SAND2016-4589C

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Correcting camera distortion including effects from temperature variation and diffraction

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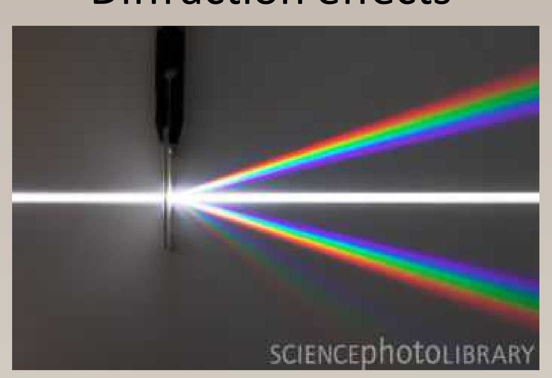
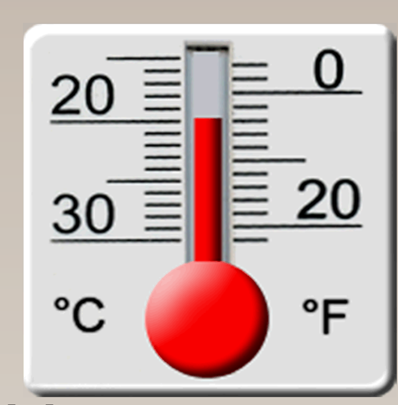
We developed an efficient and accurate model to recover line-of-sight from distorted image coordinates. This model, which incorporates effects due to temperature variation and diffraction, is based on a least squares 5th-order 3-dimensional polynomial fit. Using t-tests we determined a small number of significant terms which enabled us to reduce the