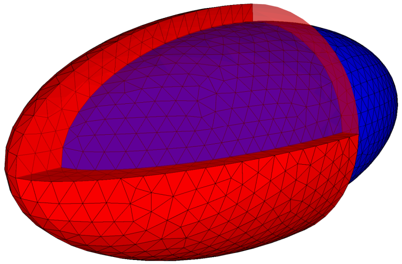
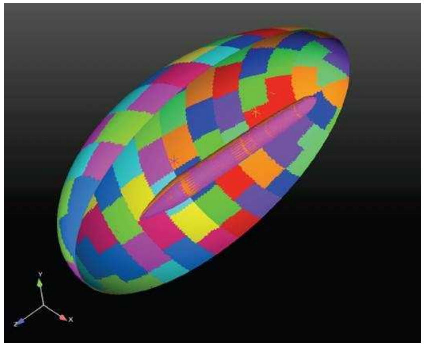
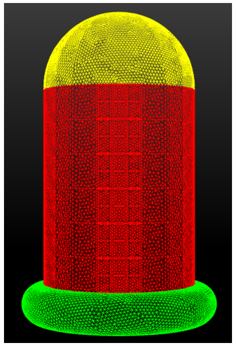
SAND2017-1299PE







Structural Acoustic Modeling Capabilities in Sierra-SD

Timothy Walsh

Overview of Sierra-SD Structural Acoustic Capabilities

- Massively parallel
- Hex, wedge, tet acoustic elements (up to order p=6), coupled with both 3D and 2D (shell) structural elements
- Linear and nonlinear acoustics
- Allows for mismatched acoustic/solid meshes
  - Mortar or multi-point constraints (MPC)'s
- Infinite elements and Perfectly Matched Layers (PML)
- Solution procedures:
  - Frequency response (frequency-domain)
  - Transient (time-domain)
  - Eigenvalue (modal) analysis
    - Linear and quadratic (complex modes)

Structural Acoustic Equations of Motion

acoustics

$$\nabla^2\phi = \frac{1}{c^2}\ddot{\phi}, \ \text{ in } \Omega_f \times (0,T)$$

$$\nabla\phi \cdot \boldsymbol{n}_f = -\rho_f \ddot{u}_n, \ \text{ on } \partial\Omega_f^N \times [0,T]$$

$$\phi = 0, \ \text{ on } \partial\Omega_f^D \times [0,T]$$

$$\phi(0,T) = 0, \ \text{ in } \Omega_f$$

$$\dot{\phi}(0,T) = 0, \ \text{ in } \Omega_f$$

solid mechanics

$$\nabla \cdot \boldsymbol{\sigma} = \rho\ddot{\boldsymbol{u}}, \ \text{ in } \Omega \times (0,T)$$

$$\boldsymbol{\sigma}\boldsymbol{n} = \boldsymbol{h}, \ \text{ on } \partial\Omega^N \times [0,T]$$

$$\boldsymbol{\sigma} = \boldsymbol{D} : \boldsymbol{\nabla}\boldsymbol{u}, \ \text{ in } \Omega \times [0,T]$$

$$\boldsymbol{u} = \boldsymbol{0}, \ \text{ on } \partial\Omega^D \times [0,T]$$

$$\boldsymbol{u}(0,T) = \boldsymbol{0}, \ \text{ in } \Omega$$

$$\dot{\boldsymbol{u}}(0,T) = \boldsymbol{0}, \ \text{ in } \Omega$$

Time domain

$$[M]\boldsymbol{a}(t) + [C]\boldsymbol{v}(t) + [K]\boldsymbol{u}(t) = \boldsymbol{f}(t)$$

Frequency domain (Helmholtz)

$$[H(\omega)]\boldsymbol{z}(\omega) = \boldsymbol{F}(\omega)$$

$$[H(\omega)] = -\omega^2[M] + i\omega[C] + [K]$$

Discretized Equations of Motion

- Fully coupled time domain formulation

$$\begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_s & L^T \\ L & -C_a \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ -f_a \end{bmatrix}$$

- Fully coupled eigenanalysis formulation

$$\lambda^2 \begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} + \lambda \begin{bmatrix} C_s & L^T \\ L & -C_a \end{bmatrix} \begin{bmatrix} u \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Fully coupled frequency-domain formulation

$$-\omega^2 \begin{bmatrix} M_s & 0 \\ 0 & -M_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} + i\omega \begin{bmatrix} C_s & L^T \\ L & -C_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & -K_a \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{bmatrix} f_s \\ -f_a \end{bmatrix}$$

