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Solving the MINLP Problem of Unit Commitment on AC Power Systems

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Overview

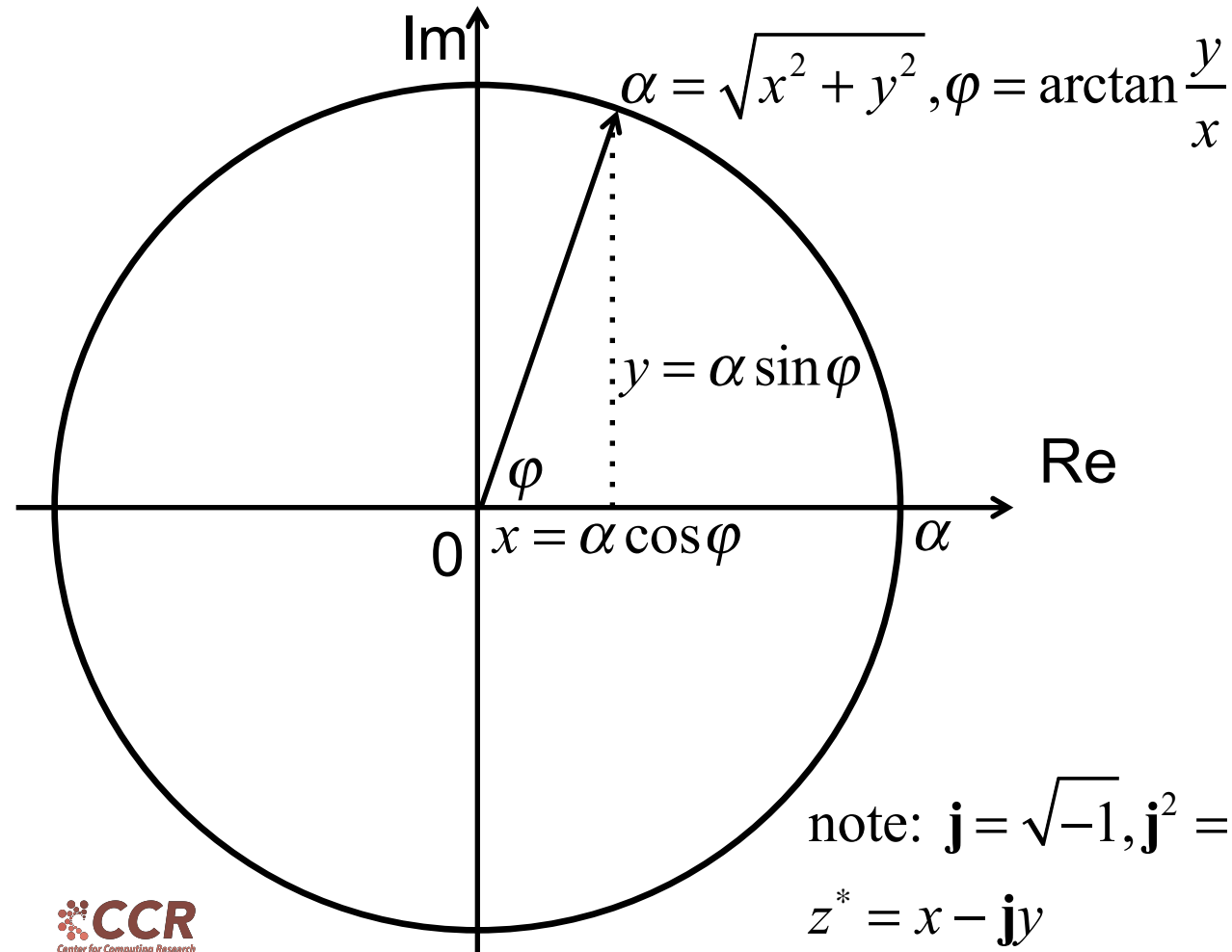
- Mathematical Preface
- Motivations & Background
- Today's Practice
- Contributions
 - Local Solution Method
 - Global Solution Method
- Ongoing Challenges

MATHEMATICAL PREFACE

Complex Number Conversion

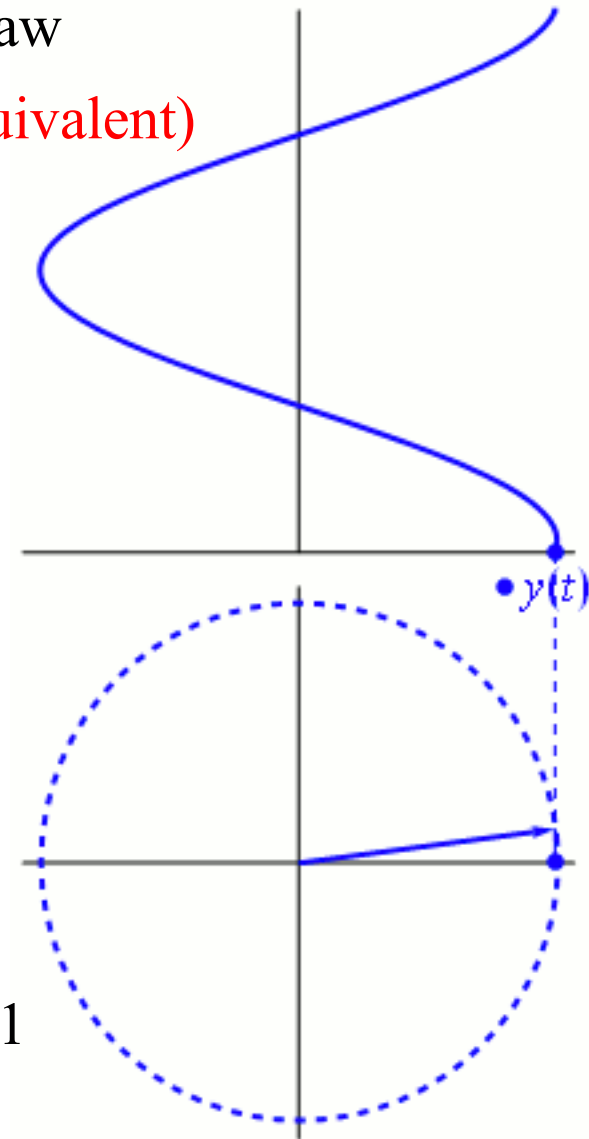
Polar: $z = \alpha \angle \varphi = \alpha e^{j\varphi} = \alpha(\cos \varphi + j \sin \varphi)$ Euler's Law

Rectangular: $z = x + jy$ **Polar $\leftrightarrow$ Rectangular (Equivalent)**



note: $j = \sqrt{-1}, j^2 = -1$

$z^* = x - jy$



Basic Notation

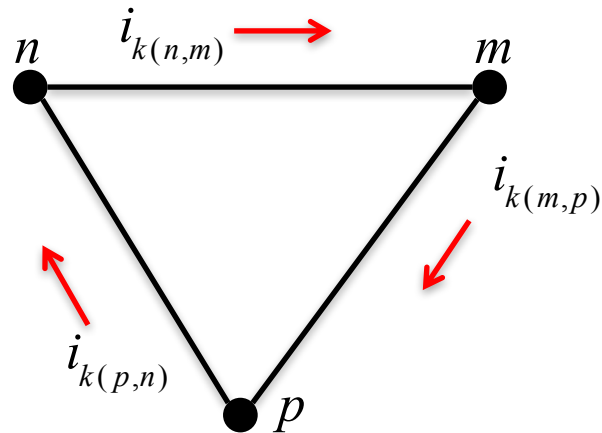
- Voltage signal: $v(t) = v^{\max} \cos(\omega t + \theta)$
- Current signal: $i(t) = i^{\max} \cos(\omega t + \phi)$
- Apparent Power: $s = p + \mathbf{j}q$
 - The measure of AC power
 - Energy/Time expressed in Volt-Ampere (VA); Real (W), Reactive (VAr)
- Admittance Matrix: $Y = G + \mathbf{j}B$
 - Equivalenced AC Network Characteristics
- RMS:
 - Voltage magnitude: $|v| = \sqrt{1/T \int_0^T v(t)^2 dt} = v^{\max} / \sqrt{2}$
 - Current magnitude: $|i| = i^{\max} / \sqrt{2}$
- Steady-State: ω constant, e.g. 60 Hertz
 - Voltage phasor (RMS, cosine-referenced): $v = |v| e^{\mathbf{j}\theta} = v^r + \mathbf{j}v^j$
 - Rate of movement of charge
 - Charge/Time expressed in Ampere (A)
 - Current phasor (RMS, cosine-referenced): $i = |i| e^{\mathbf{j}\phi} = i^r + \mathbf{j}i^j$
 - Potential electrical energy
 - Energy/Charge expressed in Volt (V)

Ohm's Law, AC Power, & Kirchhoff's Laws

- Ohm's Law: $v = iR$ $i = R^{-1}v$ DC Circuits
 $v = iZ$ $i = Z^*v = Yv$ AC Circuits
Resistance (R) in Ohms (Ω): Opposition to a steady electric DC current
Impedance (Z) in Ohms (Ω): Opposition to the flow of AC current (sinusoidal)
Admittance (Y) in Siemens (S): Ease at which AC current flows
- Power Formula (Joule's First Law):
 $p = vi$ DC Circuits
 $s = vi^*$ AC Circuits
- Kirchhoff's Circuit Laws:
Kirchhoff's Current Law (KCL)
Kirchhoff's Voltage Law (KVL)

Kirchhoff's 1st & 2nd Circuit Laws

Kirchhoff's Current Law (KCL)

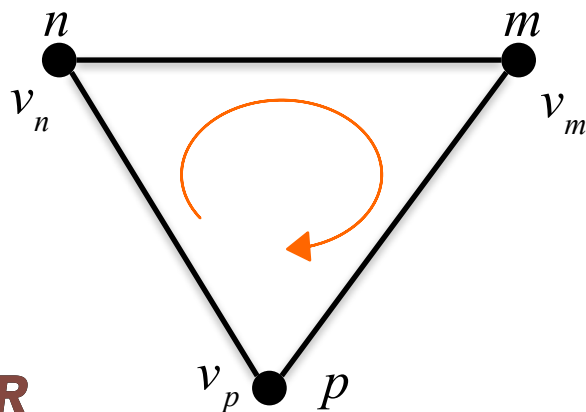


$$i_{k(n,m)} - i_{k(m,p)} = 0$$

$$i_{k(m,p)} - i_{k(p,n)} = 0$$

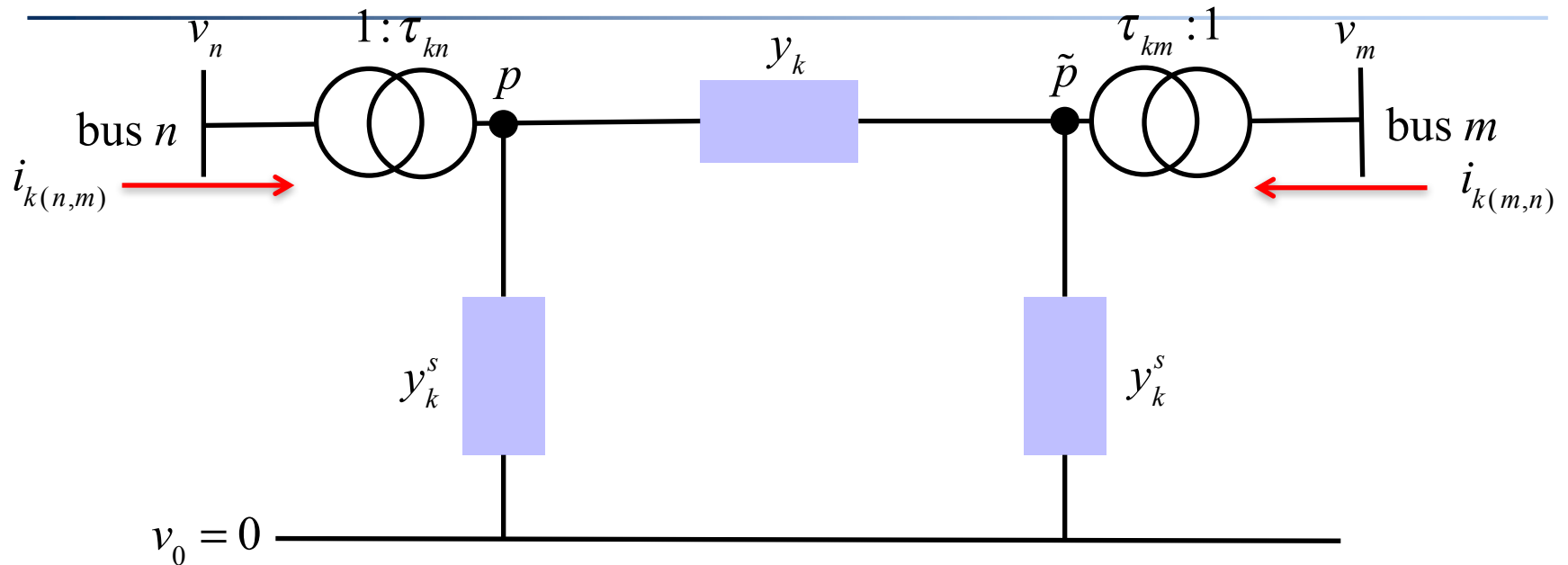
$$i_{k(p,n)} - i_{k(n,m)} = 0$$

Kirchhoff's Voltage Law (KVL)



$$(v_n - v_m) + (v_m - v_p) + (v_p - v_n) = 0$$

Generalized Branch Admittance Matrix Y^k



Extended π -model for line $k \in K$

$$\begin{bmatrix} Y_{1,1}^k & Y_{1,2}^k \\ Y_{2,1}^k & Y_{2,2}^k \end{bmatrix} := \begin{bmatrix} |\tau_{kn}|^2 (y_k + y_k^s) & -\tau_{kn}^* \tau_{km} y_k \\ -\tau_{kn} \tau_{km}^* y_k & |\tau_{km}|^2 (y_k + y_k^s) \end{bmatrix}$$

Line: If $t_{kn} = t_{km} = 1$

In-Phase (n -side): If $t_{kn} \neq 1, t_{km} = 1, y_k^s \approx 0$

Phase-shifting (n -side): If $t_{kn} \neq 1, \varphi_{kn} \neq 0, t_{km} = 1, y_k^s \approx 0$

Nodal Power Balancing Constraints

Mathematically Equivalent Approaches

- Polar Power-Voltage Power Flow Formulation (PSV)

$$v_n \sum_{m \in N} v_m (g_{nm} \cos \theta_{nm} + b_{nm} \sin \theta_{nm}) - p_n + p_n^d = 0, \forall n \in N$$

$$v_n \sum_{m \in N} v_m (g_{nm} \sin \theta_{nm} - b_{nm} \cos \theta_{nm}) - q_n + q_n^d = 0, \forall n \in N$$

- Rectangular Power-Voltage Power Flow Formulation (RSV)

$$v_n^r \sum_{m \in N} (g_{nm} v_m^r - b_{nm} v_m^j) + v_n^j \sum_{m \in N} (g_{nm} v_m^j + b_{nm} v_m^r) - p_n + p_n^d = 0, \forall n \in N$$

$$v_n^j \sum_{m \in N} (g_{nm} v_m^r - b_{nm} v_m^j) - v_n^r \sum_{m \in N} (g_{nm} v_m^j + b_{nm} v_m^r) - q_n + q_n^d = 0, \forall n \in N$$

- Rectangular Current Injection Formulation (RIV)

$$i_n^r = \sum_{m \in N} (g_{nm} v_m^r - b_{nm} v_m^j) \quad i_n^j = \sum_{m \in N} (g_{nm} v_m^j + b_{nm} v_m^r), \forall n \in N$$

$$p_n = v_n^r i_n^r + v_n^j i_n^j + p_n^d \quad q_n = v_n^j i_n^r - v_n^r i_n^j + q_n^d, \forall n \in N$$

IV-ACOPF Formulation

$$\min \sum_n C_n^{g,2} (p_n^g)^2 + C_n^{g,1} p_n^g$$

s.t.

