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subject: Dynamic, elastic-plastic response of a 2-DOF mass-spring system.

Introduction

The objective of the work presented here arose from abnormal, drop scenarios and specifically the question of how the accelerations and accumulation of plastic strains of internal components could be affected by the material properties of the external structure. In some scenarios, the impact loads can induce cyclic motion of the internal components. Therefore, a second objective was to explore differences that could be expected when simulations are conducted using isotropic hardening vs. kinematic hardening plasticity models.

The simplest model that can be used to investigate the objectives above is a two-degree-of-freedom mass/spring model where the springs exhibit elastic-plastic behavior. The purpose of this memo is to develop such model and present a few results that address the objectives.

Model

Figure 1 presents the model used in this work. It consists of two discrete masses m_1 and m_2 and two elastic-plastic springs: S_1, between a base and m_1 and S_1 joining the two masses. Here, m_1 and S_1 represent the external structure of the dropped object while m_2 and S_2 represent the internal mass and material. The two degrees of freedom are the displacements u_1 and u_2 while the base motion u_o is prescribed. For the examples presented here $u_o = 0$ and the system is excited by prescribing the initial conditions as $u_1(0) = u_2(0) = 0$ and $\dot{u}_1(0) = \dot{u}_2(0) = v_o$. Here, the dots represent derivatives with respect to time.

From the free-body-diagrams in Fig. 2, the equations of motion of the system are

$$\ddot{u}_1 = -\frac{1}{m_1}(-F_1 + F_2) \quad (1)$$

$$\ddot{u}_2 = -\frac{1}{m_2}F_2 \quad (2)$$

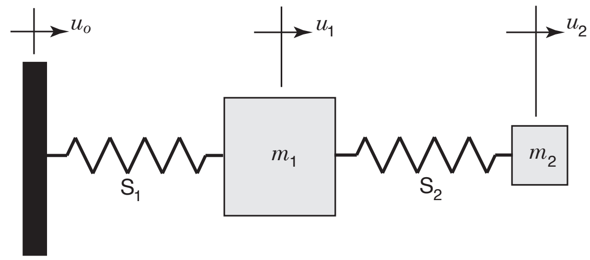


Figure 1. Schematic of the model.

where F_1 and F_2 are the forces exerted by S_1 and S_2, respectively. The generic elastic-plastic force-stretch (F-δ) response of the springs is shown in Fig. 3. It is bi-linear with elastic slope k, yield force F^o and post-yield slope k'. The relation between F and δ is written in incremental form as follows:

$$dF = dF(k_1, k_2, F^o, \rho, \iota, d\delta) \tag{3}$$

where ρ is a real state variable, and ι is an integer state variable. Here dF and $d\delta$ represent increments in F and δ. In this model, the stretch in each of the two springs is given by

$$\begin{aligned} \delta_1 &= u_1 - u_o \quad (4) \\ \delta_2 &= u_2 - u_1. \quad (5) \end{aligned}$$

In the case of isotropic hardening ρ represents the maximum force achieved during plastic deformation $F^{\max}$. While the size of the elastic region is initially $2F^o$, it grows to $2F^{\max}$ during plastic deformation. Therefore, upon reverse loading from a state with force $F^{\max}$ yielding occurs when the force is $-F^{\max}$ as shown in dashed line in Fig. 3. In kinematic hardening, on the other hand, the size of the elastic region remains fixed at $2F_o$, so upon reverse loading, yielding occurs when the force changes by $2F^o$ from the point of unloading. In this case the elastic region must translate along the F axis during plastic deformation. Here ρ represents the position of the center of the elastic region, which is shown as α in the figure and is plotted as a function of δ in dash-dot line. The F-δ response during reverse loading is shown in solid line. Finally, the integer state variable ι simply tracks whether deformation occurs elastically or plastically.

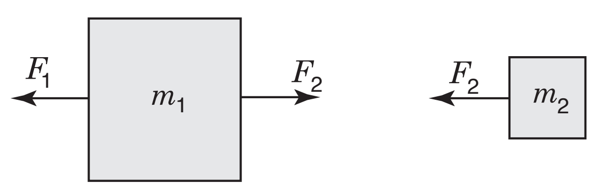


Figure 2. Free body diagrams.