Research Areas in Acoustics Sierra-SD

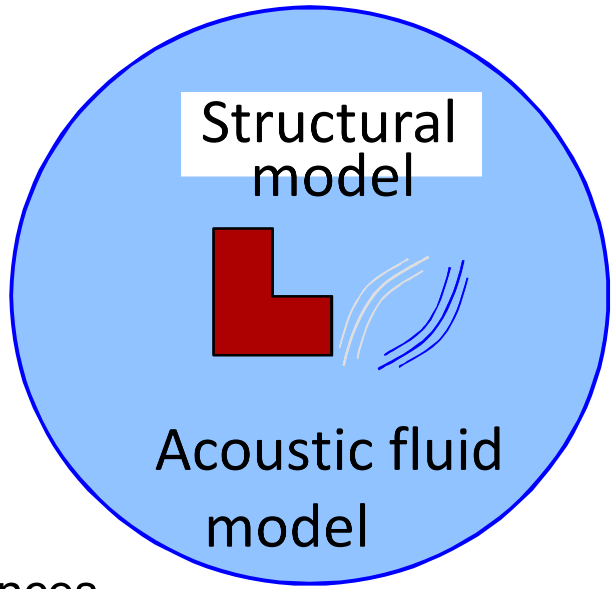
- On-going research areas in Sierra-SD-acoustics
  - Nonlinear acoustics
  - Infinite elements and Perfectly Matched Layers (PML)
  - High order ('p') finite elements
  - Inverse problems

- Upcoming research and development areas
  - Cavitating acoustic finite elements
  - Lighthill's acoustic analogy
  - Coupling with nonlinear solver (Sierra-SM)

Why Nonlinear Acoustics?

Linear acoustics is inadequate for many applications

- **Resonating cavities**
- **Large-amplitude sources**
- **Far-field of explosions**
- **Aeroacoustic noise**



<u>Assumptions of Linear Acoustic Theory</u>

- Small amplitude waves
- Linear constitutive fluid model
- No fluid convection

<u>Consequences</u>

- Resonance leads to infinite amplitude waves
- "Sine wave remains a sine wave"
- No wave distortion
- Wavespeed independent of stress state in fluid

