# Financial Portfolio Management using Adiabatic Quantum Optimization: The Case of Abu Dhabi Securities Exchange

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## 1. Introduction

Financial portfolio selection is the problem of optimal allocation of a fixed budget to a collection of assets (commodities, bonds, securities etc.) which produces random returns over time. The word optimal, however, can have different meanings. For instance, one could define optimal so that a solution to the portfolio problem maximizes expected or average value of the return. While this definition is intuitively appealing, it is also naive because the values of a random process can deviate from the expected value. A definition of optimal should therefore incorporate deviations from the expected value. In his 1952 paper [4], Harry Markowitz proposed just such a definition of optimal for the financial portfolio selection problem. Markowitz proposed that the problem has an optimal solution when an investor maximizes expected return *and* minimizes variance in the values of the return. In other words, an investor should look to maximize gain and minimize risk for a given financial portfolio.

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Markowitz's portfolio selection theory also formalizes the wisdom of portfolio diversification, where instead of allocating the largest fractions of the budget to a select number of assets with largest expected return, an investor allocates his budget to many assets with the hope that the fortune of some of these assets will not be influenced by the misfortune of some others.

Formally, consider a portfolio with $n$ assets. Let $a_i$ be the percentage allocation of the total budget to asset $i$ and let $R_i$ denote the random variable representing the return from asset $i$. Further, let $R$ denote the random variable for the return from the entire portfolio. Then

$$R = \sum_i a_i R_i$$

whereby the expected return from the the portfolio is

$$E(R) = \sum_i a_i E(R_i) \tag{1}$$

due to the linearity of expectation. Next, consider the variance of $R$ which is given by

$$\mathrm{Var}(R) = \sum_{i,j} a_i a_j \mathrm{Cov}(R_i, R_j), \tag{2}$$

where $\mathrm{Cov}(R_i, R_j)$ is the co-variance of the random variables $R_i$ and $R_j$ and which reduces to the variance of $R_i$ for $i = j$. An investor working with respect to Markowitz theory will maximize (1) while minimizing (2). The information that the investor needs to achieve these two tasks is the pairs of statistics $[E(R_i), \mathrm{Cov}(R_i, R_j)]$. In his paper, Markowitz refers to these pairs as $(E, V)$ combinations and assures that it is always possible to identify an optimal one.

More mathematically, this is a bicriteria quadratic optimization problem in which the set of all optimal solutions yield the so-called efficient frontier or efficient portfolios. Among all optimal portfolios, the decision maker will make a choice based on his/her preferences. In the literature this formulation is also known as the mean-variance model.

For a given pair of assets $i$ and $j$, the contribution to $\mathrm{Var}(R)$ is zero when either $a_i$ or $a_j$ is zero, that is, when the investor does not allocate any budget to asset $i$ or $j$. However, this contribution can be made smaller by allocating positive budget to assets $i$ and $j$ and ensuring that $\mathrm{Cov}(R_i, R_j)$ is negative. A negative value for $\mathrm{Cov}(R_i, R_j)$ will imply that the returns from asset $i$ and $j$ influence each other inversely and an adverse change in the returns from one asset will mean a favorable change in the returns from the other. Hence, not only does Markowitz theory justify the maxim of diversification mentioned earlier, but it also provides a measure for diversification.

## 2. Portfolio Selection as Unconstrained Optimization

In a relaxed form, the Markowitz portfolio model can be formulated as a quadratic unconstrained binary optimization (QUBO) problem as follows:

$$\begin{aligned}\text{Minimize}: \quad & \theta_1 \sum_i [-\alpha_i E(R_i)] \\ & + \theta_2 \sum_{i,j} \alpha_i \alpha_j \mathrm{Cov}(R_i, R_j) \\ & + \theta_3 \left( \sum_i \alpha_i A_i - B \right)^2\end{aligned}$$

subject to $\alpha_i \in \{0, 1\}$, $A_i$ is the maximum amount of money that can be invested in the $i$-th asset, and where $B$ is the total budget. In this relaxed formulation we suppose that fractional shares of stocks can not be purchased, so the decision maker either invests the total amount $A_i$ into the $i$-th asset or nothing. The objective function is composed by three terms: the expected return, the volatility of the investment, and a penalization term that takes into account the difference between the invested amount of money $\sum_{i=1} A_i \alpha_i$ and the total budget $B$. The positive weights $\theta_i, i = 1, 2, 3,$ describe the relative importance of each criterion in the decision making process.

