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Energy Based Representation of 6-DOF Shaker Shock Low-Cycle Fatigue Tests

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Carl Sisemore, Vit Babuska, Jason Booher
Sandia National Laboratories
Albuquerque, NM

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Introduction & Motivation

- Energy Spectra and Shock Response Spectra (SRS) are both valid for quantifying mechanical shock severity
  - Typically associated with a single shock event
    - Not necessarily in one axis only
  - Many environments are comprised of multiple shock events
    - Transportation – Ground or Air
- Purpose of this work was to investigate how energy spectra apply to repetitive shock events
  - Is there an energy spectra relationship similar to more traditional fatigue life estimation methods?

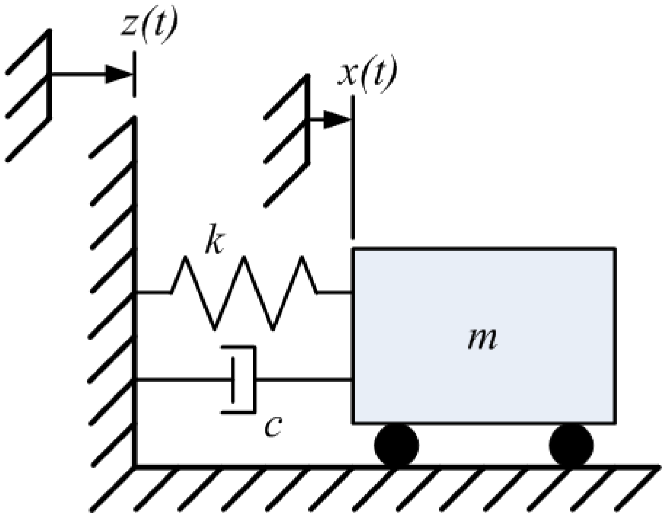
Mechanical Shock Characterization

- All response spectra are derived from the SDOF oscillator with base excitation equation of motion.
  - Absolute Acceleration Shock Response Spectrum
  - Pseudo-Velocity Shock Response Spectrum (PVSRS)
  - Energy Response Spectrum

- The equation of motion in terms of relative coordinates is:

$$m\ddot{w}(t) + c\dot{w}(t) + kw(t) = -m\ddot{z}(t)$$

$$w(t) = x(t) - z(t)$$



Energy Response Spectra

- Multiply the SDOF equation by the incremental displacement $dw = \dot{w}(t)dt$ and integrate

$$\int_{t_0}^{t_f} \ddot{w}(t)\dot{w}(t)dt + 2\zeta\omega_n \int_{t_0}^{t_f} \dot{w}^2(t)dt + \omega_n^2 \int_{t_0}^{t_f} w(t)\dot{w}(t)dt = -\int_{t_0}^{t_f} \ddot{z}(t)\dot{w}(t)dt$$

- Energy Balance Equation

$$\frac{1}{2}\dot{w}^2(t)\Big|_{t_0}^{t_f} + 2\zeta\omega_n \int_{t_0}^{t_f} \dot{w}^2(t)dt + \frac{1}{2}\omega_n^2 w^2(t)\Big|_{t_0}^{t_f} = -\int_{t_0}^{t_f} \ddot{z}(t)\dot{w}(t)dt$$

Relative Kinetic Energy E_K^R

Viscously Dissipated Energy E_D

Absorbed or Potential Energy E_A

Input or Total Energy E_I

Input Energy & Dissipated Energy

- Input energy is a function of the load vector and the response velocity vector

$$E_I = \int_{t=0}^{t=T} \boldsymbol{p}(t) \cdot \dot{\boldsymbol{x}}(t) dt$$

- For steady state cyclic loading,

$$E_I = \int_{t=0}^{N/\Omega} \boldsymbol{p}(t)\, \dot{\boldsymbol{x}}(t) dt = N \int_{t=0}^{1/\Omega} \boldsymbol{p}(t) \dot{\boldsymbol{x}}(t) dt = N\overline{E_I}$$