Line Current Flows

$$i_{k(n,m)}^r = \text{Re}(Y_{1,1}^k v_n + Y_{1,2}^k v_m), \quad i_{k(m,n)}^r = \text{Re}(Y_{2,1}^k v_n + Y_{2,2}^k v_m) \quad \forall k \in K$$

$$i_{k(n,m)}^j = \text{Im}(Y_{1,1}^k v_n + Y_{1,2}^k v_m), \quad i_{k(m,n)}^j = \text{Im}(Y_{2,1}^k v_n + Y_{2,2}^k v_m) \quad \forall k \in K$$

Network Current Balancing

$$i_n^r - \left(\sum_{k(n,\cdot)} i_{k(n,m)}^r + G_n^{sh} v_n^r - B_n^{sh} v_n^j \right) = 0 \quad \forall n \in N$$

$$i_n^j - \left(\sum_{k(n,\cdot)} i_{k(n,m)}^j + G_n^{sh} v_n^j + B_n^{sh} v_n^r \right) = 0 \quad \forall n \in N$$

Nodal Power Injections

$$p_n^g - (v_n^r i_n^r + v_n^j i_n^j) = p_n^d \quad \forall n \in N$$

$$q_n^g - (v_n^j i_n^r - v_n^r i_n^j) = p_n^d \quad \forall n \in N$$

Nodal Voltage Magnitude Limits

$$(V_n^{\min})^2 \leq (v_n)^2 \leq (V_n^{\max})^2 \quad \forall n \in N$$

Generator Limits

$$P_n^{\min} \leq p_n^g \leq P_n^{\max} \quad \forall n \in N$$

$$Q_n^{\min} \leq q_n^g \leq Q_n^{\max} \quad \forall n \in N$$

Thermal Line (Flowgate) Limits

$$(i_{k(\cdot)})^2 \leq (I_k^{\max})^2 \quad \forall k(n,m), k(m,n) \in F$$

RSV-ACOPF Formulation

$$\min \sum_n C_n^{g,2} (p_n^g)^2 + C_n^{g,1} p_n^g$$

Network Power Balancing

$$\sum_{k \in K(n)} p_{k(\cdot)} - p_n^g + p_n^d = 0 \quad \forall n \in N$$

$$\sum_{k \in K(n)} q_{k(\cdot)} - q_n^g + q_n^d = 0 \quad \forall n \in N$$

Nodal Voltage Magnitude Limits

$$(V_n^{\min})^2 \leq (v_n)^2 \leq (V_n^{\max})^2 \quad \forall n \in N$$

Thermal Line (Flowgate) Limits

$$(p_{k(\cdot)})^2 + (q_{k(\cdot)})^2 \leq (S_k^{\max})^2 \quad \forall k(n, m), k(m, n) \in K$$

Generator Limits

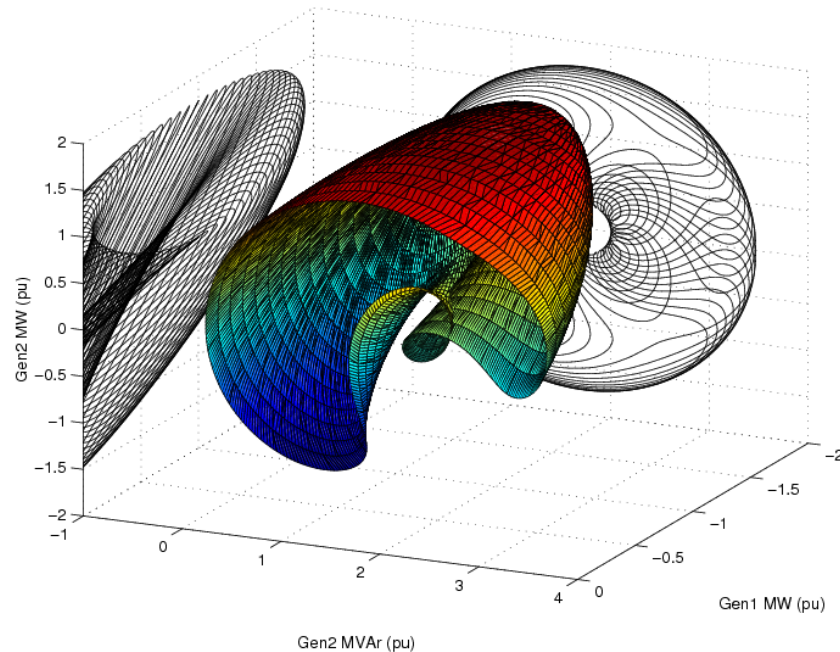
$$P_n^{\min} \leq p_n^g \leq P_n^{\max} \quad \forall n \in N$$

$$Q_n^{\min} \leq q_n^g \leq Q_n^{\max} \quad \forall n \in N$$

Line Power Flows

$$p_{k(n,m)} = v_n^r i_{k(n,m)}^r + v_n^j i_{k(n,m)}^j, p_{k(m,n)} = v_m^r i_{k(m,n)}^r + v_m^j i_{k(m,n)}^j \quad \forall k \in K$$

$$q_{k(n,m)} = v_n^j i_{k(n,m)}^r - v_n^r i_{k(n,m)}^j, q_{k(m,n)} = v_m^j i_{k(m,n)}^r - v_m^r i_{k(m,n)}^j \quad \forall k \in K$$



ACOPF Feasible Region for a Three-Bus Network

- **Nonconvex, NP-Hard**
- **Multiple Local Optima**

MOTIVATIONS & BACKGROUND

Background

- 1962: Carpentier formulates the ACOPF based upon KKT conditions
- 1960's to present day: Trends with algorithmic advancements in OR
- 21st century: Global convergence methods (Phan, Jabr, Bai, Lavaei)

Motivations

- Co-optimizes Real and Reactive Power Injections
- Changing Energy Landscape
 - More utility-scale renewables
 - More distributed resources
- Co-Optimize for Market Efficiency and Security
- Practical Application

Where are we going?

- In Industry?



- In Academia?

- Conventions/Assumptions
- Toy Networks (IEEE, MATPOWER)
- Little Knowledge of Practice

The Hype:

- N-1, N-1-1, N-k SCOPF
- _____ + ACOPF

The Bottleneck:

- ACOPF

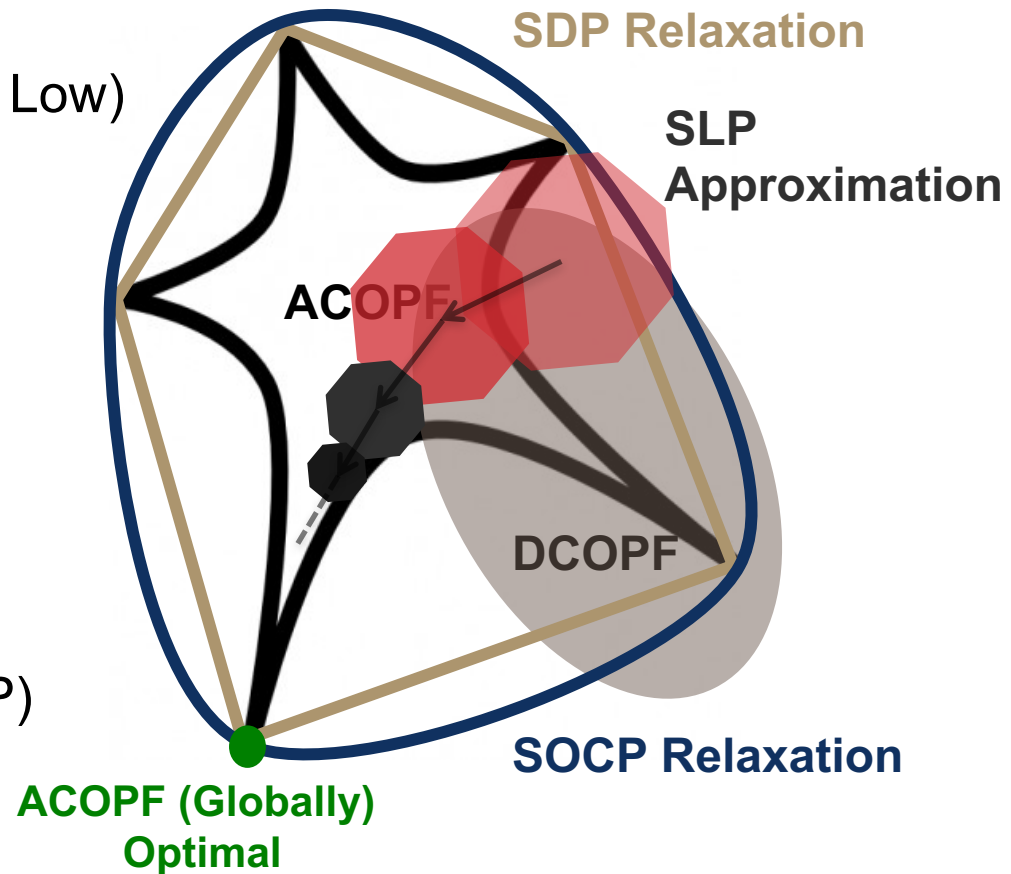
Future Work?

- Amass from 50+ Years of Work
- Collaborate and Compete
- Real World Applications



Solution Techniques

- **DCOPF**
 - Linear
- **Convex Relaxations**
 - SDP (Bai et al., Lavaei and Low)
 - SOCP (Jabr, Kocuk et al.)
 - QCP (Coffrin et al.)
 - Relaxation Tightening
 - Chen, Coffrin, Kocuk, Molzahn, etc.
- **Approximations**
 - Decoupled Methods
 - Iterative Methods (e.g., SLP)

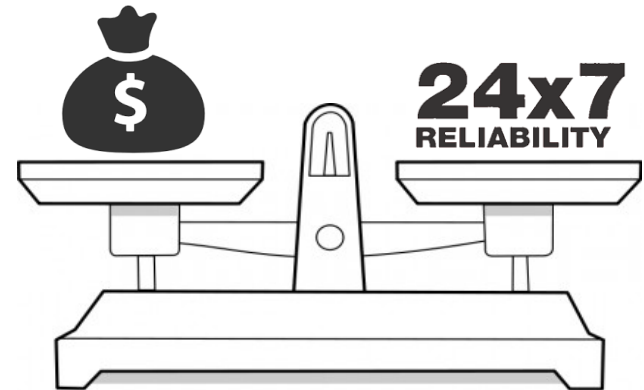


TODAY'S PRACTICE

System Operator Goals

1. Ensure Just and Reasonable Rates, Terms, and Conditions
2. Promote Safe, Reliable, Secure, and Efficient Infrastructure

... Deliver reliable and affordable electric power



Wholesale Market (Sale for Resale)

- **Federal Energy Regulatory Commission (FERC)**
Regulatory Authority:
 - Federal Power Act (FPA)
 - Public Utility Regulatory Policies Act (PURPA)

Retail Market (Sale for Use)

- **State Regulatory Commissions**
Regulatory authority varies by state statute

The Joint Board on Economic Dispatch for the Northeast Region stated in 2006 that some operational constraints are not fully represented within the current software, and the benefits of economic dispatch can be negatively impacted if all the available services and products are not accurately considered. Inaccuracies in the market software and the subsequent operator intervention cause economic inefficiencies due to suboptimal dispatch in the short-term and misplaced incentives for long-term decision-making.

