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Identification of Fragments in a Meshfree Peridynamic Simulation

David Littlewood, Stewart Silling, Paul Demmie

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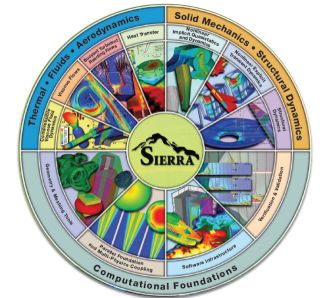
Fragment Identification in a Peridynamic Simulation

GOAL

- Develop a fragment identification algorithm for meshfree peridynamics
- Better support the simulation of fragmentation processes and comparison to experimental data

STRATEGY

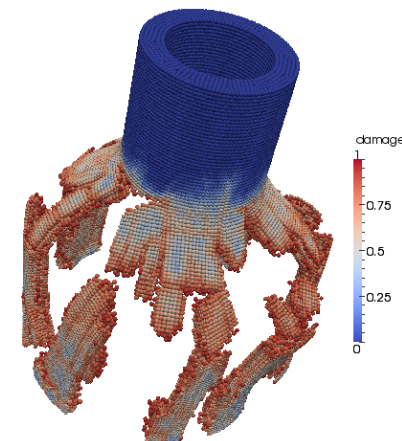
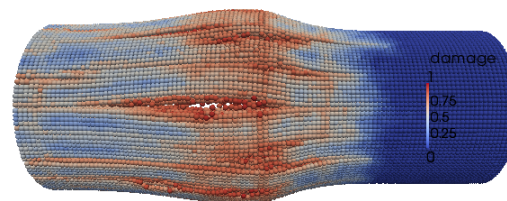
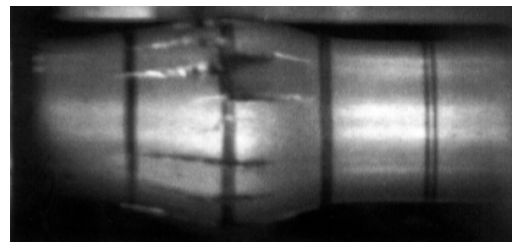
- Identify fragments by inspecting bond connectivity and bond damage
- Compute relevant fragment data, e.g., fragment masses
- Implement algorithm natively in *Sierra/SM* code base



Collaborators

Ryan Terpsma
Shivonne Haniff
Michael Tupek

Example simulation Cylinder Fragmentation



D.J. Littlewood. Simulation of dynamic fracture using peridynamics, finite element modeling, and contact. Proceedings of ASME IMCEC 2010.

T.J. Vogler, T.F. Thornhill, W.D. Reinhart, L.C. Chhabildas, D.E. Grady, L.T. Wilson, O.A. Hurricane, and A. Sunwoo. Fragmentation of materials in expanding tube experiments. *International Journal of Impact Engineering*, 29:735-746, 2003.

Peridynamic Theory of Solid Mechanics

Peridynamics is a mathematical theory that unifies the mechanics of continuous media, cracks, and discrete particles

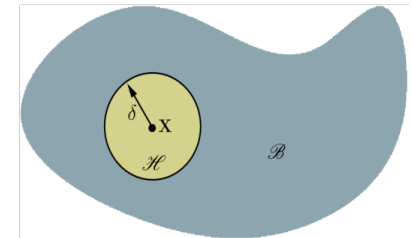
GOVERNING EQUATIONS

- Peridynamics is a *nonlocal* extension of continuum mechanics
- Remains valid in presence of discontinuities, including cracks
- Balance of linear momentum is based on an *integral equation*:

$$\rho \ddot{\mathbf{u}}(\mathbf{x}, t) = \int_{\mathcal{B}} \mathbf{f}(\mathbf{u}(\mathbf{x}', t) - \mathbf{u}(\mathbf{x}, t), \mathbf{x}' - \mathbf{x}) dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}, t)$$