number of parameters in our model by a factor of four. By exploiting symmetry between polynomial coefficients we again decreased the size of our model fourfold. In total, we solved for 9 parameters for the zero order object/image mapping and 28 parameters per wavelength for the 1st order object/image mapping. The maximum Euclidean angle error is on the order of 10^{-4} degrees.

Motivation:

- Optical lens causes distortion
  - Temperature dependence
  - Diffraction effects





- Current correction models [1], [2] :
  - No direct dependence on temperature
  - May be expensive (ex. iterative)

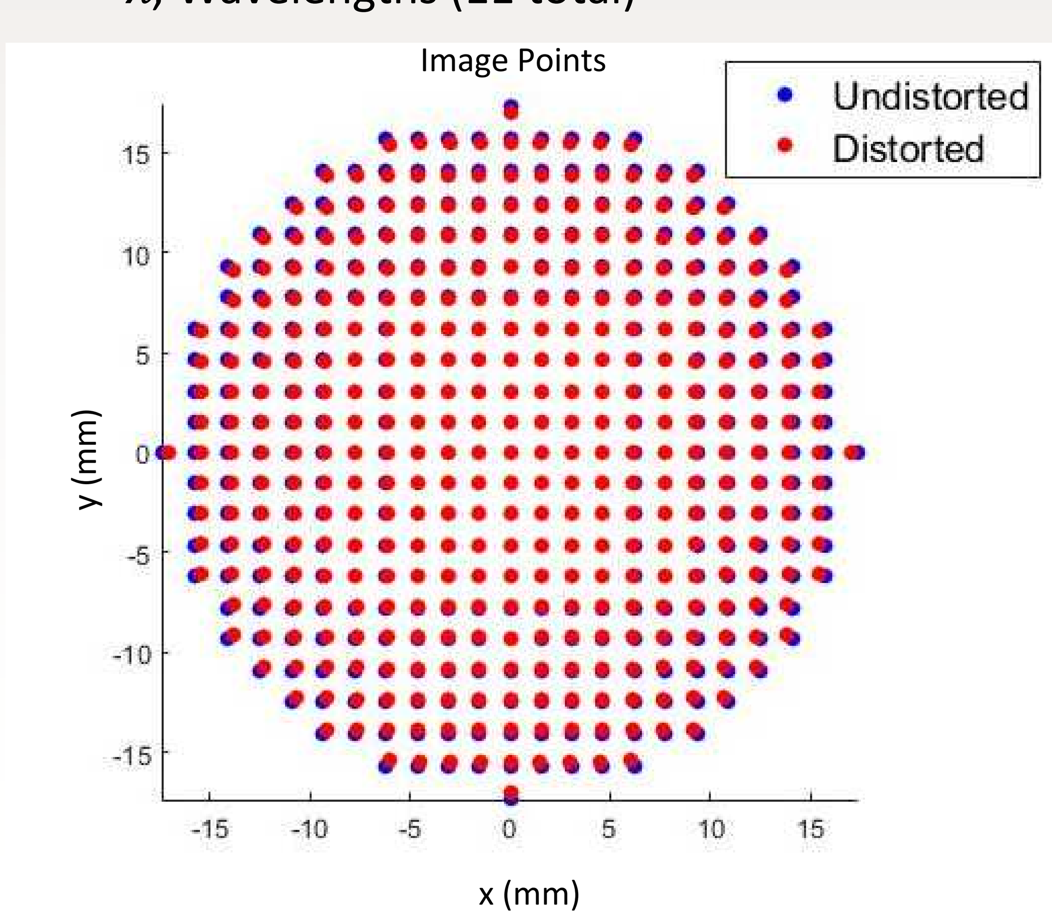
Objective:

Simple and accurate correction distortion model incorporating temperature and diffraction effects

Data:

Generated with Code V Optical Design Software

- Zero order (nondispersed)
  - x, y, Image centroids
  - θ_x, θ_y, Angles of arrival
  - t, Temperatures (11 total)
- 1st order (dispersed)
  - $x, y, \theta_x, \theta_y, t$ for each $+x, -x, +y, -y$ diffraction order
  - λ, Wavelengths (11 total)



Approach:

Least squares (LS) 5th-order 3-dimensional polynomial fit

Method 1 (Brute Force):

- One model for each θ_x, θ_y, diffraction direction, and wavelength
- 5th order polynomial (all 56 cross-terms)
- $\theta_x = a_1 + a_2x + a_3y + a_4t + a_5x^2 + a_6xy + a_7xt + \dots + a_{36}x^5 + a_{37}x^4y + a_{38}x^4t + a_{39}x^3y^2 + \dots + a_{56}\,t^5$
- $\theta_y = b_1 + b_2x + b_3y + b_4t + b_5x^2 + b_6axy + b_7xt + \dots + b_{36}x^5 + b_{37}x^4y + b_{38}x^4t + b_{39}x^3y^2 + \dots + b_{56}\,t^5$

Rewrite in matrix form:

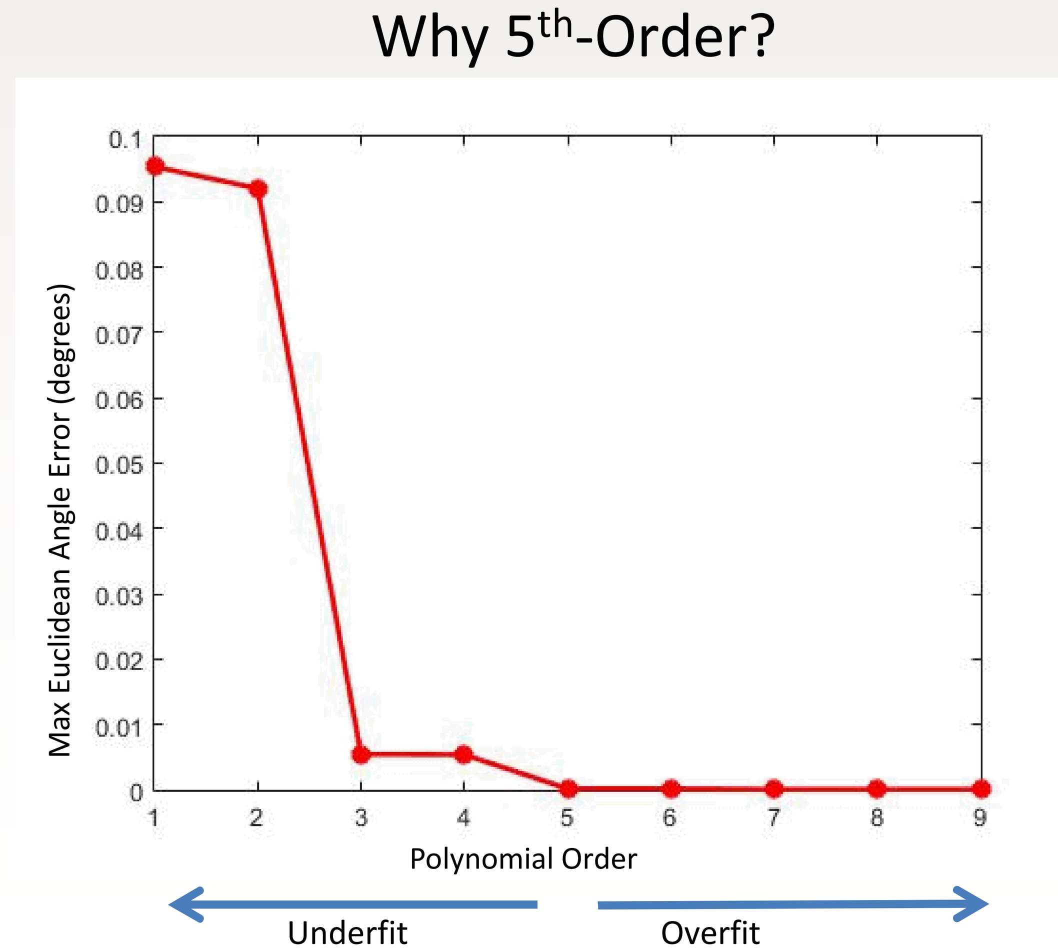
LS: $\Theta_x = V\boldsymbol{a}, \quad \widehat{\boldsymbol{a}} = (V^TV)^{-1}V^T\Theta_x, \quad \widehat{\Theta_x} = V\,\widehat{\boldsymbol{a}}$

$\Theta_y = W\boldsymbol{b}, \quad \widehat{\boldsymbol{b}} = (W^TW)^{-1}W^T\Theta_y, \quad \widehat{\Theta_y} = W\widehat{\boldsymbol{b}}$

Results 1:

- Max Error = $\sqrt{(\Theta_x - \widehat{\Theta_x})^2 + (\Theta_y - \widehat{\Theta_y})^2}$ = O(10^{-4}) degrees
- Too many parameters!
  - 90 LS equations with 56 parameters each!