Real-World OPF Applications

**Day-Ahead
Market**

**Residual Unit
Commitment**

**Real-Time
Market Look-
Ahead**

**Real-Time
Economic
Dispatch**

**Operations and Pricing
(Market Layer)**

DCOPF

**DCOPF
with
Losses**

**Decoupled
OPF**

**AC
Feasibility**

ACOPF

**Mathematical Representation of Operational/Physical Constraints
(Software Layer)**

**Transmission
Elements**

**Generation
Resources**

**Controllable
Devices**

**Operational
and Thermal
Limits**

Demand

**Power Grid
(Physical Layer)**

Unit Commitment in the Day-Ahead Market

Current Practices

UC

- Copper-plate (no network/single node)
- Ignores congestion; requires cutsets to proxy capacity limits on network
- Most tractable

SCUC+DCOPF

- Real power flows only (proportional to current)
- B θ (full) or PTDF (compact) approach

Extensions:

- Accounts for losses (extension)
- Incorporates AC feasibility; requires nomograms/cutsets to proxy reliability requirements

Proposed Approach

SCUC+ACOPF

- Co-optimizes real and reactive power dispatch
- Accounts for commitments needed for blackstart service, reactive support, voltage support, and interface control
- Nonlinear, nonconvex on meshed networks (need to approximate)

The link between physics and prices

- Locational marginal pricing (LMP) is the spot price of electricity
- Dual variable/Lagrange multiplier (λ_n) to real power balancing at all buses

$$p_n - p_n^d + p_n^g = 0 \quad (\lambda_n)$$

ACOPF

$$p_n = |v_n| \sum_{m \in \mathcal{N}} |v_m| (G_{nm} \cos \theta_{nm} + B_{nm} \sin \theta_{nm})$$

DCOPF

$$p_n = \sum_{m \in \mathcal{N}} (B_{nm} \theta_{nm}) \approx |\tilde{v}_n| \sum_{m \in \mathcal{N}} |\tilde{v}_m| (B_{nm} \theta_{nm})$$

DCOPF with losses

$$p_n = \sum_{m \in \mathcal{N}} \left(G_{nm} (\theta_{nm})^2 / 2 + B_{nm} \theta_{nm} \right)$$

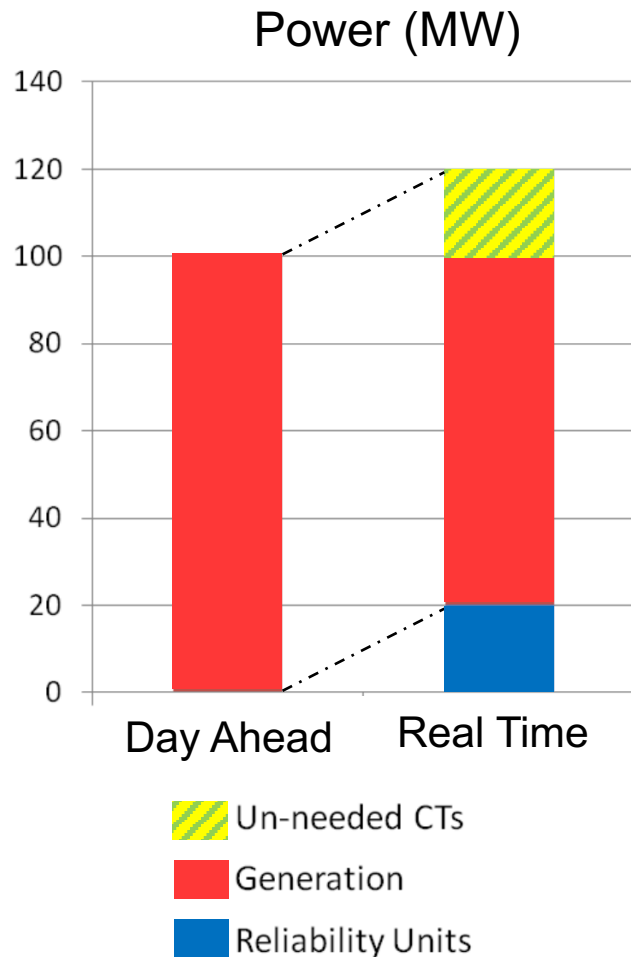
The LMP incorporates the marginal cost of supplying the next MW of load for a given location in time; includes

1. marginal unit cost,
2. cost of network congestion (due to thermal line limits), and
3. cost of real power losses on the network

- Operational Challenges
 - Committing Least Cost + Maintaining Reliability
 - Out-of-Merit Reliability Commitments
 - Better convergence between day-ahead and real-time prices

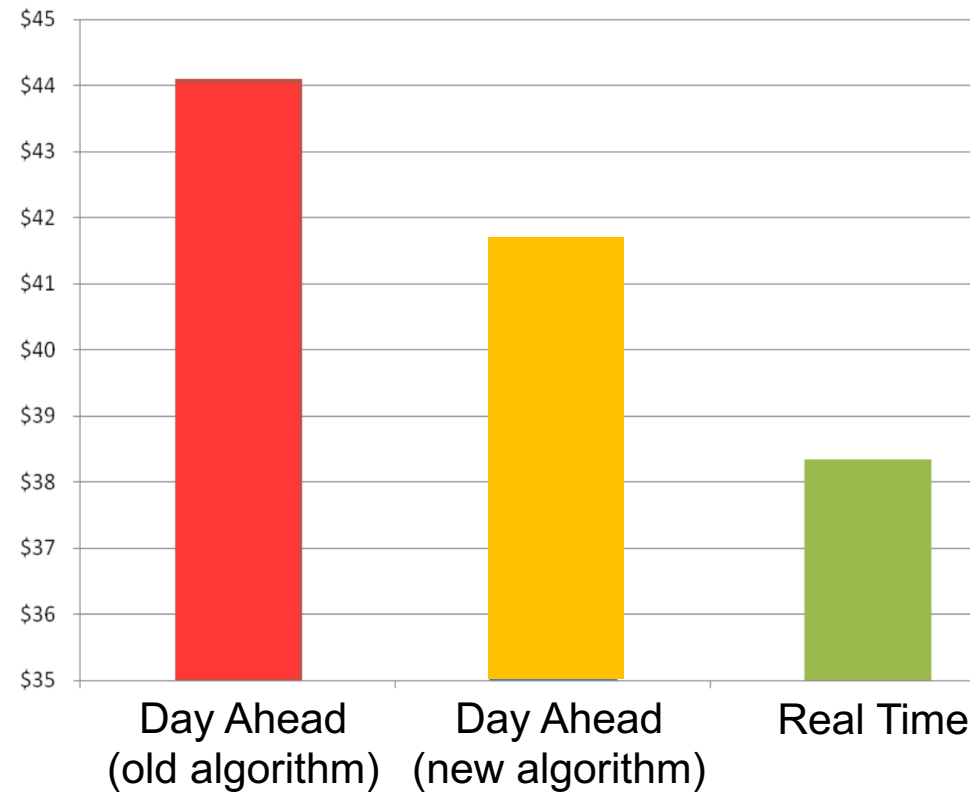
- Algorithmic Challenges
 - Accounting for reliability needs in dispatch and pricing optimization
 - Better physical representation of the generating units and underlying network

Reliability Commitment Example



Load Forecast: 100 MW

Average LMPs (\$)



PJM
October 3, 2012

Source: PJM "Impact of Reliability Units Being Included in the Day-Ahead Market" (2013)

CONTRIBUTIONS

CONTRIBUTIONS

LOCAL SOLUTION METHOD

SLP Linear Subproblem

MIN Piecewise linear cost function with penalty factors

s.t.

Line Current Flows

$$i_{k(n,m)}^r = \text{Re}\left(Y_{1,1}^k v_n + Y_{1,2}^k v_m\right), i_{k(m,n)}^r = \text{Re}\left(Y_{2,1}^k v_n + Y_{2,2}^k v_m\right) \quad \forall k \in K$$

$$i_{k(n,m)}^j = \text{Im}\left(Y_{1,1}^k v_n + Y_{1,2}^k v_m\right), i_{k(m,n)}^j = \text{Im}\left(Y_{2,1}^k v_n + Y_{2,2}^k v_m\right) \quad \forall k \in K$$

Network Current Balancing

$$i_n^r - \left(\sum_{k(n, \cdot)} i_{k(n,m)}^r + G_n^{sh} v_n^r - B_n^{sh} v_n^j \right) = 0 \quad \forall n \in N$$

$$i_n^j - \left(\sum_{k(n, \cdot)} i_{k(n,m)}^j + G_n^{sh} v_n^j + B_n^{sh} v_n^r \right) = 0 \quad \forall n \in N$$

Nodal Power Injections

First-order Taylor series

Generator Limits

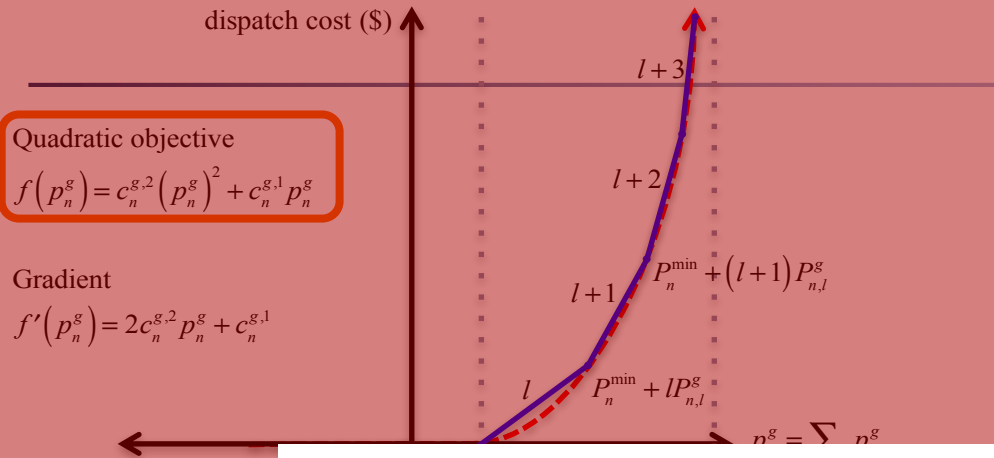
Inequality constraints with
slack variables

Nodal Voltage Magnitude Limits

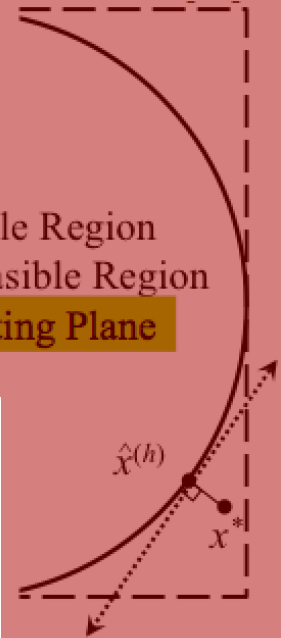
Outer approximation,
First-order Taylor series,
Step-size bounds,
Tangential cutting planes, &
Inequality constraints with
slack variables

Thermal Line (Flowgate) Limits

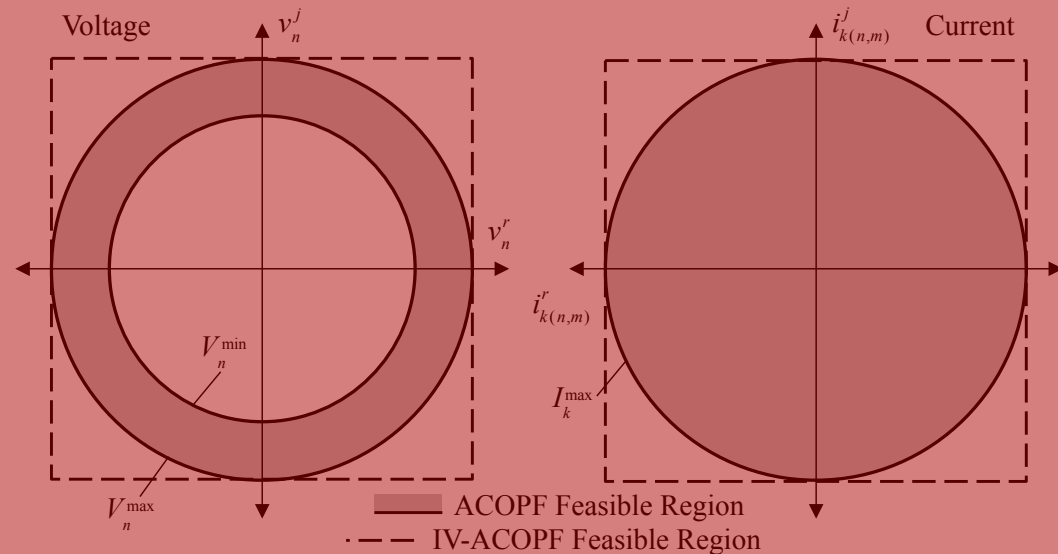
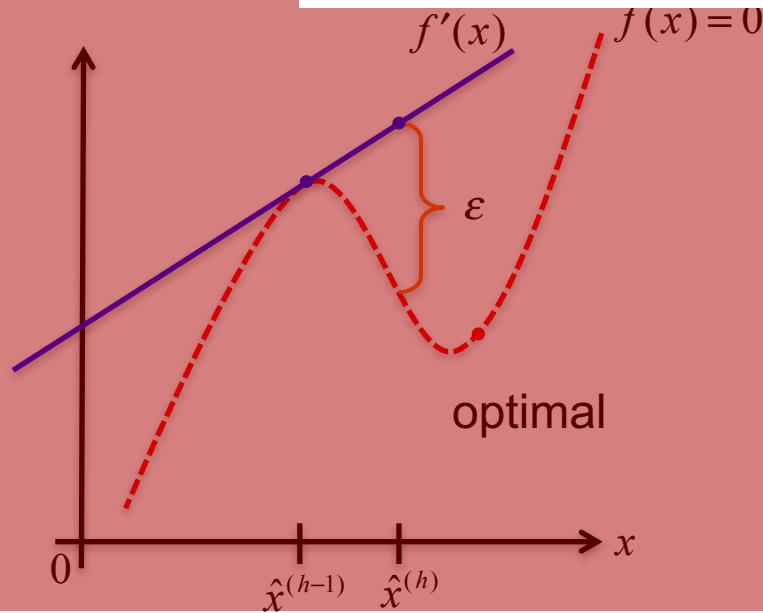
Set reduction, Outer approximation,
First-order Taylor series,
Tangential cutting planes, &
Inequality constraints with
slack variables



— ACOPF Feasible Region
 - - - IV-ACOPF Feasible Region
 Tangential Cutting Plane



Linearization Techniques



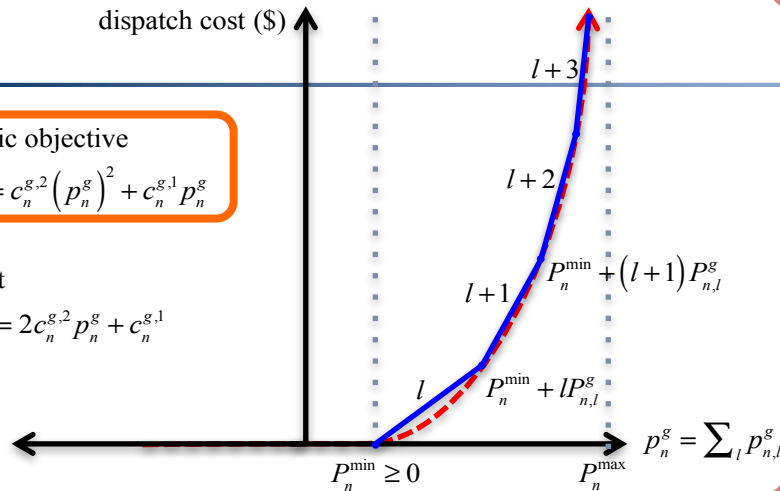
dispatch cost (\$)