- Material failure* is modeled through the breaking of peridynamic bonds

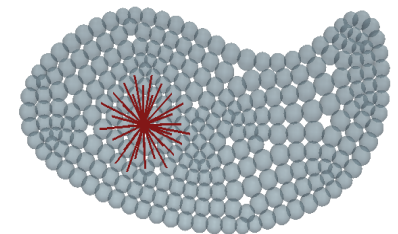
The point X interacts directly with all points within its horizon



DISCRETIZATION OF A PERIDYNAMIC BODY

Direct discretization of the strong form of the governing equation

$$\rho \ddot{\mathbf{u}}_i^n = \sum_p \mathbf{f}(\mathbf{u}_p^n - \mathbf{u}_i^n, \mathbf{x}_p - \mathbf{x}_i) V_p + \mathbf{b}_i^n$$



S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids*, 48:175-209, 2000.

S.A. Silling and E. Askari. A meshfree method based on the peridynamic model of solid mechanics. *Computers and Structures*, 83:1526-1535, 2005.

Constitutive Models

- Bond-based materials: pairwise forces are functions of the initial positions, current positions, and displacement histories of the bonded material points
- Relative position, relative displacement, and bond stretch:

$$\xi = \mathbf{x}' - \mathbf{x}$$

$$\eta = \mathbf{u}(\mathbf{x}') - \mathbf{u}(\mathbf{x})$$

$$s = \frac{|\xi + \eta| - |\xi|}{|\xi|}$$

- Bond force magnitude: $f = g(s) (1 - \mu(t))$

Microelastic Material¹

$$g(s) = \frac{18k}{\pi\delta^4} s$$

Microplastic Material²

$$g(s) = \frac{18k}{\pi\delta^4} (s - \bar{s})$$

$$\bar{s}(0) = 0$$

$$\dot{\bar{s}} = \begin{cases} \dot{s} & \text{if } |s - \bar{s}| \geq s_Y \\ 0 & \text{otherwise} \end{cases}$$

Critical Stretch Bond Failure³

$$\mu(t) = \begin{cases} 0 & \text{for } s_{\max} < s_{\text{crit}} \\ 1 & \text{for } s_{\max} \geq s_{\text{crit}} \end{cases}$$

1. S.A. Silling. Reformulation of elasticity theory for discontinuities and long-range forces. *Journal of the Mechanics and Physics of Solids*, 48:175-209, 2000.
2. R.W. Macek and S.A. Silling. Peridynamics via finite element analysis, *Finite Elements in Analysis and Design*, 43:1169-1178, 2007.
3. S.A. Silling and E. Askari. A meshfree method based on the peridynamic model of solid mechanics. *Computers and Structures*, 83:1526-1535, 2005.

Process for Identifying Fragments

ASSUMPTION

- Fragments are completely isolated from each other

A FRAGMENT NUMBER IS ASSIGNED TO EACH NODE IN THE MODEL

- Computational domain is traversed to identify networks of unbroken bonds
- Process is iterative, converges when fragment numbers are no longer changing
- A fragment number is assigned to every node in the model
- All nodes in an isolated subdomain have the same fragment number

SUPPLEMENTAL OPERATIONS

- Tiny fragments are (optionally) combined into a common group and assigned a common fragment number
 - Tiny fragments are defined as those containing less than T_* nodes
- Fragments are sorted by mass and renumber from 1 to N_{frag}
- Related quantities of interest are computed for each fragment
 - Mass, center of mass, linear and angular momentum, moments of inertia, block names

Algorithm for Assigning Fragment Numbers

SERIAL CODE EXECUTION

```
DO initialize fragment numbers to node ids
REPEAT until fragment numbers stop changing
  FOR every node  $i$ 
    FOR all neighbors  $j$  of node  $i$ 
      IF the bond between nodes  $i$  and  $j$  is unbroken
        DO assign  $\max( F_i, F_j )$  to nodes  $i$  and  $j$ 
```