- Input energy per cycle has another interpretation—as the energy dissipated per cycle

$$E_I = E_D \qquad \overline{E}_I = \overline{E}_D$$

Basquin's Equation for Fatigue Life

- Basquin's equation is a well known model of cyclic high-cycle fatigue

$$NS^b = C$$

  - *N* is number of cycles, *S* is stress amplitude, *b* is the fatigue strength coefficient, and *C* is an empirically determined constant
  - Fatigue strength coefficient is determined from the *S*-*N* data where $-1/b$ is the slope of the *S*-*N* line in log-log space
- Basquin's equation is a model based on test data
  - Typical fatigue strength coefficients range from 3 – 25
  - 6.4 is a common average value for structural materials
- Postulate that a similar relationship exists for input energy and shock induced failure

Dissipated Energy & Fatigue Life

- Damping energy is a non-linear function of stress amplitude

$$\bar{E}_D \propto S^{\beta} \begin{cases} \beta \sim 2.4, S < 0.8S_{yield} \\ \beta \sim 8 \ \ , S \geq 0.8S_{yield} \end{cases}$$

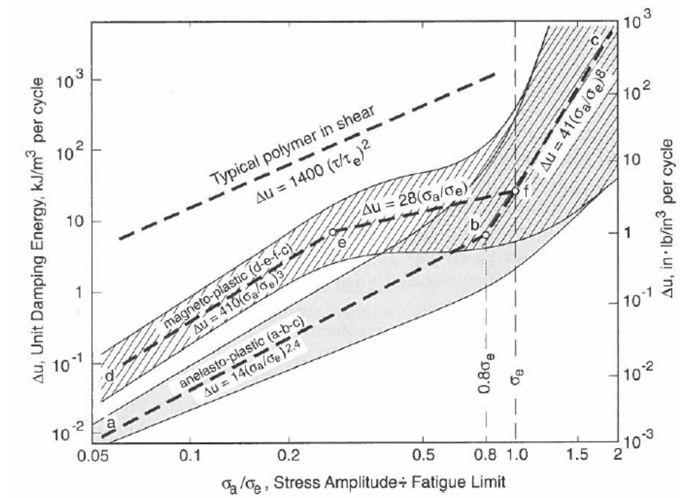


Figure 15.40, page 832, Dowling, N.E., Mechanical Behavior of Materials

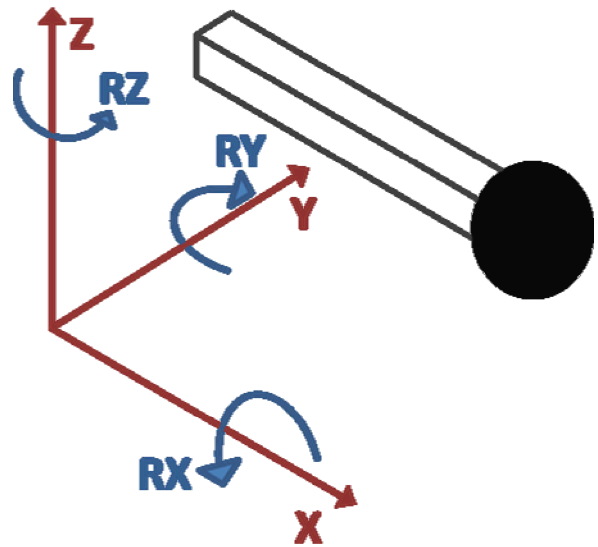
- Hypothesize that total input energy and fatigue life are related by

$$E_I = C_D^{\frac{\beta}{b}} N^{\left(1-\frac{\beta}{b}\right)}$$

where N is the number of shocks and C_D and $\frac{\beta}{b}$ are experimentally determined parameters

Composite Energy Spectra

- What is the correct methodology for predicting failure from multi-axis shock?
  - Three translational and three rotational spectra
  - Failure should not be equal to any single axis response but to the composite of all inputs loading the structure
- Since there is a known relationship between pseudo-velocity and stress, assume a stress based solution
  - Each of the six shocks develops stress in the structure, bending or axial.



$$PV_{xeff} = PV_x$$

$$PV_{yeff} = PV_y + PV_{rz}l + PV_{rx}h$$

$$PV_{zeff} = PV_z + PV_{ry}l$$

Composite Input Energy Spectrum

- Combing stresses, and hence PVSRS yields:

$$PV_{composite} = \frac{c}{4l} PV_{xeff} + \sqrt{\left(PV_{yeff}\right)^2 + \left(PV_{zeff}\right)^2}$$