Quadratic objective

$$f(p_n^g) = c_n^{g,2} (p_n^g)^2 + c_n^{g,1} p_n^g$$

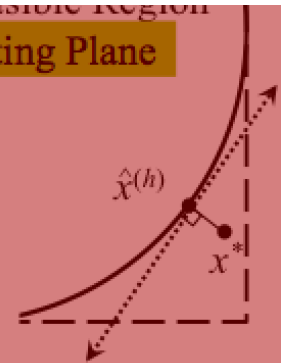
Gradient

$$f'(p_n^g) = 2c_n^{g,2} p_n^g + c_n^{g,1}$$

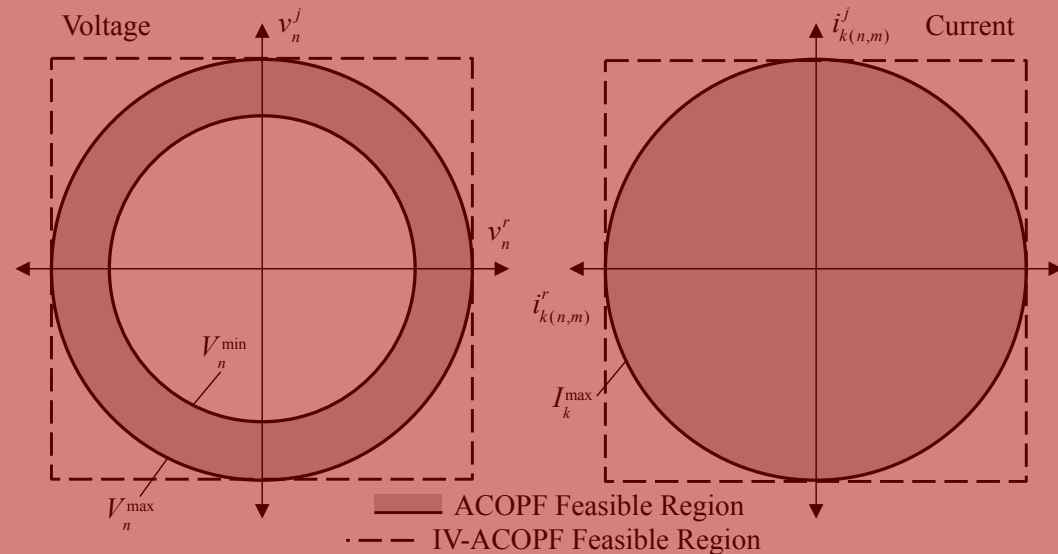
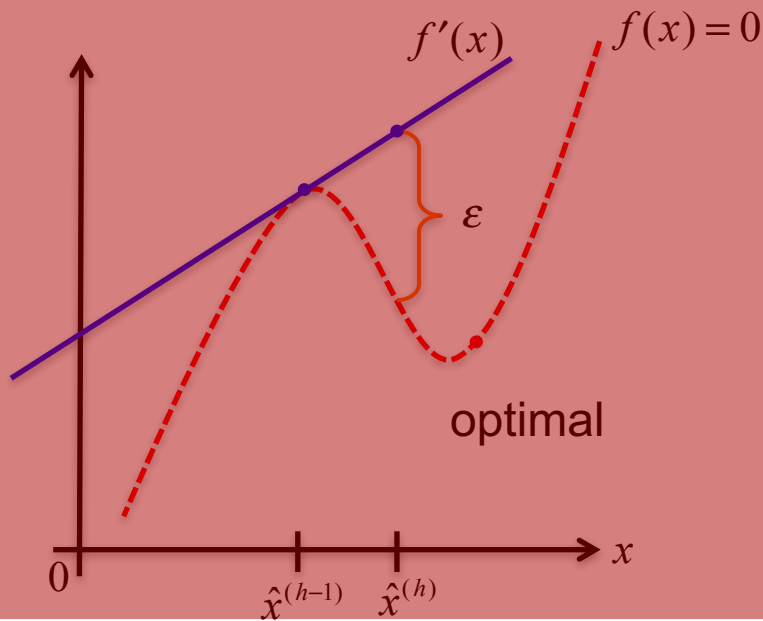


Upper Bound Cost Approximation

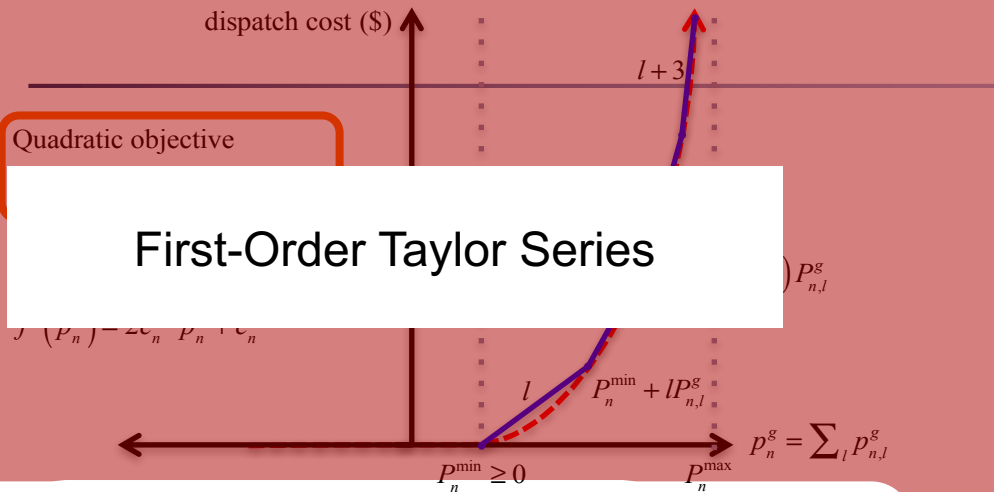
Tangential Cutting Plane



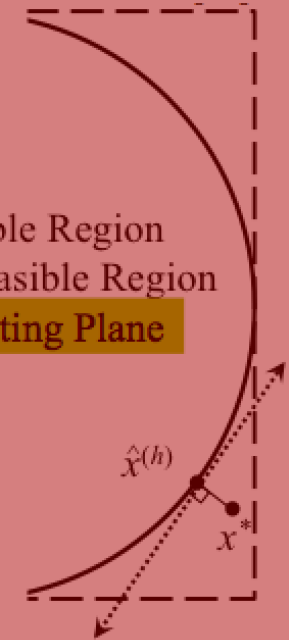
$$f(x) := f(\hat{x}^{(h)}) + J_f(\hat{x}^{(h)})(x - \hat{x}^{(h)}) + o(\|x - \hat{x}^{(h)}\|)$$



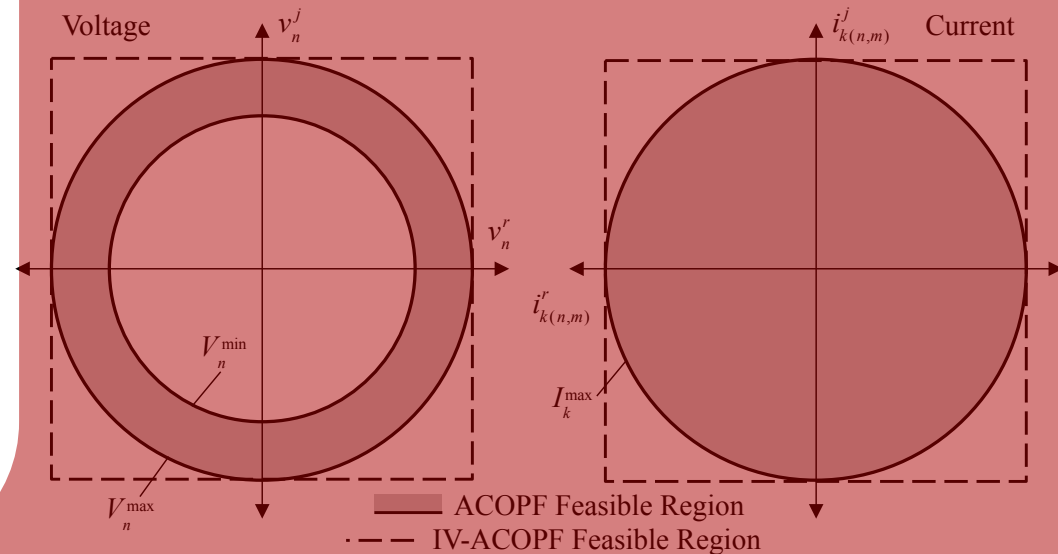
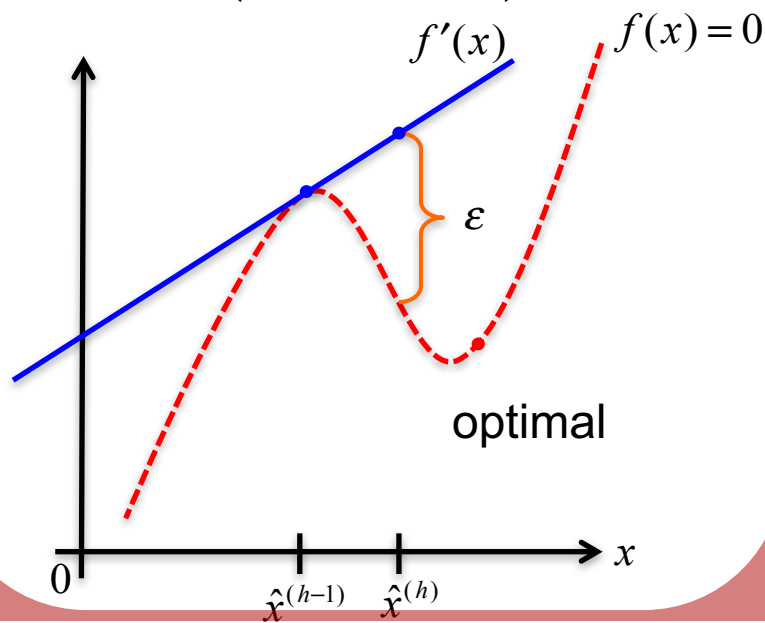
First-Order Taylor Series

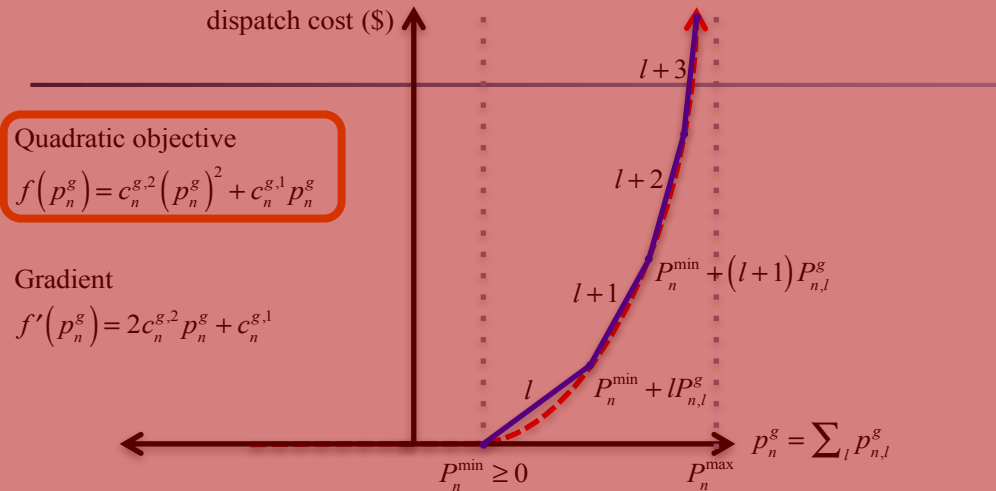


— ACOPF Feasible Region
 - - - IV-ACOPF Feasible Region
 Tangential Cutting Plane

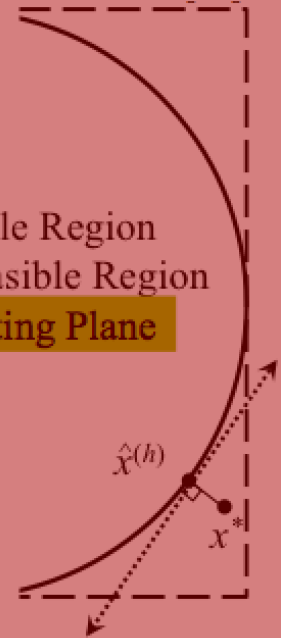


$$f(x) := f(\hat{x}^{(h)}) + J_f(\hat{x}^{(h)})(x - \hat{x}^{(h)}) + o(\|x - \hat{x}^{(h)}\|)$$

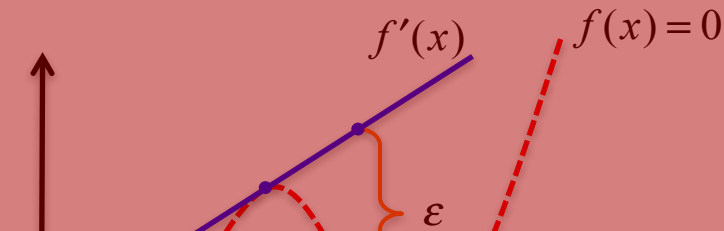




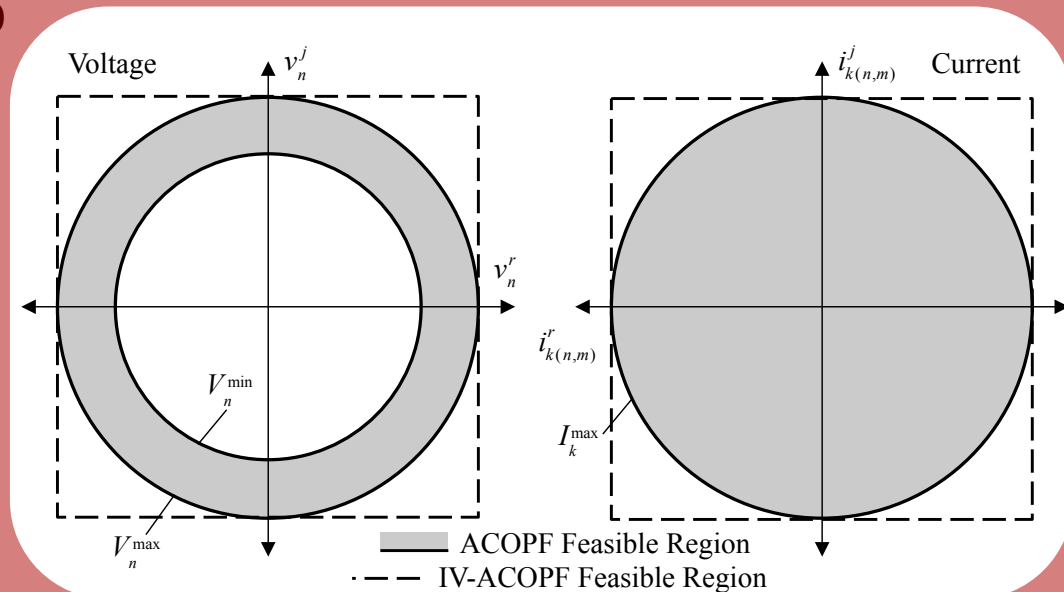
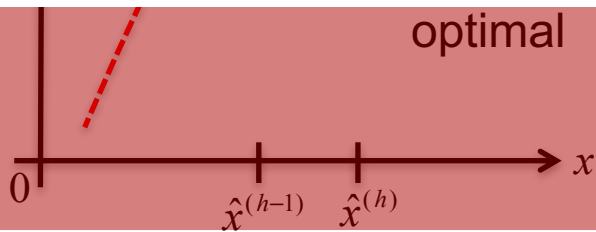
— ACOPF Feasible Region
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 Tangential Cutting Plane