Algorithm for Assigning Fragment Numbers

PARALLEL CODE EXECUTION

DO initialize fragment numbers to node ids

REPEAT until fragment numbers stop changing across all processors

REPEAT until on-processor fragment numbers stop changing

FOR every node i


FOR all neighbors j of node i

IF the bond between nodes i and j is unbroken


DO assign $\max(F_i, F_j)$ to nodes i and j

FOR every node i

DO assign $\text{global_max}(F_i)$ to node i on all processors



Additional loop for
global convergence



Synchronization of fragment
numbers across processors

Characterization of Fragmentation Process

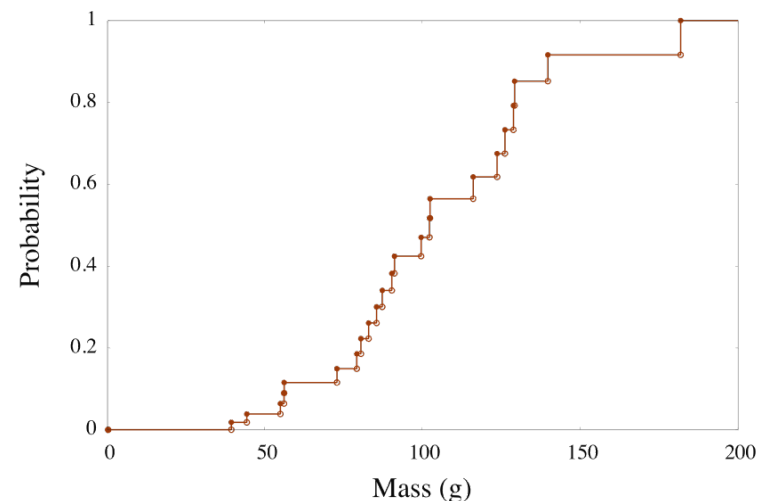
CUMMULATIVE DISTRIBUTION FUNCTION (CDF)

- A CDF can be created for any quantity of interest
- Provides insight into the fragmentation process
- Allows for comparison with experimental data

$$P(X) = \frac{1}{M} \sum_{\substack{i=1 \\ X_i \leq X}}^{N_{\text{frag}}} m_i$$

$$M = \sum_{i=1}^{N_{\text{frag}}} m_i$$

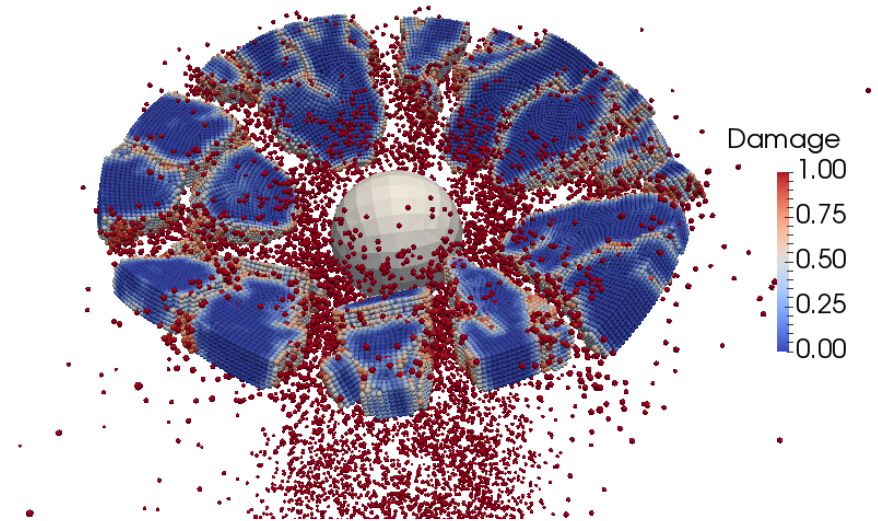
Example: CDF for fragment mass



$P(X)$ is the probability that a given material point belongs to a fragment whose property value X_i is less than X