Eulerian Formulations for Nonlinear Acoustics

- The linear acoustic wave equation

$$\frac{1}{c^2}\phi_{tt} - \Delta\phi = 0$$

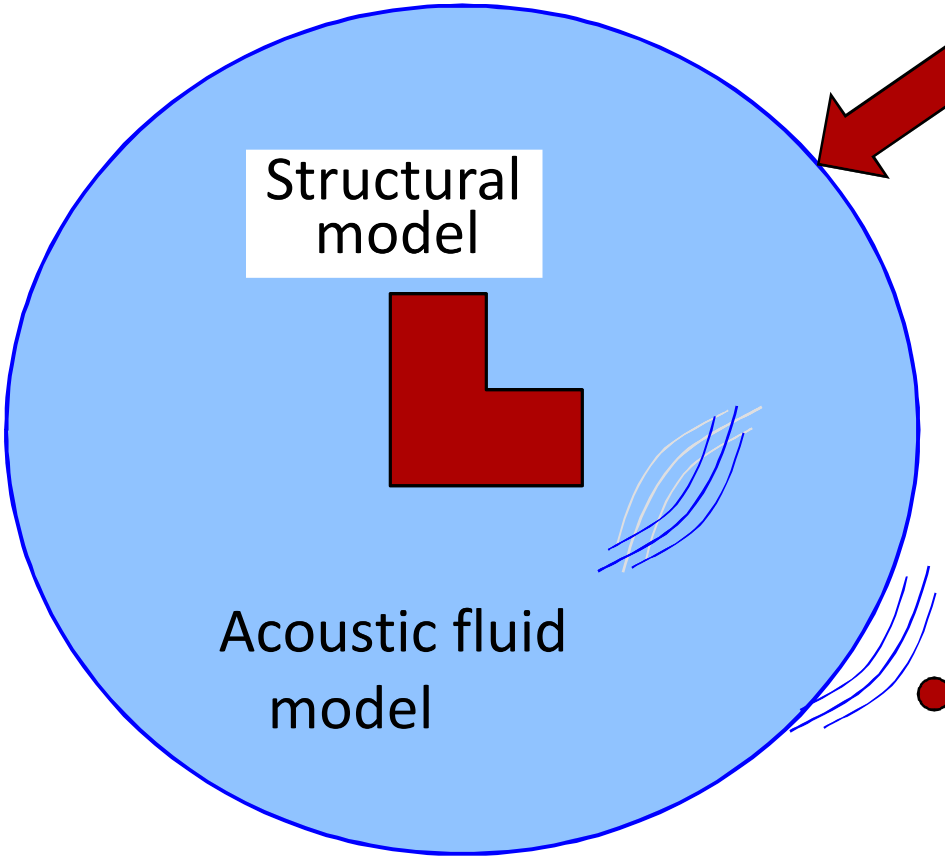
- The 2nd order Kuznetsov Equation

$$\frac{1}{c^2}\phi_{tt} - \Delta\phi + \frac{1}{c^2}\frac{\partial}{\partial t}\left[(\nabla\phi)^2 + \frac{B/A}{2c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 + b\nabla^2\phi\right] = 0$$

- High order nonlinear acoustic equation

$$\frac{1}{c^2}\phi_{tt} - \Delta\phi + \frac{1}{c^2}\frac{\partial}{\partial t}\left[(\nabla\phi)^2 + b\nabla^2\phi\right] + \frac{1}{2c^2}\nabla\psi \bullet \nabla(\nabla\phi)^2 + \frac{\gamma-1}{c^2}\left(\frac{\partial\psi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right)\Delta\psi = 0$$

Far-Field Acoustics



Common Requirement:
far-field boundary conditions for finite element analysis

- **Infinite Elements**
- **Perfectly Matched Layers (PML)**

microphone:
compute far-field response

Comparison of Infinite Elements and PML

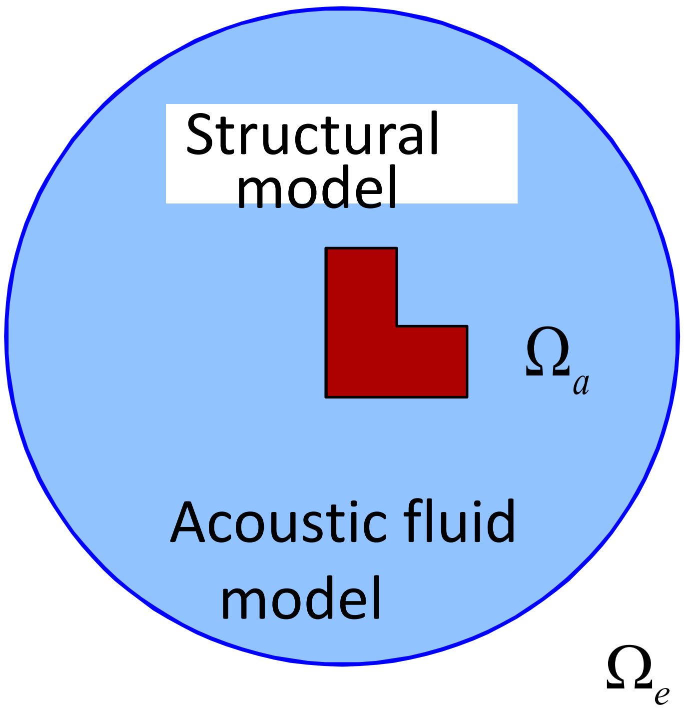
Infinite Elements

- Time and frequency domain formulations are identical (same matrices)
- Restricted to homogeneous media on ellipsoidal domains
- Built-in capability for computing far-field pressures (outside of acoustic mesh)

PML

- Originally restricted to frequency domain solutions
- Works on arbitrarily shaped convex domains (with corners)
- Can also absorb evanescent waves, and in some cases works on heterogeneous domains
- No capability for computing far-field pressure

Infinite Element Formulation



$$\Omega = \Omega_a + \Omega_e$$

Acoustic wave equation for fluid

$$\frac{1}{c^2} p_{tt} - \Delta p = 0 \quad \Omega x [0, T]$$

$$\frac{\partial p}{\partial n} = g(x,t) \qquad \Gamma x [0, T]$$

Weak formulation on exterior domain

$$\int_{\Omega} \frac{1}{c^2} \ddot{p}\, q\, dV + \int_{\Omega} \nabla p \bullet \nabla q \, dV = \int_{\Gamma} g q \, dS$$