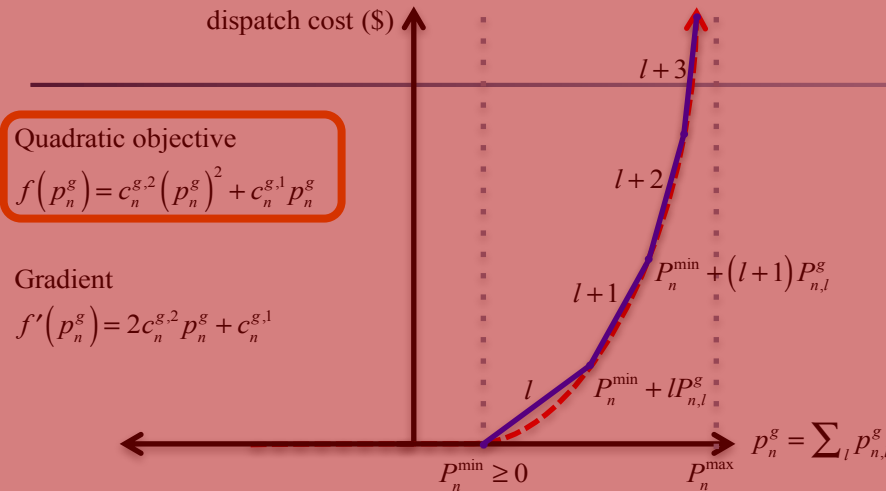


$$f(x) := f(\hat{x}^{(h)}) + J_f(\hat{x}^{(h)})(x - \hat{x}^{(h)}) + o(\|x - \hat{x}^{(h)}\|)$$

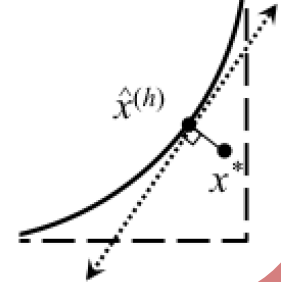


Outer Approximation of Phasors

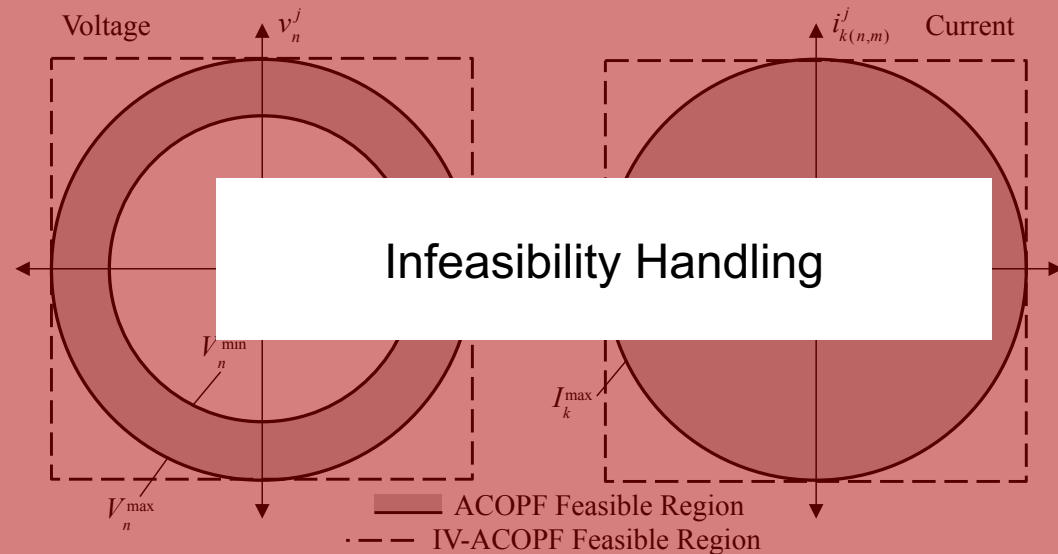
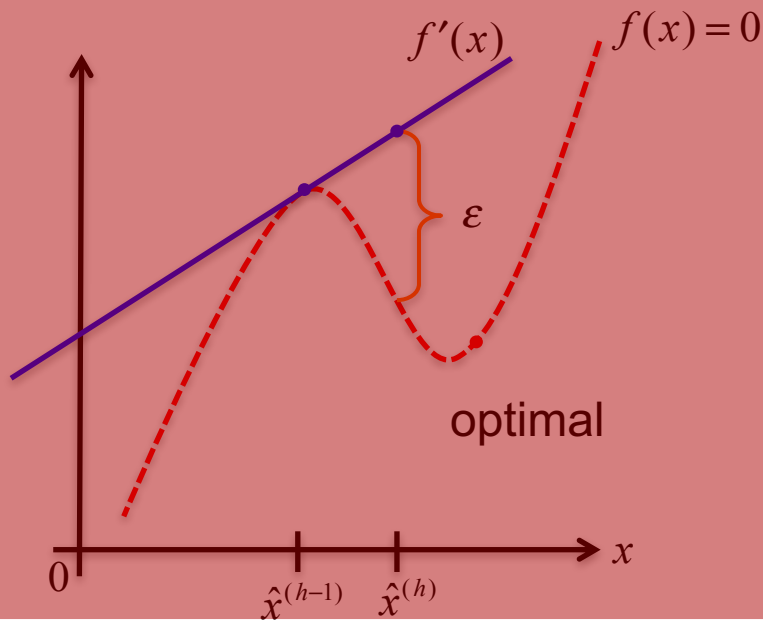




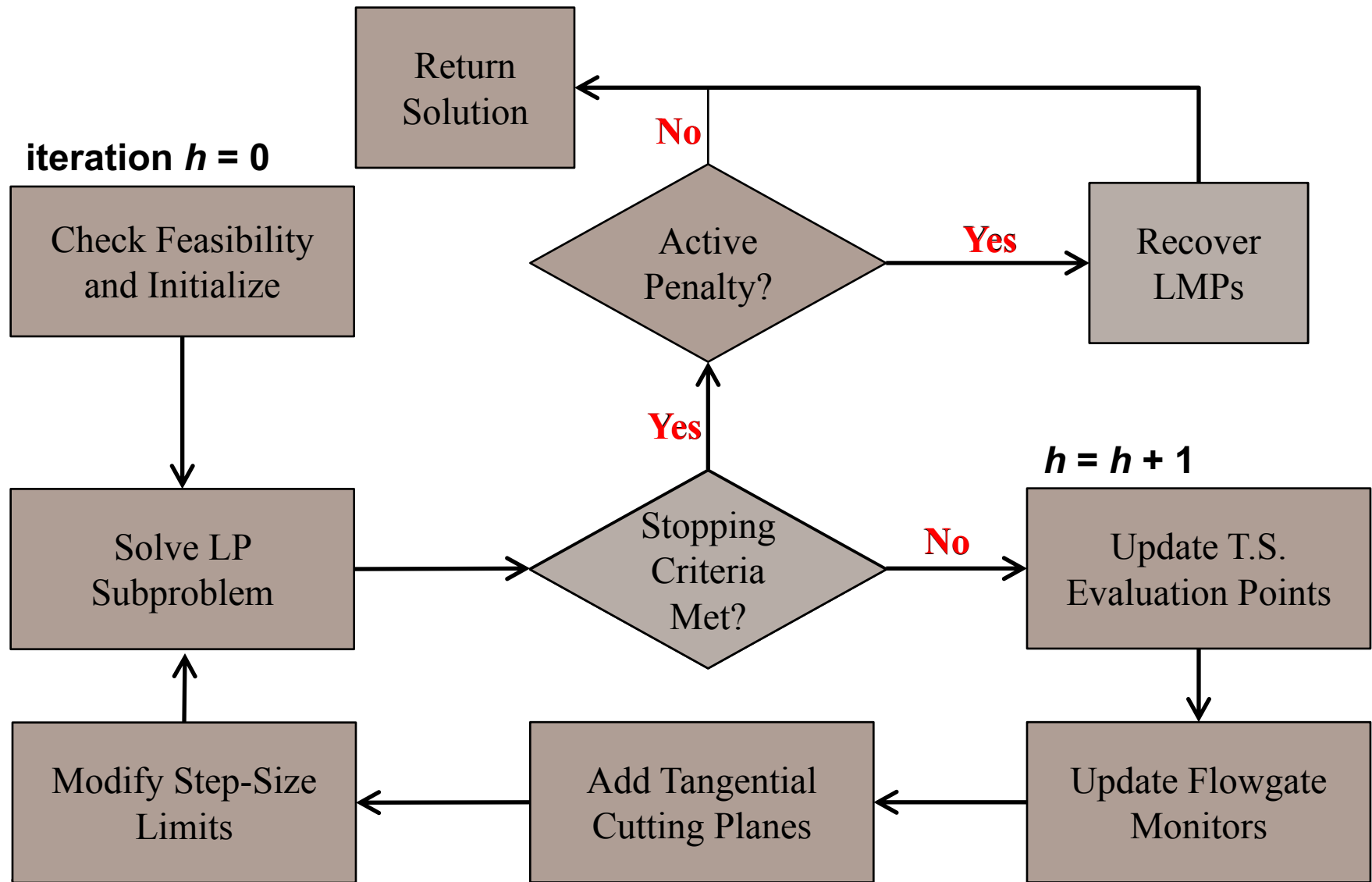
— ACOPF Feasible Region
 - - - IV-ACOPF Feasible Region
 **Tangential Cutting Plane**



$$f(x) := f(\hat{x}^{(h)}) + J_f(\hat{x}^{(h)})(x - \hat{x}^{(h)}) + o(\|x - \hat{x}^{(h)}\|)$$



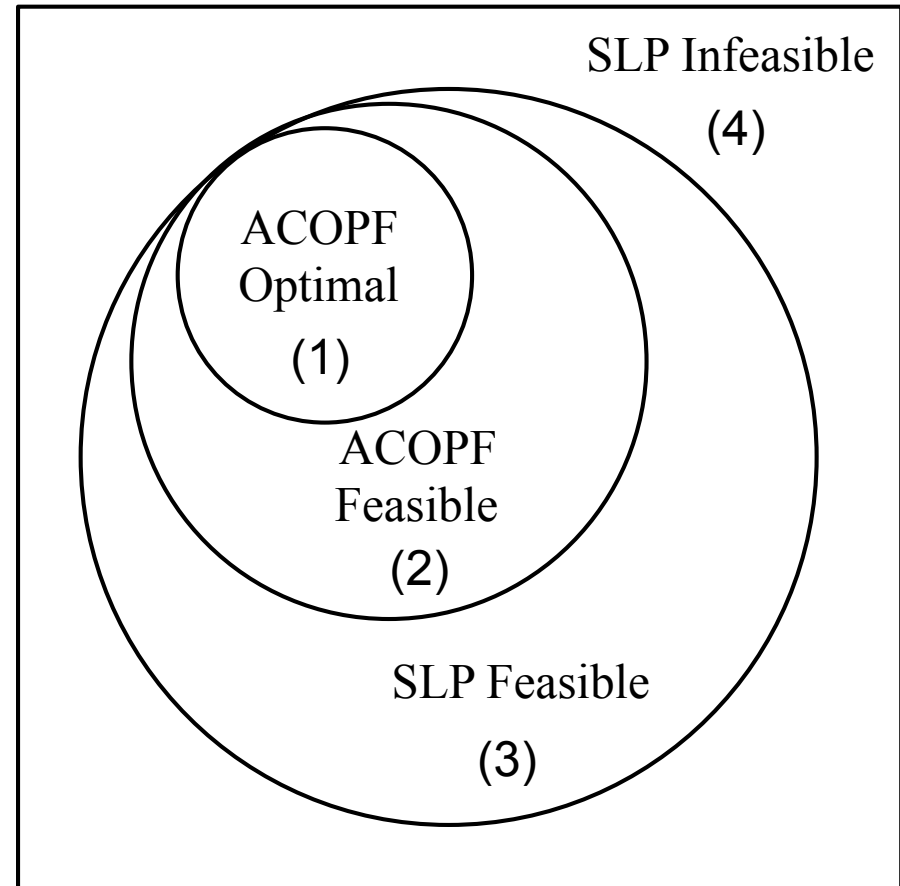
Successive Linear Program (SLP)



SLP Convergence Properties

Extends Theorem 10.3.1 of Bazaraa et al. [7]

- (1) A KKT point to the ACOPF is found
- (2) The SLP optimal solution is ACOPF feasible but not optimal
 - Still a useful solution; may be better than a DCOPF with AC feasibility or decoupled OPF solution
- (3) The SLP optimal solution is ACOPF infeasible
 - Active penalties present
 - Solution may be useful depending upon whether the violated limits are “soft” or “hard”
- (4) The SLP is infeasible
 - The ACOPF may have no solution
 - The SLP requires a better initialization