Example Simulation: Fragmenting Disk

- Elastic sphere impacting a brittle elastic disk
- Projectile modeled with classical FEM
 - Elastic material model
 - Radius 5.0 mm
 - Initial velocity 35.0 m/s
 - ~1,000 hexahedral elements
- Target modeled with peridynamics
 - Bond-based microelastic material model
 - Critical stretch bond failure rule
 - Radius 17.0 mm, height 2.5 mm
 - ~75,000 nodal volumes



Material parameters
for projectile

Parameter	Value
Density ρ	993.1 kg/m ³
Bulk modulus k	1.0 GPa
Poisson's ratio ν	0.3

Material parameters
for target

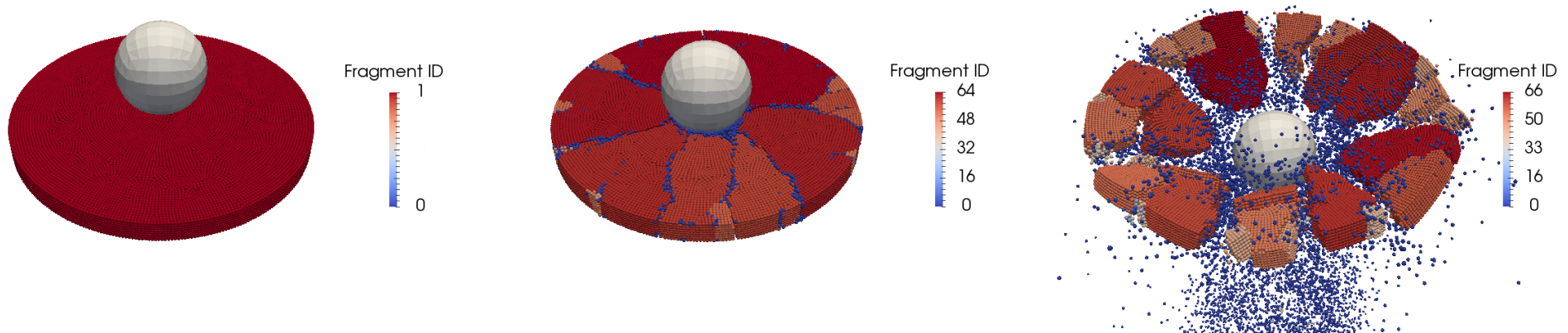
Parameter	Value
Density ρ	2200.0 kg/m ³
Bulk modulus k	14.9 GPa
Horizon δ	1.0 mm
Critical stretch s_{crit}	0.0005

Parameters for
fragment identification

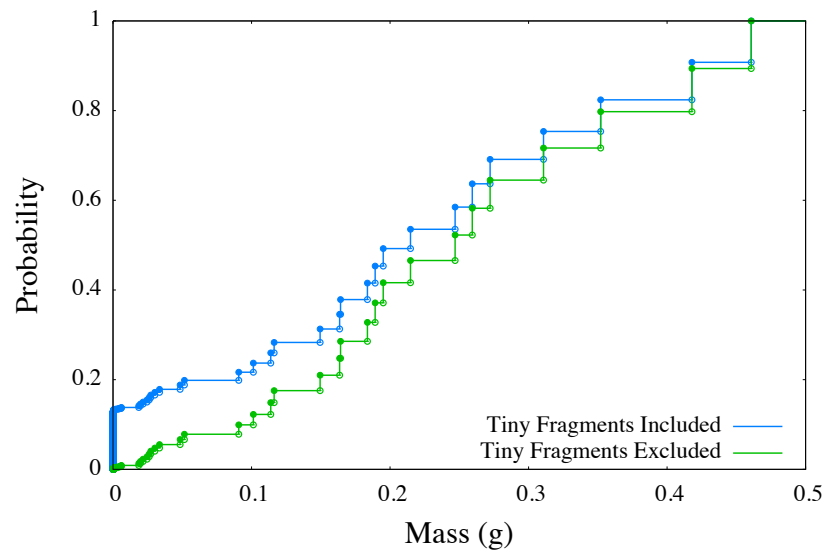
output file = frag_data.csv
increment = 4.0e-5
minimum fragment size = 5