Trial and weight functions

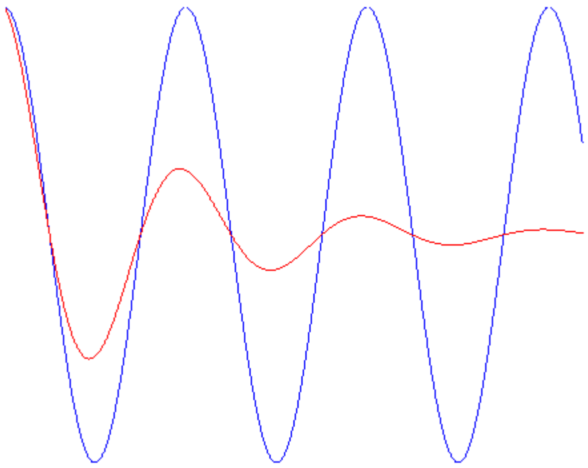
$$\phi(x,\omega) = P(x) e^{-ik\mu(x)} \qquad q = D(x) P(x) e^{ik\mu(x)}$$

$$\left(-\omega^2 M + i\omega C + K\right) p = f$$

Overview of PML

WTF4

- Undamped solution of wave equation: e^{ikx}
  - this wave will propagate indefinitely in the x direction
- Complex Coordinate System:
  - $\tilde{x} = a(x) + ib(x)$
- Wave Equation becomes:
  - $e^{ik\tilde{x}} = e^{i(-ka(x)+ikb(x))} = e^{-kb(x)}e^{ika(x)}$
  - Damped Wave Equation



Slide 11

WTF4 I would just set a(x) = x

Walsh, Timothy Francis, 7/24/2014

General Formulation for PML

Complex coordinate stretching $$\tilde{x} = x - \frac{i}{\omega}\int_x^a \sigma(\xi)d\xi \quad a < x < \bar{a}$$

Helmholtz equation over complex coordinates $$-\tilde{\Delta}p - k^2 p = 0$$

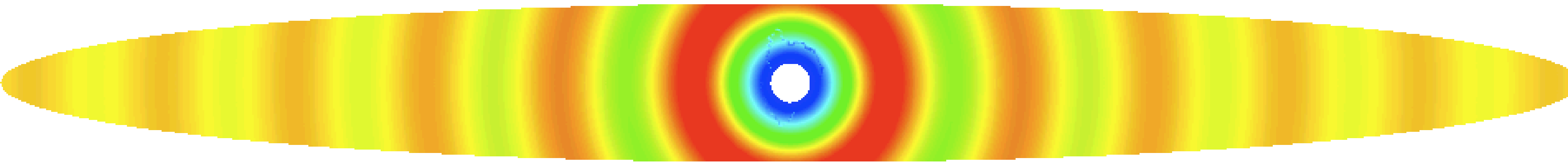
Weak form over complex coordinates $$\int_{\tilde{\Omega}_I} \langle\tilde{\nabla}p, \tilde{\nabla}q\rangle - k^2 pq \; d\Omega_I = \int_{\tilde{\Gamma}_S} gqdS$$

Mapped weak form back to real coordinates $$\int_{\Omega_I} \left[(\boldsymbol{J}^{-1}\nabla p)\cdot(\boldsymbol{J}^{-1}\nabla q) - k^2 pq\right] J(x,y,z)d\Omega_I = \int_{\Gamma_S} gqdS$$

Re-write as Helmholtz equation with variable coefficients $$\int_{\Omega_I} \tilde{\boldsymbol{A}}\langle\nabla p, \nabla\bar{q}\rangle - k^2\tilde{J}p\bar{q} \; d\Omega_I = \int_{\Gamma_S} g\bar{q}d\Gamma_S$$

$$\tilde{\boldsymbol{A}} = \tilde{J}\tilde{\boldsymbol{J}}^{-1}\tilde{\boldsymbol{J}}^{-T}$$

