

**Final Report for University of Texas Participation in Project:  
Collaborative Research: Center for Nonlinear Simulation for Energetic Particles in Burning  
Plasmas"**  
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The University of Texas research group, consisting of its PI, H. L. Berk, with two additional senior members B. N. Breizman and L. Zheng, guided a post-doc, Ge Wang, in the development of a simplified model for describing chirping behavior in fusion related plasmas. Chirping behavior is difficult to observe in straight forward numerical simulations as chirping requires accurate fine tuning in phase space that primitive 3-D particle codes, in 5-D phase space, have difficulty in producing. On the other-hand, chirping behavior is characterized by a dominant wave particle resonance, that can be approximated by the construction of a 2- dimensional phase space in action-angle space, if one discards the non-resonant interaction terms. By making use of this approximation, we successfully demonstrated this chirping in numerical simulation experiments. In the figure below we simulated the build-up of the energetic particle population. Initially, the TAE mode was excited. Evidence of limited chirping can be seen at early time, with the observation that the chirping does not penetrate in the continuum.

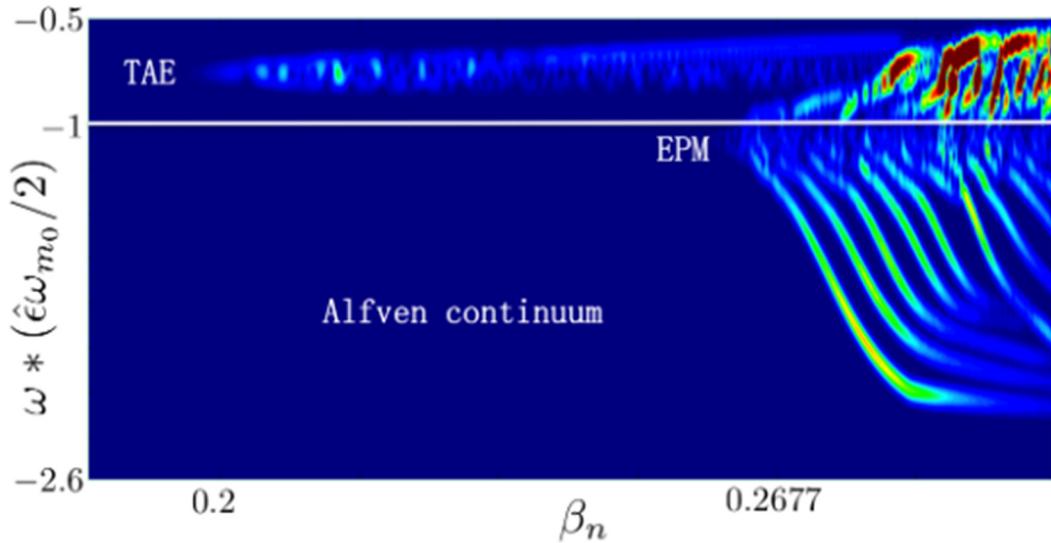
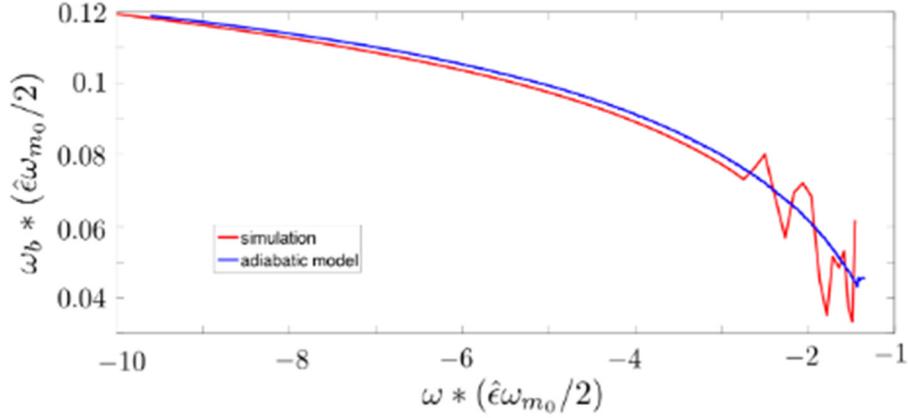