Example Simulation: Fragmenting Disk

ALGORITHM CAPTURES EVOLUTION OF FRAGMENTATION PROCESS



EXCLUSION OF TINY FRAGMENTS HAD A SIGNIFICANT EFFECT

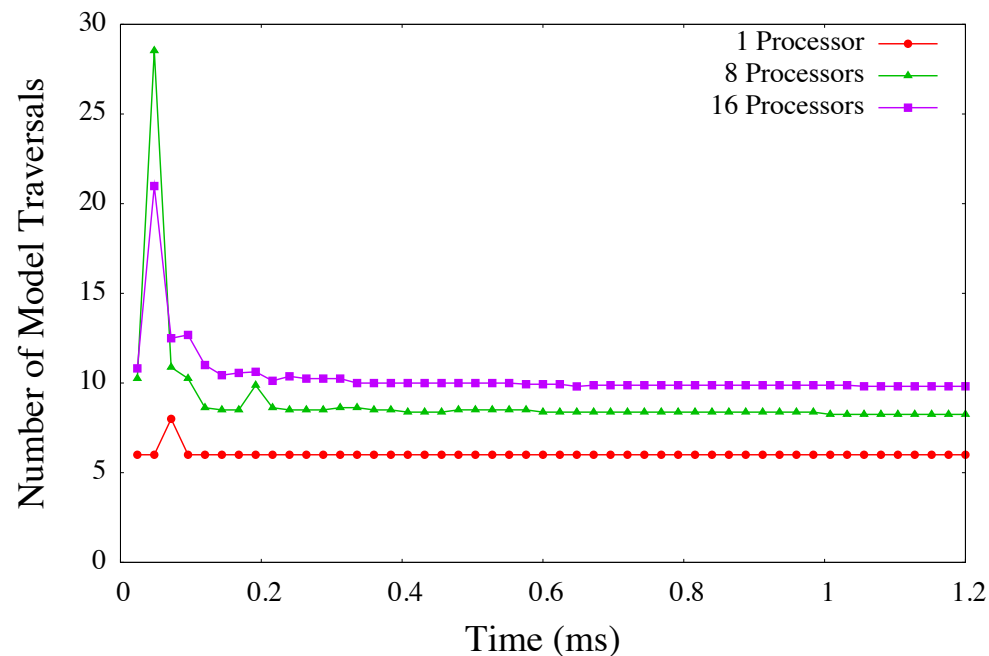


Threshold Fragment Size	Total Mass of Tiny Fragments
1	0.000 g
2	0.531 g
3	0.613 g
4	0.641 g
5	0.651 g

