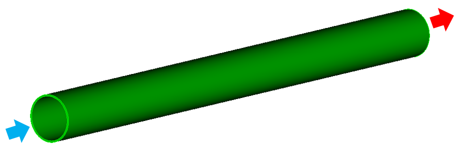

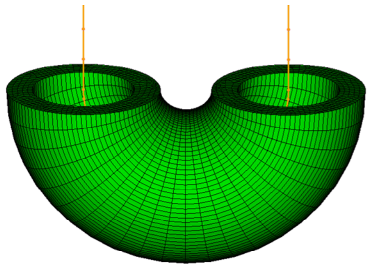

Verification of Advective Bar Elements Implemented in the Sierra/Aria Thermal Response Code

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Albuquerque, NM, USA

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Outline

- Objectives and Motivation
- Advective Bar Element Model
- Verification Strategy
- Streamwise Mesh Mapping
- Solution Convergence
- Method of Manufactured Solutions
- Summary

Objectives and Motivation

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Objective

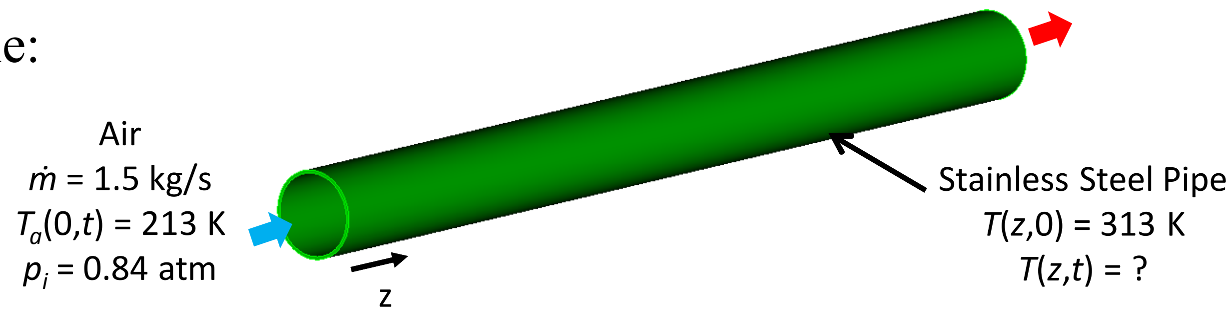
- Provide supporting evidence of proper implementation of the advective bar element model in the Sierra/Aria, a finite element heat transfer code developed at Sandia, through code verification activities

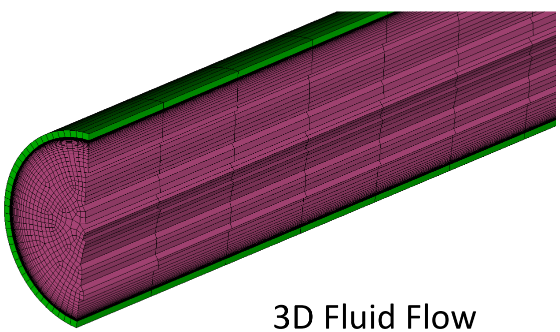
Motivation

- 1D advective bar is a good approximation when conjugate heat transfer is needed in a model where 3D CFD is too expensive
- Provide confidence to analysts that the advective bar element model is properly implemented

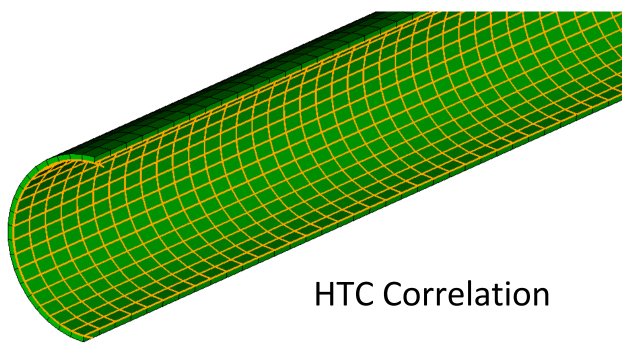
Advective Bar Element Model

- A reduced order finite element model for 1D fluid flow convectively coupled with a surrounding 3D solid
- Energy advection in fluid flows is integrated with very large existing 3D models without a significant increase in setup or computation time with acceptable physical accuracy
- Example:

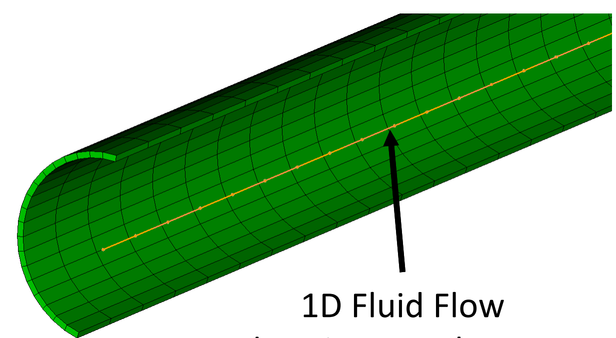




3D Fluid Flow



HTC Correlation



1D Fluid Flow
Advective Bar Elements

Governing Equations

- The 1D mass, momentum, and energy conservation equations for fluid flow:

$$\frac{\partial \rho_f}{\partial t} + \frac{\partial(\rho_f w)}{\partial z} = 0$$

$$\rho_f \frac{\partial w}{\partial t} + \rho_f w \frac{\partial(w)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\rho_f c_{v,f} \frac{\partial T_f}{\partial t} + c_{p,f} \frac{\partial(\rho_f w T_f)}{\partial z} - \frac{\partial}{\partial z}\left(k_f \frac{\partial T_f}{\partial z}\right) = S$$

- The 3D heat equation for the surrounding solid volume:

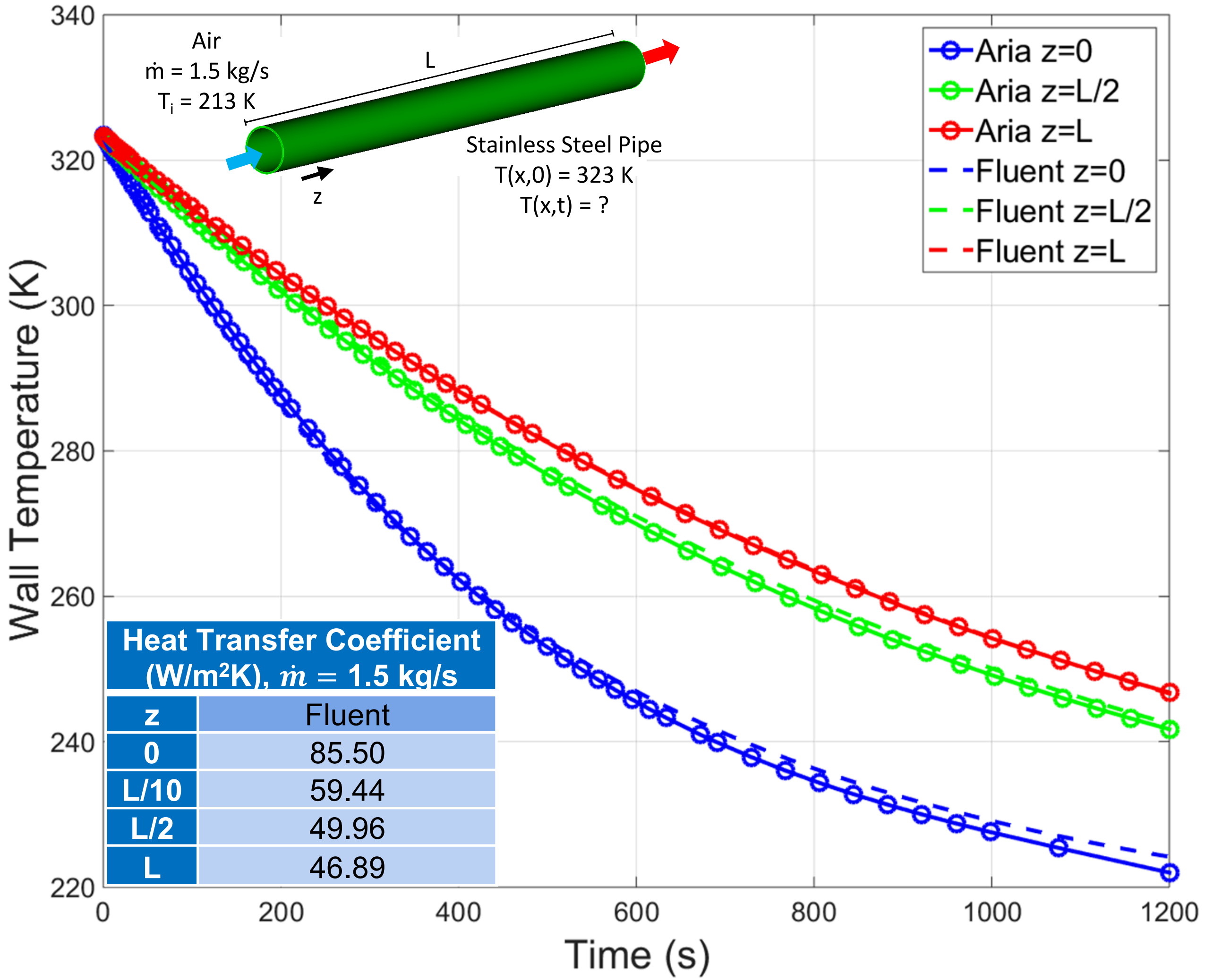
$$\rho_s c_{p,s} \frac{\partial T_s}{\partial t} - \nabla \cdot (k_s \nabla T_s) = 0$$

- The equations are coupled though the source term *S* and BCs:

$$S = h(T_s - T_f) \qquad q_n = h(T_f - T_s)$$

- Presently, the mass and momentum equations are simplified in the model and only solve the steady-term in the mass equation

Comparison with ANSYS Fluent®



Heat Transfer Coefficient (W/m²K), $\dot{m}$ = 1.5 kg/s	
z	Fluent
0	85.50
L/10	59.44
L/2	49.96
L	46.89

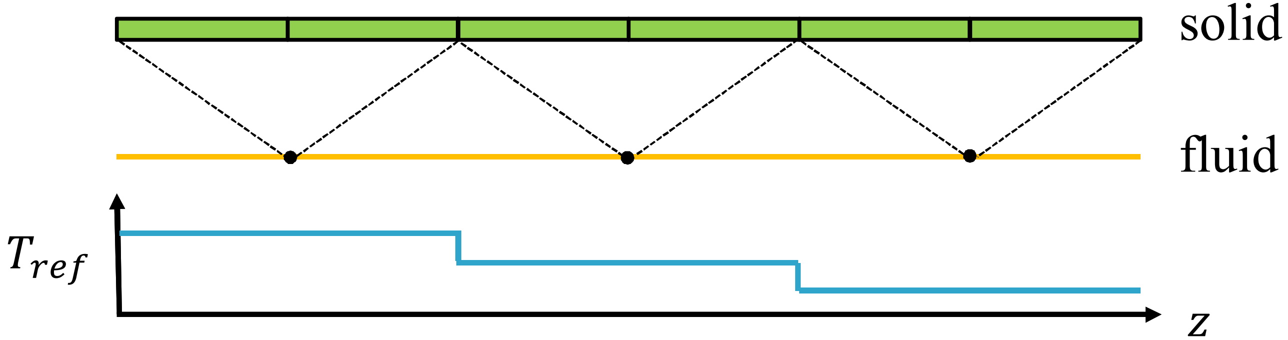
- An advective bar model for pipe flow was created and compared with a model using ANSYS Fluent® 15.0
- Using HTCs calculated from ANSYS Fluent, the temperature profiles in the pipe were in close agreement between the models

Verification Strategy

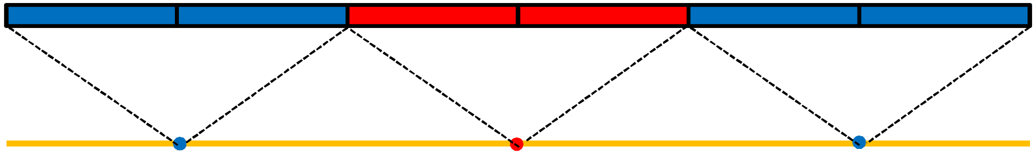
- Confirm the proper mapping of bar element nodes to the surrounding surfaces for several meshes through visual inspection
- Mesh refinement study to demonstrate solution convergence in local temperatures for a simple problem
- Use the Method of Manufactured Solutions (MMS) to confirm the appropriate order of convergence in L^2 and H^1 norms using an exact solution:
  - Uncoupled fluid and solid
  - Coupled fluid and solid
  - Include temperature-dependent material properties (k and c_P)
  - Include an empirical heat transfer coefficient correlation
    - Dittus-Boelter equation

Streamwise Mesh Mapping

- To reduce computational overhead, only values at the fluid nodes are used to couple the two models



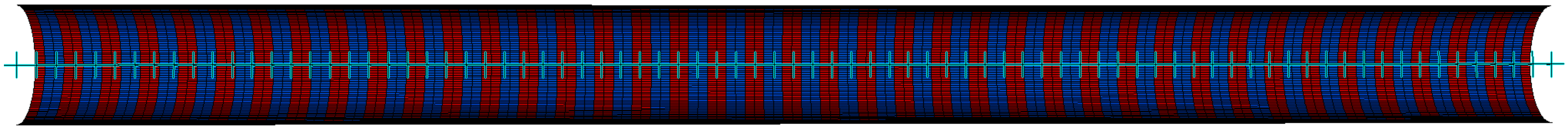
- To assure appropriate mapping between the nodes and the surrounding surfaces, the mapping is visually compared
- Nodes and the corresponding mapped surfaces were colored in alternating bands