The solution of the equations of motion was accomplished using the function `ode45` in Matlab. Verification exercises included writing a separate script assuming that both springs were

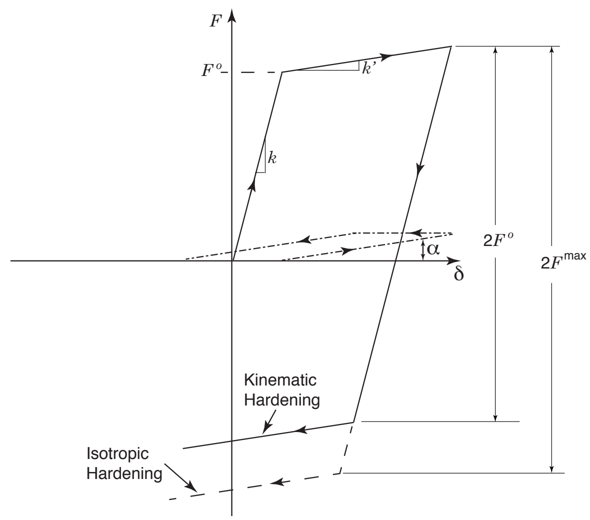


Figure 3. Bi-linear spring behavior for isotropic and kinematic hardening.

linearly elastic, and then comparing its results to those of the elastic-plastic scripts when no plastic deformation occurred. In addition, the scripts of the material response were exercised separately to verify that they produced the expected responses. The size of the time step was also chosen by comparing results from several values.

Results

Even with only two degrees of freedom and simple bi-linear force-displacement relations for the springs, the model contains 8 parameters. Since the objective of this work was purely demonstrative, no physical system was used to set the parameters of the model. So, the sizing of the arbitrary parameters was conducted as follows: The mass m_2 was taken to be one half of m_1. They were given the values of 5 and 10 respectively[1]. The values of k_1 and k_2 were taken to be equal so that the fundamental frequency of the system was about 300. Making $k_1 = k_2 = 50 \times 10^6$ gave the natural frequencies for the two modes values of 270 and 670. The yield forces F_1^o, F_2^o and the post-yield slopes k_1', k_2' were chosen to give approximately realistic elastic-plastic responses, and their values are given in Table 1 for the nominal case and the variations to be considered. The parameters that deviate from the nominal case are shown in bold font. Finally, the initial conditions of the system were taken as $u_1(0) = u_2(0) = 0$ and $\dot{u}_1(0) = \dot{u}_2(0) = -20$.

Figure 4 shows five plots of the response of the system for the nominal parameter set. Figure 4(a) shows the displacements of the masses as functions of time. The simulation time was 0.015, sufficient to achieve a steady-state with minimal additional plastic deformation. Note that the kinematic hardening model damps the motion more than the isotropic hardening model. The same can be seen in Fig. 4(b) for the acceleration response. In both plots it

[1]Since no physical system was modeled, the units of the model parameters and variables are irrelevant. Hence, no units will be presented.

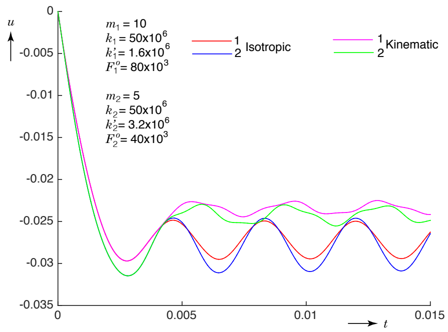
Table 1. Model parameters. The properties of m_2 and S_2 remain the same for all cases.

Case	m_1	k_1	k_1'	F_1^o	m_2	k_2	k_2'	F_2^o
Nom.	10	50×10^6	1.6×10^6	80×10^3	5	50×10^6	3.2×10^6	40×10^3
Var. 1	10	50×10^6	1.6×10^6	$\mathbf{100 \times 10^3}$	5	50×10^6	3.2×10^6	40×10^3
Var. 2	10	50×10^6	$\mathbf{2.0 \times 10^6}$	80×10^3	5	50×10^6	3.2×10^6	40×10^3
Var. 3	10	50×10^6	$\mathbf{2.0 \times 10^6}$	$\mathbf{100 \times 10^3}$	5	50×10^6	3.2×10^6	40×10^3

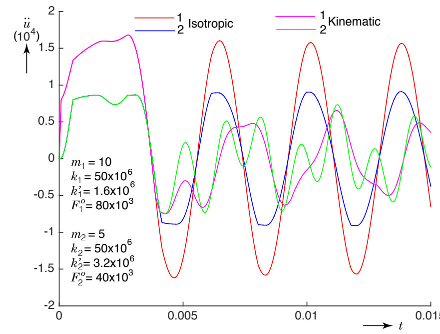
can be seen that, as expected, the predictions of the two elastic-plastic models are identical for the first acceleration pulse. Figure 4(c) shows the force-stretch response of the springs. Note that the responses are identical up until the first unloading, but then S_1 yields upon reverse loading for kinematic hardening, but remains linearly elastic for isotropic hardening. Figure 4(d) shows the behavior of S_2 in more detail. Note the higher forces in the isotropic hardening case. Finally 4(e) Shows the accumulation of plastic deflection in S_2, given by δ_2^p as function of time. In this case, the isotropic model gives a higher accumulation of plastic deflections.