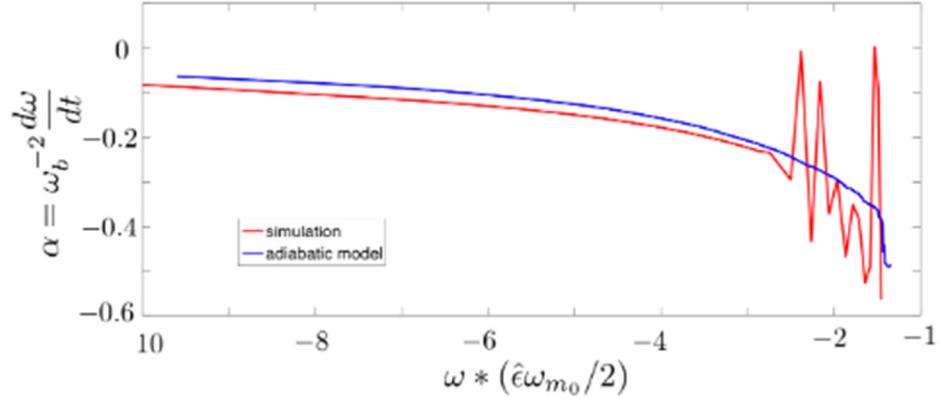
FIG. 9: With increasing driving  $\beta_n$ , the TAE mode, whose frequency lies in the TAE gap ( $-1 < \omega < 1$ ), becomes unstable, causing moderate upward and downward frequency chirps. Above a critical value, an EPM in the continuum ( $|\omega| > 1$ ) is excited, initiating a rapid frequency chirping to lower frequencies. Wave damping has been produced by choosing a complex value for  $\bar{\Delta} = 0.42 - .016i$ .

is excited, producing a large and chirping signal. The response of this signal was studied in detail in order to understand its theoretical basis. We found that the downward response of the frequency chirp, can be due to either the formation of a hole or a clump with the clump moving

outward towards the tokamak walls and the hole inward toward the center. The time varying frequency,  $\omega(t)$ , is found to be locked to the frequency  $n\omega_\phi - p\omega_\theta$ , with  $n$  and  $p$  integers that characterize the resonance and  $\omega_\phi$  and  $\omega_\theta$  the unperturbed mean rate of toroidal and poloidal circulation of resonant particles around the tokamak. An analytic theory was developed to describe the long-time chirping rate, and its results were found to be compatible with simulation as is shown in the figure below.



(a) the bounce frequency  $\omega_b$



(b) the normalized chirping rate  $\alpha = \omega_b^{-2} \frac{d\omega}{dt}$

FIG. 13: The results from the simulation and adiabatic theory are compared.

The early noise on the right-hand side of the figure was due to noise from competing secondary chirping structures, which cleaned up at later time, when a sufficiently large phase space separation developed from the competing secondary structures, which resulted in clean signals, as seen in the smooth part of the red curves in the above figure.

Included in the secondary material being enclosed, is a copy of the paper that describes this work. This paper is currently under review by the journal Nuclear Fusion. It received favorable

reviews from the referee, but with comments that needed to be addressed before acceptance. The currently enclosed draft contains the changes that accounts for the referees' comments.

I consider above work as an important accomplishment of the project. It shows the power of the reduced Hamilton model and many other problems can be formulated that can build on the methods of this work. Further, the relative simplicity of the physics being investigated allows the development of analytic methods that explain the phenomena being observed. As there are considerable opportunities available to extend the scope of this model. It is unfortunate that there was not enough monetary support to continue the investigations that can be carried on with methods developed in this project .