Results: 10-to-1 Prolate Spheroid



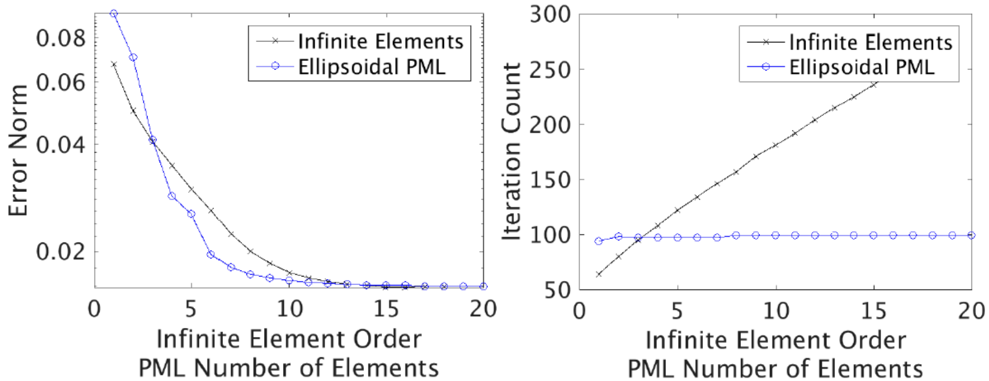
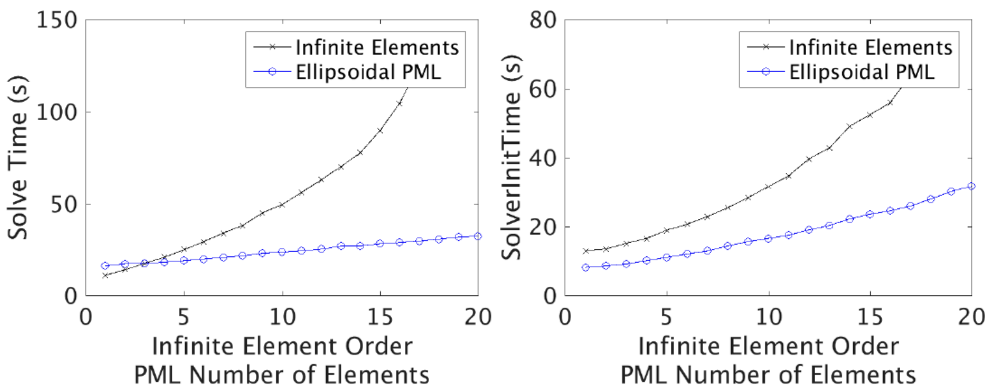


Figure 9: Comparison Between IE and PML (100 Hz)



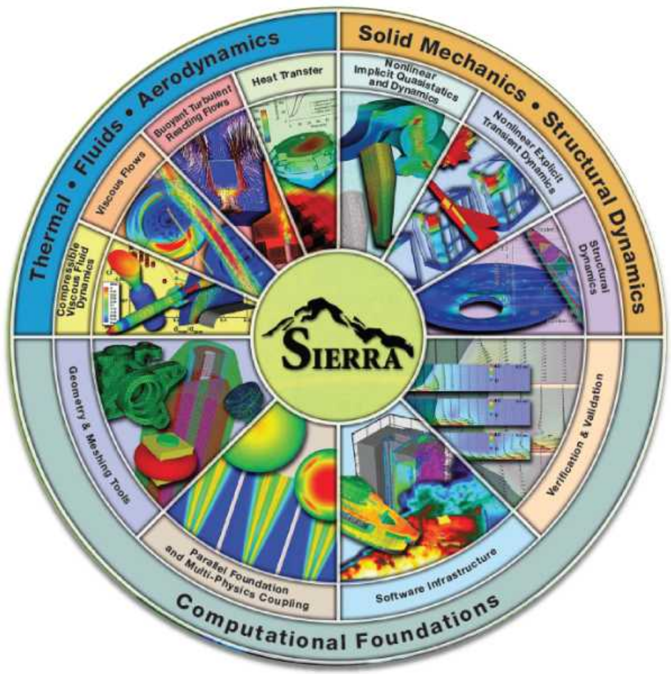
For a fixed level of accuracy

- PML required many less iterations than infinite elements
- PML solution times were much faster
- In frequency domain, PML is clear winner over infinite elements

Inverse Problems - Motivation

Forward Solver

Material properties
Geometry
Boundary conditions
Loads
Residual stresses
etc



Displacement
Pressure
Temperature
Flow field
etc

State Variables (outputs)

System parameters

Experimental data + inverse solution = missing link!