Figure 5 shows the same five plots but for the first variation, consisting of raising the value of F_1^o by 25% to 100×10^3. The main difference resulting from raising the value of F_1^o can be seen in Figs. 5(c) through (e). Note the larger magnitudes of the forces in the springs as well as the more severe plastic deformation in S_2. The kinematic hardening model predicted a more severe reverse load in S_2 than the isotropic model. For this reason, the accumulated plastic deflection of S_2 is more severe for kinematic hardening than for isotropic hardening, the reverse of the nominal case.

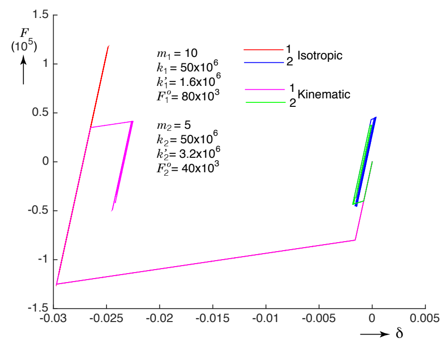
Figure 6 shows what happens when F_1^o has the value of the nominal case, but k_1' increases by 25% to 2.0×10^6 in variation 2. In this case the springs also see higher loads and deformations than in the nominal case, and the isotropic hardening model gives higher plastic deformation accumulation in S_2. Finally, Figure 7 shows the case when both k_1' and F_1^o are 25% higher than in the nominal case. This combination causes the highest loads and deformations in the springs as well as the highest amount of plastic deformation. Note that the kinematic hardening model yields higher values of accumulated plastic deflection.



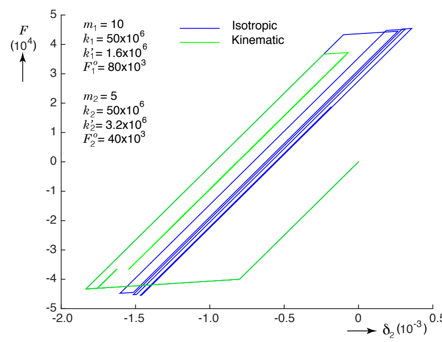
(a)



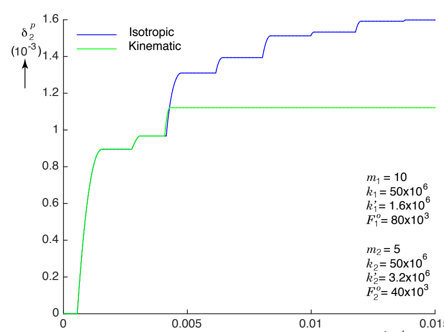
(b)



(c)



(d)



(e)

Figure 4. Results for the nominal case. (a) Displacements vs. time, (b) accelerations vs. time, (c) force vs. stretch for both springs, (d) force vs. stretch for S_2 only and (e) accumulation of plastic deflection in S_2.

