LA-UR-17-22938
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Intended for:	Pipeline Simulation Interest Group 2017 Annual Meeting, 2017-05-09/2017-05-12 (Atlanta, Georgia, United States)
Issued:	2018-02-06 (rev.1)

PSIG 1715

Economic Optimization of Intra-Day Gas Pipeline Flow Schedules using Transient Flow Models

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This paper was prepared for presentation at the PSIG Annual Meeting held in Atlanta, Georgia, 9 May – 12 May 2017.

ABSTRACT

As dependence of the bulk electric power system on gas-fired generation grows, more economically efficient coordination between the wholesale natural gas and electricity markets is increasingly important. New tools are needed to achieve more efficient and reliable operation of both markets by providing participants more accurate price signals on which to base their investment and operating decisions.

Today's Electricity energy prices are consistent with the physical flow of electric energy in the power grid because of the economic optimization of power system operation in organized electricity markets administered by Regional Transmission Organizations (RTOs). A similar optimization approach that accounts for physical and engineering factors of pipeline hydraulics and compressor station operations would lead to location- and time-dependent intra-day prices of natural gas consistent with pipeline engineering factors, operations, and the physics of gas flow.

More economically efficient gas-electric coordination is envisioned as the timely exchange of both physical and pricing data between participants in each market, with price formation in both markets being fully consistent with the physics of energy flow. Physical data would be intra-day (e.g., hourly) gas schedules (burn and delivery) and pricing data would be bids and offers reflecting willingness to pay and to accept. Here, we describe the economic concepts related to this exchange, and discuss the regulatory and institutional issues that must be addressed. We then formulate an intra-day pipeline market clearing problem whose solution provides a flow schedule and hourly pricing, while ensuring that pipeline hydraulic limitations, compressor station constraints, operational factors, and pre-existing shipping contracts are satisfied. Furthermore, in order to support the practical application of these concepts, we provide a computational example of gas pipeline market clearing on a small interpretable model, and validate the results using a commercial pipeline simulator. Finally, we validate the modeling by cross-verifying simulations with SCADA data measured on a real pipeline system.

INTRODUCTION

The growing reliance of the bulk electric power system on gas-fired generation has made organized coordination between the wholesale natural gas and electricity markets an increasingly pressing need. Replacement of coal fired and nuclear plants with gas-fired generating capacity significantly increases the amount of natural gas used as fuel for power generation. In parallel, the variability of electric generation from wind and solar increases the variability of pipeline deliveries to gas-fired generators used to balance the electric grid. The resulting intra-day and even sub-hourly swings in demand for natural gas as a fuel for electric generation create new challenges for pipeline operators, and may pose reliability risks for both gas pipelines and electric systems.

The need to better coordinate the two sectors to mitigate these risks is well recognized, and is reflected in the recent orders 787 and 809 by the Federal Energy Regulatory Commission (FERC), which regulates access to pipeline capacity[1,2]. Coordination mechanisms proposed to date are based on widening the scope of operational information exchanged by the two sectors and on adjusting the timing of when these exchanges occur[3]. While these measures are helpful, a truly efficient coordination should be based on timely exchange of both physical and pricing data with price formation in both markets being fully consistent with the physics of energy flow.

Electricity prices consistent with the physical flow of electric energy in the power grid are the outcome of economic optimization of power system operation in organized electricity markets administered by Regional Transmission Organizations (RTOs)[4, 5]. A similar optimization approach that accounts for physical and engineering factors of pipeline hydraulics and compressor station operations would lead to location- and time-dependent economic value of natural gas consistent with the physics of gas flow.

Our goal is to formulate and solve a transient pipeline optimization problem that maximizes total market surplus over supply and offtake schedules. Market Suprlus in this context is defined as the sum of the producer/supplier surplus and consumer/buyer surplus. Producer surplus is derived whenerver the price the producer receives exceeds the value they are willing to accept for the goods they sell. Similarly, consumer surplus is derived whenever the price the consumer ends up paying for good is below the value they are prepared to pay. Market surplus is the sum of individual surpluses over all consumers/buyers and producers/sellers participating in the market.

The appropriate transient optimization solution dynamically allocates pipeline capacity among transactions between suppliers and consumers based on the economic value of these transactions. Compressor operations and line pack are optimized in conjunction with the selection of location-dependent offers to sell, and bids to buy, natural gas. Location-based (nodal) prices of natural gas are computed as dual variables corresponding to the nodal flow balance constraints in the optimal solution, and reflect the time- and location-dependent economic value of gas in the network.

More economically efficient gas-electric coordination is then envisioned as the timely exchange of both physical and pricing data between participants in each market, with price formation in both markets being fully consistent with the physics of energy flow. Physical data would be intra-day (e.g., hourly) gas schedules (burn and delivery) and pricing data would be bids and offers reflecting willingness to pay and to accept. Location-based gas prices would be obtained using optimization of transient pipeline flow models. Inputs to the pipeline optimization problem include prices that power plants are willing to pay for gas, as derived from nodal electricity prices that are produced by power system optimization.

In this paper, we define the pricing concept in terms of Locational Trade Values for natural gas (LTVs) that are obtained using the single-price two-sided auction mechanism while accounting for the physics of natural gas flows and engineering factors of pipeline networks. In contrast to previous studies[6,7], we do not linearize gas flow equations and thus retain the impact of non-linearities on LTV formation. We adopt a modeling approach developed for large-scale control system modeling of gas pipelines[8,9], so that constraints on flow and energy usage by compressors can be described, and an optimization formulation that maximizes market surplus is presented. While marginal pricing and economic spot markets for gas have been studied[10], the LTVs described here provide price signals that reflect the physical ability to transport gas through a pipeline system. We describe a preliminary engineering economic analysis of LTV basis differentials created through the proposed market mechanism. We also describe properties of the mechanism, including revenue adequacy for the market administrator, which have been shown in the case of power systems to make practical implementation possible.

To illustrate the economic transient optimization concept, we provide computational results for a small test system that is optimized for maximal allocation of capacity by dynamic scheduling of deliveries, compressor operations, and corresponding LTVs. The optimization problem is solved using a simplified modeling approach, and the feasibility of the obtained solution is then verified by a simulation performed using a high fidelity commercial solver. In addition, we demonstrate that the physical and engineering modeling used in the transient optimization prototype approaches an adequate representation of actual pipeline behavior. This is shown by cross-verification of SCADA data measurements from simulations of a real pipeline under highly transient conditions with pipeline simulations using the reduced model with nodal parameters specified using a subset of the SCADA data as the set of inputs. The concepts, models, computational methods, and validations described here are preliminary. Although they provide a promising path for integrating and automating markets, scheduling, and operations of gas pipelines, we expect that numerous multi-year studies and development activities will be required to bring the methodology into the field.

Background

Significant and rapid growth in the use of natural gas for power generation in the United States is greatly increasing demand for transportation of gas through large-scale interstate pipelines[11]. Among other factors this is being driven by environmental regulations, the transition to cleaner electric power sources, the abundance of inexpensive natural gas, and improvements in gas turbine efficiency[12]. Coal-fired and nuclear power plants therefore continue to be replaced primarily by gas-fired generating units throughout the United States[13]. Because power production by gas turbines can be ramped up and down easily, gas-fired generators are widely used to compensate for fluctuations caused by variable and non-dispatched sources including wind and solar[14]. Increased reliance on gas-fired generation is transferring the demand for electric energy onto natural gas pipeline infrastructure[14,15]. Moreover, that demand is increasingly variable by hour within the day.

Market structures for interstate pipeline transportation services in the United States are at present constrained within a regulatory framework that was not designed to support market responsive price formation[16]. Access to pipeline capacity is provided at rates regulated by the Federal Energy Regulatory Commission (FERC). Holders of firm physical rights are allowed to sell unneeded capacity on a daily basis through a release mechanism. Released capacity is bundled with gas supply and traded bilaterally in a locational spot market for natural gas. Trading platforms such as the Intercontinental Exchange (ICE) serve as major vehicles for price formation. Reported price indices for several dozen locations in North America change daily with Friday prices prevailing over the weekend. These daily prices do not reflect intra-day demand variations.

Historically, intra-day demand variations were primarily caused by changes in residential and commercial loads. These changes are typically weather driven, predictable, and reasonably well managed by pipeline operators. In contrast, significant intra-day and even sub-hourly swings in demand for natural gas as a fuel for electric generation create new challenges for pipeline operators, and pose reliability risks for gas pipelines and electric systems. Better coordination is needed between the two sectors to mitigate these risks[13,17]. The implications of these regulatory changes on coordinating operations of gas pipeline and electric power grids have been recently examined[18].

Coordination mechanisms proposed to date are based on widening the scope of operational information exchanged by the two sectors and on adjusting the timing of when these exchanges occur. In addition to such changes, new economic tools are needed for gas-electric coordination that provides financial incentives for market participants to change behavior in a way that would result in more efficient and reliable operation of both infrastructures. Intra-day locational prices of natural gas that are consistent with the physics and engineering constraints of pipeline operation could provide such a tool. However, this complexity is highly challenging to account for in physical operation, and current approaches can only roughly estimate capacities for intra-day market clearing[19]. Even today, price formation on the natural gas spot market is based on bilateral trading[20,21], and pricing of capacities relies on statistical analysis of historical data[22].

In the electric power industry, standard practice is use of optimization to price electric energy based on the physical ability of the electric network to deliver it from producers to consumers[4,5]. In contrast, with the exception of a market in the Australian province of Victoria[6], the use of physics-based optimization to clear natural gas markets remains a topic of research. Developing locational pricing mechanisms for natural gas is challenging because of complex physical and engineering factors of pipeline hydraulic modeling and optimization[23,24]. Thus, in addition to the different physical and operational aspects of gas pipelines and electric power grids, there is also a disparity in market mechanisms that complicates attempts to bridge the gap in coordination between these sectors[17].

Auction-based pricing mechanisms for pipeline capacity that are similar to what is used in wholesale electricity markets have been of interest for nearly 30 years, and were explored in a 1987 FERC report[25]. In that report, a linear programming model for auctioning pipeline transportation rights was proposed, with primary auctions to be conducted as often as daily. More frequent secondary auctions for re-selling of capacity rights were envisioned as well. Many of the ideas in the 1987 proposal remain relevant and deserve to be re-examined in light of noted trends in the natural gas industry, improved optimization techniques[9,24], and the significant experience gained through successful implementation of auction-based market mechanisms over the past two decades in the power industry worldwide.

Optimization Formulation

In this section we present an overview of transient optimization, describe our approach to simplified pipeline modeling for the purpose of transient optimization of large-scale systems, and explain the optimization formulation suggested for use as a market mechanism. The mathematical nomenclature and formulation is presented in an appendix.

Transient Optimization Overview

Many transient optimization approaches have been proposed for creating operational plans that satisfy expected dynamically changing loads while keeping operation within contractual and operating constraints and equipment limitations[26-29]. Such methods aim to provide time-dependent schedules for compressor discharge pressures by looking ahead and repositioning line pack to optimal locations in advance of expected upcoming load fluctuations. In addition to finding feasible operational plans under challenging circumstances, these techniques can be tasked with objectives such as minimizing operational costs, achieving user specified line pack targets in critical regions, or determining maximum possible time-integrated deliveries. Transient optimization problems are typically computationally intensive yet depend on accurate and timely information. Solutions must also be computed rapidly enough to support real-time decision-making, while human interfaces and work flow must aid operators and marketers in that decision making. Timely solutions are complicated by the nature of pipeline control engineering, which includes continuous and discrete control variables, and which are highly challenging to optimize under dynamic conditions. Nevertheless, development of transient optimization tools is needed for pipelines to effectively deal with the difficulties of interacting with electric transmission systems. We restrict the present work to continuous optimization formulations without explicit treatment of discrete variables, and suggest that this is an acceptable approximation for intra-day optimization of large (e.g. continental) scale transmission pipeline systems.

Formulations that employ recourse to account for uncertain upcoming system loads have been developed[30], and provide an important capability. However, as with most previously proposed transient optimization concepts, the actual intra-day load profiles are considered as parameters, which are possibly uncertain, rather than optimization variables. The major obstacle to fielding such approaches is the use of predictions for load profiles, so that there is no guarantee that the expected conditions will actually take place. In contrast, the paradigm presented here proposes an organized mechanism for shippers and operators of a pipeline system to make optimal decisions about what the upcoming system loads should be. If implemented, such a decision-making system would eliminate substantial uncertainty for all parties involved in pipeline system operations. In this paper, we propose a transient optimization method that, with further development, could enable day-ahead or rolling horizon flow scheduling and compressor operation optimization based on an economic market concept. The computation can be rapid enough to produce timely results on a commodity computing platform using general optimization solvers even for large pipelines[31,32]. A fielded system would be able to utilize high performance computing, as done in power systems operations to further reduce solution times.

Simplified Modeling of Pipeline Systems

Here we are interested in modeling the large-scale, system-wide dynamics of a pipeline network for the purpose of studying engineering economics on a regional or continental scale. Although we employ several simplifications for the purpose of this proof-of-concept study, the modeling can be extended to capture more complex physical and engineering aspects. Specifically, we assume isothermal flow through a horizontal pipeline with constant gas composition, and where gas compressibility is specified using the CNGA method in the equation of state[23,33]. We also assume that flow changes are sufficiently slow so as not to excite waves or shocks, so that second order terms may be removed from the dynamic equations, and relatively coarse discretizations in both space and time may be used. The important parameters for a pipe are length, diameter, and the Colebrook-White friction factor. The dynamics of gas flow within the pipe can then be modeled using the isothermal Euler equations in one dimension, with the inertia and gravity terms omitted[8,38].