Experimental Design

- Networks
 - IEEE 14, 30, 57, 118, and 300-bus
 - Polish (MATPOWER) 2383, 2736, 2746, 3012, 3120, and 3375-bus
- Initialization Techniques
 - Flat Start
 - 10 Uniform Starts (Uniformly distributed voltage magnitude)
 - 10 DC-OPF “Hot Starts”
 - 10 AC-OPF “Hot Starts”
- Transmission Constraints
 - None, tight, and loose (network-wide fixed parameter)
- Solvers
 - Gurobi 5.6.2 and Cplex 12.5.1: Solves SLP IV-ACOPF
 - Ipopt 3.11.4 with linear solver MA27: Solves nonlinear IV-ACOPF with piecewise cost function including penalties and slack variables on the inequalities (NLP)
- Software Platform
 - Python 2.7 with Pyomo 3.2 (Sandia API for math. programming)

Time Complexity Performance

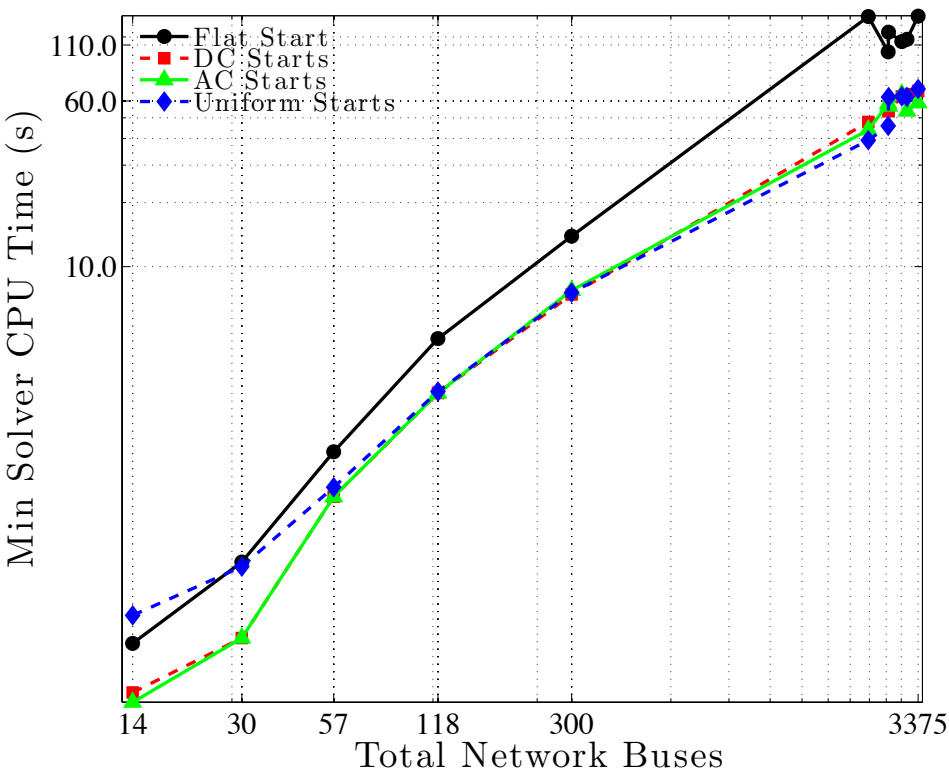
$$\Theta(|\mathcal{N}|^p)$$

Baseline	Best-Case Simulations			All Converged Simulations		
	p	R^2	RMSE (s)	p	R^2	RMSE (s)
NLP/KNITRO	1.42	0.83	1.46	1.47	0.82	1.40
NLP/IPOPT	1.13	0.95	0.60	1.34	0.97	0.50
SLP/CPLEX	0.97	0.99	0.20	1.01	0.98	0.33
SLP/Gurobi	1.01	0.99	0.21	1.03	0.98	0.33
Thermally Constrained						
NLP/KNITRO	1.39	0.88	1.13	1.39	0.89	1.08
NLP/IPOPT	1.11	0.98	0.36	1.22	0.97	0.50
SLP/CPLEX	0.99	0.99	0.17	1.00	0.98	0.31
SLP/Gurobi	1.06	0.99	0.23	1.05	0.97	0.36

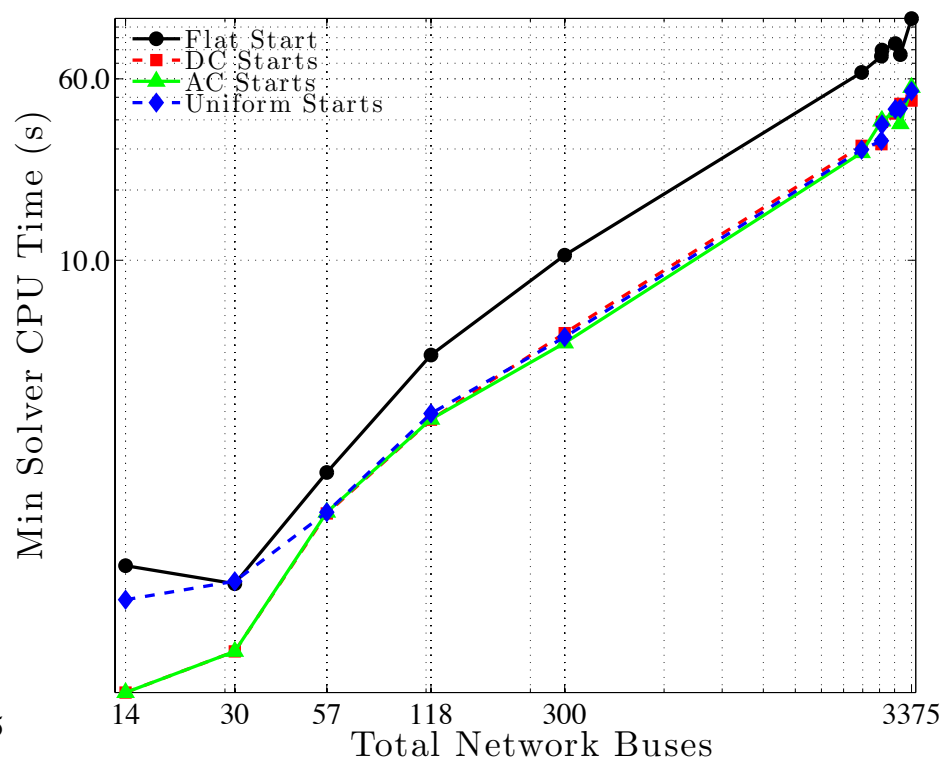
- Running time increases linearly with the network size ($p=1$ corresponds to a linear algorithmic scaling) for the SLP algorithm
- *Potentially applicable in the strict time frames of the real-time markets*

Effects of Taylor Series Initialization

Gurobi



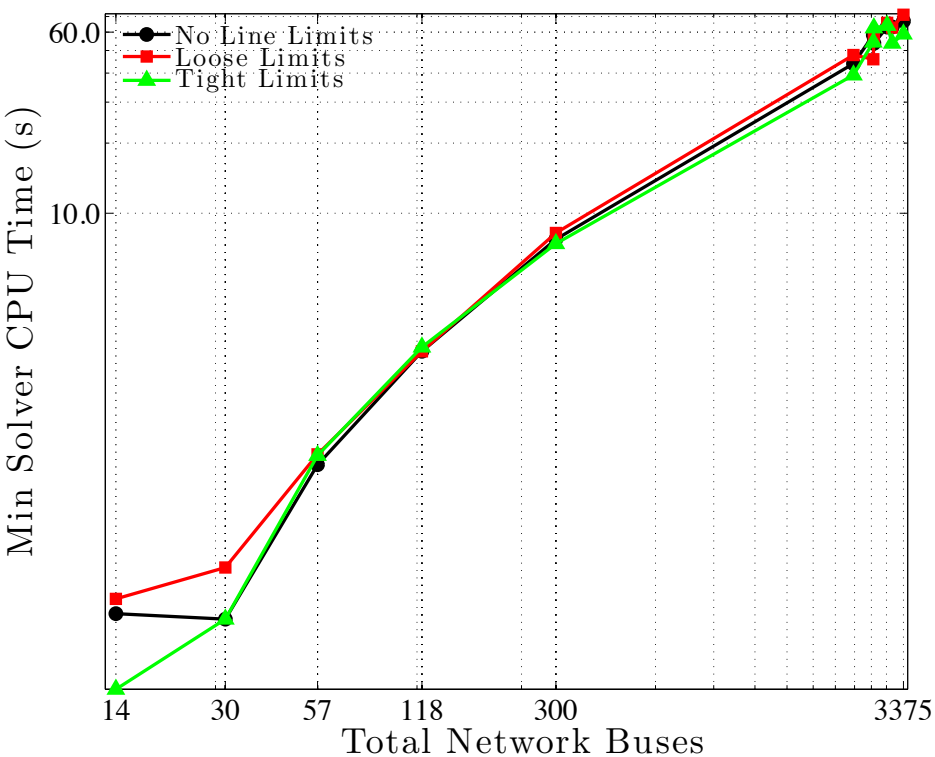
Cplex



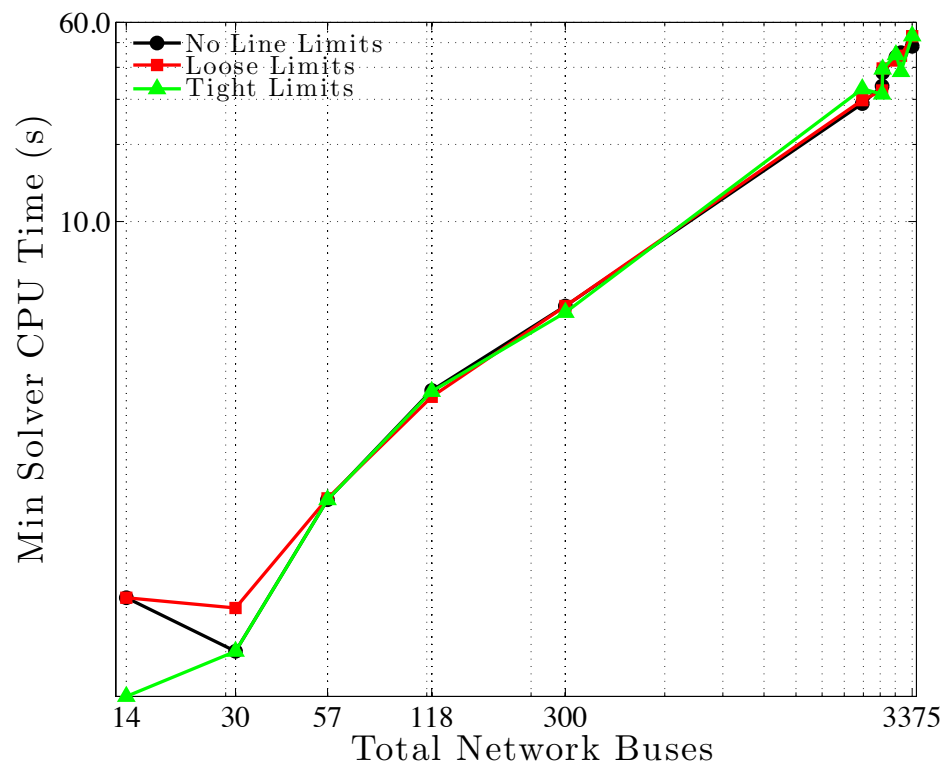
Uniform (randomized) starts perform robustly on the SLP,
as compared to the “hot starts” (DC and AC starts)

Effects of Network Congestion

Gurobi



Cplex



Congestion does not substantially
increase run-time for solving the SLP

Quality of Convergence

- SLP Performance Summary:
 - Real power costs within 0.5% (across all converged runs) of the lowest recorded cost
 - Power factors (pf) within 0.64% (across all converged runs) from the highest recorded

$$\text{pf} = \sum_n p_n^g / \left((p_n^g)^2 + (q_n^g)^2 \right)$$

- SLP with randomized starts perform robustly
- Congestion does not substantially increase the SLP runtime
- SLP active penalties in 2 cases—removed with fine-tuning

CONTRIBUTIONS GLOBAL SOLUTION METHOD

Second-Order Cone Relaxation (SOCR) (Jabr, 2006; Kocuk, 2015)

$$c_{b,b,t} = (v_{b,t}^r)^2 + (v_{b,t}^j)^2 = v_{b,t}^2$$

$$c_{b,k,t} = v_{b,t}^r v_{k,t}^r + v_{b,t}^j v_{k,t}^j = |v_{b,t}| |v_{k,t}| \cos \theta_{b,k,t}$$

$$s_{b,k,t} = v_{b,t}^r v_{k,t}^j - v_{k,t}^r v_{b,t}^j = -|v_{b,t}| |v_{k,t}| \sin \theta_{b,k,t}$$

Power Balance

$$\sum_{l \in \mathcal{L}_b^{in}} P_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} P_{l,t}^f + G_b^{sh} c_{b,b,t} + P_{b,t}^D - \sum_{g \in \mathcal{G}_b} P_{g,t}^G = 0 \quad \forall b, t$$

$$\sum_{l \in \mathcal{L}_b^{in}} Q_{l,t}^t + \sum_{l \in \mathcal{L}_b^{out}} Q_{l,t}^f - B_b^{sh} c_{b,b,t} + Q_{b,t}^D - \sum_{g \in \mathcal{G}_b} Q_{g,t}^G - \sum_{sc \in \mathcal{SC}_b} Q_{sc,t}^{SC} = 0 \quad \forall b, t$$

Power Flow

$$P_{l,t}^f = G_l^{ff} c_{b,b,t} + G_l^{ft} c_{b,k,t} - B_l^{ft} s_{b,k,t} \quad \forall l, t$$

$$Q_{l,t}^f = -B_l^{ff} c_{b,b,t} - B_l^{ft} c_{b,k,t} - G_l^{ft} s_{b,k,t} \quad \forall l, t$$

$$P_{l,t}^t = G_l^{tt} c_{k,k,t} + G_l^{tf} c_{b,k,t} + B_l^{tf} s_{b,k,t} \quad \forall l, t$$