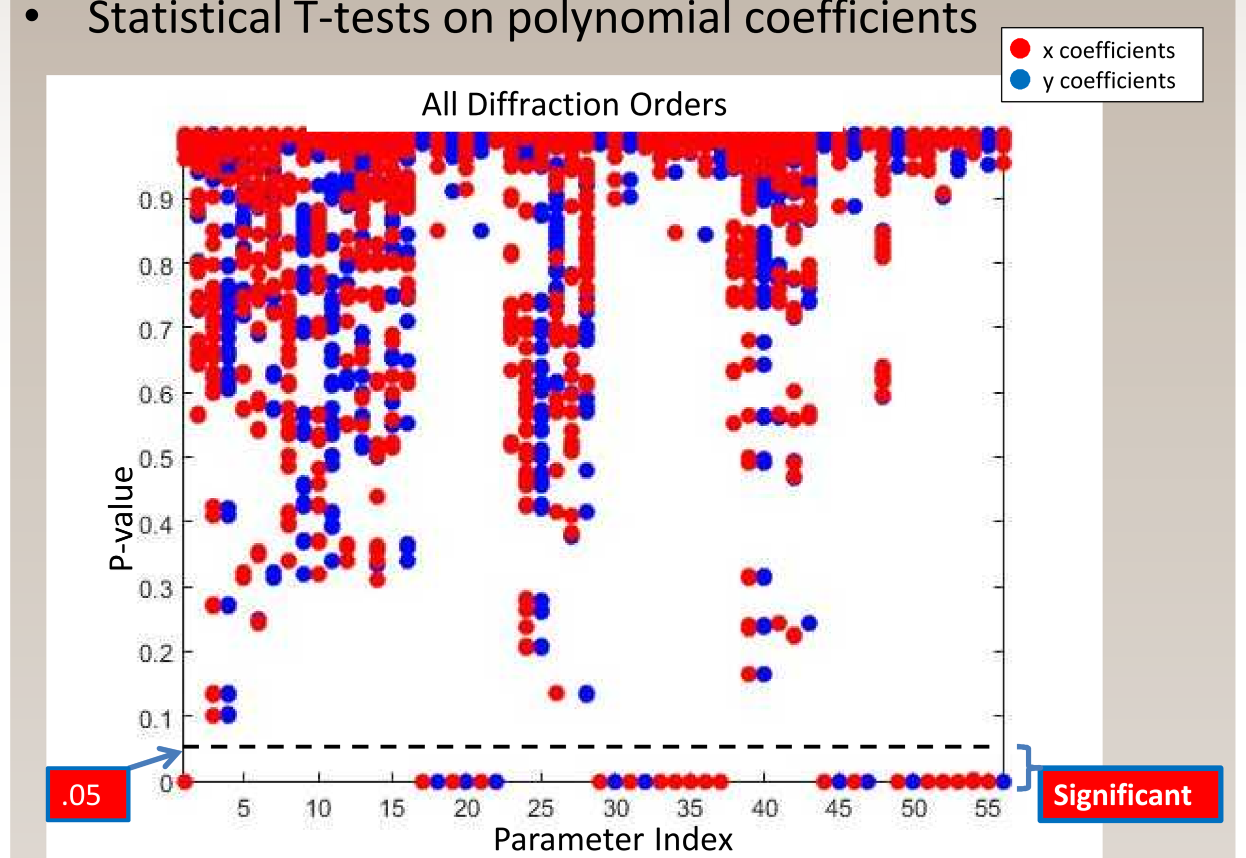
Why 5th-Order?



Method 2 (Fast, Accurate):

Determine significant terms

- Statistical T-tests on polynomial coefficients



- V, W: 56 columns to 9, 12, or 16 columns
- $\boldsymbol{a}, \boldsymbol{b}$: 56 entries to 9, 12, or 16 entries

Incorporate Symmetry

Symmetry Between:

- $\boldsymbol{a}$ and $\boldsymbol{b}$, zero order
- $\boldsymbol{a}$, $\pm$x diffraction order and $\boldsymbol{b}$, $\pm$y diffraction order
- $\boldsymbol{b}$, $\pm$x diffraction order and $\boldsymbol{a}$, $\pm$y diffraction order

Antisymmetry Between:

- $\boldsymbol{a}$, + diffraction orders and $\boldsymbol{a}$, - diffraction orders
- $\boldsymbol{b}$, + diffraction orders and $\boldsymbol{b}$, - diffraction orders