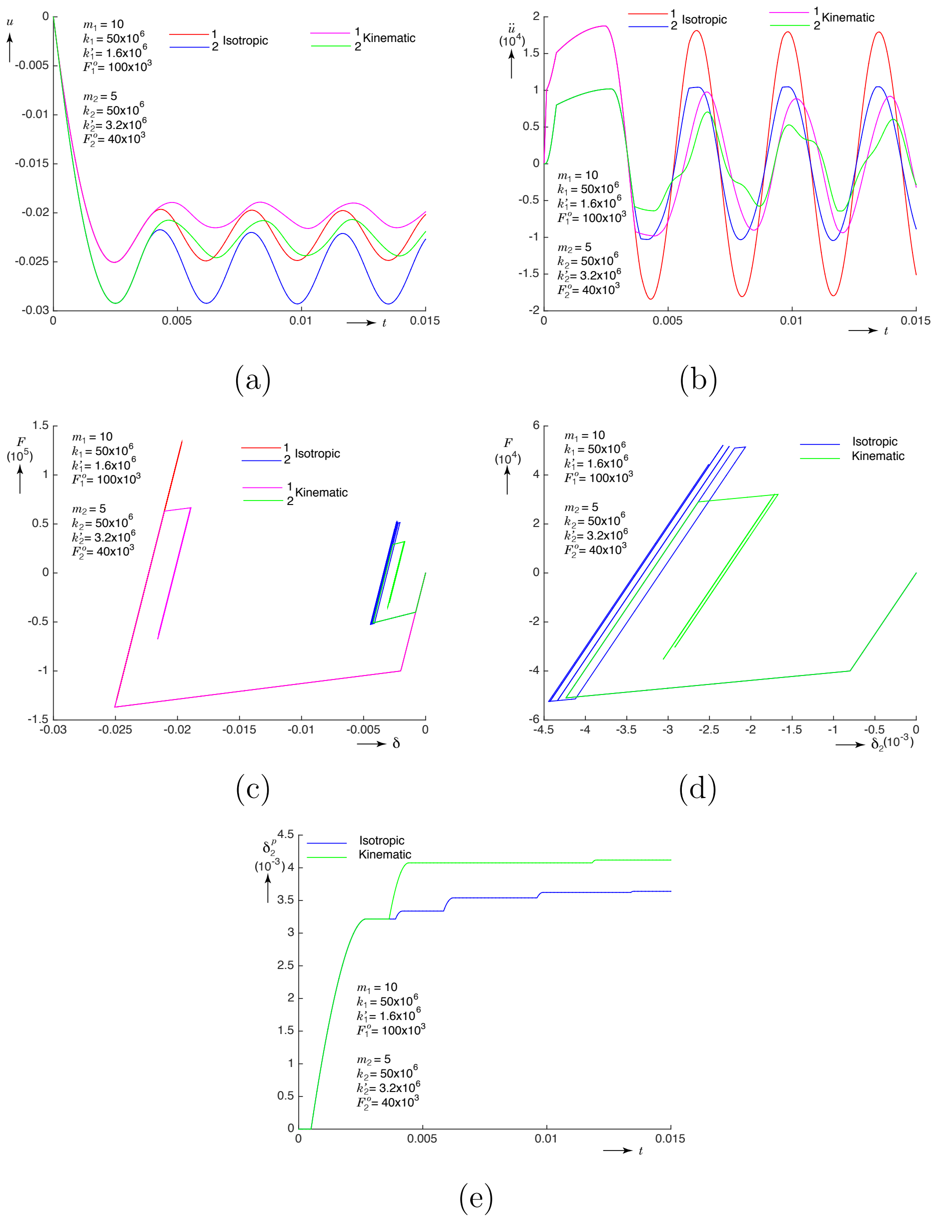