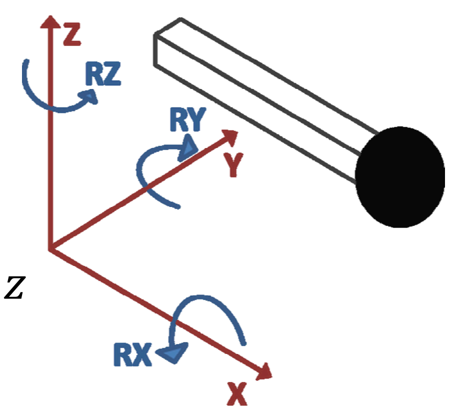
  - Where the PV_{xeff} scale factor is derived from the axial to bending stress ratio $\frac{\sigma_{Axial}}{\sigma_{Bending}} = \frac{F/_A}{Mc/_I} = \frac{c}{4l}$

- A direct relationship exists for Absorbed Energy

$$E_{Am} = \tfrac{1}{2} PV^2 \qquad PV_j = \sqrt{2E_{Amj}}, \quad j = x, y, z$$

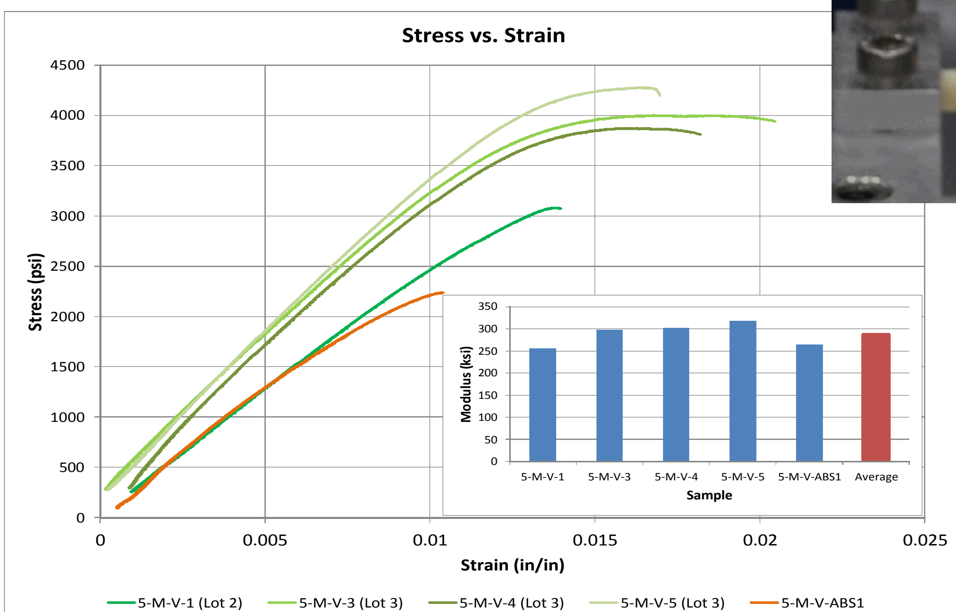
- Similarly for Input Energy

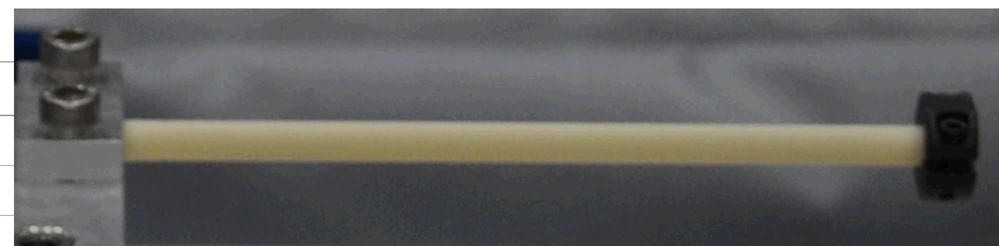
$$E_I = \frac{1}{2}\left[E_{Ix}^* \frac{c}{4l} + \sqrt{\left(E_{Iy}^*\right)^2 + \left(E_{Iz}^*\right)^2}\right]^2 \qquad E_{Ij}^* = \sqrt{2E_{Ij}}, \quad j = x, y, z$$