Example Simulation: Fragmenting Disk

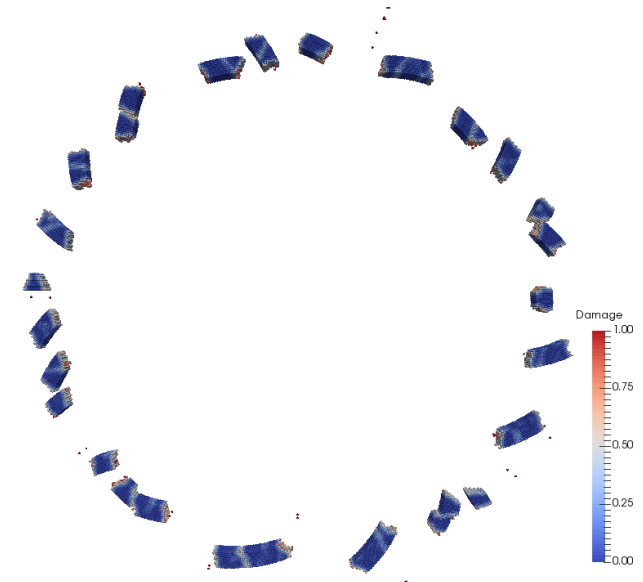
COMPUTATIONAL EXPENSE OF FRAGMENT IDENTIFICATION ALGORITHM

- Overall computational expense of algorithm was low
 - Fragment identification algorithm called 50 times
 - Computational expense between 0.2% and 0.3% of overall simulation time
- Additional processors resulted in modest increase in number of required iterations
- Number of iterations is highest when fragmentation is occurring
 - Possible result of fragments that are connected by a small number of bonds

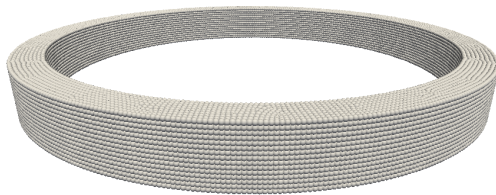


Example Simulation: Expanding Ring

- Fragmentation of an expanding ductile ring
- Bond-based microplastic material model
- Critical stretch bond failure rule
- Inner radius 110.0 mm, outer radius 125.0 mm, height 25.0 mm
- Initial outward radial velocity 100.0 m/s
- ~60,000 nodal volumes



Discretization of ring



Material parameters

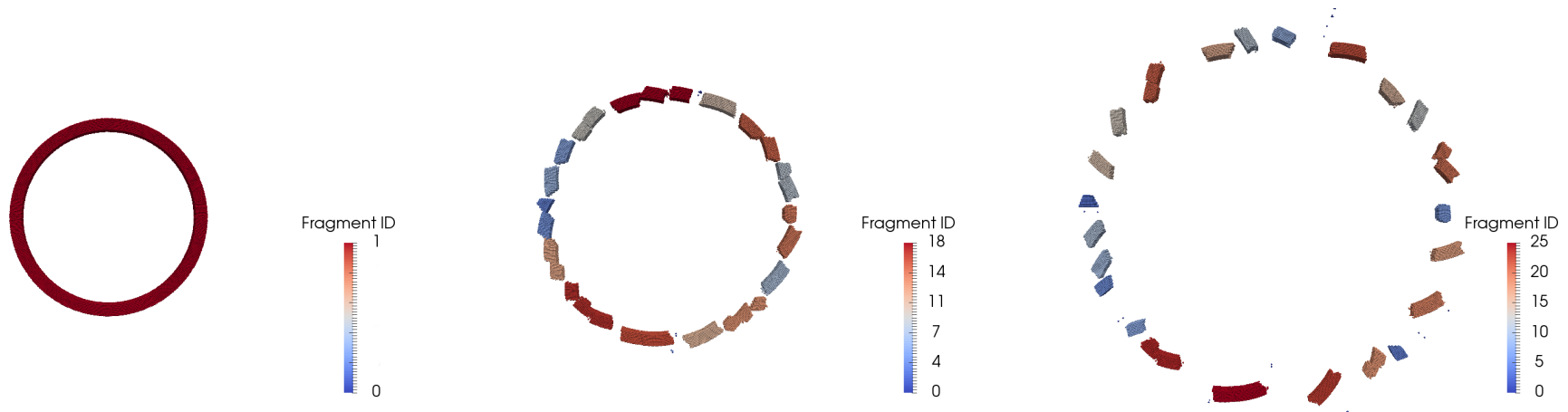
Parameter	Value
Density ρ	7850.0 kg/m ³
Bulk modulus k	140.0 GPa
Horizon δ	5.025 mm
Yield stretch s_y	0.000988
Critical stretch s_{crit}	0.02