For simplicity in this study, compressor stations and regulator elements are modeled as two-ended flow devices that can enforce the given time-dependent pressures on a specified side, such as the discharge pressure. Theoretical power for compressors is computed as a simple function of volumetric flow rate ϕ and compression ratio α, given by $|\phi(t)|(\max\{\alpha(t),1\}^h-1)$, where $h=(\gamma-1)/\gamma$, and γ is the specific heat capacity ratio of the gas. In this paper we do not model removal of gas from the pipeline to fuel compressor station operation, as it is a relatively small quantity of the through-flow (e.g. 0.25%) and does not significantly affect marginal prices.

We consider a system of pipes, compressors, and regulators that are connected at nodes. Within the pipes, the mass flux and density evolve according to the simplified Euler equations. This collection of elements connected at nodes is considered as a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where each segment $e=\{i,j\}\in\mathcal{E}$ is an edge that connects two nodes i and j in the set of nodes $\mathcal{V}$. The instantaneous state within an edge is characterized by the pressure p_{ij} and flow ϕ_{ij}, which for pipes are functions of both time on an interval [0, T] and space on an interval $[0, L_e]$, where T is the optimization horizon and L_e is the length of pipe segment e. We assign a positive flow direction on each pipe, and then derive equations that relate the pressure and flow at the boundaries of a pipe segment to the conditions at a node. Each node is classified as either a pressure (slack) node $j\in\mathcal{V}_P$, where a pressure profile p_j in time is specified and flow is a free variable, or a flow node $j\in\mathcal{V}_F$, where the time-dependent flow d_j entering or leaving the network is specified and pressure is free. At least one pressure node must be included in the model so that there is a degree of freedom in flow to ensure that the initial value problem in simulations used to validate the optimization solution is well-posed. This will typically be a large source point, such as a supply interconnection or storage unit, where the pressure is a given boundary condition. An illustration and a more detailed description of the variables used in such reduced nodal modeling are illustrated in Figure 1. Each node must satisfy the Kirchhoff-Neumann flow balance condition that requires mass moving through the node to be conserved. This stipulates that the sum of incoming flows is equal to the sum of outgoing flows plus any consumption d_j at that node. Each specified flow node $j\in\mathcal{V}_D$ is also assigned an internal nodal pressure, p_j which serves as an auxiliary variable. A compressor can boost the pressure difference between pipe segments attached at its inlet and outlet nodes. This induces extra compatibility equations into the description of the coupled system of differential equations.

Economic Pipeline Optimal Control Problem

We formulate an optimal control problem (OCP) subject to partial differential equation (PDE) constraints for gas pipeline networks, for which the edge dynamics and nodal conditions described above form the dynamic constraints. The aim is to maximize an economic objective function in the form of the market surplus. This market surplus is maximized in total over the optimization horizon $[0,T]$ which may be a 24-hour day or longer. At each point in time, market surpluse is computed as the difference between the the economic value consumers (buyers) are placing on (willing to pay for) gas purchases $\hat{d}_j(t)$ at nodes j minus the value of gas which producers (sellers) are placing on (willing to accept for) gas sales $\hat{s}_j(t)$ at nodes j. The inputs to the problem consist of the bid and offer prices $c_j^d(t)$ and $c_j^s(t)$, respectively that buyers or sellers at a node j are willing to pay or accept at time t within the optimization horizon $[0,T]$. In addition to price bids, quantity bids are also supplied in the form of pre-existing contracts $\bar{q}_j(t)$, minimum and maximum offtake curves $d_j^{min}(t)$ and $d_j^{max}(t)$ of buyers, and minimum and maximum supply curves $s_j^{min}(t)$ and $s_j^{max}(t)$ of suppliers. The economic objective is maximized subject to a collection of constraints that describe pipeline system operation, and where the control variables include compression ratios $\underline{\alpha}_{ij}(t)$ of gas compressors or compression ratios in the system. The PDE dynamics for gas flow on each pipe (i,j) are enforced, as well as flow balance at each node j and pressure changes caused by compression. Inequality constraints include minimum and maximum limits on pressure on each pipe, maximum power limits of each compressor, and maximum and minimum withdrawals or injections for offtakers and suppliers. For simplicity, we choose terminal conditions on the state and control variables to be time-periodic. Alternative initial and terminal conditions such as mass balance over the optimization period on certain subsystems could be included instead.

Crucially, we assume that no discrete changes to the network topology occur during the optimization period. Thus, no discrete variables, such as binary on/off switches, are included in the formulation. While compressor stations are in reality subject to complex operational limitations, we demonstrate that, in principle, nonlinear station constraints can be included in a computationally tractable manner as long as the modeling does not include on/off variables. For instance, a large compressor station with multiple (e.g. a dozen or more) units that receive flow from a common feeder and deliver flow to a common header can be modeled as a single theoretical boost ratio for the purpose of optimization. Modern compressor stations often have control systems that can be set to track a set point or reference signal for discharge pressure or horsepower. Thus we suppose that the management of individual units is automated, and focus on the large-scale system effects of control actions while supposing that subsystems can be taken care of at a local level. The optimal control formulation for the two-sided auction market and the mathematical nomenclature are given in Figures 2 and 3.

LOCATIONAL PRICING

In this section we review the concept of locational pricing, which is now in widespread use throughout the world in organized wholesale electricity markets. We then review recent preliminary results on extending this concept to natural gas pipeline networks.

LOCATIONAL MARGINAL PRICING OF ELECTRICITY

Locational Marginal Prices (LMPs) for electricity emerged in the United States in late 1990s – early 2000s with the formation of organized electricity markets such as PJM Interconnection[5] and Independent System Operators (ISO) of New York, New England[4] and California followed later by Midcontinent ISO, Southwest Power Pool and ERCOT. In these systems, LMPs are defined for thousands of electric network nodes (busses) and are used to price electricity sales and purchases on a locational basis. Most electricity markets use a two-settlement system in which electricity is first traded in the day-ahead (DA) and then in the Real-Time (RT) markets. Transactions cleared in the DA market are represented by hourly power injection and withdrawal schedules and corresponding hourly DA LMPs defining economic values of these schedules that are location-specific and changing hourly. Outcomes of the DA market are financially binding.

Transactions cleared in the RT market are typically represented by schedules and LMPs determined in real time (i.e. changing every 5 minutes). RT LMPs are determined *ex post* consistently with actual economic dispatch of the electric system and are used to price deviations between actual electricity injections and withdrawals and schedules cleared in the DA market.

Economically, LMPs reflect the incremental cost to the system of serving an infinitesimal incremental demand imposed at a specific location (node) in the network at a specific point in time. In the absence of binding transmission constraints (and ignoring marginal transmission losses), LMPs at all nodes are identical and equal to the short-run operating and fuel cost of the marginal generating resource. Each binding transmission constraint adds one additional marginal resource such that the total number of marginal resources equals number of simultaneously binding constraints plus one. This is because serving an incremental load at a given node becomes a balancing act of maintaining power flow through each binding constraint equal to that constraint limit. As result, at each location, LMP equals a linear combination of short-run operating and fuel costs of marginal resources with coefficients specific for that location.

While serving to price transactions between electricity market participants, electric LMPs can provide information that is critical for the market-based coordination of gas and electric networks. For a gas-fired generating unit, electric LMPs effectively determine a ceiling on the price that unit will be willing to pay for natural gas. Indeed, to avoid operating at a loss, a generator would be willing to pay for fuel no more than

$$C_{\max} = (LMP - VOM) /\ H + R$$

where $C_{\max}$ is the gas price ceiling, LMP is the electric LMP at the generator's node, *VOM* is the non-fuel variable operating and maintenance costs of generator, and H is the generator's heat rate. The term R reflects an additional risk premimum generators would factor into their willinges to pay for gas to avoid excess charges they may face in the real-time electritiy market and potentially high non-performance penalties during scarcity events.

LOCATIONAL PRICING OF GAS

Combined with electric LMPs, locational pricing of natural gas may become another critical economic tool for the efficient coordination of gas and electric network operation. To avoid confustion of electric LMPs and with spot prices for natural gas already in place, we will use the term Locational Trade Value (LTV) for natural gas. In a similar manner to the information provided by electric LMPs, LTVs would reflect the incremental cost to a natural gas supply system of serving an infinitesimal incremental demand for natural gas imposed at a specific location (node) in the network at a specific point in time. Another important similarity between electric LMPs and gas LTVs is their consistency with the physical operation of the respective network. That property contrasts LTVs from daily cleared regional gas prices. Daily prices reflect anticipated constraints in the gas transportation network based on the previously *allocated* pipeline capacity determined in daily throughput quantities. Locational difference in such daily prices known as *basis differentials* are driven by the expectation that the demand for throughput capacity needed to move gas from one location to another will exceed the total allocated capacity limit and that capacity therefore needs to be rationed. Thus, the basis differential is effectively related to the allocated limit of the maximum daily throughput of a pipeline or its segment.

This representation of pipeline transportation capacity, and the pricing scheme associated with it, over-simplify the capabilities of the pipeline network and assume away non-linear relationships between gas flows, pressure and compressor horsepower limitations. In the ensuing discussion of illustrative numerical examples, we demonstrate that even for a single pipe, basis differentials may not be directly attributable to constrained throughput because the static capacity allocation mechanism does not capture the transient nature of the mechanics of gas movement within the pipeline network. In contrast, LTVs accurately capture the physics of pipeline flow in both space and time. They reflect the noted non-linear relationships between gas flows, pressure, the capabilities of compressor stations, transient phenomena. To illustrate this, we first briefly discuss LTVs in non-linear gas networks under the assumption of steady state gas flows and then present an illustrative analysis of dynamic LTV behavior reflective of transient effects.

Non-linear Network, Steady State Flow. Following a recent preliminary study[34], two types of constraints could cause the difference in LTVs at the two ends of a pipe – a pressure constraint and compression constraint. The first type occurs if the pressure in the pipe reaches the Maximum Allowable Operating Pressure (MAOP) level, and the second when a compressor operates at maximum horsepower limit or maximum compression ratio. Analysis of these conditions further indicates that for the LTVs to differ, the pipe must be simultaneously constrained both at the sending and at the receiving end. At the receiving end, the pressure must fall to the low limit. At the sending end, either the maximum pressure or the maximum compression constraint must be binding. The pressure congestion would uniquely define the constrained pipe flow by

$$\phi_{\max} = \frac{\sqrt{p_{\max}^2 - p_{\min}^2}}{\sqrt{\beta}}.$$

where, $p_{\max}$ represents MAOP, $p_{\min}$ is the minimum pressure requirement at the receiving end of the pipe, and β is the constant that depends on pipe diameter and friction factor and which reflects resistive losses.

However, when the sending end of the pipe is constrained due to the compression limitation, the pipe flow is not uniquely determined and may vary depending on the compression ratio α according to

$$\phi = \frac{E^{\max}}{\varepsilon(\alpha^h + 1)},$$

where $E^{\max}$ and ε are compressor's horsepower limit and efficiency, respectively, and α is the compression ratio. The latter is dependent on the suction pressure at the compressor. Therefore, although the pipe is constrained, and its throughput may be different from the predetermined allocated capacity, it could be below it or exceed it.

LTVs in the Dynamic Case. Here we consider a dynamic two-node example as depicted in Figure 4 below. As shown in that figure, two nodes are connected by a single pipe with a compressor located at node 1 and a single gas off-taker located at node 2 with the demand profile ranging between 100 MMcfd and 300 MMcfd. This offtaker (demand) node can be served by three suppliers, two of which are located at node 1 and one at node 2. Node 1 suppliers offer gas at prices of $2/Mcf and $3/Mcf. Supplier at node 2 offers gas at $5/Mcf. The objective here is to satisfy the demand profile while minimizing the total supply costs over the 24-hour period. With the offtaker effectively willing to take gas at any price, the objective of market surplus maximization is equivalent to minimizing supply costs.

Here we solve the capacity allocation and pricing problem under six scenarios effectively that represent six different single-pipe systems operating under the same supply and demand conditions but differing from each other by the level of MAOP within the pipe connecting nodes 1 and 2. We solve the optimal control problem described in Figure 2 using the computational methodology that was developed in previous work[8,34] and which is summarized below. We consider six scenarios with MAOP limits ranging between 500 psi and 1000 psi. A comparison of solutions for these six scenarios is presented in Figures 5 and 6. In all the example computations, gas flows from node 1 to node 2.

Figure 5 shows the dynamics of LTVs and pressure levels at both nodes for each scenario. Solid lines represent LTV dynamics expressed in $/Mcf with values in the left vertical scale. Dashed lines represent pressure dynamics expressed in psi with the value in the right vertical scale. The blue color corresponds to the LTV and pressure at node 1, and the orange color to those same variables for node 2. Pressure at node 1 is taken at the discharge end of the compressor.

Figure 6 presents LTV differences between nodes 1 and 2 shown as solid blue line and the line pack dynamics shown as orange bars. In this figure, the line pack is represented as the difference between the pipe's incoming gas flow and outgoing flow. A positive value reflects packing of the pipe, while a negative value reflects unpacking of the pipe.

