Final Technical Report

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Accelerated Simulation of Kinetic Transport Using Variational Principles and Sparsity

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ABSTRACT

This project is centered on the development and application of techniques of sparsity and compressed sensing for variational principles, PDEs and physics problems, in particular for kinetic transport. This included derivation of sparse modes for elliptic and parabolic problems coming from variational principles. The research results of this project are on methods for sparsity in differential equations and their applications and on application of sparsity ideas to kinetic transport of plasmas.

REPORT

This project is centered on the development and application of techniques of sparsity and compressed sensing for variational principles, PDEs and physics problems, in particular for kinetic transport. This included derivation of sparse modes for elliptic and parabolic problems coming from variational principles. In this context, sparsity is in the spatial dependence of the modes; i.e., the modes have compact support. We have also developed a hierarchical basis which comes from a variational principle. The resulting basis functions have compact support and are comparable to wavelets, in that they are multiscale with localization in both position and wavenumber.

We also developed applications to kinetic theory, as well as to additional PDEs of mathematical physics. In particular, we expect to use the variational approach as a connection between microscopic and macroscopic components of a multiscale method. In the kinetic theory context, the microscopic component will be particle interactions and the macroscopic component will be a continuum (fluid) description. The resulting velocity distribution function f will be represented as a hybrid combination $f=M+k$ of a Maxwellian distribution M (the thermal or fluid component) and a set of discrete particles (the kinetic component) k.

In previous use of this hybrid representation, the particles were all real particles so that the kinetic component is nonnegative; i.e., $k \geq 0$. This severely limits the usefulness of the representation. In the past year we developed a new approach that allows for particles of negative weight, so that k can be both positive and negative. Earlier methods using negative particles suffered from growth in the total number of particles (since positive and negative particles can approximately cancel), which made the methods computationally demanding, and completely intractable for Coulomb collisions. Our new approach greatly reduces the problem with growth of the number of particles, so that the method is computationally efficient. Most important, the method works as well for Coulomb collisions, as it does for non-charged particles. This method has been developed for both spatially homogeneous and spatially inhomogeneous problems. This has the promise to be a significant breakthrough in Monte Carlo methods for plasma kinetics.

Personnel

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Publications

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