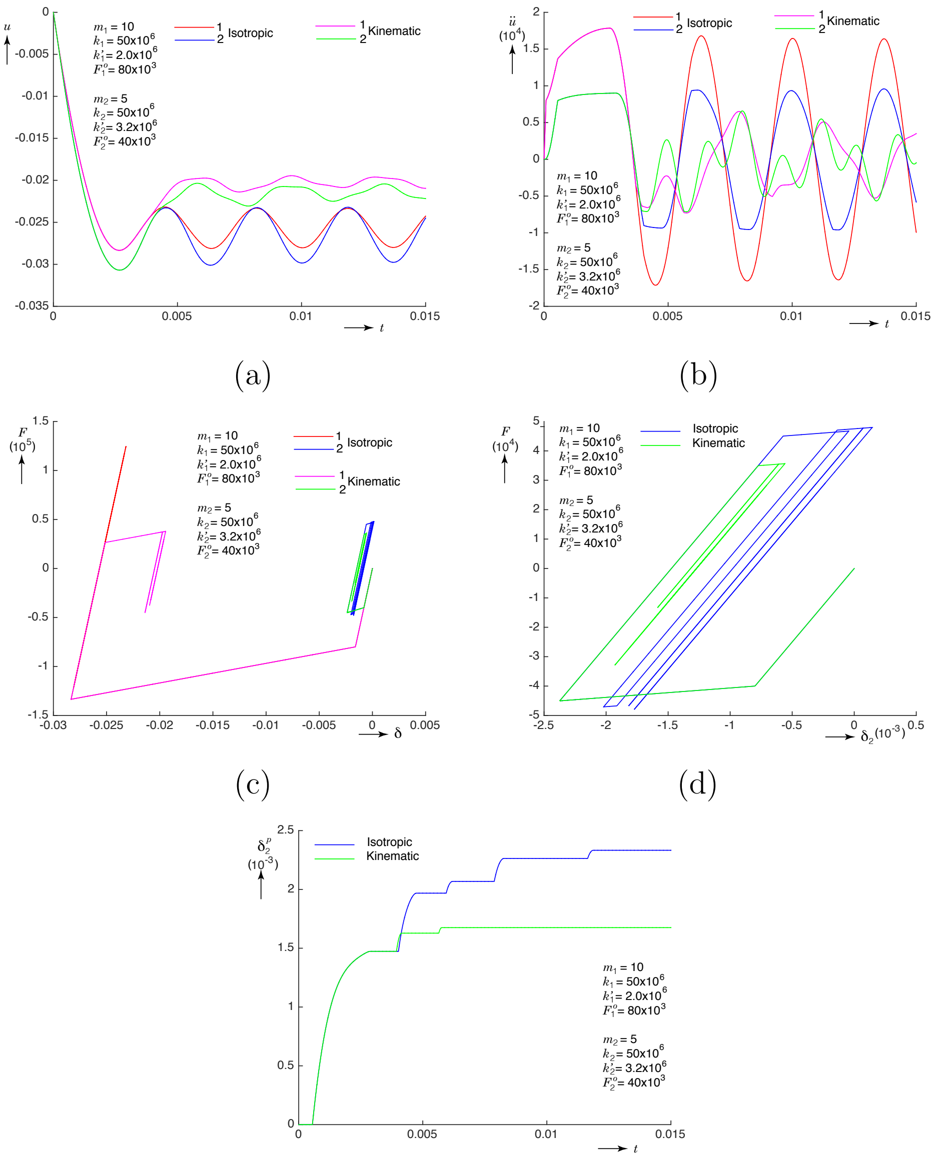