For scenarios with low MAOP limits such as MAOP = 500 psi and MAOP = 550 psi, we observe a $3/Mcf difference in LTV values between nodes 1 and 2 dominating most of the 24-hour period. The constant (over time) LTV difference corresponds to the steady state regime within a pressure constrained pipe: the MAOP pressure level is maintained at node 1 and the minimum pressure of 300 psi is maintained at node 2 (note in Fig. 6 the absence of any packing or unpacking of the pipe at that time). During this constrained regime, the $2/Mcf supply at node 1 remains constant and below the supply maximum of 220 MMcfd, the $3 supply at node 1 is not used at all, and the balance of the demand at node 2 is met by the $5/Mcf supply at that node. For the MAOP = 500 psi scenario, this steady state regime lasts approximately 16 hours. In the system with MAOP of 550 psi, the duration of the steady state regime is shorter and lasts only 13 hours. At very low demand levels the LTVs at both nodes converge at $2/Mcf. At intermediate demand levels, the LTV at node 2 rapidly diverges from the LTV at node 1.

The MAOP = 575 psi scenario is structurally similar to the MAOP = 500 psi and MAOP = 550 psi scenarios except that with a higher MAOP the system can actually tap into the \$3/Mcf supply at node 1. As a result, the price at that node predominantly settles at two levels: \$2/Mcf at low demand and \$3/Mcf during the steady state regime. The price at node 2 transitions between \$2/Mcf and \$5/Mcf at intermediate demand levels and stays at \$5/Mcf during the steady state regime.

Everything else being equal, the system with MAOP = 600 psi is special as it reflects the MAOP level at which LTVs at nodes 1 and 2 begin to converge at \$5/Mcf during high demand hours. As seen on Figures 5 and 6, during this period of LTV convergence, the pressure and flow regimes still very much resemble the steady state outcome with very little line pack activity taking place and pressures at both ends staying close to their respective limits.

In the scenarios with higher MAOP limits such as 800 and 1000 psi the system exhibits no steady state behavior. In these cases, the regime is transient for the entire 24-hour period, packing gas in the pipe during lower demand hours and unpacking during high demand hours. In the MAOP = 800 psi scenario, the LTVs converge at \$2/Mcf for approximately one-third of the time, at \$5/Mcf for another one-third of the time, and LTVs at nodes 1 and 2 diverge from each other in transitions between high and low levels during the remaining one-third of the optimization period. For some time during that LTV divergence in the early hours of the day, the discharge pressure binds at the MAOP limit. Similarly, for some time during the LTV divergence in the later hours of the day, the pressure at node 2 binds at the lower limit of 300 psi. Note that we observe no constraints to the flow of gas through the pipe during the period of LTV divergence. The optimal regime in the scenario with MAOP = 1000 psi looks similar to that of the system with MAOP = 800 psi except that the discharge pressure never binds at MAOP and the compressor horsepower becomes a constraining element at the sending end of the pipe. As with the MAOP = 800 psi scenario, LTV differences occur during intermediate hours and not during hours of high or low demand.

The above analysis of LTVs leads to several important observations.

1. Economic congestion (or congestion-based LTV differentials) in the pipeline is not necessarily driven by limitations on the pipeline *throughput*.
2. In a pipeline system with sufficient line pack potential, economic congestion is non-monotonic with respect to demand: LTV differentials can occur at intermediate load levels but may disappear at high and low demand levels.
3. LTV differentials may be essentially a transient phenomenon associated with LTVs migrating between higher and lower levels but at a different pace depending on the location.
4. Using LTVs as a pricing mechanism instead of, or in addition to, regional daily prices might have significant financial implications for market participants. For example, if paid according to LTVs, gas suppliers may enjoy high gas prices at the time of high demand due to the observed convergence of LTVs, whereas daily prices based on linear capacity allocation would tend to reduce payments to producers located *upstream* of such a capacity constraint. Similarly, consumers who pay according to LTVs may enjoy lower payments for the part of the day with lower demand and during the price transitions between lower and higher levels, whereas daily prices based on linear capacity allocation would tend to increase payment by all consumers located *downstream* of such a capacity constraint.
5. Under the dynamic LTVs, precise hour-by-hour coordination in price and supply/demand scheduling is important as it has major financial implications for market participants. It is therefore essential that prices and physical schedules are developed through a formalized mechanism that guarantees that developed schedules are feasible and binding, and that LTVs formed through this mechanism are consistent with engineering limitations, pipeline operations, and the physics of gas flows.

GAS-ELECTRIC COORDINATION

We envision LTVs becoming instrumental in improving coordination of gas and electric systems. Conceptually, a coordination mechanism could be based on an iterative direct exchange of electric LMPs and gas LTVs between the corresponding market clearing mechanisms. Gas-fired generating units would use hourly LTVs at precise locations on the gas pipeline system where they take gas as a fuel and convert these hourly LTVs into hourly and real-time offer prices they submit to their electric market operators. Once the electricity market clears based on that information, gas-fired units would receive their generation schedules and electric LMPs. Generation schedules would then be converted into gas burn sheets and electric LMPs would be used to develop gas purchase bids indicating the generators' willingness to pay for gas. That information would be submitted to the gas market operator and the iterative process repeats.

This conceptual scheme, even if it were proven to converge mathematically, be tractable computationally, and reflect realistic engineering operations, cannot currently be implemented because of barriers of an operational and institutional nature. Operational barriers are apparent from a side-by-side comparison of timelines of scheduling decision processes in the natural gas and electric systems as presented in Figure 7. As one can see in this timeline, there exists a highly intricate succession of decision cycles on the electric side and natural gas side. The timings of the day-ahead price formation for natural gas and power do not coincide. First, regional forward prices of natural gas emerge in bilateral trading and capacity release mechanism. These prices, although not backed up by delivery confirmation, are then used by electric generators to bid in the day-ahead (DA) electricity market. The DA market run by the electric system operator is a fairly complicated process which includes not only a complex mixed integer optimization task, but also a number of post optimization verification steps assuring the feasibility of the optimization solution. Within the timing allotted to the DA market process; there is little room for any envisioned iterative processes to exchange gas and electric prices and schedules back and forth.

Once the DA market clears and the financially binding operational schedules for electric generators are determined, generators have just enough time to make delivery nominations with the pipeline for the next gas day. If the nominations are confirmed in the Timely and Evening cycles on the gas side, daily delivery quantities are essentially guaranteed. If they are not confirmed due to pipeline capacity limitations, generators will face significant financial exposure as they are obligated to deliver power but have no gas to produce it. Even if the daily delivery quantity is confirmed, generators typically need non-ratable gas deliveries that pipelines typically cannot guarantee.

Furthermore, most fast-start combined cycle generators and gas turbine peaking facilities are not committed in the DA market. Instead those units are typically scheduled through the hourly reliability updates or close to the real-time market. These “last-minute” decisions do not fit into the existing decision cycles on the gas side. What is really needed here is an hourly natural gas balancing market that would work after the completion of the Evening Cycle and allow market participants to trade deviations from approved schedules in the Timeline and Evening Cycles. These deviations could be traded through the formal optimization based auction-type market mechanism as described above. Such an auction could be run on an hourly basis using a rolling horizon approach, such that each hour the auction would optimize the system for multiple hours (e.g. 24 hours or even more). Such a balancing market would provide a repeated forward-looking price discovery mechanism to help the gas and electric sectors to efficiently coordinate their operations.

Indeed, if the anticipated operation of the electric system produces forward looking gas burn schedules that cause operational problems on the pipeline side, a gas balancing market will reveal these operation difficulties through high LTVs at the location of gas-fired generators that are causing the problem. Once receiving this information, generators would adjust upward their real-time offers to produce electricity and the electric system operator will likely re-dispatch these generators by displacing them with other resources that are either not gas constrained or even not gas fired. This coordination approach will quickly and efficiently relieve constraints on the gas side, reduce consumer prices in both sectors and improve reliability of energy delivery.

Detailed implementation of such a mechanism is a topic of on-going research. An extensive program of research and development would be required to standardize and validate technology based on existing proof-of-concept work. In addition, its adoption by the industry will likely require a complex stakeholder process and regulatory reform.

If implemented, the proposed short-term coordination mechanism will have major long-term implications for both the electric and gas industry as it will help to resolve the ongoing debate on the extent to which gas-fired generators should rely on long-term contracts for firm transportation capacity. Generating companies, especially merchant independent power producers, are not willing to enter such agreements because of a perceived high risk of such arrangements. Specifically, this risk is associated with contracting variable generation profiles that are translated into non-ratable gas use profiles. The current lack of a transparent and liquid market and associated price discovery mechanisms for non-ratable gas use profiles presents risk and uncertainty in attempting to sell under-utlizied capacity on an hourly basis. The proposed gas balancing market will fill this void and help generation owners to make an informed economic decision on the level of firm tranportiation capacity to acquire to mitigiate the financial risk associated with the volatility of two energy markets they are exposed to on the supply and demand side.

COMPUTATIONAL METHODOLOGY

Substantial research and development has been done on computational methods for transient optimization of gas pipeline systems, resulting in two general classes of methods. One set of existing *“simulation-based”* methods relies on repeated executions of high-fidelity simulations[26,27,35,36]. Such methods accommodate highly detailed models that yield solutions of accurate physical feasibility, and adjoint-based gradients for use in optimization codes can be obtained at little extra computational cost. While these methods allow exploitation of sparsity and parallelization, higher order derivatives and Jacobians of the active constraints, both of which would accelerate convergence and aid robustness, are computationally costly.

Alternatively, *“discretize-then-optimize”* approaches allow rapid evaluation of constraint Jacobians for the entire optimization period. Starting with an optimal control formulation that includes a cost objective and all equality and inequality constraints on state variables, algebraic approximations of partial differential equations (PDEs) describing the physical behavior of the system are incorporated directly as constraints within the optimization problem, rather than as independent simulations. Model reduction may be used to simplify the complexity of PDE representation in space. The problem is discretized in time using approximations (such as finite differences) of the functions evaluated at time- and space-collocation points. This results in a nonlinear program (NLP) with purely algebraic objective and constraint functions. Although this type of formulation may become very large-scale, it can be solved by taking advantage of special structure[28] or by recently developed general optimization tools for problems with sparse constraints[9]. While entire problem must be discretized on a coarser grid than in a simulation-based approach for computational tractability, thus potentially reducing accuracy, the induced error remains local and can be shown to be acceptable.

The approach used in the computational studies presented here utilizes the *“discretize-then-optimize”* approach[8], in which a large scale NLP is produced and solved using the IPOPT interior point solver[31]. The results of the optimization can be used to produce an initial value problem (IVP) that can then be solved using numerical methods designed for pipeline simulation based on the reduced modeling approach. The same IVP can also be solved using a commercial simulation engine for the purpose of solution verification, as in the computational study presented in the next section. We note that because the maximum market surplus based pipeline optimization problem is nonlinear and nonconvex, no guarantee is given on whether an interior point optimization method reaches a global solution. Thus, it is important to investigate the optimality gap and determine whether the solutions obtained are indeed global, and thus to verify whether the dual variables provide the desired Lagrange multipliers and thus the correct values of LTVs for the pricing mechanism.

COMPUTATIONAL STUDY

We briefly describe the problem that is solved by a prototype methodology for transient optimization of large scale gas pipeline systems that was recently developed to solve the optimal control problem given in Figure 2. Beyond obtaining the optimal control solution, we then create an IVP that we simulate using the reduced model methodology and a commercial solver. This provides a proof-of-concept of how transient optimization solutions can be automatically validated using a commercial solver, which would be a critical intermediate step before intra-day optimization could be used in the field.

OPTIMAL CONTROL CASE STUDY

We examine a simple 6-node model that is illustrated in Figure 8. There is a single supply point at node 1, and five offtakers located at nodes 2, 3, and 4. There are two offtakers at node 3 and at node 4 that represent different types of customers – a local distribution company (LDC) and a gas-fired generator). Two compressors are used to boost pressure, located between nodes 1 and 5 and nodes 2 and 6.

Inputs to the problem in Figure 2 consist of physical and economic information in the form of pre-existing contracted flows and price-quantity bids for the secondary market. The physical market inputs consist of flow bids as functions of time throughout a 24 hour day, which are shown in Figure 9. In this example the pre-existing contracts for gas injections/withdrawals (mmscfd) are constant, and the purchases and sales by market participants are variations from these steady rated profiles. Each purchaser provides maximum (solid) and minimum (dashed) bounds on the variations that they are willing to make from their pre-existing steady take. These are given separately for increments (as demand) and decrements (as supplies) to the secondary market. The smoother curves represent variations that LDCs may expect from their expected total load, and the more rapidly changing curves represent activation of gas-fired generators. The economic market inputs consist of price bids as functions of time throughout a 24 hour day, which are shown in Figure 10. The supplier at node 1 offers at a constant price, while the offtakers at nodes 2, 3, and 4 bid at constant price if an LDC, and bid using the expected electricity price if a gas-fired generator.

Solving the optimal control problem given in Figure 2 produces physical and economic market outputs that determine how the pipeline system is operated and the prices paid by shippers. The time-dependent physical flow solution is shown in Figure 11, and consists of the total physical flows in or out of each node, as well as the purchases and sales in the secondary market. The physical solution also includes protocols for compressor operation, which are specified as compression ratios, discharge pressures, and power expended, and are shown in Figure 12. The discharge pressures are the control variables used as time-dependent set points in operating the system. The economic market outputs are given in Figure 13, and consist of the LTVs throughout the system. A price is obtained at each location in the pipeline network, including within pipeline segments between custody transfer locations where price bids would be provided. Our focus is on nodal pricing, in order to provide prices at metered custody transfer points that reflect the capacity of the entire pipeline system.