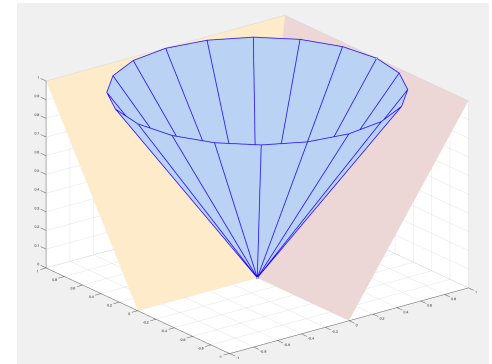
$$Q_{l,t}^t = -B_l^{tt} c_{k,k,t} - B_l^{tf} c_{b,k,t} + G_l^{tf} s_{b,k,t} \quad \forall l, t$$

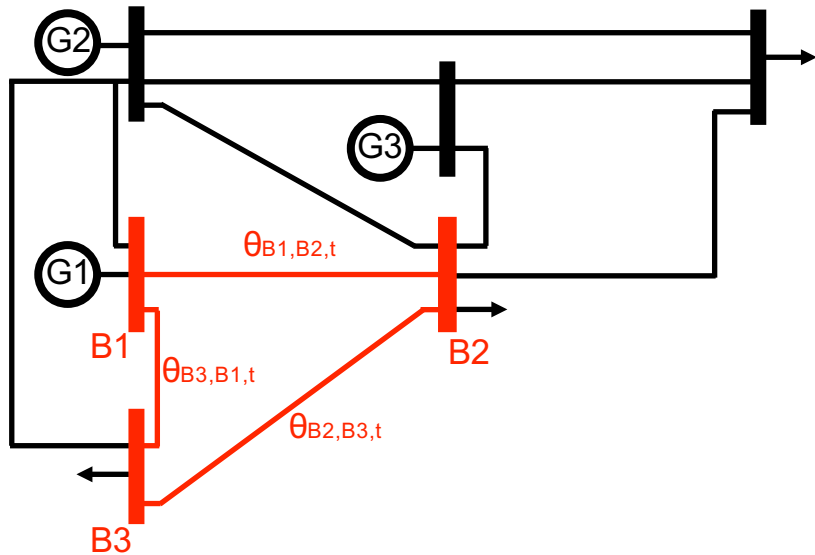
Bounds

$$(v_b^{min})^2 \leq c_{b,b,t} \leq (v_b^{max})^2 \quad \forall b, t$$

Second-Order Cone Constraints

$$c_{b,k,t}^2 + s_{b,k,t}^2 \leq c_{b,b,t} c_{k,k,t} \quad \forall l, t$$





Convex Relaxation of arctan:

Linear Over- and Under-Estimators

Optimality-Based Bound Tightening (OBBT)

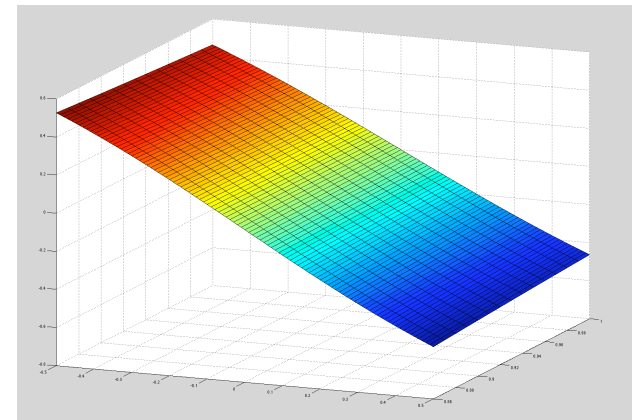
Gradually Adding Cycle Constraints

Cycle Constraints:

the sum of angle differences on each cycle equals to zero

$$\theta_{B1,B2,t} + \theta_{B2,B3,t} + \theta_{B3,B1,t} = 0$$

$$\theta_{b,k,t} = -\arctan\left(\frac{s_{b,k,t}}{c_{b,k,t}}\right)$$



Global ACOPF Solution Method Performance

Case Name	Optimal Solution	Optimality Gap (%)	CPU Time (s)	Iteration Number
Case6ww	3126.36	0.008	0.26	4
Case14	8081.52	0.003	0.43	3
Case30	574.52	0.000	0.95	5
Case39	41864.18	0.005	1.21	3
Case57	41737.79	0.006	7.29	12
Case89	5817.60	0.009	46.2	44
Case118	129660.69	0.006	18.5	14
Case300	719725.10	0.009	82.7	49
NESTA Case6ww	3143.97	0.000	0.74	7
NESTA Case14	244.05	0.003	0.22	3
NESTA Case30	204.97	0.000	0.57	4
NESTA Case39	96505.52	0.009	3.00	8
NESTA Case57	1143.27	0.006	9.62	20
NESTA Case89	5819.81	0.009	55.8	57
NESTA Case118	3718.64	0.000	93.7	55
NESTA Case300	16891.28	0.000	138.2	26

CONTRIBUTIONS

UC+ACOPF

System Parameterizations

Nodal voltage limits

Reserve requirements

Real and reactive power load

Transformer tap ratio and phase-shifters

Thermal line limit and line resistance, reactance, and susceptance

Shunts

Generator Characteristics

Synchronous condensers

Power generated and unit-on state in T0

Minimum/maximum real and reactive power outputs

Minimum up/down time

Ramp up/down limits

Startup/shutdown ramp limits

Startup lags

Startup/shutdown costs

min Production Costs + Start-up Costs + No-Load Costs

s.t. **AC Network Limits¹**

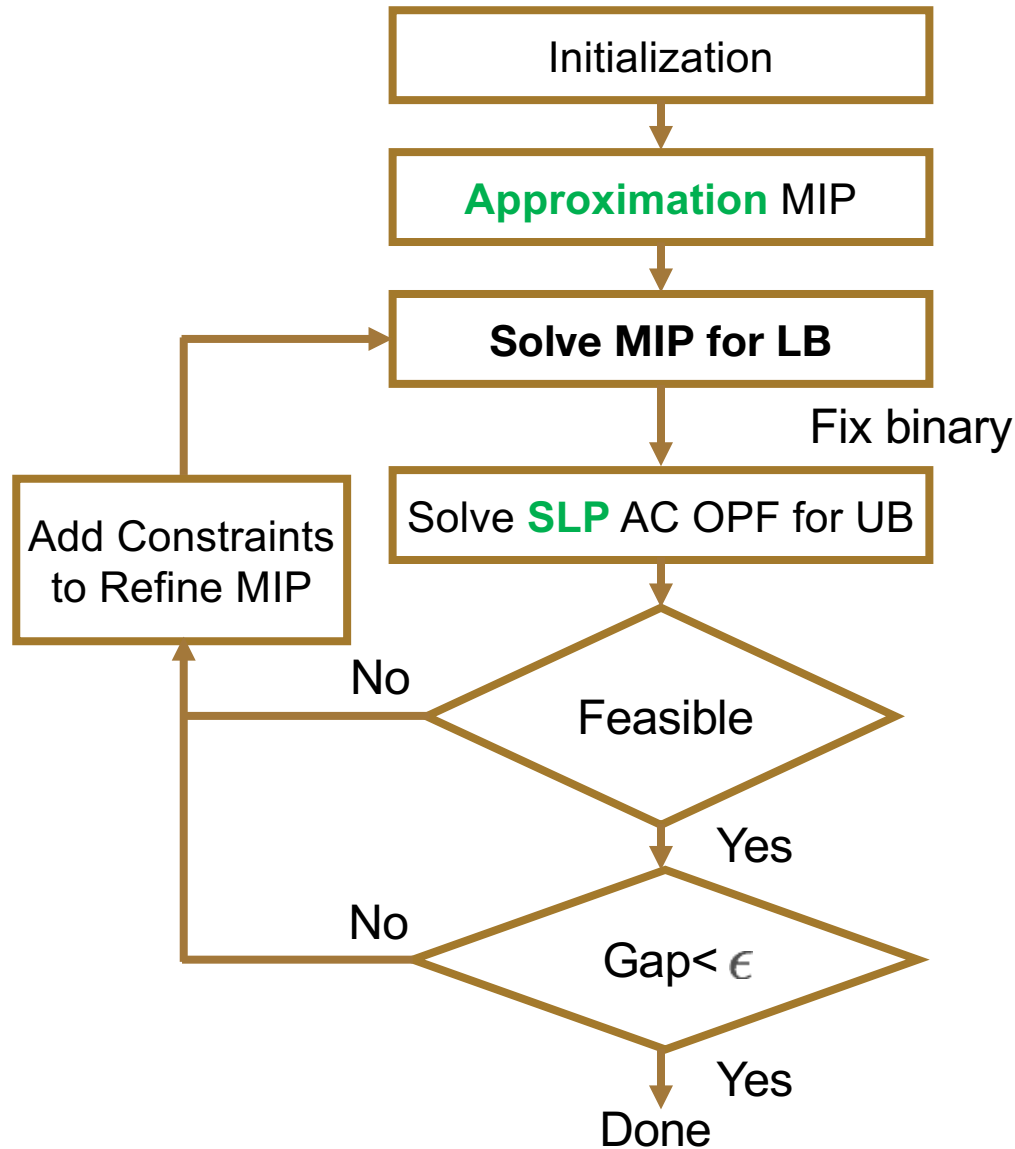
- Real power balancing
- Reactive power balancing
- Voltage magnitude bounds
- Thermal line limits
- Spinning reserves

Apparent Power Production Limits²

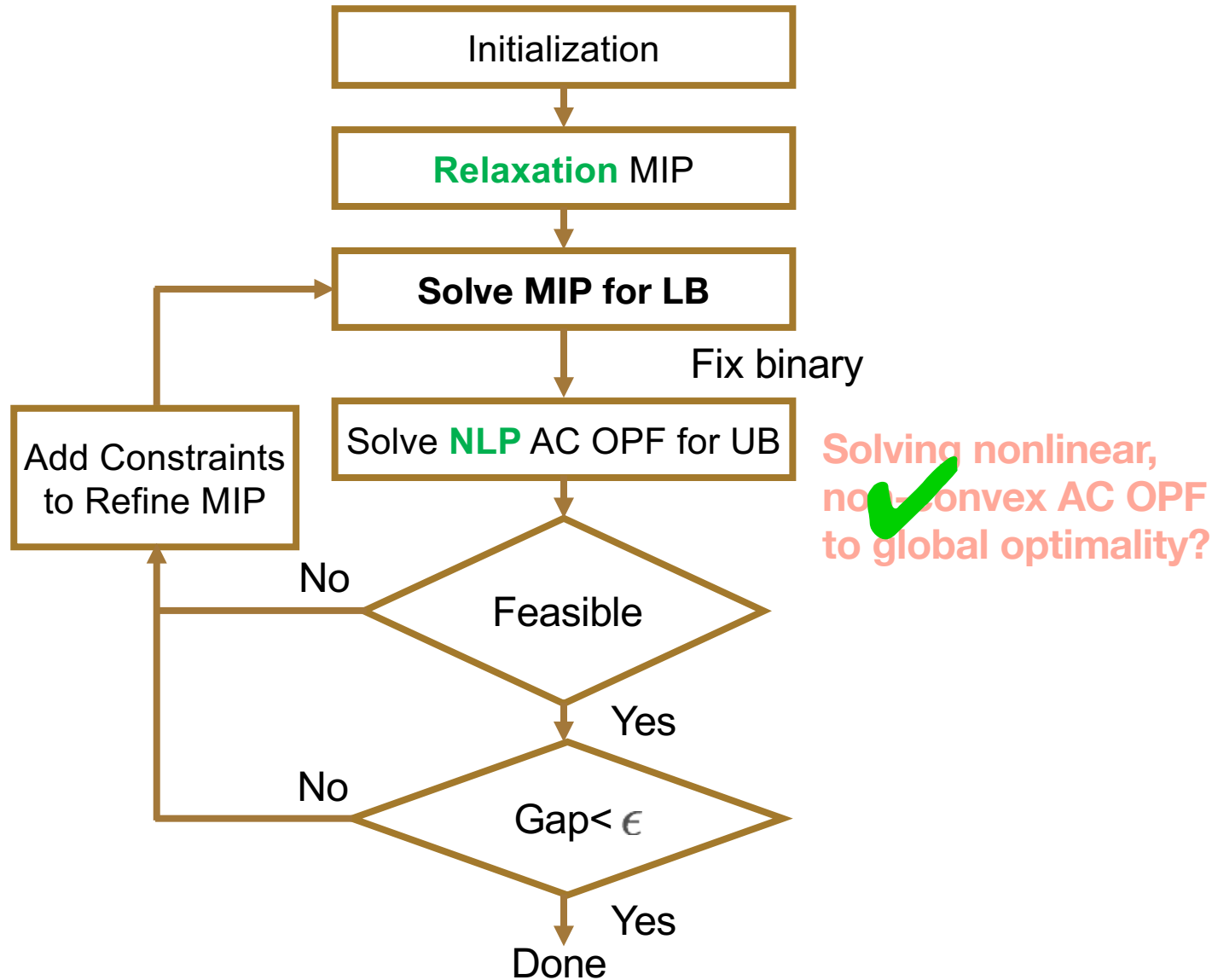
- Maximum/minimum real power generation
- Maximum/minimum reactive power generation
- Ramp up/down rates on real power
- Minimum up/down time

1. Extends Castillo, Lipka, Watson, Oren, and O'Neill. "A successive linear programming approach to solving the IV-ACOPF," *IEEE Trans. On Power Syst.*, 2016, DOI: 10.1109/TPWRS.2015.2511010.
2. Extends Morales-España, Latorre, and Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Trans. on Power Syst.*, vol. 28, no. 4, pp. 4897–4908, 2013.