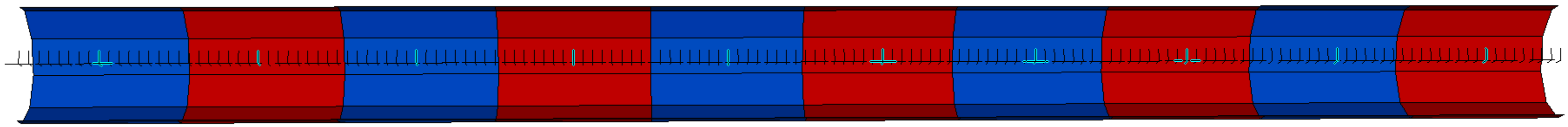
- For the pipe flow problem, this pattern should be consistent azimuthally

Streamwise Mesh Mapping

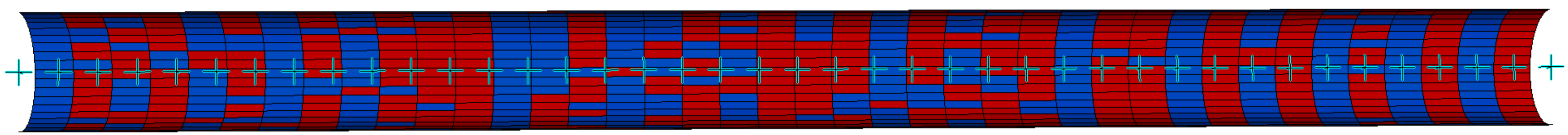
- 160 elements axially in the pipe, 80 bar elements



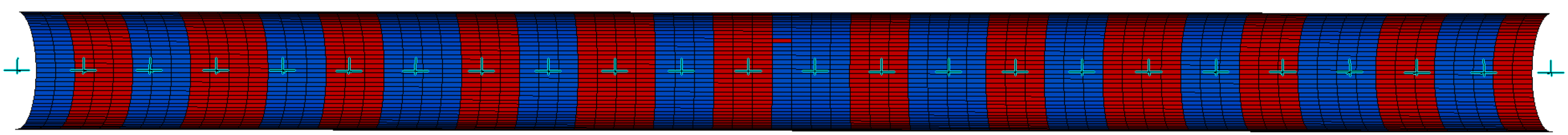
- 10 elements axially in the pipe, 160 bar elements



- 40 elements axially in the pipe, 40 bar elements

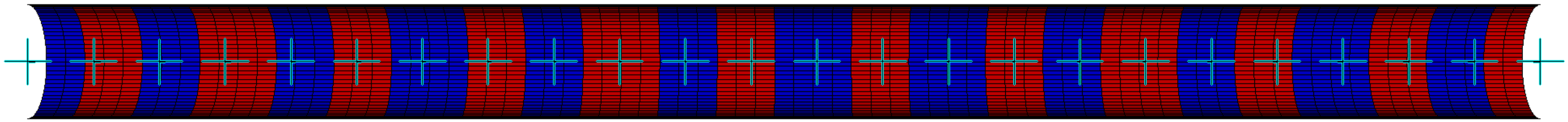


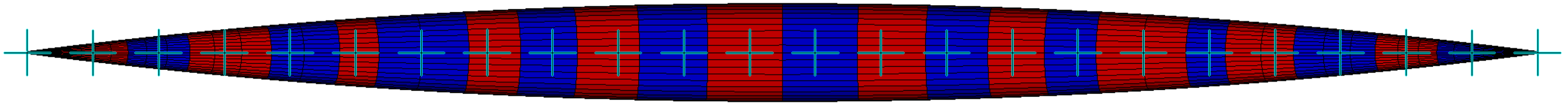
- 77 elements axially in the pipe, 23 bar elements



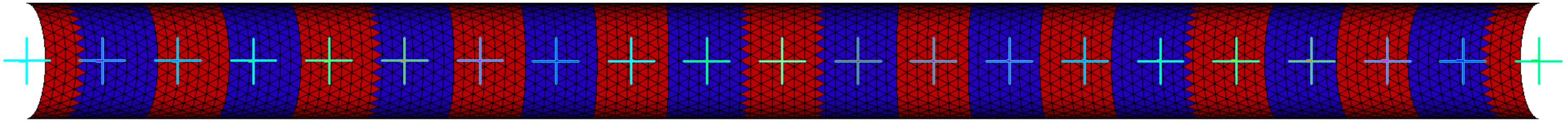
Streamwise Mesh Alignment

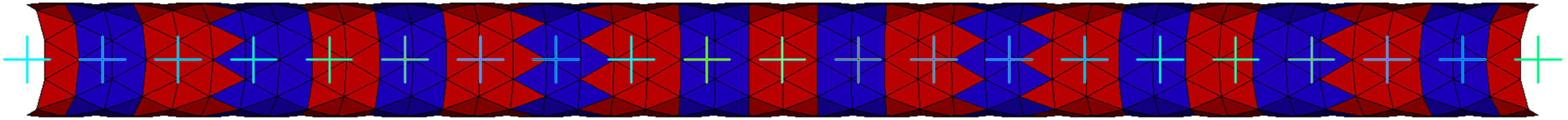
- Annular flow:
  77 elements in the pipe, 48 inner volume elements (variable), 23 bar elements





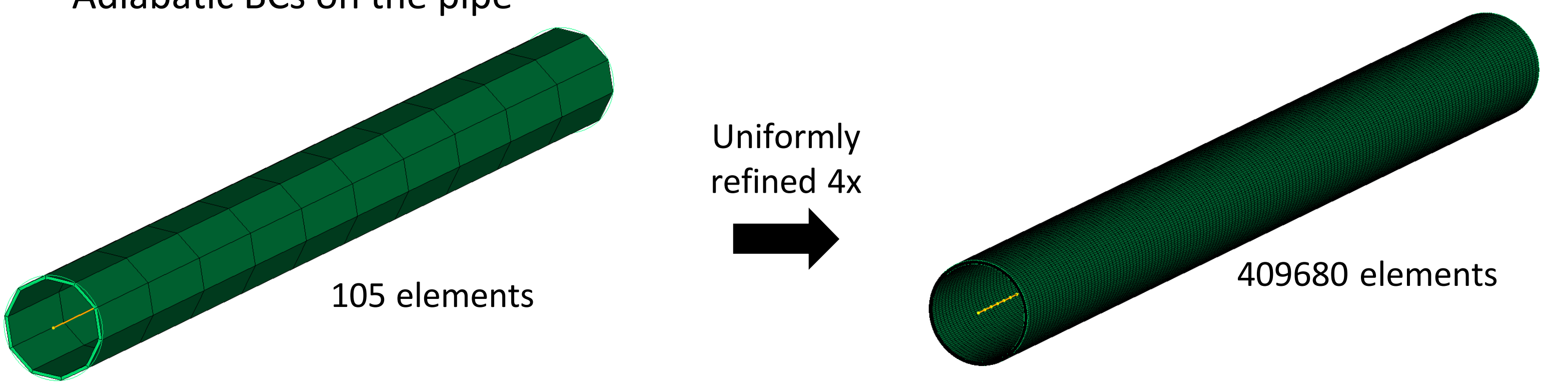
- Tetrahedral elements (pipe flow):