CROSS-VALIDATION BY COMMERCIAL SIMULATION

In order for transient optimization to be used in the manner illustrated in Figure 7, there must be assurance that the solutions provided using a coarse-grained optimization solver are sufficiently accurate to produce a feasible flow schedule and compressor operations within all required limitations. Here, we provide a proof-of-concept to demonstrate how such an intermediate step could be done to enable use of intra-day optimization of pipeline transients in the field.

After solving the optimal control problem in Figure 2, the physical solution given in Figures 11 and 12 can be automatically validated using a commercial solver. First, an initial value problem (IVP) is constructed from the time periodic solution produced by the optimization. A well-posed IVP requires pressure or physical flow out of the network to be specified for each network node. The boundary conditions are pressure at the slack node 1 and the physical flows leaving the nodes 2, 3, and 4. The actions of compressors are specified using the discharge pressures. To parameterize the problem in the commercial simulator, the IVP must be initiated from a steady state. This steady initial condition is provided by solving an auxiliary steady-state optimization problem where the inputs are averaged inputs of the transient problem. The boundary conditions are then interpolated between those corresponding to the steady solution and the initial values of the periodic transient solution. The periodic boundary conditions are then applied for several cycles.

The same IVP constructed using the above method was then solved by integrating a differential-algebraic equation (DAE) system produced using reduced modeling approaches[8,37], and also solved using a commercial simulator.

Because either pressure or physical outflow was specified for each network node, the validation can be made by examining the dependent variable that is obtained by solving the IVP. Thus, we examine physical inflow at node 1 where pressure was specified, and pressures at the nodes 2, 3, and 4 where physical outflow was specified. The resulting comparison is shown in Figure 15. A close match is obtained between the reduced simulation and the commercial solver.

Real Data Validation

Beyond using a commercial solver to demonstrate the potential of the prototype transient optimization approach, we present the result of a case study in which the reduced system modeling formalism was validated using the combination of a planning model for a real pipeline system and temporal SCADA data measured from the same system during the course of one calendar month.

The static network model was simplified from a model used for capacity planning, typically with steady-state optimization. The simplification procedure requires several assumptions. First, passive components and connections in the system such as valves were removed, and their status (open/closed) was used to determine any modifications to the topology. Second, although multiple compressor units make up a compressor station, the suction and discharge of the entire station occur through common headers. Therefore we model the entire station as a single theoretical compressor with an aggregate power, and assume that the individual compressor units can be controlled locally to maintain a desired discharge pressure of the entire station. The subsystem that was extracted is illustrated in Figure 16, and consists of 78 reduced model nodes, 95 pipes with total length of 444.25 miles, and 4 compressors. For each pipe, physical parameters used were length, diameter, and friction factor and were taken directly from the planning model. However, the friction factor was scaled down by an engineering factor of 0.85 to compensate for pipe efficiency factors commonly used by commercial software packages but not considered in the reduced modeling approach.

The temporal network model consists of measurements from a SCADA system used for operation of the pipeline from which the test system model was extracted. This system provides hourly measurements of pressure (psig), temperature (degrees F), and volumetric flow (mcfh) out of the system at 31 metered custody transfer meter and check measurement locations, as well as average gas gravity and thermal content (mBTU/mmscfd). Check measurements at the 4 compressor stations include suction and discharge pressure (psig), suction and discharge temperature (degrees F), and volumetric through-flow (mcfh). Using this information, we computed mass flow (mmscfd) at each reduced network model location where flow leaves or enters the system and pressure at the slack node. These independent boundary conditions are shown in Figure 17.

The quantities of interest for the validation are then the corresponding variables at those nodes, i.e., pressures measured at meter locations used as flow nodes and inflow measured at the slack node location. In a manner similar to what was done for validation of the computational study, the IVP was constructed by producing a steady-state initial solution using optimization, and interpolating to the start of the temporal data. A DAE system used for pipeline simulation of the reduced model was produced using the static network data, the initial conditions, and the boundary conditions.

The IVP simulation solution produces pressures at all reduced model nodes and inflow to the system at the slack node, so a comparison can be made at meter locations. Figure 18 shows the SCADA data of these dependent boundary conditions that are used as the basis for comparison, and Figure 19 shows the dependent boundary conditions computed using the reduced model simulation of the constructed IVP. Finally, we compare the SCADA data in Figure 18 to the simulation result in Figure 19. Specifically, we consider the relative distance (where distance here is in the sense of L_1 norm, i.e., absolute value of the difference) between the values of each variable at each time point as a percentage (%) value. This result is shown in Figure 20. Observe that the relative distance is minimal. The overall mean is 4.1746%, while the most extreme discrepancies occur at the lateral with locations U, V, and W, namely, 48.16%, 78.70%, and 86%. The mean excluding meters at V, W, and U becomes 2.9425%, and the maximum discrepancy excluding meters at U, V, and W is 25.01%. The relative difference in flow at the pressure node A has a mean of 2.4557% and maximum value of 23.77%.

We see in Figure 20 that the greatest discrepancy between the data and modeling occurs at the locations U, V, and W in the network shown in Figure 16, with differences between simulation and data of up to 85%. Specifically, the simulation overestimates the pressure at locations U, V, and W at certain times. Notably, this overestimation does not occur all of the time; there are time periods of several days when the discrepancy is under 3%. This indicates that the modeling used in the simulation does match the data for these time periods. The discrepancy could be caused by additional flows leaving the system at certain time periods (when the discrepancy is observed) on the lateral containing meters U, V, and W upstream of location T. We note that although there is a difference between pressure readings and the simulation at these locations on one lateral, the bulk of the flows through the examined subsystem and pressures on the main line were captured accurately by the reduced system modeling.

Conclusions

In this paper, we have introduced a market-based formulation of the transient pipeline optimization problem using the economic criteria of maximization of the market surplus. We have demonstrated that the RNF-based methods of transient pipeline optimization[8] perform well for solving this problem and offer robust and scalable solutions. We were able to verify the feasibility of these optimization solutions in comparison with an industry standard commercial solver. In addition, we validated the reduced modeling approach vis-à-vis a planning model and actual SCADA measurements for a real pipeline subsystem.

In addition to optimizing operational decisions, the proposed methods yield economic value of natural gas in the form of Locational Trade Values (LTVs). In contrast to the regional daily prices prevailing in today's markets, LTVs are consistent with the physics of gas flow in the pipeline networks subject to essential engineering constraints. This makes LTVs an important instrument for improved gas-electric coordination, especially if used for intra-day coordinated scheduling of non-ratable supplies and deliveries. Preliminary illustrative analysis of LTVs reveals the shortcomings of daily prices that are disconnected from the physics of pipeline operations and indicates how market participants both on the supply and demand side could benefit from using LTVs as an intra-day pricing mechanism.

ACKNOWLEDGEMENT

The authors wish to thank Michael Chertkov, Russell Bent, John Rozenkranz, Russ Philbrick, Michael Goodman, Hubert Eldridge, and Alex Hollis for productive discussions and suggestions. We also wish to express our gratitude to DNV-GL and Kinder Morgan Corporation for supporting this study. This work was carried out as part of Project GECO for the Advanced Research Project Agency-Energy of the U.S. Department of Energy under Award No. DE-AR0000673. Work at Los Alamos National Laboratory was conducted under the auspices of the National Nuclear Security Administration of the U.S. Department of Energy under Contract No. DEAC52-06NA25396, and was supported by the Advanced Grid Modeling Research Program in the U.S. Department of Energy Office of Electricity.

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FIGURES

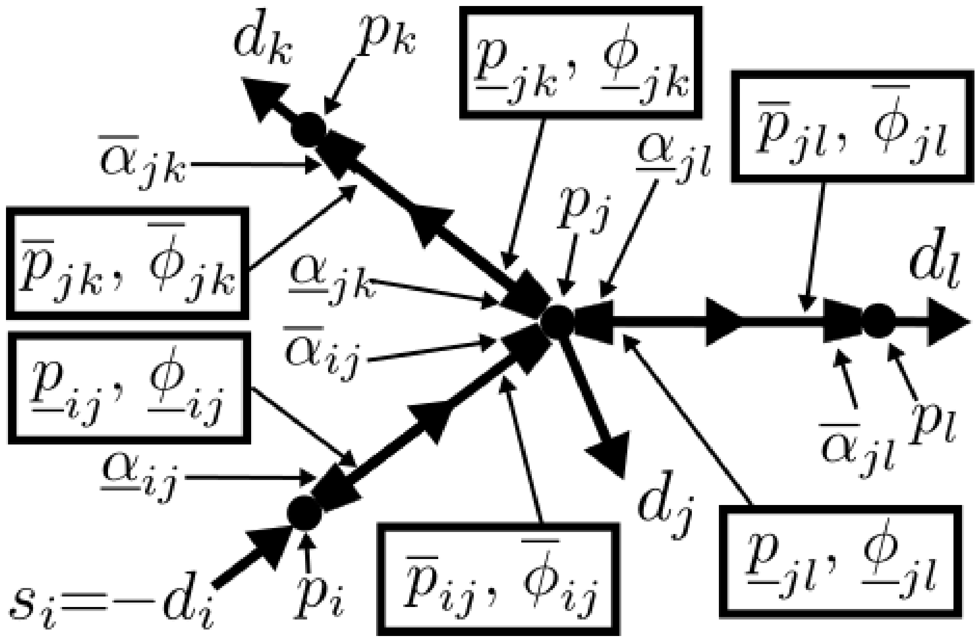

Figure 1 – Diagram of nodal control system modeling for large-scale gas transmission pipelines. Given a directed graph that represents the pipeline network, $\underline{p}_{ij}$ and $\overline{p}_{ij}$ represent pressures at the sending and receiving ends of each pipe, while $\underline{\phi}_{ij}$ and $\overline{\phi}_{ij}$ represent mass flux at the sending and receiving ends of each pipe. The quantities $\underline{\alpha}_{ij}$ and $\overline{\alpha}_{ij}$ represent pressure boost ratios of compressors that are, without loss of generality, located at every interface between a node and a pipe. Thus, nodal pressures p_i and p_j are related to pipe endpoint pressures $\underline{p}_{ij}$ and $\overline{p}_{ij}$ according to $\underline{p}_{ij} = \underline{\alpha}_{ij} p_i$ and $\overline{p}_{ij} = \overline{\alpha}_{ij} p_j$. The withdrawal from the network at a node j is denoted by d_j, which is constructed from pre-existing contracts $\overline{q}_j(t)$ and secondary supply and demand profiles $\hat{s}_j(t)$ and $\hat{d}_j(t)$, or the supply injected at a node i is denoted by s_j. See Figure 2 for the appropriate nodal flow balance relation.

$$\max \quad \text{Market Surplus:} \quad \sum_{j\in\mathcal{V}} \int_0^T c_j^d(t)\hat{d}_j(t)\mathrm{d}t - \sum_{j\in\mathcal{V}} \int_0^T c_j^s(t)\hat{s}_j(t)\mathrm{d}t$$

$$\begin{aligned}
\text{s.t.} \quad \text{Mass conservation:} \quad & \partial_t\rho_{ij} + \partial_x\phi_{ij} = 0, && \forall (i,j)\in\mathcal{E},\\
\text{Momentum conservation:} \quad & \partial_t\phi_{ij} + \partial_x p_{ij} = -Z(p_{ij})RT_w \frac{f_{ij}}{2D_{ij}}\frac{\phi|\phi|}{p}, && \forall (i,j)\in\mathcal{E},\\
\text{Equation of State:} \quad & p_{ij} = Z(p_{ij})RT_w\rho_{ij}, && \forall (i,j)\in\mathcal{E},\\
\text{Nodal flow balance:} \quad & \sum_{k\in\partial_- j} A_{jk}\underline{\phi}_{jk}(t) - \sum_{i\in\partial_+ j} A_{ij}\overline{\phi}_{ij}(t) - \bar{q}_j(t) &&\\
& -(\hat{s}_j(t) - \hat{d}_j(t)) = 0, && \forall j\in\mathcal{V},\\
\text{Compressor boost:} \quad & \underline{p}_{ij}(t) = \underline{\alpha}_{ij}(t)p_i(t), && \forall (i,j)\in\mathcal{E},\\
& \overline{p}_{ij}(t) = \overline{\alpha}_{ij}(t)p_j(t), && \forall (i,j)\in\mathcal{E},\\
\text{Pressure limits:} \quad & p_{ij}^{\min} \le p_{ij}(t,0) \le p_{ij}^{\max}, && \forall (i,j)\in\mathcal{E},\\
& p_{ij}^{\min} \le p_{ij}(t,L_{ij}) \le p_{ij}^{\max}, && \forall (i,j)\in\mathcal{E},\\
\text{Boost upper limits:} \quad & \underline{\varepsilon}_{ij}|\underline{\phi}_{ij}(t)|\left((\underline{\alpha}_{ij}(t))^h - 1\right) \le \underline{E}_{ij}^{\max}, && \forall (i,j)\in\mathcal{E},\\
& \overline{\varepsilon}_{ij}|\overline{\phi}_{ij}(t)|\left((\overline{\alpha}_{ij}(t))^h - 1\right) \le \overline{E}_{ij}^{\max}, && \forall (i,j)\in\mathcal{E},\\
\text{Boost lower limits:} \quad & \underline{\alpha}_{ij}(t) \ge 1, \quad \overline{\alpha}_{ij}(t) \ge 1 && \forall (i,j)\in\mathcal{E},\\
\text{Supply limits:} \quad & s_j^{\min}(t) \le s_j(t) \le s_j^{\max}(t) && \forall j\in\mathcal{V},\\
\text{Demand limits:} \quad & d_j^{\min}(t) \le d_j(t) \le d_j^{\max}(t) && \forall j\in\mathcal{V},
\end{aligned}$$

Figure 2 – Optimal control formulation for two-sided pipeline auction market. The objective is to maximize the market surplus for the pipeline system, subject to flow physics, mass flow balance at nodes, and actions of gas compressors – constraints that specify the dynamics of the system. In addition, the problem must include inequality constraints that reflect operational limitations of the system – these include minimum and maximum limits on pressure (which are enforced on each pipe), maximum power limits on compressor stations, and a requirement that compression ratios are positive (to reflect compressor bypass in the case when no pressure boost is needed or flow is in the opposite direction of compressor orientation). Finally, minimum and maximum constraints on supply and demand at each node are generated based on physical injection or offtake capabilities as well as the financial positions of shippers bidding into the market at that location. Additional constraints that require the total mass (and thus energy) in the system to return to the initial value at the end of the optimization interval may be added. In the present study, we enforce time-periodicity of the solution, i.e., the entire system state (all flows and pressures) at the time T is equal to that at time 0.

