction of regions of high Reynolds averaged Navier Stokes uncertainty,” *Physics of Fluids*, (2015).
- **J. Ling**, “Using Machine Learning to Understand and Mitigate Model Form Uncertainty in Turbulence Models,” *IEEE ICMLA*, (2015).
- **J. Ling**, A. Ruiz, G. Lacaze, and J. Oefelein, “Uncertainty analysis and data-driven model advances for a Jet-in-Crossflow,” *Journal of Turbomachinery*, (2016).
- **J. Ling**, R. Jones, and J. Templeton, “Machine Learning Strategies for Systems with Invariance Properties,” *J. Comp. Phys.*, (2016).
- **J. Ling**, F. Coletti, S. Yapa, J. Eaton, “Experimentally informed optimization of turbulent diffusivity for a discrete hole film cooling geometry,” *International Journal of Heat and Fluid Flow*, (2013).

# Acknowledgements

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# Questions