Abstract Optimization Formulation

Abstract optimization formulation

$$\underset{u,p}{\text{minimize}} \quad J(u,p)$$

$$\text{subject to} \quad g(u,p) = 0$$

Objective function

PDE constraint

$$\mathcal{L}(u,p,w) := J + w^T g$$

Lagrangian

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + g_u^T w \\ J_p + g_p^T w \\ g \end{Bmatrix} = \{0\}$$

First order optimality conditions

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & g_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & g_p^T \\ g_u & g_p & 0 \end{bmatrix} \begin{Bmatrix} \delta u \\ \delta p \\ w^\star \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ g \end{Bmatrix}$$

Newton iteration

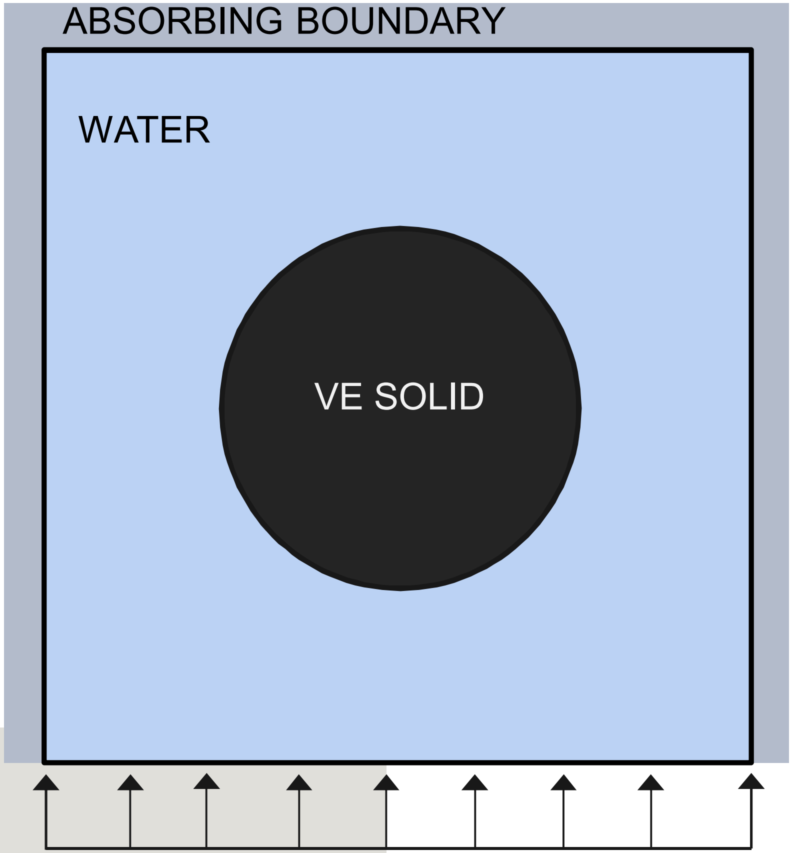
$$W \Delta p = -\hat{J}',$$

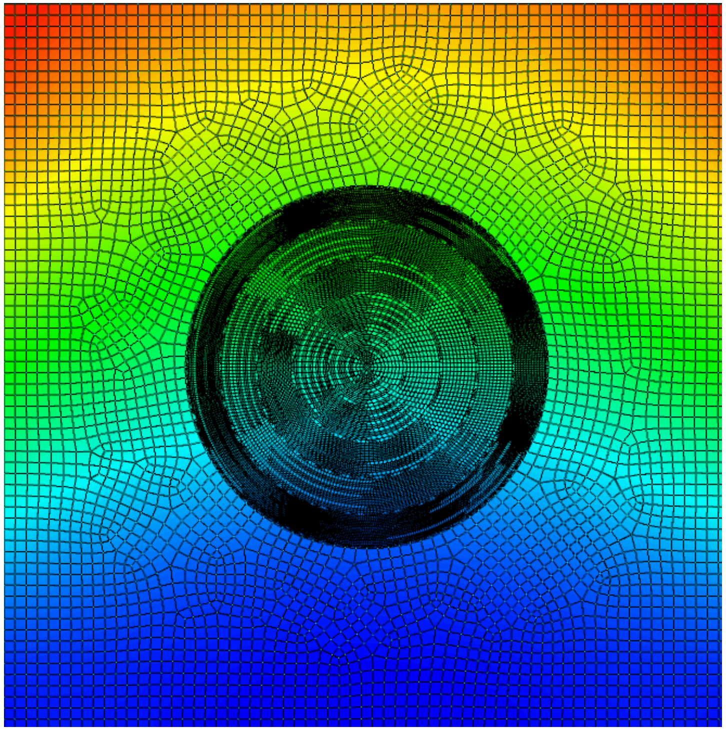
$$W = g_p^T g_u^{-T} (\mathcal{L}_{uu} g_u^{-1} g_p - \mathcal{L}_{up}) - \mathcal{L}_{pu} g_u^{-1} g_p + \mathcal{L}_{pp}$$