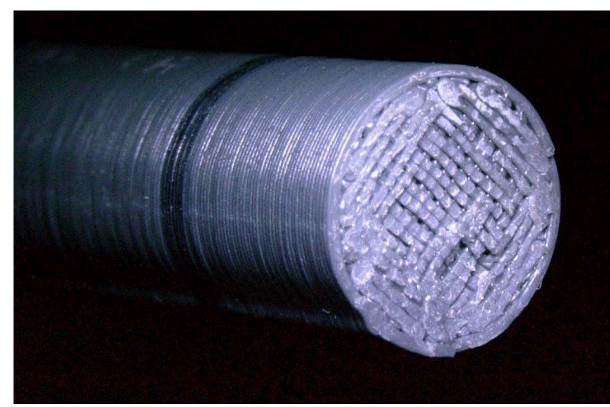


Test Article Description

- ABS plastic 3-D printed cantilever beams used for this study
  - All beams printed with a cross-wise raster
  - Elliptic cross section 0.25in major axis, 0.188in minor axis
  - 0.010in stress concentration notch near the fixed end
  - Steel collar weight at free end to lower frequency and increase stress

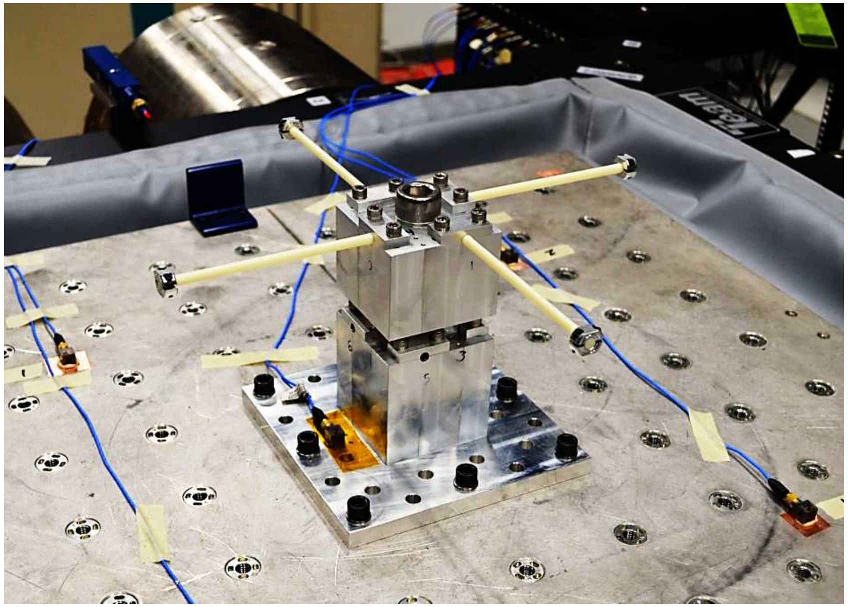


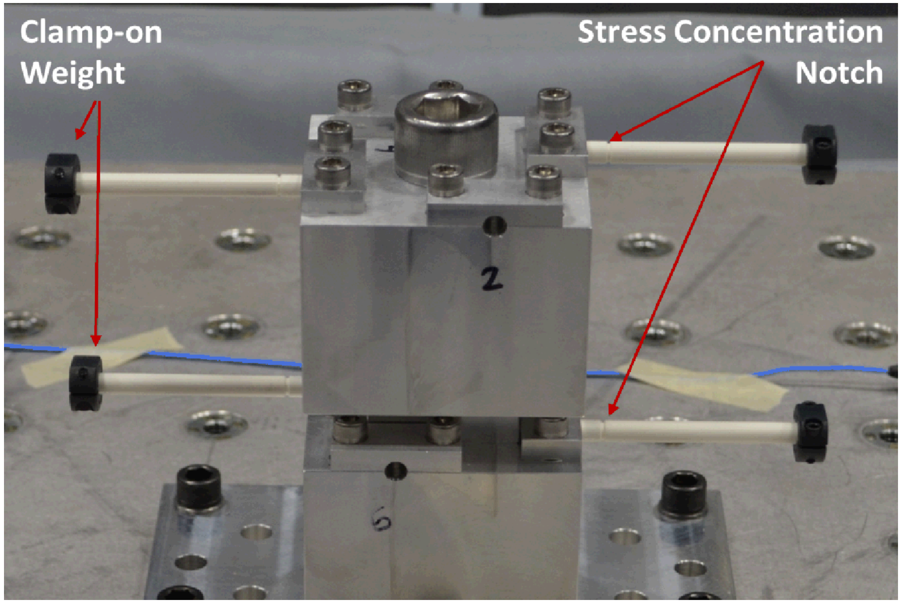




Shock Test Setup

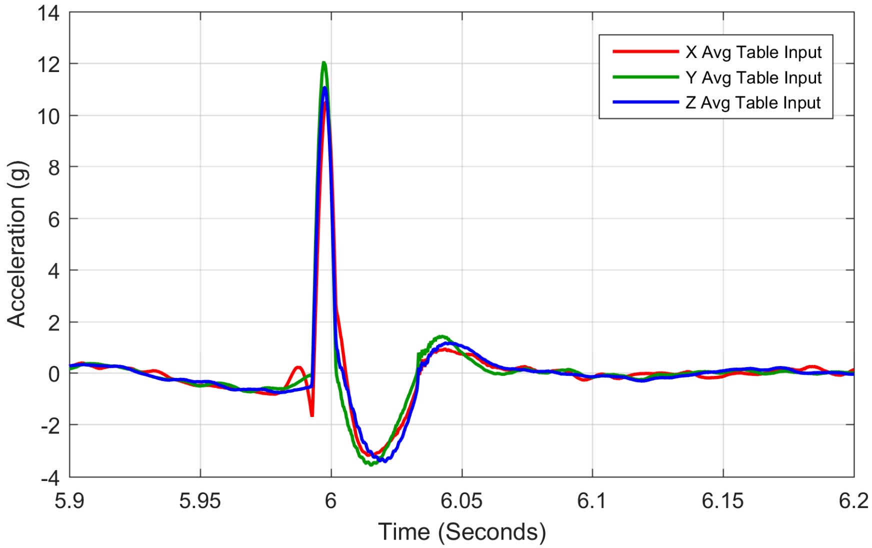
- Tested numerous cantilever beams on the SNL large 6-DOF shaker table in sets of four beams per tests
  - A few first passage tests to determine failure level
  - Fatigue testing to lower levels until all beams failed
  - 68 beams tested using 3,424 shock events