Parameters for
fragment identification

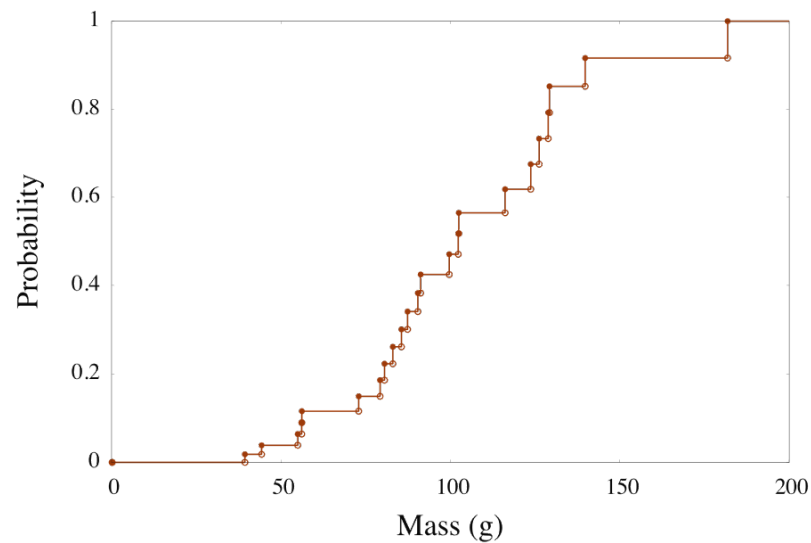
output file = frag_data.csv
increment = 2.4e-5
minimum fragment size = 0