Reduced Equations

Zero order

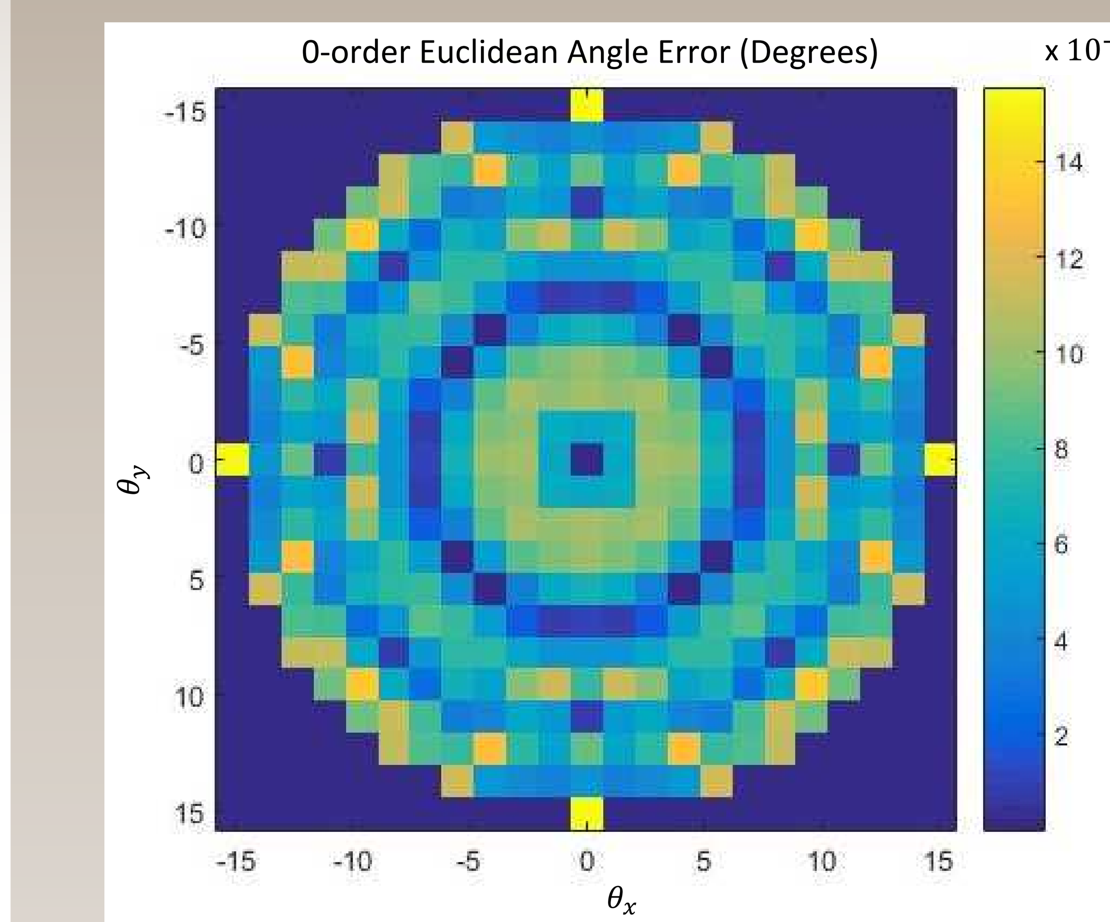
$$\begin{bmatrix} V \\ WP \end{bmatrix} \boldsymbol{a} = \begin{bmatrix} \Theta_x \\ \Theta_y \end{bmatrix}$$

1st order (+ - denote diffraction order)

$$\begin{bmatrix} V_+ \\ W_+P \\ V\_S \\ W\_SP \end{bmatrix} \boldsymbol{a} = \begin{bmatrix} \Theta_x \\ \Theta_y \\ \Theta_x \\ \Theta_y \end{bmatrix}$$

- P permutation matrix, S sign matrix

Results 2:



- Max Error = O(10^{-4})
- Small number of parameters
  - Zero order: 1 LS with 9 params
  - 1st order: 2 LS per λ with 12 and 16 params
- Reduced model (Method 2) has similar accuracy as full model (Method 1)

Conclusion:

- Model accurately predicts angle of arrival
  - Error = O(10^{-4})
- Corrects for temperature and diffraction
- Reduced model
  - requires about 1/16 the parameters as full model
  - Similar Accuracy

Model	# LS (zero order)	# params	# LS (1st order)	# params	Total # params	Max Error
Full (Method 1)	2	56 per LS	8 per λ	56 per LS	5040	$2.5\ x\ 10^{-4}$
Reduced (Method 2)	1	9 per LS	2 per λ	12, 16 per LS	315	$2.6\ x\ 10^{-4}$

References:

[1] Janne Heikkila and Olli Silven. 1997. "A Four-Step Camera Calibration Procedure with Implicit Image Correction." IEEE Computer Society Conference Proceedings on Computer Vision and Pattern Recognition.

[2] Wang et. al. 2008. "A new calibration model of camera lens distortion." The Journal of the Pattern Recognition Society, Vol. 41, 607-615.

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SAND No. 2011-XXXXP