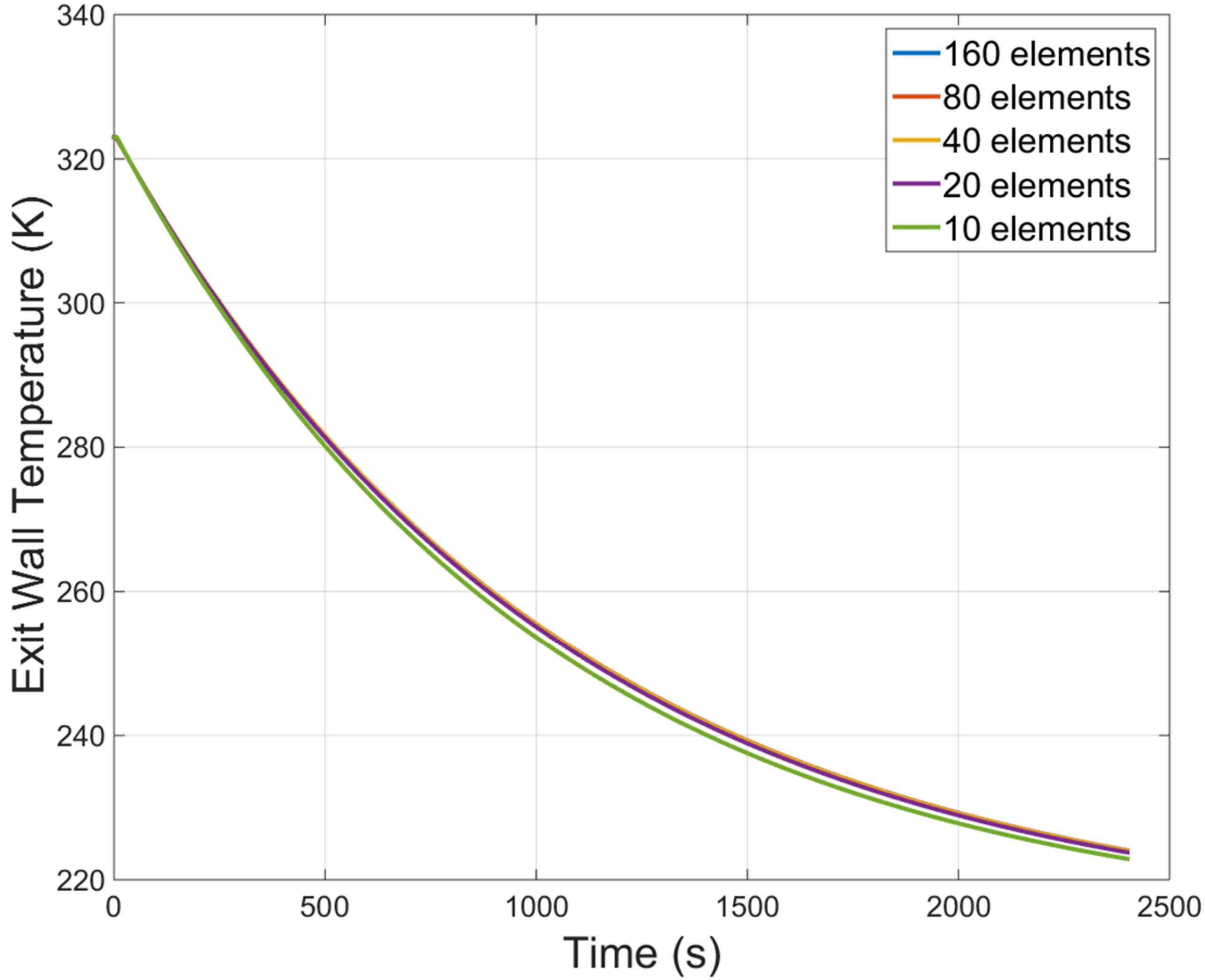


Mesh Refinement

- Five meshes were generated with increasing resolution for the simple pipe flow problem depicted earlier
  - 10 elements axially/circumferentially in the pipe, 5 bar elements
  - Each successive mesh uniformly refined by a factor of 2
- Problem parameters (t = 0→2400 s):
  - Stainless steel 304 pipe, **L = 3.6576 m, D_o = 0.3 m, D_i = 0.28 m**
  - Air mass flow rate, **$\dot{m}$ = 1.5 kg/s**
  - Inlet temperature of the air, **$T_a(0,t)$ = 213.15 K**
  - Initial Pipe Temperature, **$T(z,0)$ = 323 K**
  - A constant HTC coefficient (45 W/m^{2}K) was used on the inner pipe surface
  - Adiabatic BCs on the pipe