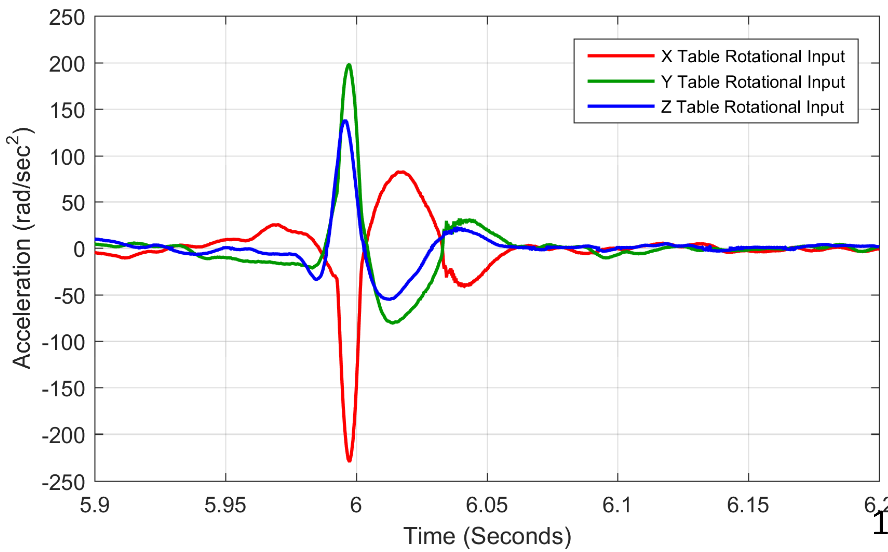




Typical 6-DOF Shock Input

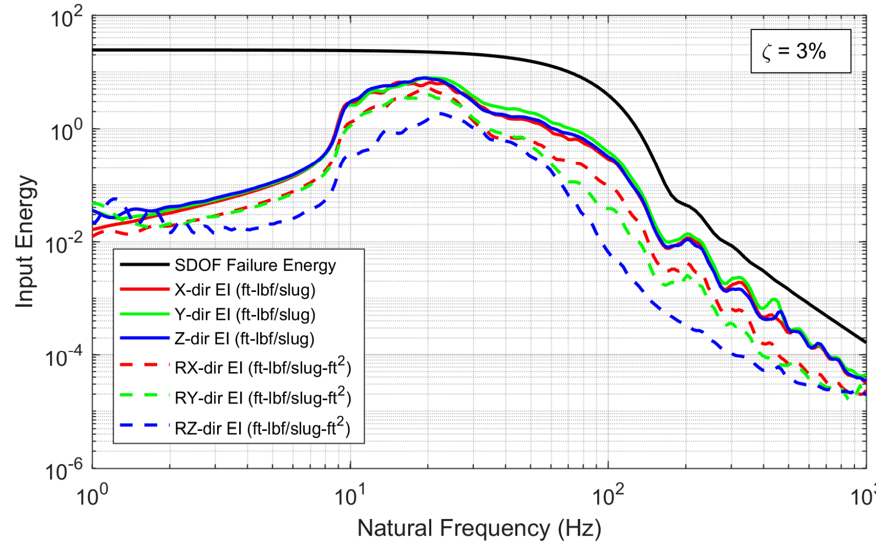
- Typical table translational and rotational acceleration
  - Pulse width and shape were constant throughout testing
  - Amplitude was varied to increase shock severity
- Translational acceleration was calculated as the average of the four corners
- Rotational acceleration was derived from tri-axial accelerometers mounted on the table corners