Figure 5. Results for variation 1. (a) Displacements vs. time, (b) accelerations vs. time, (c) force vs. stretch for both springs, (d) force vs. stretch for S_2 only and (e) accumulation of plastic deflection in S_2.



(a) (b) (c) (d) (e)

Figure 6. Results for variation 2. (a) Displacements vs. time, (b) accelerations vs. time, (c) force vs. stretch for both springs, (d) force vs. stretch for S_2 only and (e) accumulation of plastic deflection in S_2.

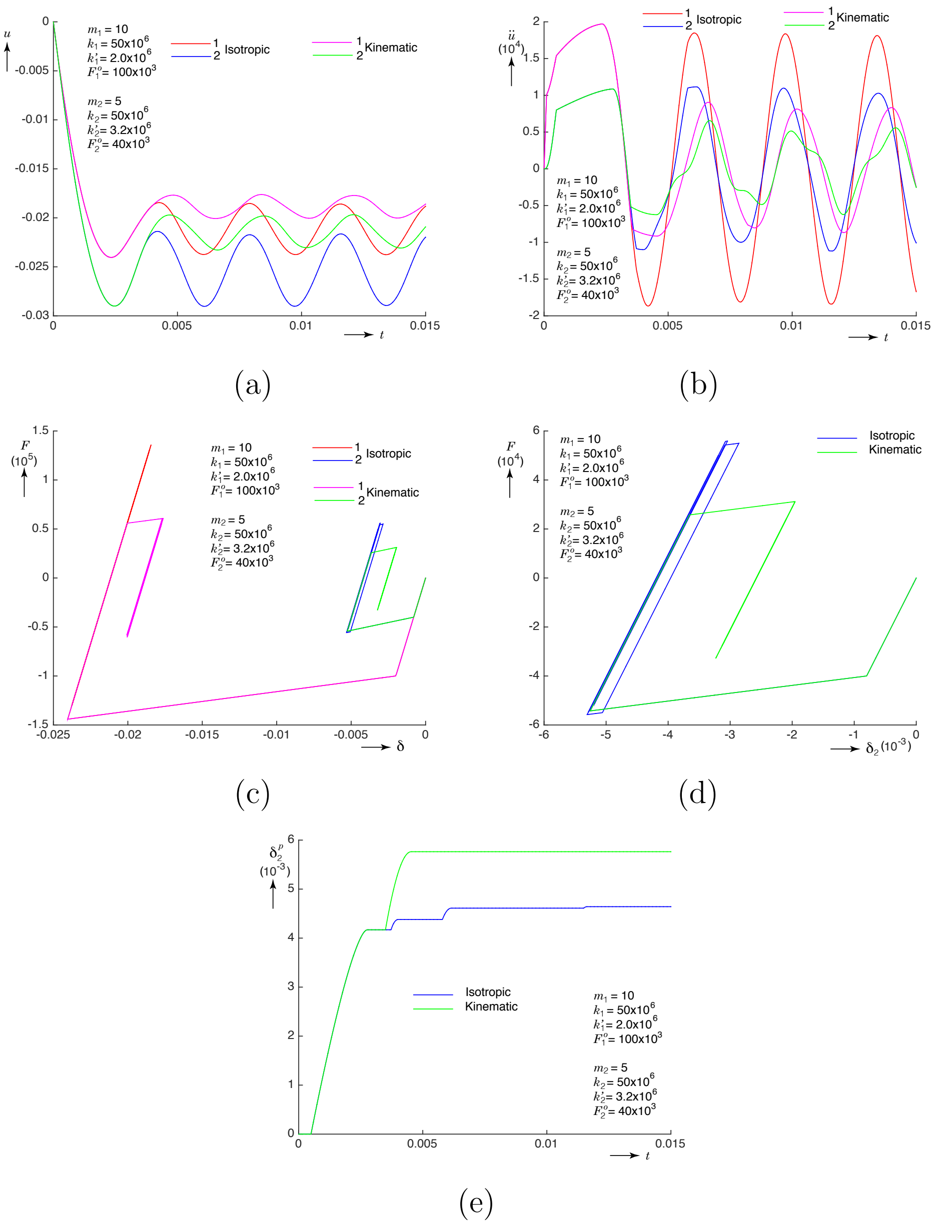


Figure 7. Results for variation 3. (a) Displacements vs. time, (b) accelerations vs. time, (c) force vs. stretch for both springs, (d) force vs. stretch for S_2 only and (e) accumulation of plastic deflection in S_2.

Summary

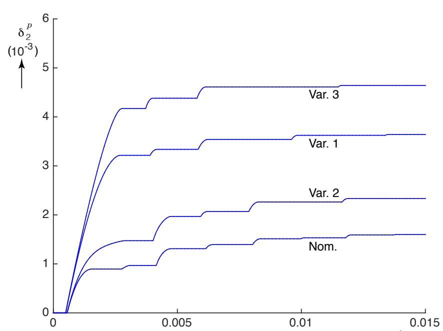
- Changing the properties of S_1 could have significant impact on the plastic deflection of S_2. Figures 8(a) and (b) compare the accumulated plastic deflection in S_2 for isotropic and kinematic hardening, respectively. Enhancing the strength of S_1 causes more severe plastic deflections in S_2 since S_1 dissipates less energy. In fact, making S_1 strictly elastic induces final accumulated plastic deflections in S_2 in the order of 0.03 and 0.075 for the isotropic and kinematic models, respectively.

- Comparing the results in Figs. 4(b), 5(b), 6(b) and 7(b) shows that the effect of the variation in the parameters on the acceleration of m_2 was not as significant as it was on the accumulated plastic deflections. The highest acceleration of m_2 on variation 3 was 27% higher than in the nominal case, whereas the maximum accumulated plastic deflection increased essentially five-fold.

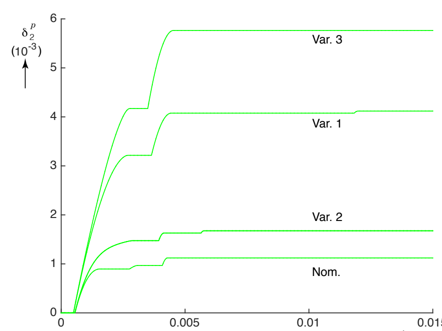
- Using kinematic hardening can yield results that have more or less accumulation of plastic deflection in S_2 depending of the value of the parameters of S_1.

- Kinematic hardening damped the motion more severely, with plastic deformation essentially ceasing after one load reversal.

- Some of the accumulation of plastic deflection occurred in compression. The impact of accumulated plastic strain in compression on ductile failure has not been investigated.



(a)



(b)

Figure 8. Comparison of accumulated plastic deflection in S_2 for all cases considered. (a) Isotropic hardening and (b) kinematic hardening.

The results presented here are based on a very limited exploration of the parameter space of the problem. They show that, under impact loads, the accumulation of plastic strain in internal components may be sensitive to moderate changes in the properties of the outer structure. Clearly, in simulating the response of actual structural systems of interest, models of those structures should be constructed and loaded by realistic loads to explore sensitivities.

Acknowledgments

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