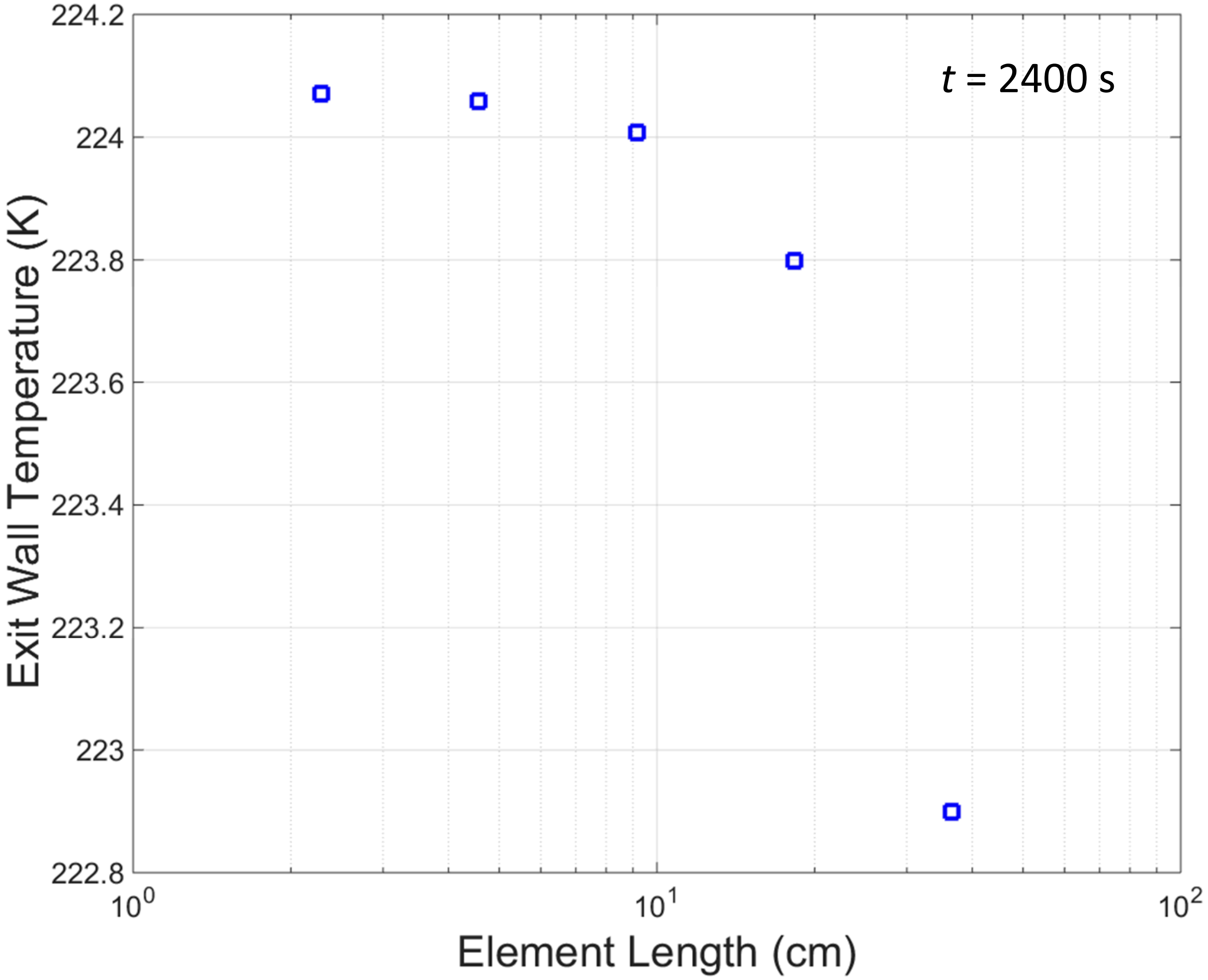


Solution Convergence



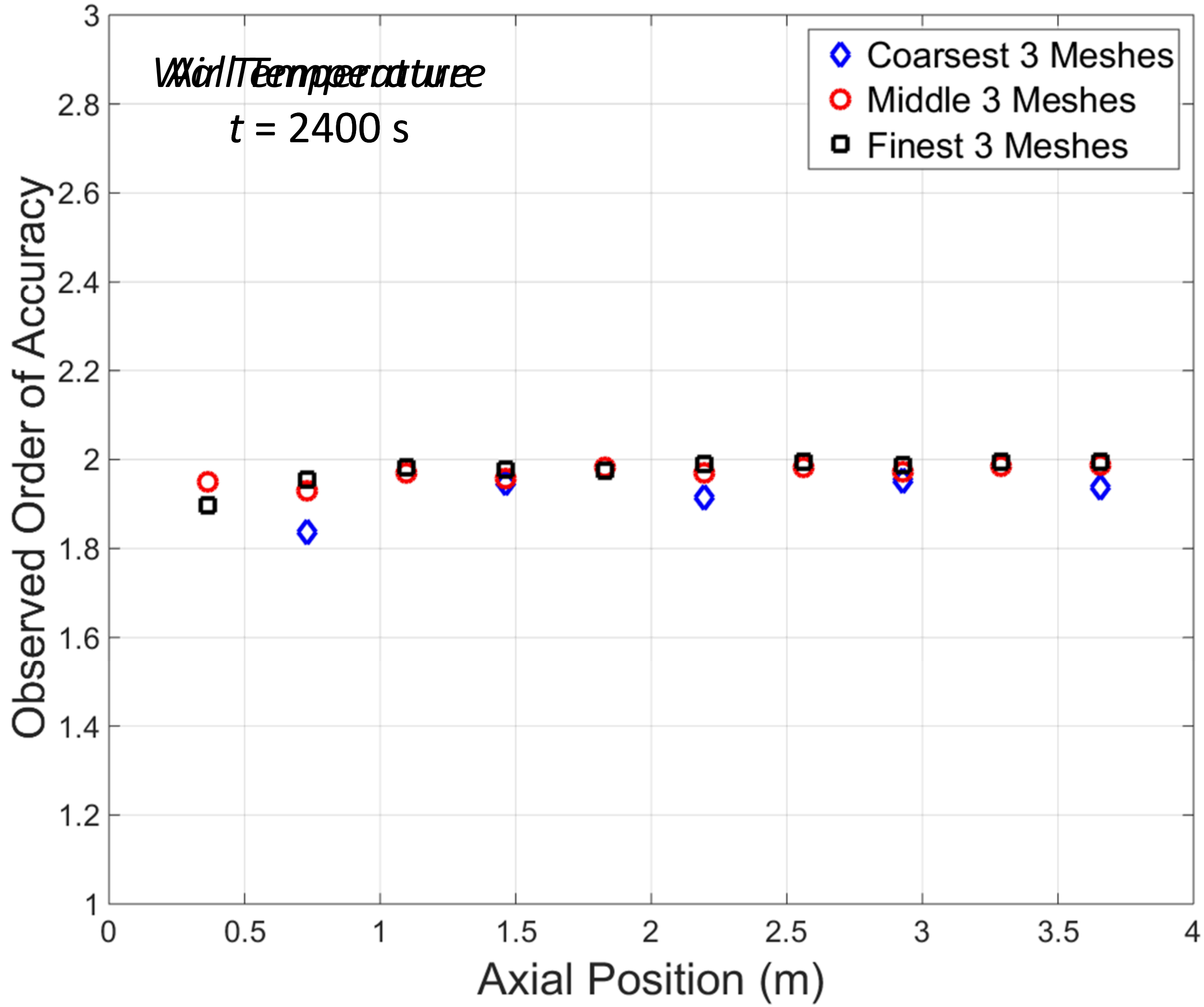
- The exit air temperature and exit pipe wall temperature are plotted as a function of time for each mesh
- Solution convergence is observed in both the air and the pipe temperature

Solution Convergence



- At t = 2400 s, the exit air temperature and exit pipe temperature are plotted as a function of axial element length
- Solution convergence is observed in both the air and the pipe temperature