$\mathcal{V}$	set of nodes (j)
$\mathcal{E}$	set of pipes (i,j) for i and j in $\mathcal{V}$
T	optimization time length; optimization interval is $[0,T]$
R	gas constant (depends on gas gravity)
T_w	working temperature (assumed constant throughout the system)
$Z(\cdot)$	gas compressibility as function of pressure (working temperature)
f_{ij}	Colebrook-White friction factor on pipe (i,j)
D_{ij}	inner diameter of pipe (i,j)
A_{ij}	cross-sectional area of pipe (i,j)
L_{ij}	length of pipe (i,j)
$c_j^d(t)$	demand bid at node j at time t
$c_j^s(t)$	supply offer at node j at time t
$\hat{d}_j(t)$	variable demand at node j at time t
$\hat{s}_j(t)$	variable supply at node j at time t
$\rho_{ij}(t,x)$	density on pipe (i,j) at time t and location x
$\phi_{ij}(t,x)$	mass flux on pipe (i,j) at time t and location x
$p_{ij}(t,x)$	pressure on pipe (i,j) at time t and location x
$p_i(t)$	pressure at node j at time t
$\underline{p}_{ij}(t), \overline{p}_{ij}(t)$	pressure at start and end of pipe (i,j)
$\underline{\phi}_{ij}(t), \overline{\phi}_{ij}(t)$	mass flux at start and end of pipe (i,j)
$p_{ij}^{\min}, p_{ij}^{\max}$	minimum and maximum pressure on pipe (i,j)
$\underline{\varepsilon}_{ij}, \overline{\varepsilon}_{ij}$	compressor energy usage factor of compressors at start and end of pipe (i,j)
$\underline{\alpha}_{ij}, \overline{\alpha}_{ij}$	boost ratios of compressors at start and end of pipe (i,j)
h	compressor energy function exponent (depends on gas specific heat capacity ratio)
$\underline{E}_{ij}^{\max}, \overline{E}_{ij}^{\max}$	maximum energy (horsepower) of compressors at start and end of pipe (i,j)
$s_j^{\min}(t), s_j^{\max}(t)$	minimum and maximum supply from node j at time t
$d_j^{\min}(t), d_j^{\max}(t)$	minimum and maximum demand at node j at time t

Figure 3 – Mathematical nomenclature for optimal control formulation in Figure 2

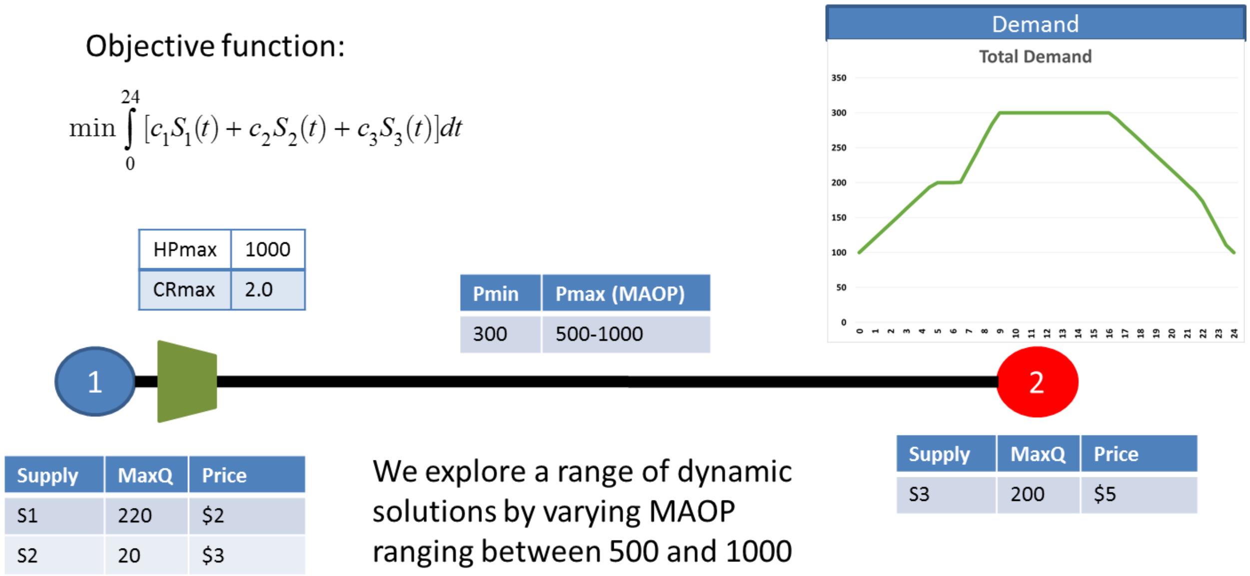

Figure 4 – Optimization problem set-up for the 2-node system LTV case study.

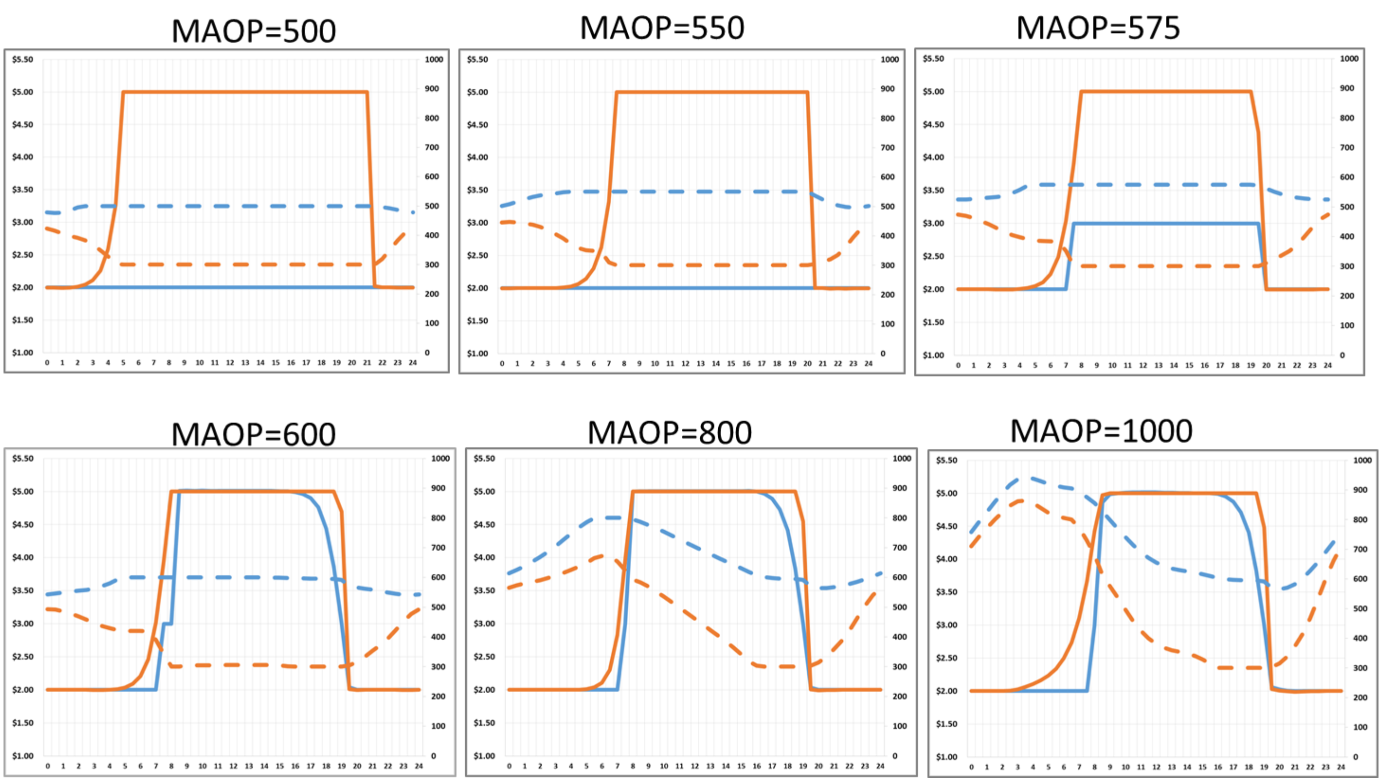

Figure 5. LTVs and Pressure Dynamics by Node by Scenario for the 2-node system LTV case study.

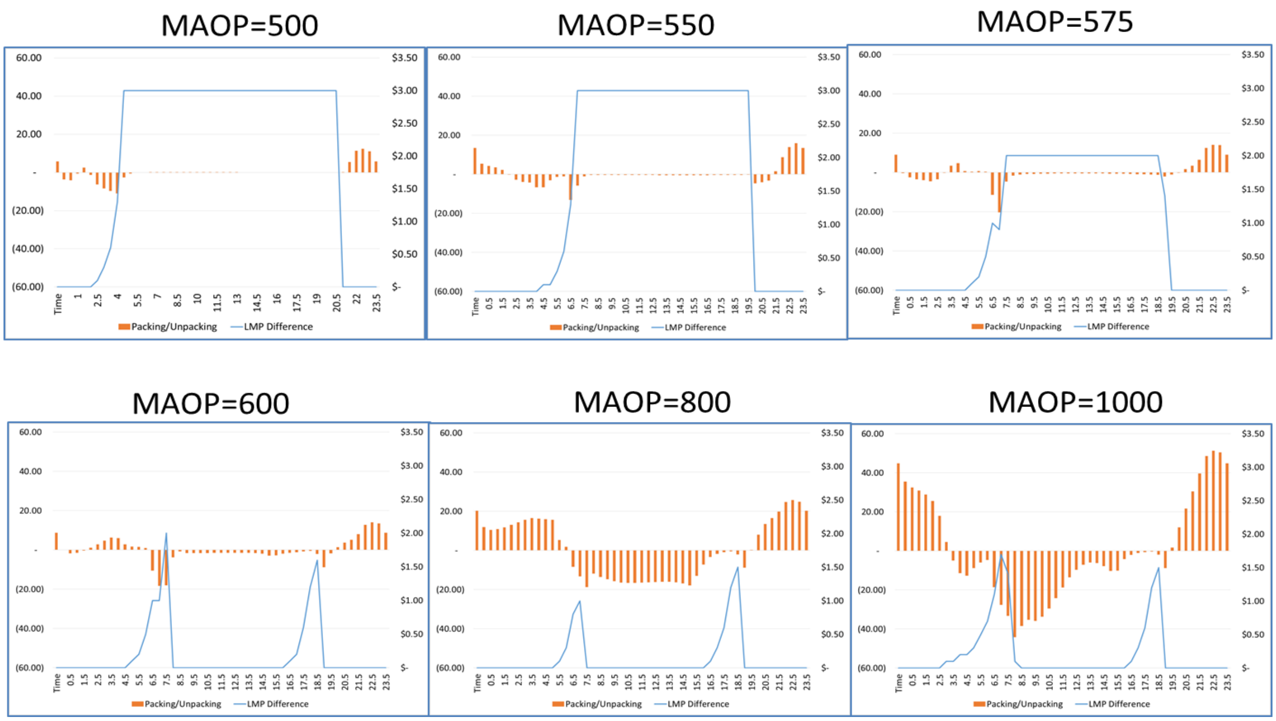

Figure 6. LTV Differentials and Line Pack Dynamics by Scenario for the 2-node system LTV case study.