The QUBO form for the Markowitz portfolio model is ideally suited for running on the specialized quantum computer available from the Canadian technology company, D-Wave Systems. D-Wave Systems has developed a specialized quantum processor that uses the principles of quantum information processing to solve unconstrained optimization problems. The D-Wave processor uses superconducting technology and the physics of quantum annealing to find the minimum energy eigenstate of the Ising Hamiltonian operator, which corresponds to the optimal strategy in the Markowitz model. Unlike a conventional computer, a quantum computer utilizes quantum bits, or *qubits*, to encode and process information. The principles of quantum physics allow qubits to store information in superposition of logical basis states. For example, a single qubit can store any normalized superposition of the logical 0 and logical 1 states simultaneously. A quantum computer manipulates superposition across multiple qubits to process information. Measurements with a quantum computer causes these superposition states to collapse into definite logical states, either 0 or 1, with probabilities given by the respective coefficients.

We show how quantum superposition states within the special purpose D-Wave processor may be used to effectively sample the space of potential portfolios. Computation then corresponds to finding the optimal portfolio as defined by the objective function above. The D-Wave processor uses quantum dynamics, namely quantum annealing, to isolate the portfolio selection, and we develop an implementation of portfolio selection using the D-Wave processor to recover the optimal portfolio. Our approach tests whether the adoption of the D-Wave quantum computer allows for a meaningful increment in computational performance for solving the Markowitz portfolio. We use D-Wave's quantum optimizer to find the optimal allocation of funds.

We map the QUBO form of Markowitz's portfolio selection problem into the well-known Ising model.

$$H = -\sum_j h_j Z_j - \sum_{i,j} J_{i,j} Z_i Z_j + \gamma \tag{3}$$

where the real-valued coefficients $h_i$ and $J_{i,j}$ define, respectively, the bias for qubit $i$ and the coupling between qubits $i$ and $j$ and the Pauli $Z_i$ operator is for the $i$-th qubit.

$$J_{i,j} = \frac{-1}{4} \left( \theta_2 \mathrm{Cov}(R_i, R_j) + \theta_3 A_i A_j \right) \tag{4}$$

$$h_i = \frac{-1}{2}\left(\theta_2 \mathrm{Cov}(R_i, R_i) + \theta_3 A_i^2 - \theta_1 E(R_i) - 2B\theta_3 A_i\right) \tag{5}$$

and $\gamma = \theta_3 B^2$.

Our goal here is to use D-Wave's quantum computer to explore the solution to portfolio optimization problems for data gathered from Abu Dhabi Stock Exchange website in Abu Dhabi, United Arab Emirates. Abu Dhabi is the capital city and the largest Emirate of the United Arab Emirates. It excels as a bridge between the East and the West in tourism, trade, financial services, to name a few. Abu Dhabi Securities Exchange offers opportunities to invest in businesses and industries in the Gulf Cooperation Council region. We have collected and processed the data from Abu Dhabi Securities exchange and used it to optimize the portfolio using MATLAB's Genetic Algorithm solver. The MATLAB solution has been validated using QBSOLV, D-Wave's simulator for its actual quantum optimizer. The next step is to validate the solutions obtained from MATLAB and QBSOLV with the actual quantum computer.