Solution Convergence



- The observed order of accuracy is calculated locally over the length of the pipe for the fluid and pipe temperature

$$\hat{p} = \frac{\ln\left(\frac{T_3 - T_2}{T_2 - T_1}\right)}{\ln(2)}$$

- Local temperatures are converging at second order as the mesh is uniformly refined

Applying MMS to Advective Bars

- The Method of Manufactured Solutions (MMS) is applied to the model for a code verification test:
  1. $L(u) = 0$ where $L(\bullet)$ is the differential operator, u is the dependent variable
  2. Create a manufactured solution $\hat{u}$
  3. Create a source term $s_M = L(\hat{u})$
  4. Obtain the modified model $L(\hat{u}) = s_M$
- The method is applied to the coupled energy and heat equations for the fluid and the solid in the advective bar model:

$$\rho c_v \frac{\partial T_f}{\partial t} + c_p \frac{\partial(\rho w T_f)}{\partial z} - \frac{\partial}{\partial z}\left(k \frac{\partial T_f}{\partial z}\right) = S + S_{M,f}$$

$$\rho c_p \frac{\partial T_s}{\partial t} - \nabla \cdot (k \nabla T_s) = S_{M,s}$$

$$S = h(T_s - T_f) \qquad q_n = h(T_f - T_s) + q_M$$

Applying MMS to Advective Bars

- For a smooth solution, we expect the L^2 error to follow:

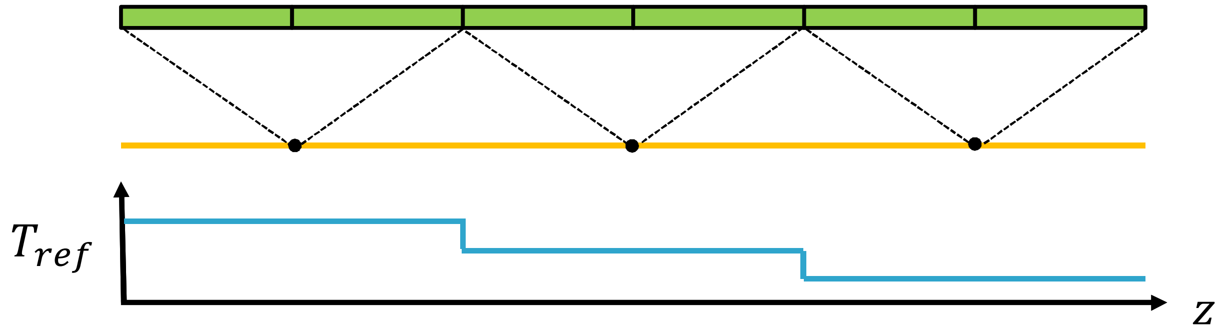
$$\|u - u_h\|_0 \le ch^{r+1}\|u\|_{r+1}$$

and the H^1 error to follow:

$$\|u - u_h\|_1 \le ch^{r}\|u\|_{r+1}$$

where c is a positive constant
r is the element order ($r = 1$)

- That is, L^2-norms should converge at h^2 and H^1-norms should converge at h^1 as the mesh is refined
- Solid-fluid coupling may also affect the order of convergence

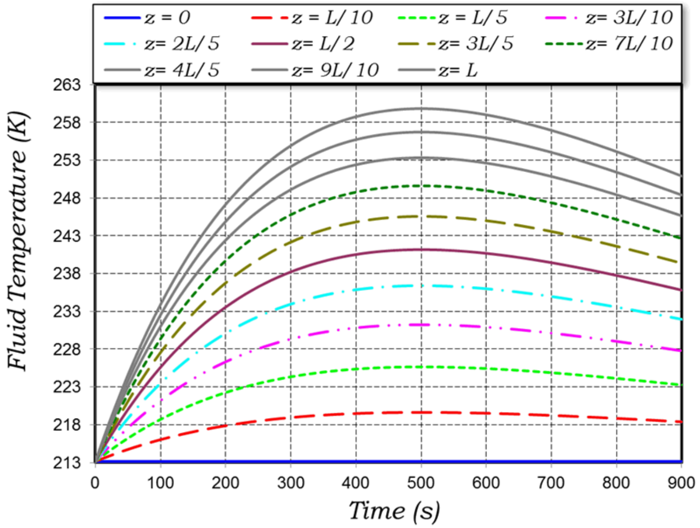


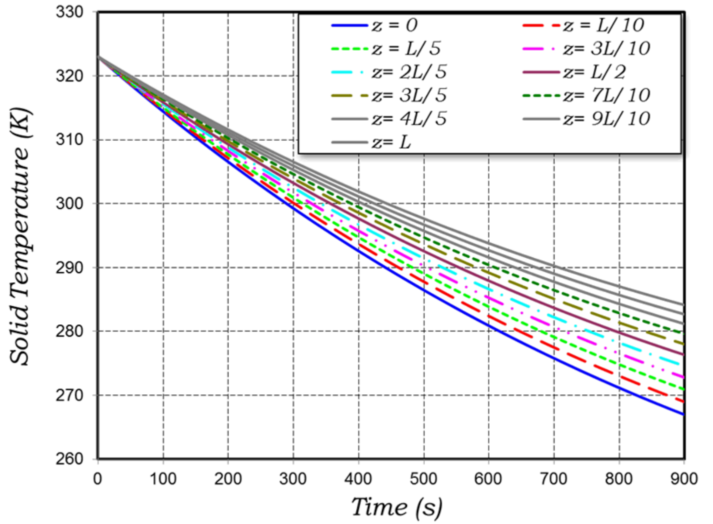
Manufactured Solutions

- Manufactured solutions for the fluid and solid, respectively:

$$\hat{T}_f = T_{i,f} + A \cdot t \cdot z \cdot e^{-Bt} \cdot e^{-Cz}$$

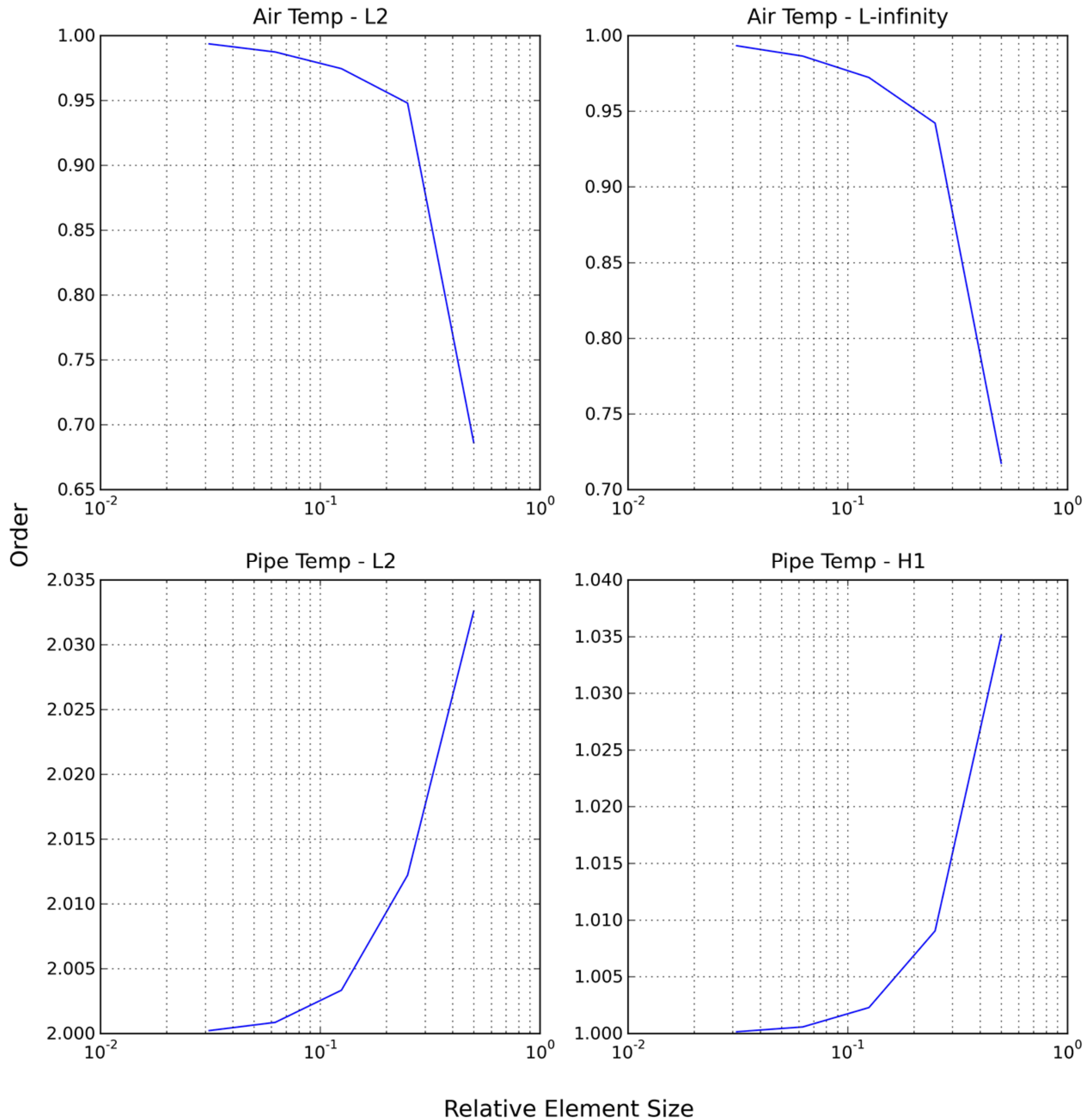
$$\hat{T}_s = T_{i,s} - D \cdot t \cdot e^{-Et} \cdot e^{-Fz} \cdot e^{-Gr}$$





- For each refinement, h → h/2 (Δt/2 and Δx/2)
- The model is simulated for a transient problem for $t = 0 \rightarrow 900\ s$
- Norms are computed using *Encore*, another Sierra application developed at Sandia used for verification and post-processing

Uncoupled Fluid and Solid

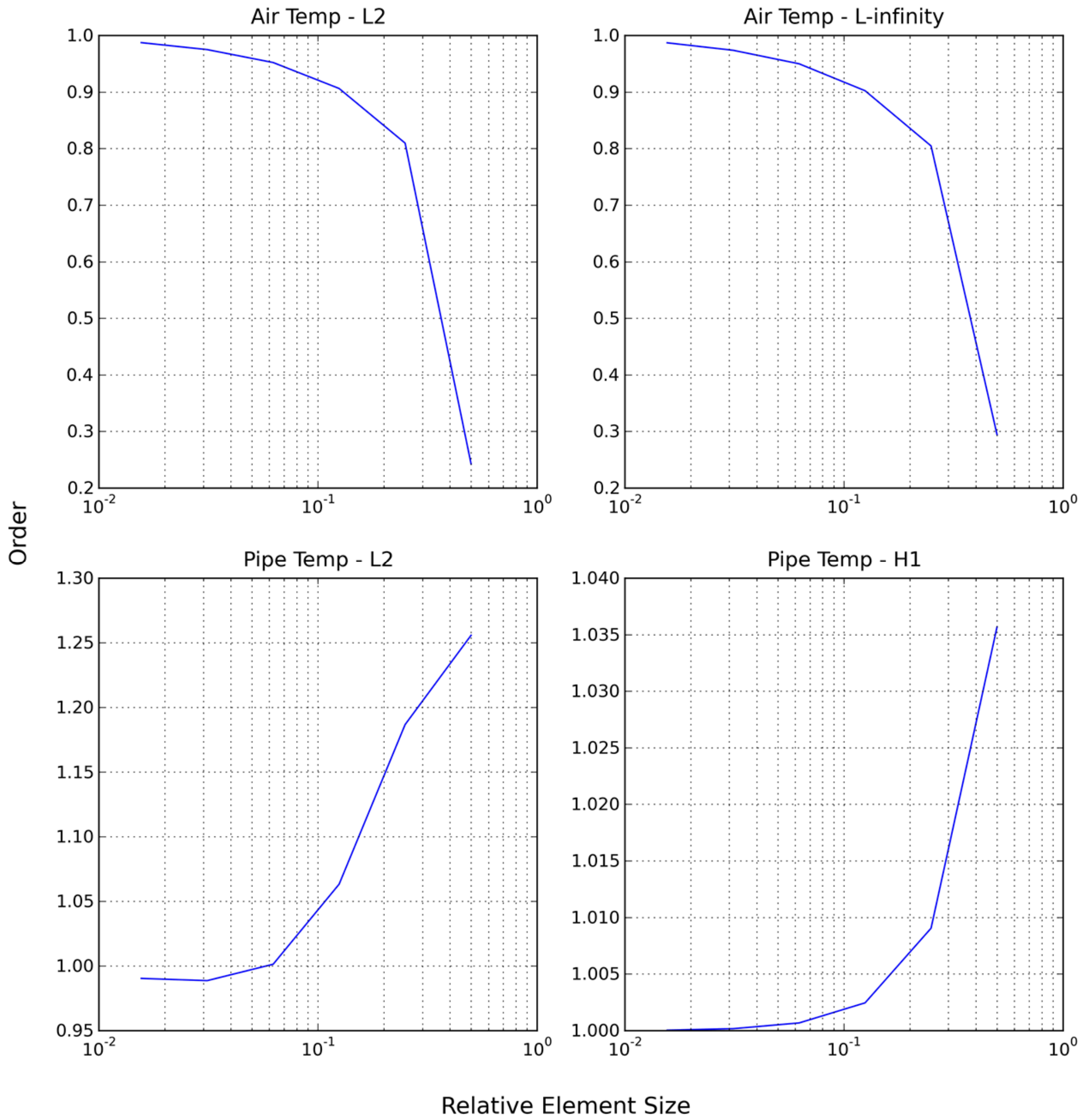


- For each successive mesh refinement, the order of convergence $\hat{p}$ is calculated:

$$\hat{p} = \frac{ln\left(\frac{\|\varepsilon_{2h}\|}{\|\varepsilon_h\|}\right)}{ln(2)}$$