Example Simulation: Expanding Ring

ALGORITHM CAPTURES EVOLUTION OF FRAGMENTATION PROCESS



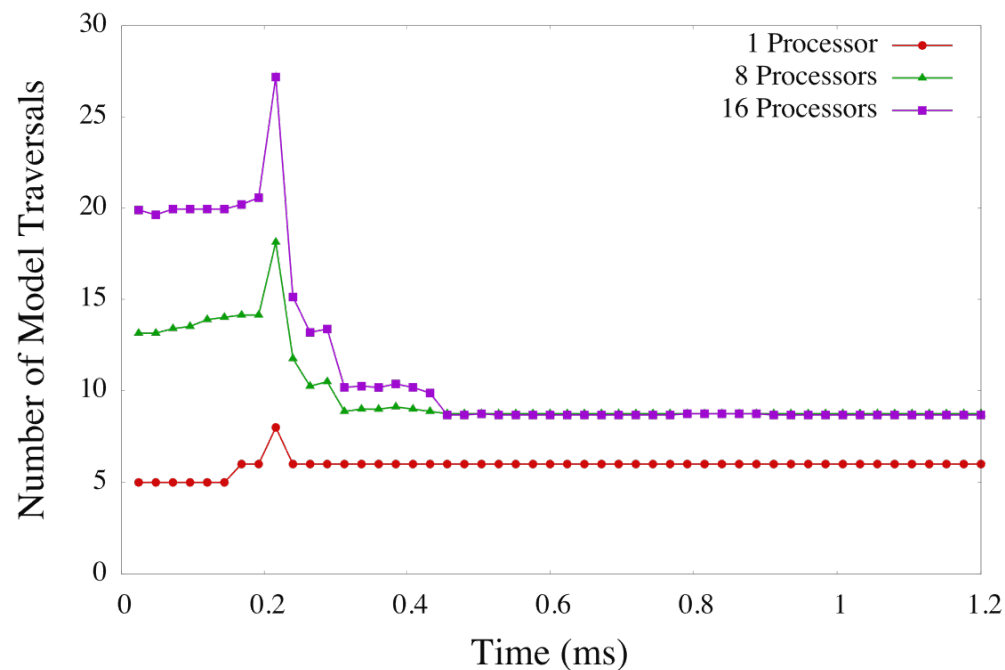
EXCLUSION OF TINY FRAGMENTS HAD VIRTUALLY NO EFFECT



Example Simulation: Expanding Ring

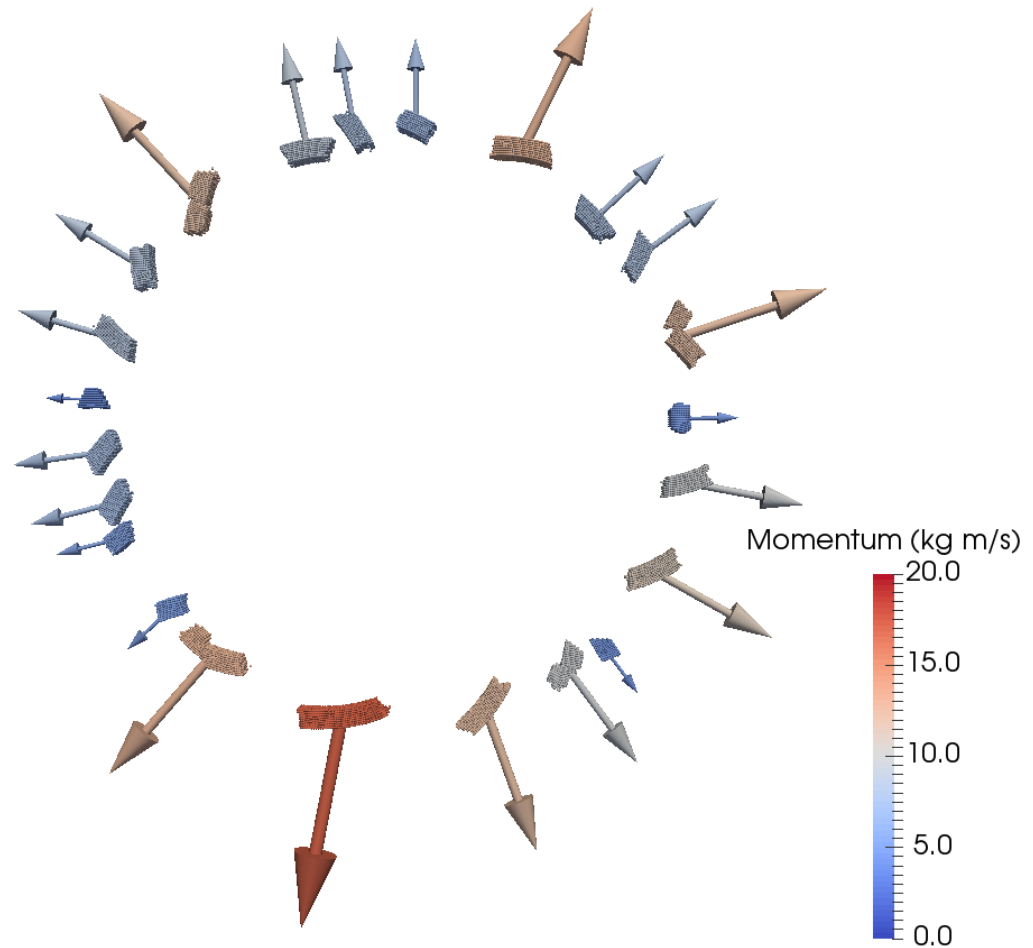
COMPUTATIONAL EXPENSE OF FRAGMENT IDENTIFICATION ALGORITHM

- Overall computational expense of algorithm was low
 - Fragment identification algorithm called 50 times
 - Computational expense between 1.0% and 2.0% of overall simulation time
- Additional processors resulted in increase in number of required iterations
- Number of iterations is highest when fragmentation is occurring



Possibilities for Additional Post-Processing

Visualization of
fragment momentum



Questions?

David Littlewood

djlittl@sandia.gov

Conference paper: IMECE2016-65400