## 2.1. Quantum Programming and Execution

Programming the D-Wave processor requires first reducing the Markowitz model to the Ising form in Eq. (3) [8]. This logical representation of the optimization function must then be encoded into the processor hardware. The D-Wave processor has a unique hardware connectivity graph called Chimera that corresponds to a rectangular array of bipartite unit cells. As shown in Eq. (4), the $i$-th and $j$-th qubits generally posses non-zero coupling between them. Mapping logical problems into the Chimera hardware therefore requires an embedding function that uses chains of coupled qubits to develop equivalent representations of the problem structure. There are a variety of embedding methods available [11]. The D-Wave Solver API (SAPI) provides an implementation of an embedding algorithm search from Cai, Macready, and Roy [12], which uses a probabilistic search to find a valid embedding. We use the SAPI-provided embedding routine provided to program the dense Ising model into the Chimera hardware. The resulting parameters for the embedded physical Ising model are then given in terms of the original Ising coefficients and an intra-chain coupling constant $J$ [10]. We set the value of $J$ to be approximately an order of magnitude greater than the largest value of an coefficient; this setting is optimized based on empirical observation.

Execution of the embedded physical program is based on the time-dependent Hamiltonian $H(t) = A(t)H_0 + B(t)H_1$, where $H_0$ is the transverse field term and $H_1$ is the physical Ising model. The annealing schedules $A(t)$ and $B(t)$ have a fixed form within the hardware but a scalable duration $T$. Larger values of $T$ imply a slower schedule. We use an annealing time $T = 20\mu s$. A single program execution returns a binary string representing the candidate solution for the ground state of the physical Ising model. We decode this string into the solution for the QUBO problem. We collect an ensemble of such strings and generate a list of these candidate solutions. We then order the list of these solutions based on the calculated energy and pick the solution with the lowest energy. Our implementation of these programming steps rely on the XACC programming framework [13]. The XACC framework is designed to provide an extensible software framework for integrating novel computational accelerators, like quantum processors, into conventional scientific application codes.

Our implementation of portfolio optimization uses the D-Wave processor to find the optimal selection based on real data from the Abu Dhabi Securities Exchange. The prototype open-source code and data set are available online [14]. As seen in the data files for the averages and covariances, there are a wide array of values for the selected stocks. As a simplification we have performed portfolio optimization using a current price $A_i$ for stock $S_i$ that matches the corresponding average $\mathrm{E}(S_i)$. We have also assumed a total budget for investment to be \$100. Finally, we have found the solution to this multi-objective optimization function to depend on the specific choices for $\theta_i$. This is a well-known feature of using the weighted sum method to represent multi-objective optimization. For our demonstrations, we have settled on the use of $\theta_i = 1/3$ for all $i$.

For the above definition, we have used adiabatic quantum optimization with the D-Wave processor to search for the optimal portfolio. Repeated runs of the program for the same anneal time of 20 $\mu$s returns the same result of a portfolio selection that costs \$121.176. This cost does not strictly match our budget because the weight sum method provides only a tradeoff between the different constraints. For example, changing the weights to $\theta_1 = 0.8$, $\theta_2 = 0.$, and $\theta_3 = 0.2$ produces a portfolio that costs \$119.007. However, this choice has clearly ignored the influence of the covariance on minimizing risk. In addition, longer anneal times (up to 2 ms) yield very similar portfolios but with slightly lower costs. We find that some selections cannot be unambiguously determined from the program. This traces back to the embedded chains that return a strictly even distribution of the 0 and 1 outcomes. We separate these stocks from the remainder of the portfolio and compute the uncertain cost separately. For our problem set, this spread is typically from \$4 to \$40.

This data set is too large for direct embedding onto the D-Wave 128-qubit simulator, and was submitted to *q*bsolv, a heuristic QUBO solver, which is designed for problems too large and/or too dense to be embedded on a D-Wave quantum computer. *q*bsolv divides QUBO problems into chunks and iterates on subQUBOs until either the best solution is found, or the input time limit runs out. The *q*bsolv solution, which made use of the D-Wave simulator, is compared to the MATLAB-derived solution, with good agreement.

In summary, we have performed Markowitz portfolio optimization using the D-Wave processor.

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