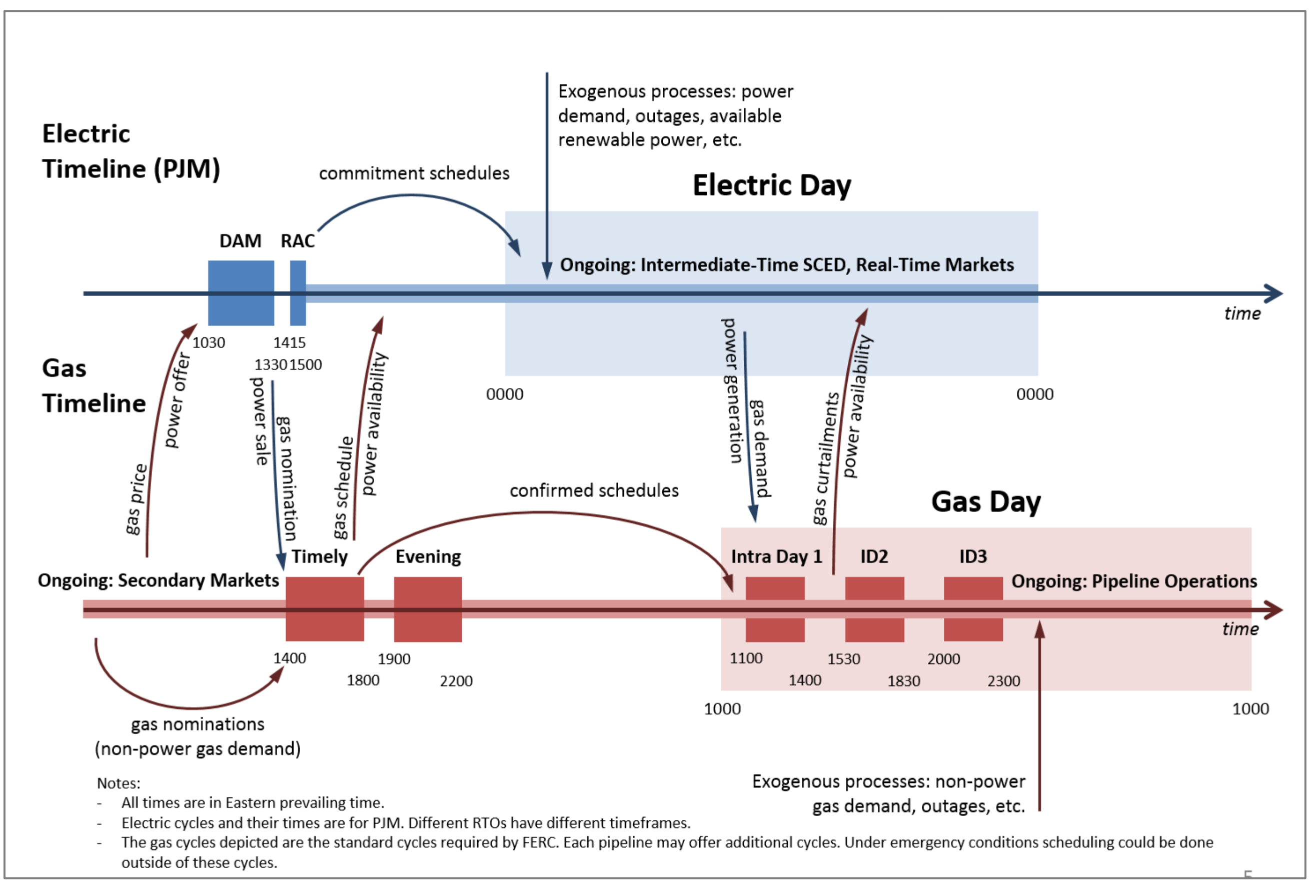

Figure 7. Description of current gas–electric decision cycles.

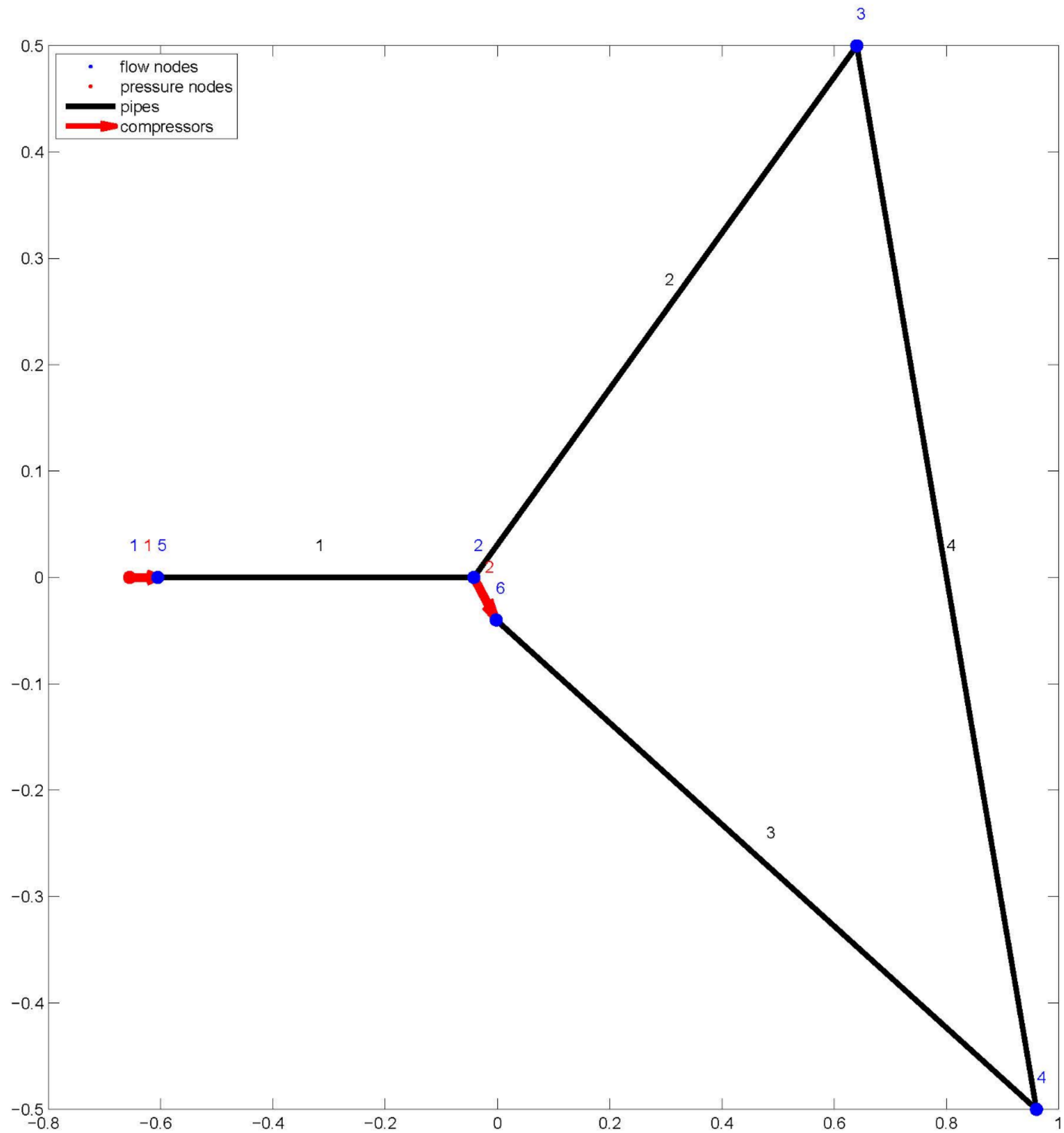

Figure 8 – Simple network used in the 6-node computationa

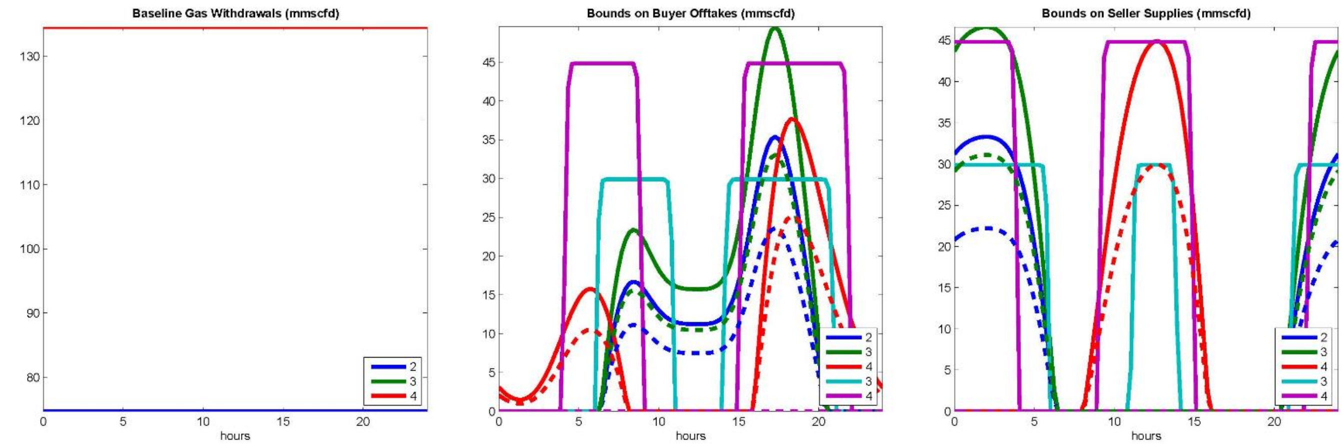

Figure 9 – Computational study physical market inputs – flow bids. Left: pre-existing contracts for gas injections/withdrawals (mmscfd); Center: Maximum (solid) and minimum (dashed) bounds on purchases (withdrawal variations) by participants (mmscfd); Right: Maximum (solid) and minimum (dashed) bounds on sales (injection variations) by participants (mmscfd).

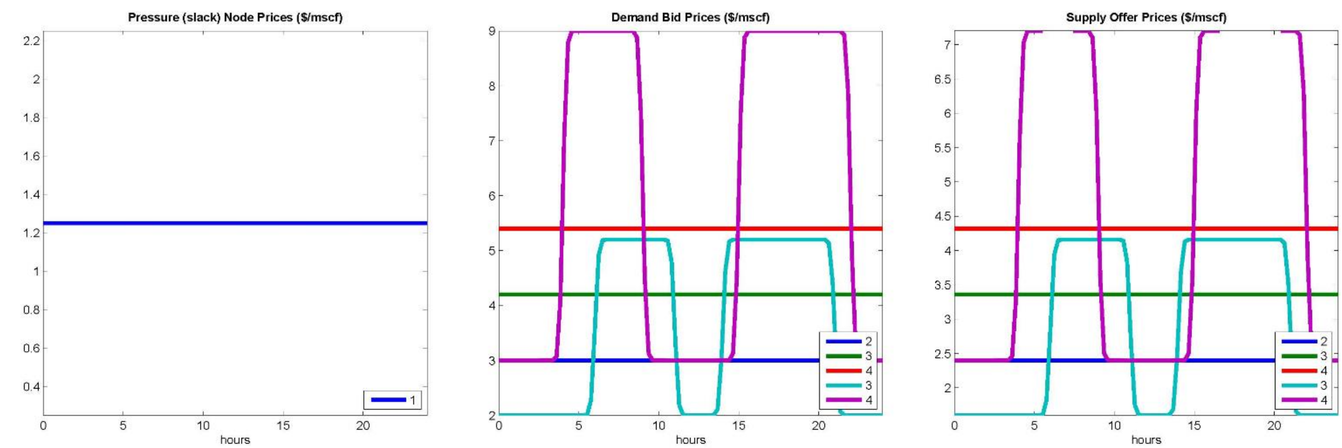

Figure 10 – Computational study economic market inputs – price bids. Left: price at a slack (pressure) nodes ($/mscf); Center: purchase prices by participants ($/mscf); Right: offer prices for sales by participants ($/mscf).

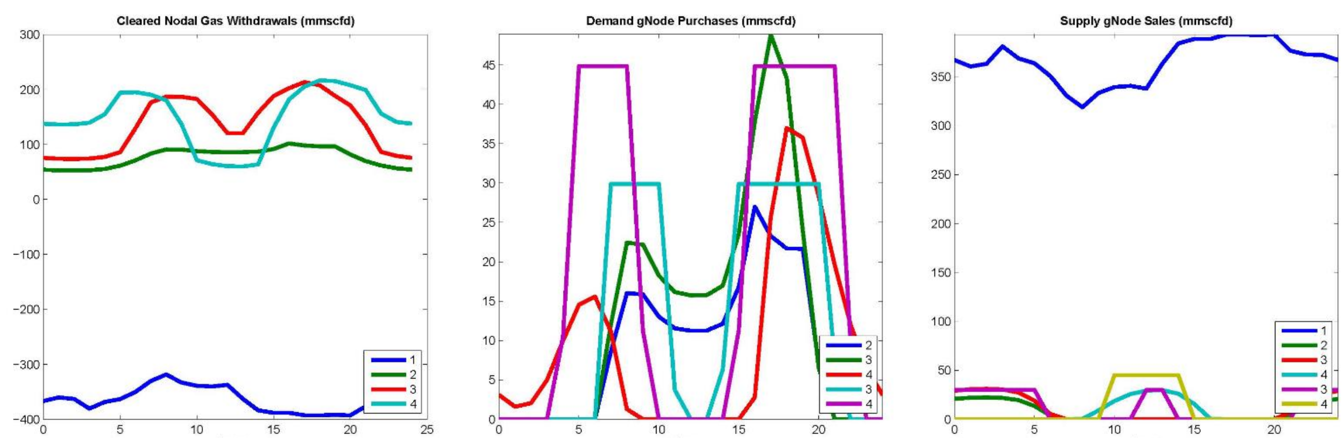

Figure 11 – Computational study physical market outputs – flow schedule solution. Left: cleared nodal gas withdrawals (mmscfd); Center: purchase by participants (mmscfd); Right: offers for sales by participants (mmscfd).