Hessian calculation

Inverse Problems: *Acoustic Cloaking*

- 2-D fluid region with circular VE solid inclusion
- Inclusion consists of concentric rings w/ distinct material properties
- Periodic acoustic load applied to end
- Match forward problem pressure distribution by adjusting **VE material parameters**

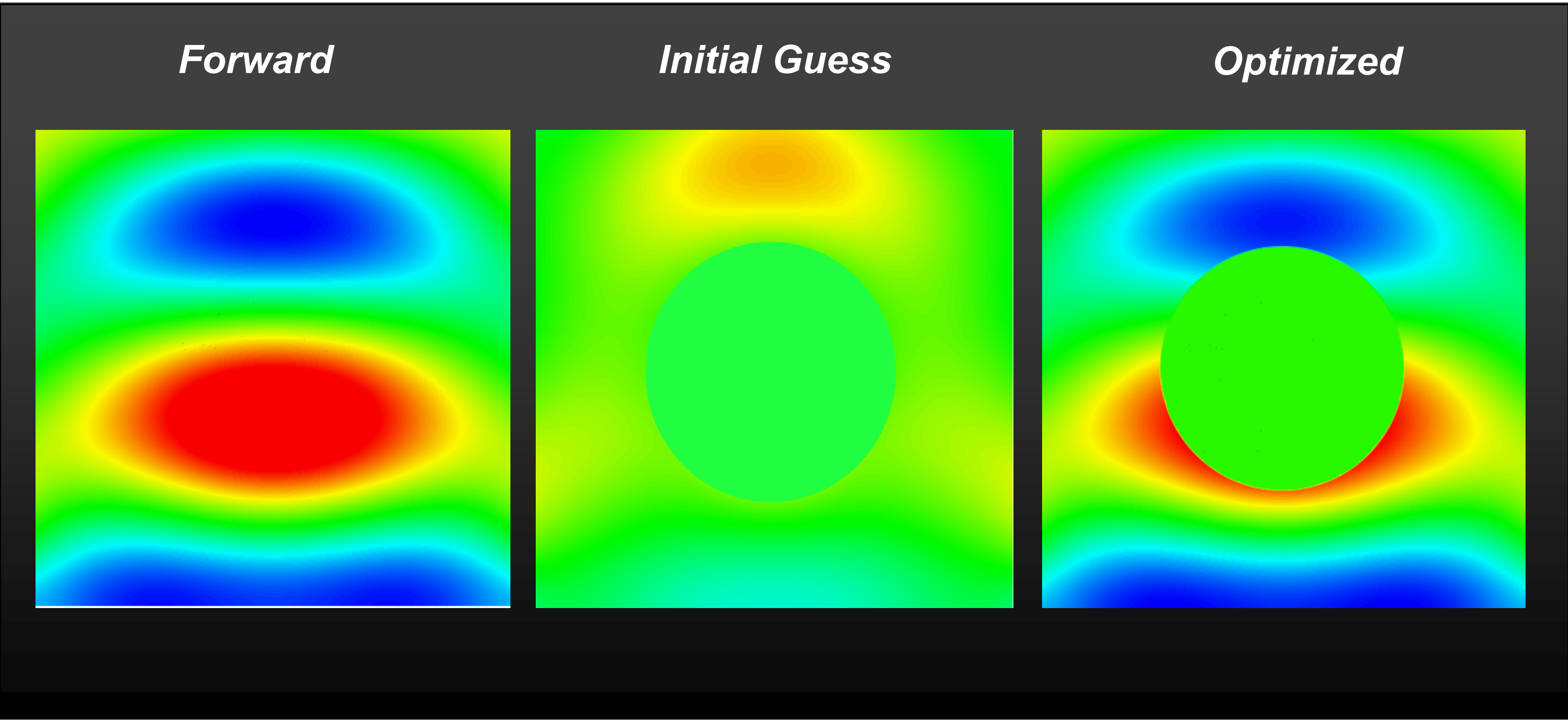




Left: Model Set up
Right: Forward problem pressure distribution (500 Hz loading) in model with 50 layers