- First, the uncoupled fluid and solid in the model is performed
- First order convergence is observed in the fluid, but second order convergence is observed in the solid

Coupled Fluid and Solid

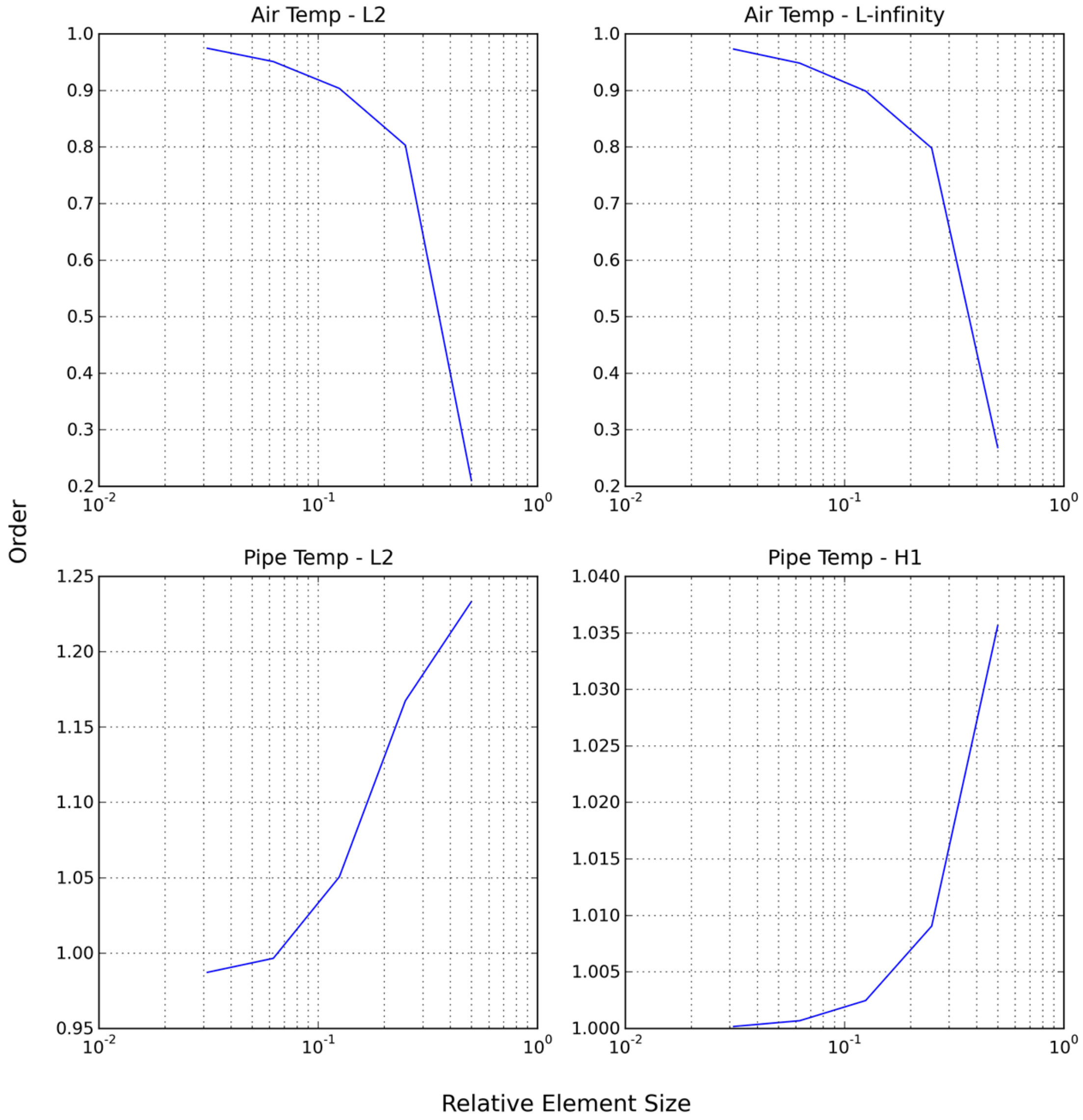


- For each successive mesh refinement, the order of convergence $\hat{p}$ is calculated:

$$\hat{p} = \frac{ln\left(\frac{\|\varepsilon_{2h}\|}{\|\varepsilon_h\|}\right)}{ln(2)}$$

- The fluid and solid are coupled through the source term and the boundary condition
- The L^2-norm in the solid begins to show first order convergence

Empirical HTC & Temp. Dep. Props



- For each successive mesh refinement, the order of convergence $\hat{p}$ is calculated:

$$\hat{p} = \frac{ln\left(\frac{\|\varepsilon_{2h}\|}{\|\varepsilon_h\|}\right)}{ln(2)}$$

- Including temperature-dependent properties as third-order polynomials and the empirical Dittus-Boelter equation does not change the convergence

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4}$$

Summary

- A reduced order finite element model for 1D fluid flow convectively coupled with a surrounding 3D solid has been implemented into the Sierra/Aria thermal response code

- Code verification concepts were applied to the advective bar elements in Aria to assess its implementation

- Mapping the bar nodes to the surrounding surfaces was implemented correctly, but some anomalous behavior can occur which doesn't affect results when typical meshes are used

- Solution convergence was observed in the temperature for increasing mesh resolution

- The method of manufactured solutions was applied to the model, but the appropriate order of convergence was not observed in the fluid, but model can still be used for predictions since it does converge to correct answer

Acknowledgements

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Questions?

References

1. Oberkampf, W. and Roy, C., Verification and Validation in Scientific Computing, 2010, University Press, Cambridge

2. Grossman, C. and Roos, H., Numerical Treatment of Partial Differential Equations, Translated and Revised by Stynes, M., 3rd Edition, 2005, Teubner

3. Salari, K. and Knupp, P., "Code Verification by Method of Manufactured Solutions," SAND2000-1444, Unlimited Release, Printed June 2000

4. Dobranich, D., "SAFSIM Theory Manual – A Computer Program for the Engineering Simulation of Flow Systems," SAND92-0693, Unlimited Release, Printed November 1993