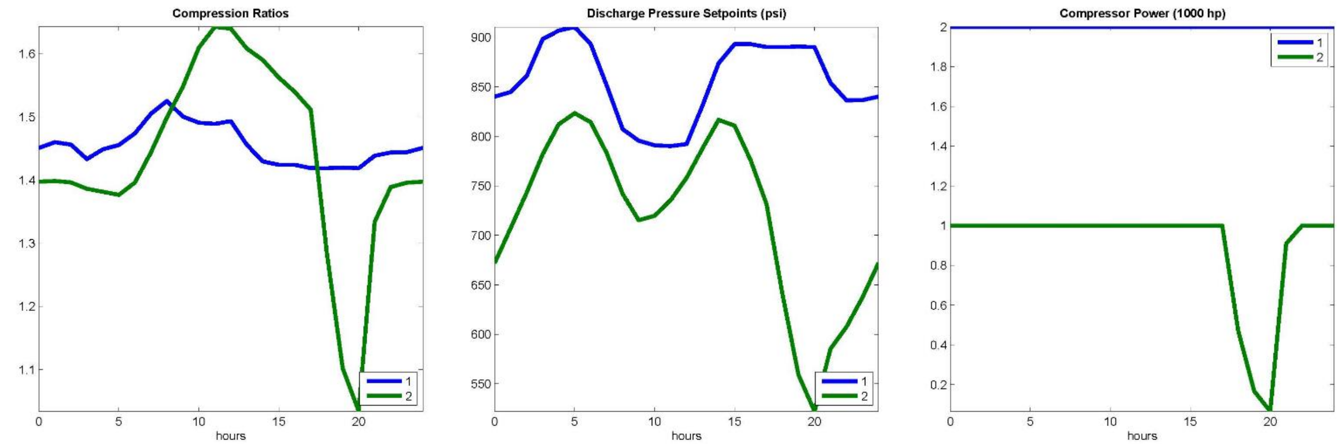

Figure 12 - Computational study physical market outputs – compressor operation solution. Left: Compression ratios; Center: Discharge Pressures (psi); Right: Power (hp).

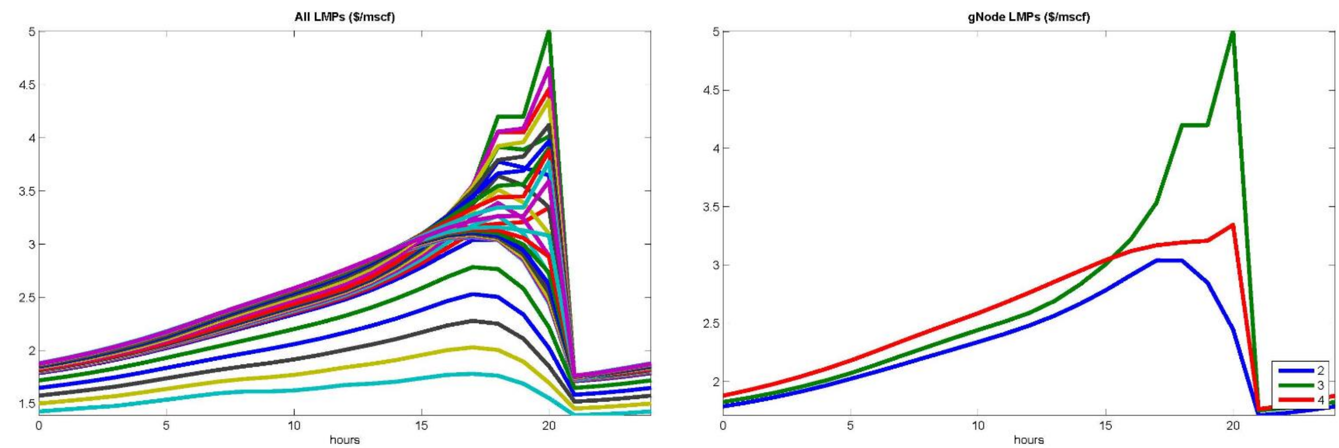

Figure 13 – Computational study economic market outputs – nodal pricing (LTV) solution. Left: time-dependent marginal price at all spatial discretization points ($/mscf); Right: marginal price at purchaser nodes ($/mscf).

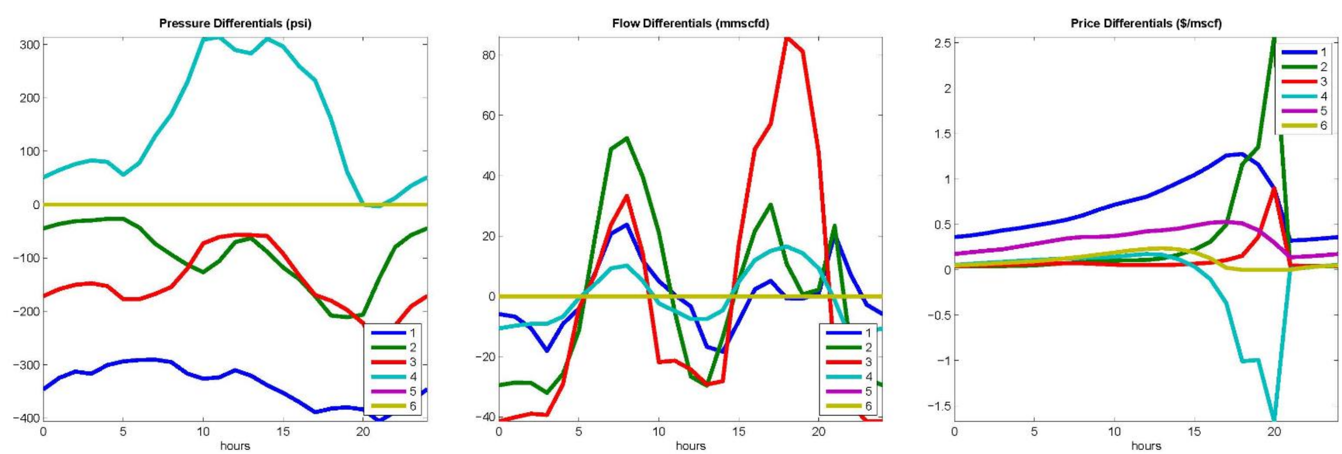

Figure 14 – Computational study solution – physical and economic differentials. Left: Pressure differentials across pipes; Center: Flow differentials across pipes; Right: Price differentials across pipes.

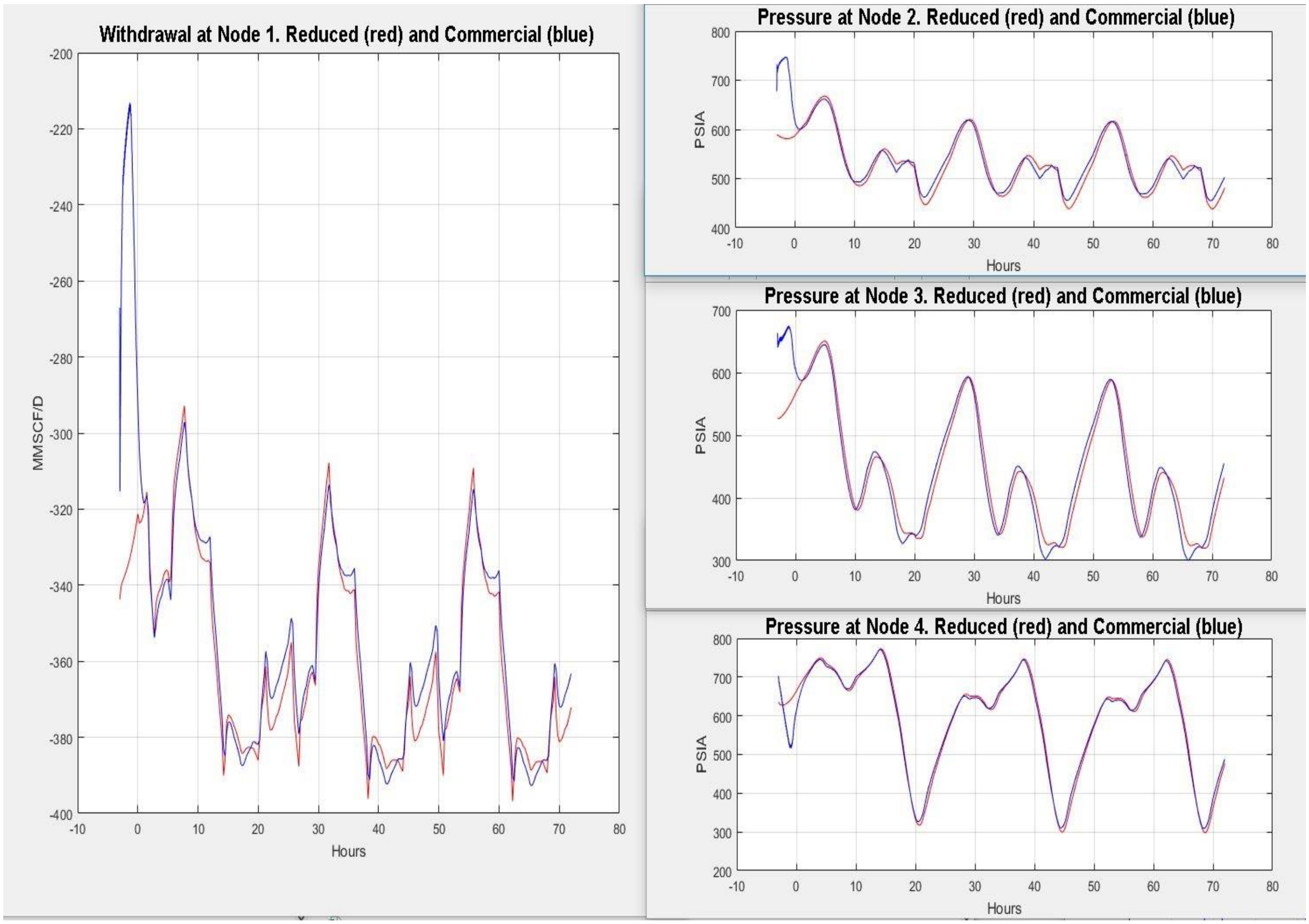

Figure 15 – Validation of modeling by solution of an initial value problem produced using transient optimization outputs by using a commercial simulator. Left: comparison of physical flow into the system at Node 1; Right: comparisons of nodal pressures at Nodes 2, 3, and 4.

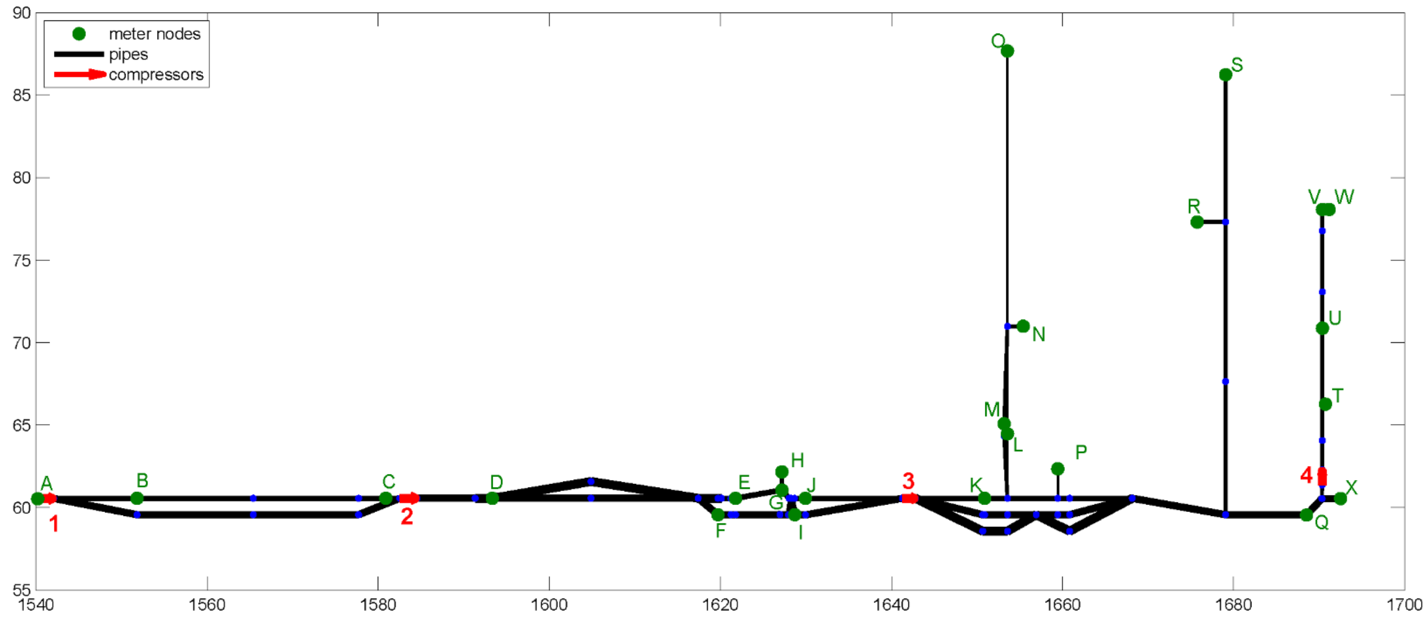

Figure 16 – Validation of modeling using real data – reduced network model. Asection of a large gas transmission pipeline system with a total of 444.25 miles of pipe represented using 78 reduced model nodes, 95 pipes, 23 metered nodes (labelled B to X), and 4 compressors (labelled 1 to 4). Major inflow is at the suction of compressor 1 and outflow is at node X, with smaller offtakes throughout the system and at laterals.