Acoustic Cloaking

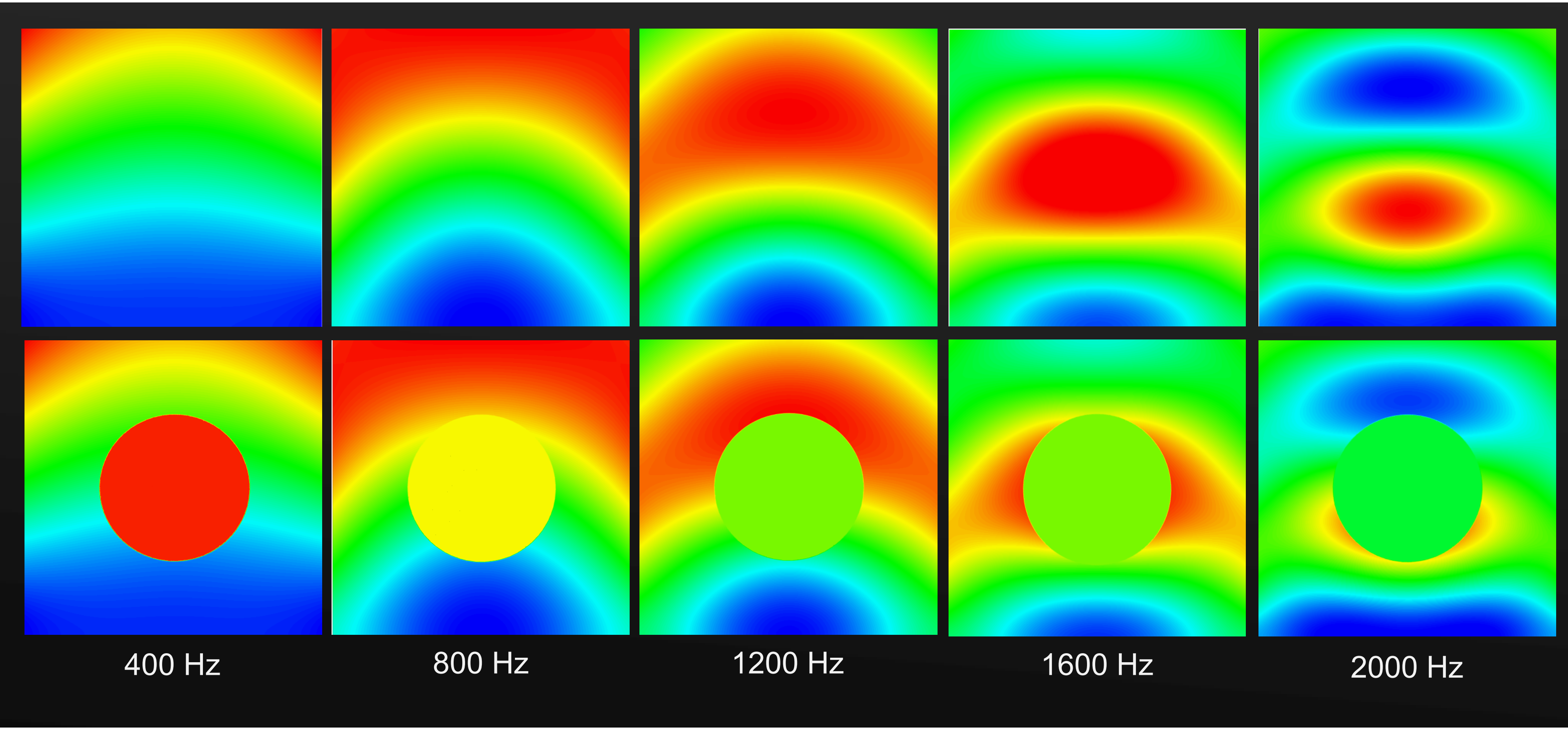
- Optimized VE foams allow recovery of desired pressure distribution



Left: Target acoustic pressure distribution, from forward problem
Center: Acoustic pressure distribution with initial material guess (2000 Hz Loading)
Right: Pressure distribution after convergence to optimized design

Acoustic Cloaking

- Optimized VE foams allow recovery of desired forward pressure distribution
  - **Top :** Acoustic pressure from forward analysis
  - **Bottom :** Acoustic pressure around optimized solid inclusion



Conclusions

- Massively parallel finite element structural acoustics capability Sierra-SD has been developed for large-scale analysis
- Applicable to large-scale models with many degrees of freedom.
- Sierra-SD and optimization codes have been loosely coupled for the solution of source and material inversion problems.
- Capability has been applied to a variety of problems inside and outside of Sandia