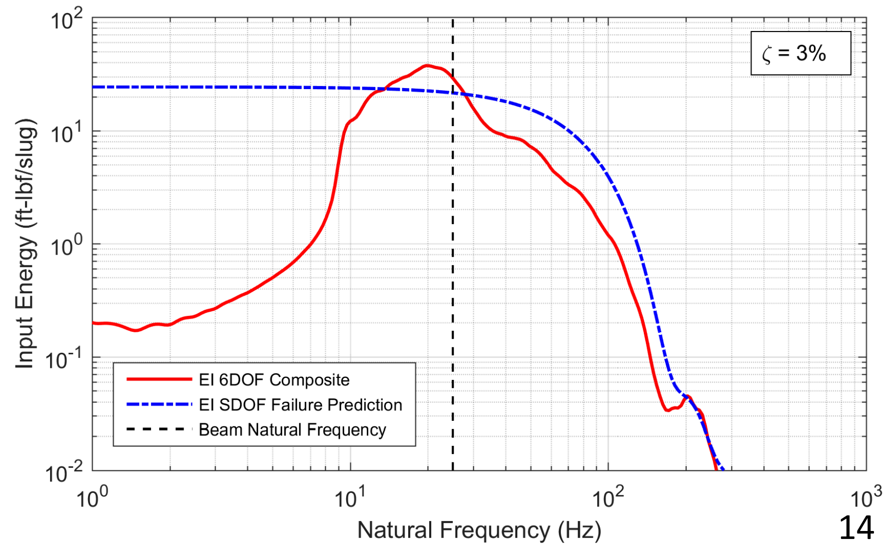




6-DOF Shock Input Energy Calculation

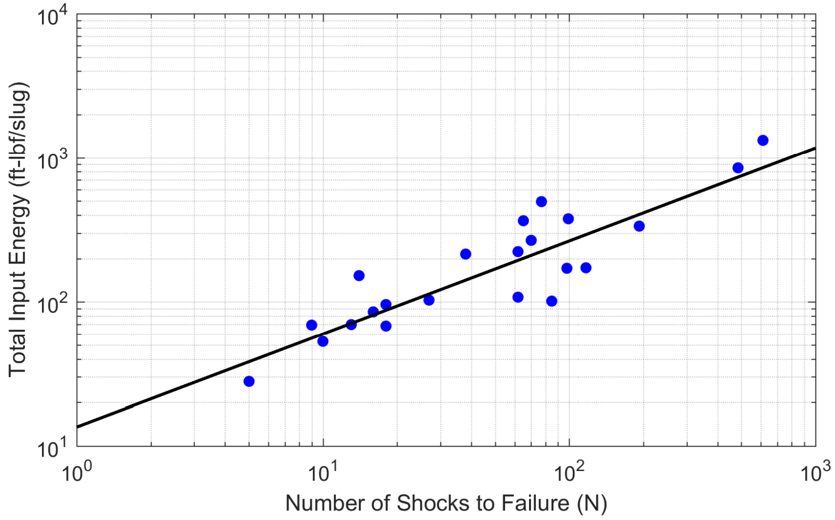
- Input Energy from 6-DOF time histories compared to SDOF failure prediction
  - SDOF failure validated with drop table testing
- Developed composite input energy spectrum calculation methodology
  - Shows composite 6-DOF energy spectrum predicts failure at beam's first natural frequency
- Composite input energy spectrum used for fatigue calculations