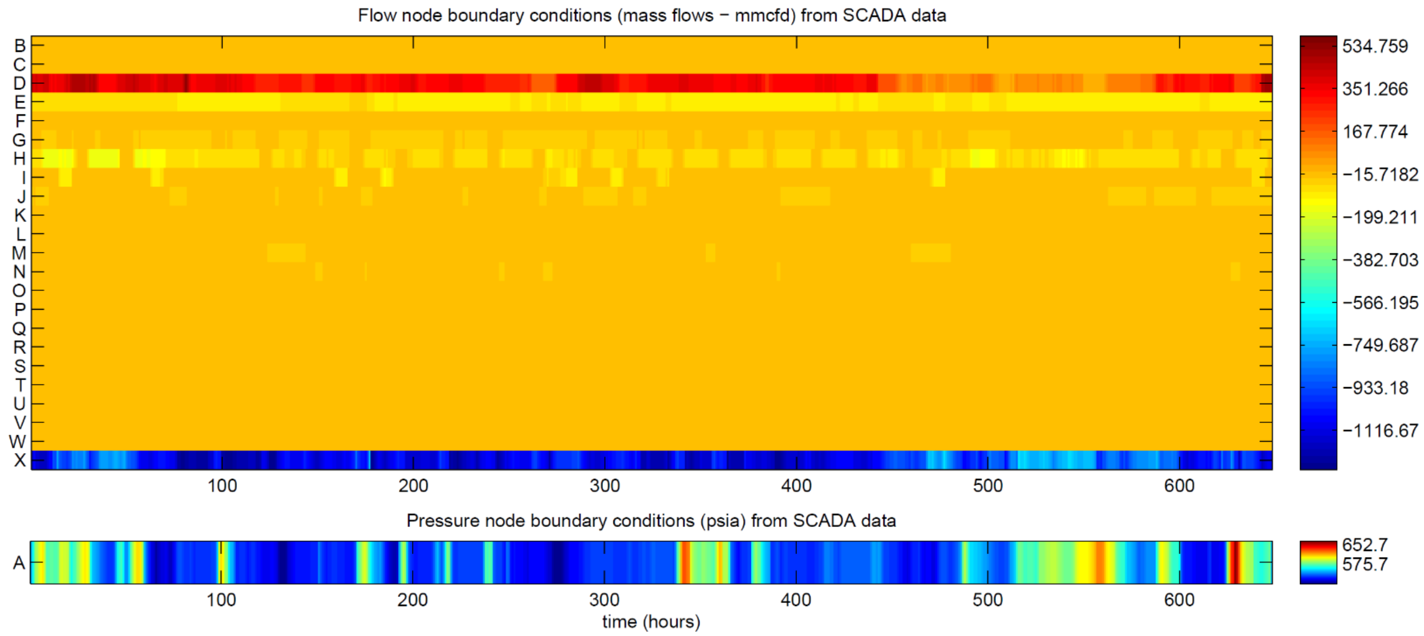

Figure 17 – Validation of modeling using real data - independent boundary conditions. These temporal parameters were synthesized from SCADA data and were used to set up the IVP simulation are given as time series with time in hours on the x-axis and location on the reduced model labelled on the y-axis. Magnitude is given in color as indicated on the bars at right. Mass flow into the system is provided at locations B to X (top) and pressure is given at location A (bottom), which acts as a slack node.

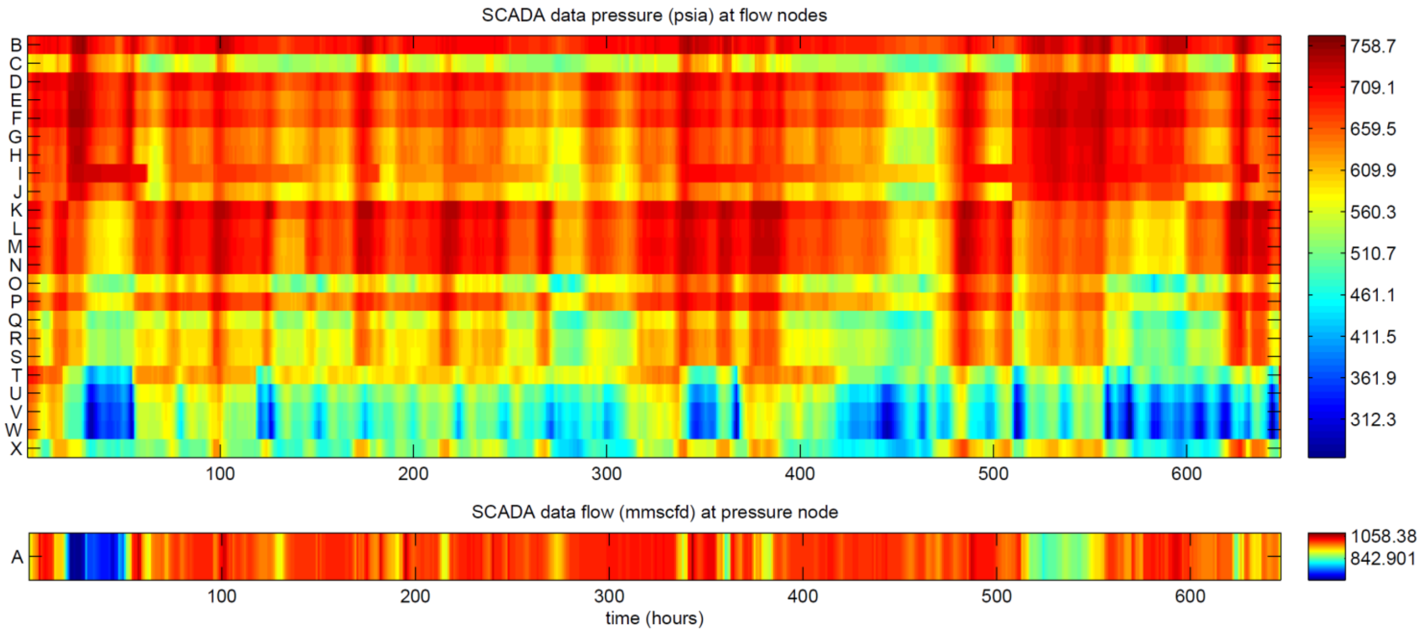

Figure 18 – Validation of modeling using real data – dependent boundary conditions from SCADA. SCADA data to be compared with dependent boundary conditions obtained by simulation using the reduced model approach are given as time series with time in hours on the x-axis and location on the reduced model labelled on the y-axis. Magnitude is given in color as indicated on the bars at right. Pressure is taken at locations B to X (top) and mass flow into the system is considered at location A (bottom).

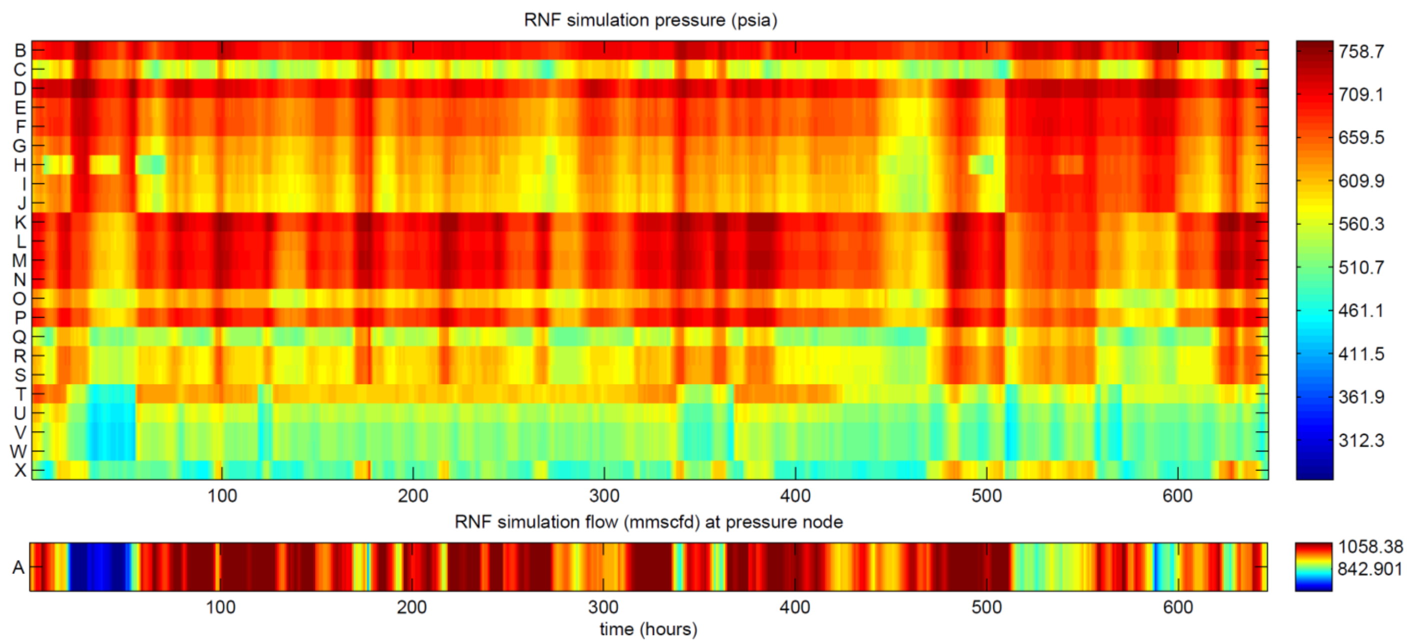

Figure 19 – Validation of modeling using real data – dependent boundary conditions from simulation. Simulation solution for dependent boundary conditions obtained using the reduced model approach are given as time series with time in hours on the x-axis and location on the reduced model labelled on the y-axis. Magnitude is given in color as indicated on the bars at right. Pressure at locations B to X (top) and mass flow into the system at location A (bottom).

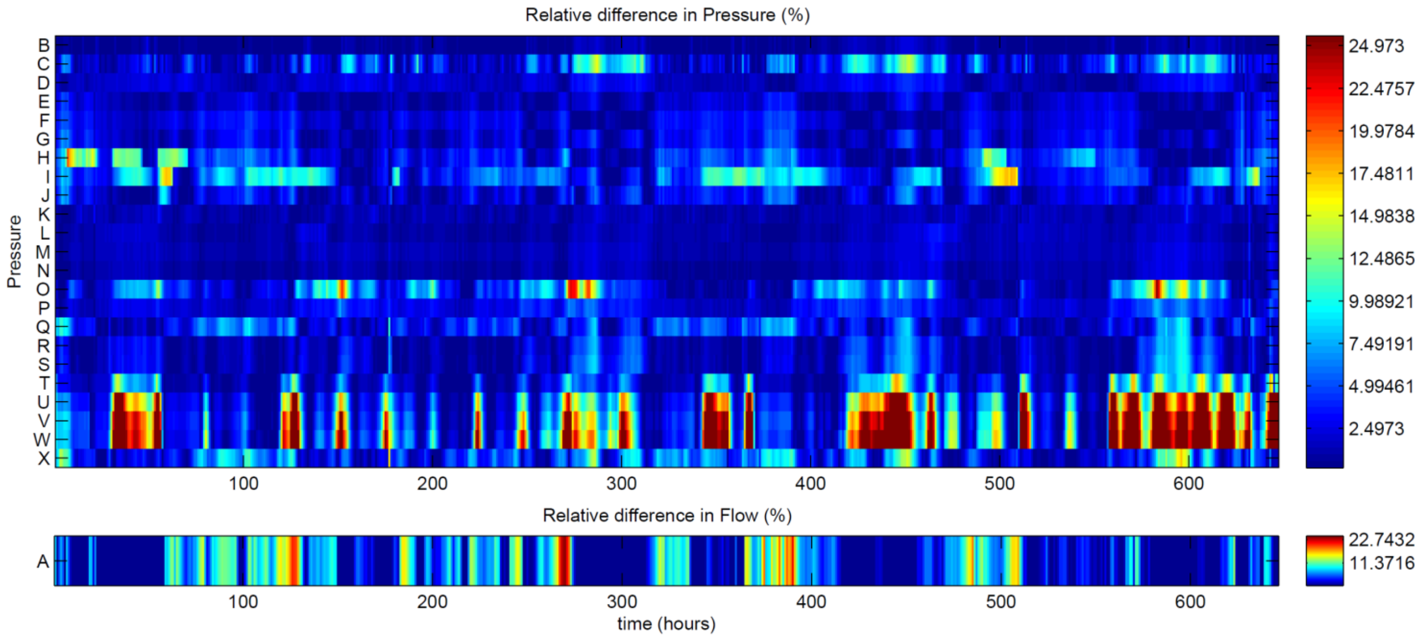

Figure 20 – Validation of modeling using real data – dependent boundary conditions from simulation. Relative distance between SCADA data in Figure 18 and the simulation outcome in Figures 19 are given as time series with time in hours on the x-axis and location on the reduced model labelled on the y-axis. Magnitude is given in color as indicated on the bars at right. Top: Difference in SCADA and simulation pressure at flow nodes; Overall mean: 4.1746%; Max at U, V, and W: 48.16%, 78.70%, and 86%; Mean excluding meters at U, V, and W: 2.9425%; Max excluding meters at U, V, and W: 25.01%. Bottom: Relative Flow difference at pressure (slack) node A; Mean: 2.4557%; Max: 23.77%.