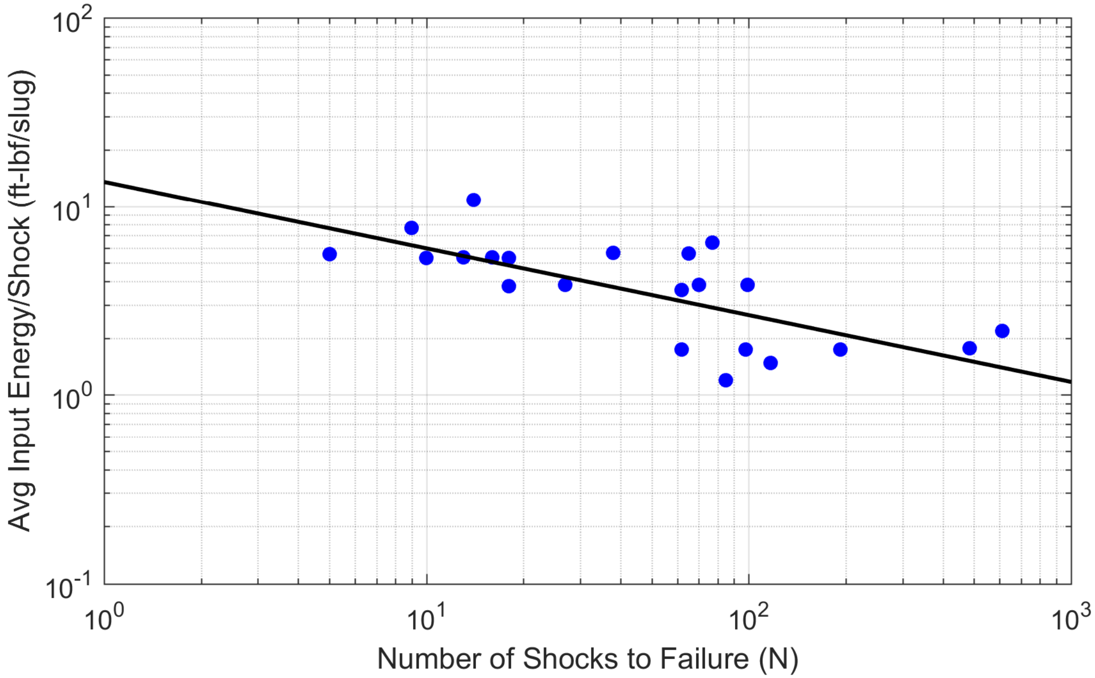
Total Input Energy from Fatigue Tests

- Calculation of the total input energy applied during the fatigue tests
  - The composite Input Energy spectrum was calculated from each shock
  - The Input Energy at the beam's first natural frequency was extracted
  - Input Energy was summed over the number of shocks to failure
  - Total Input Energy versus number of shocks to fail is calculated for each beam in the test series and plotted
  - Least squares fit to data
    - Actual slope = 0.65



6-DOF Shock Fatigue Study Results

- Same fatigue data as before plotted to show number of shocks versus average input energy per shock
  - Slope here is –0.35 which is almost equal to the expected –0.36
  - The significance is not the closeness of the two slopes but rather that it is in–family with general structural materials



Conclusions

- A power-law relationship between input energy and number of shocks to failure was derived and tested
- The relationship depends on experimental data to define the empirical coefficients, similar to the traditional *S-N* relationship
- The empirically determined coefficients are in family with those expected for general structural materials