Local Optimization Framework



Global Optimization Framework

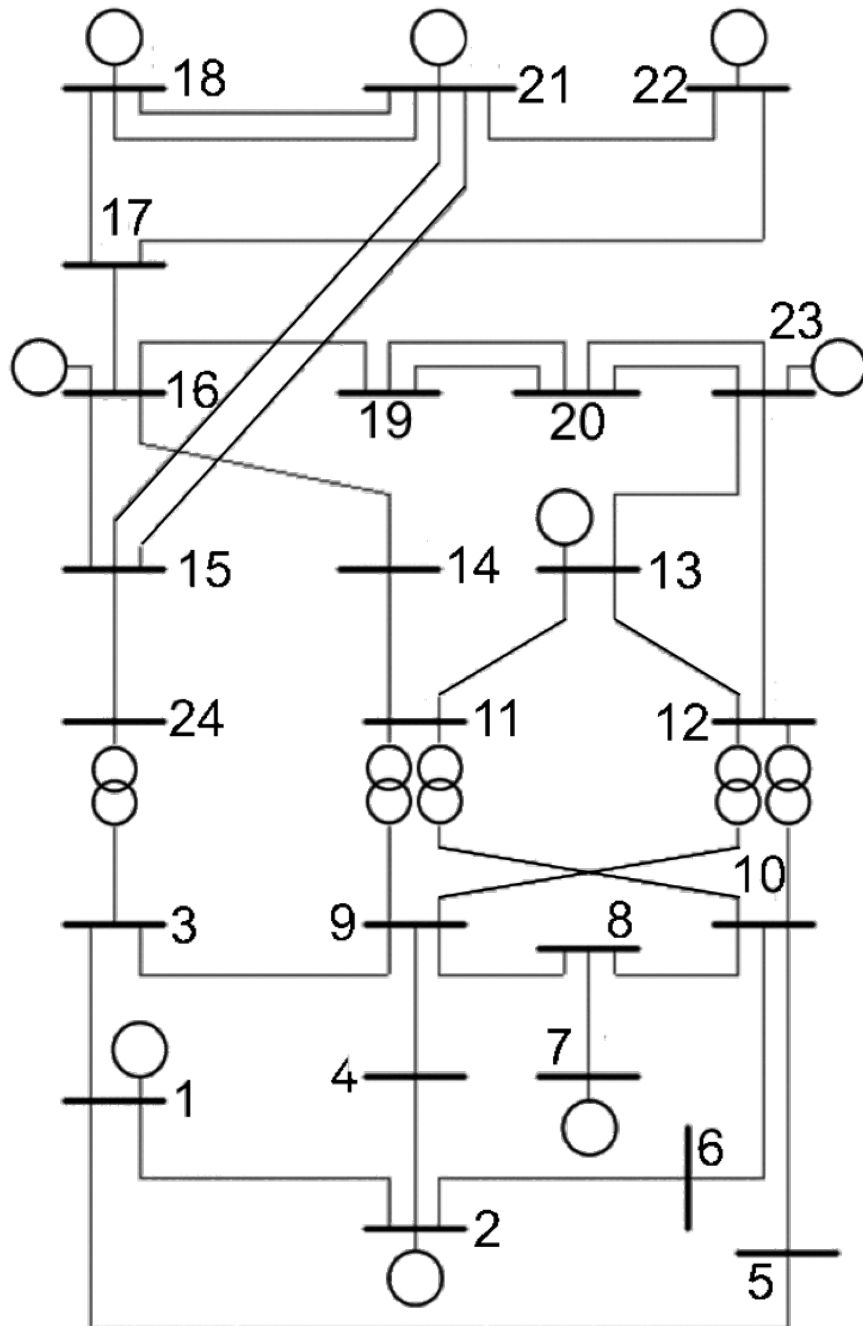


Comparative Case Studies

UC	UC+DCOPF	UC+ACOPF	UC+DCOPF+RUC
No capacity line limits No reactive power dispatch	Capacity line limits determined as $P_{\max} = V I_{\max}$ where $V = 1$ p.u. No reactive power dispatch	Capacity line limits determined as thermal line ratings ($I_{\max}$) on the current magnitude <i>Compared to DCOPF:</i> $V > 1$ p.u. → Higher power transfers $V < 1$ p.u. → Lower power transfers	Initially a SCUC+DCOPF is solved to determine the commitment schedule; if the solution is not AC feasible, then solve the SCUC+ACOPF with the specified commitment schedule in order to determine residual (add'l) unit commitments and change in dispatch.

Note: Thermal limits different in global solution method (i.e., apparent power thermal limit) and local solution method (i.e., current thermal limit) so a direct comparison (above) is inexact

RTS-79^[9]



24 nodes

32 generators

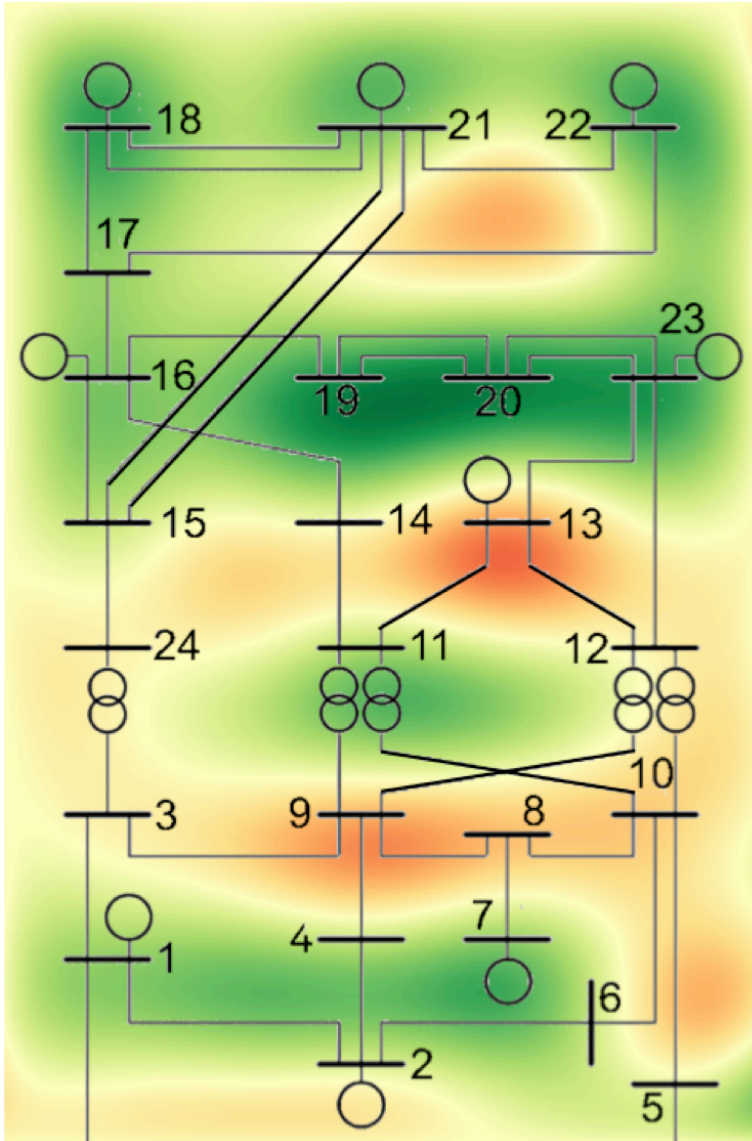
17 loads

1 synchronous condenser

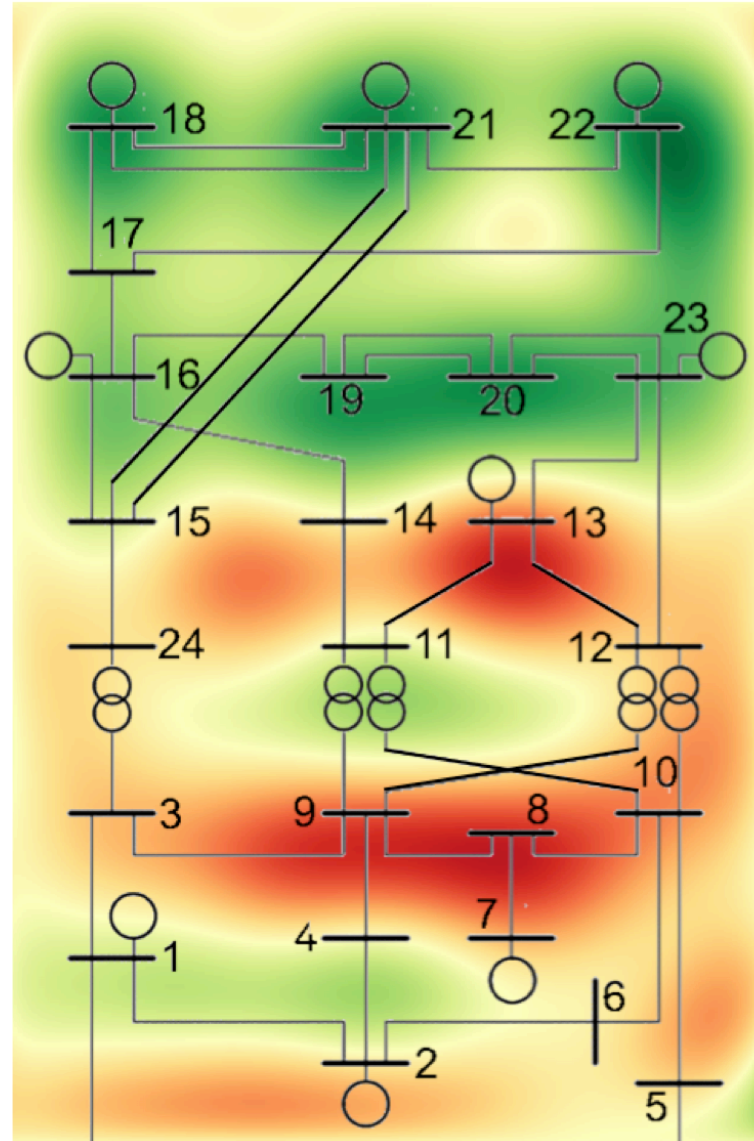
38 network elements/lines

	Cost (\$)	AC Feasible?
UC	823,145 (base)	NO
UC+DCOPF	823,894 (+0.1%)	NO
Local UC+ACOPF	895,281 (+8.8%)	YES
UC+DCOPF+RUC	896,169 (+8.9%)	YES
Global UC+ACOPF	895,096 (+8.7%)	YES

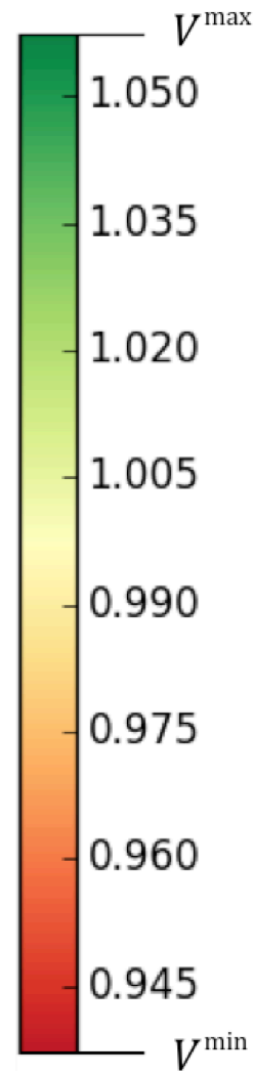
RTS-79 Voltage Levels

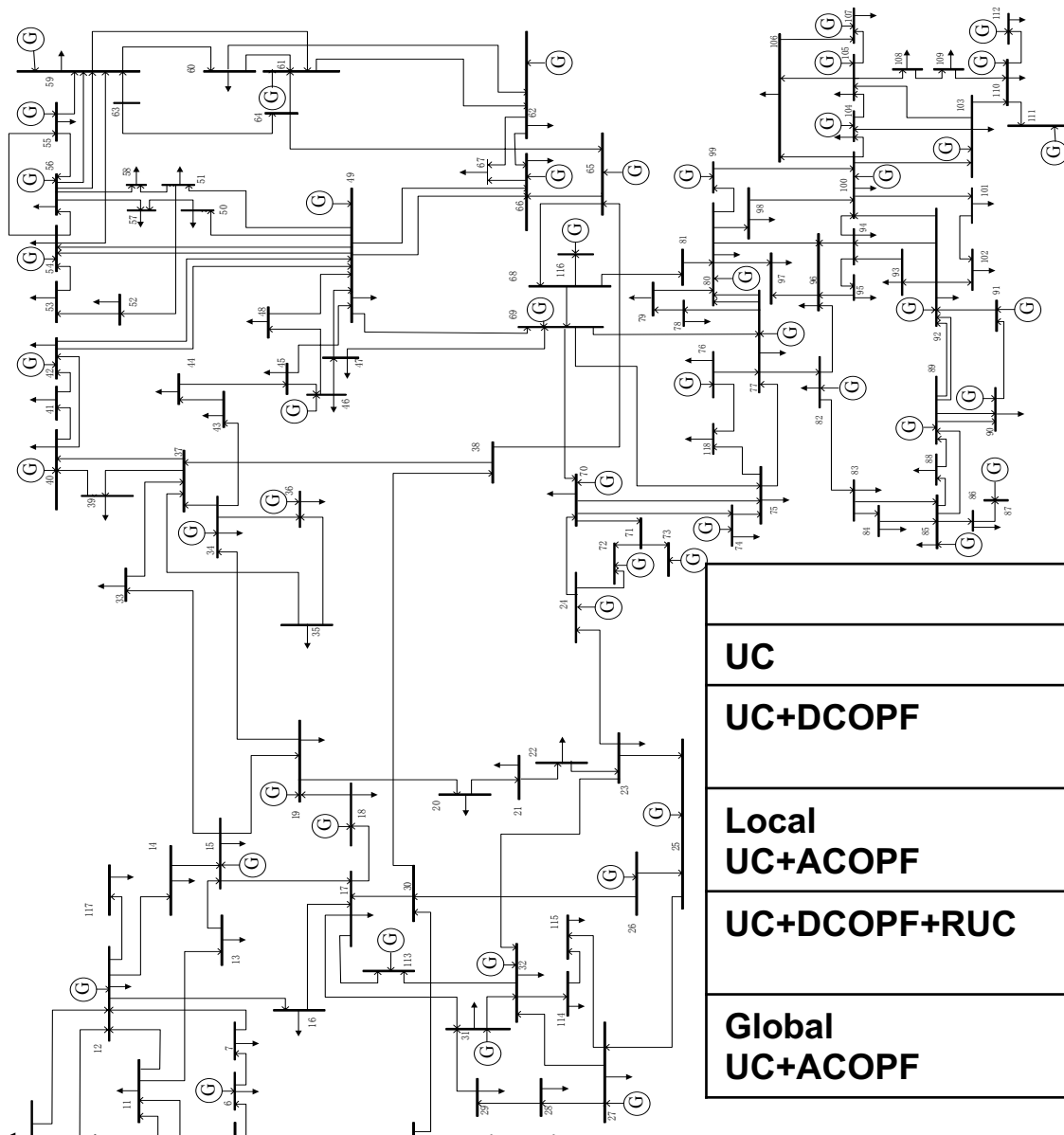


UC+ACOPF
(ACOPF FEASIBLE)



UC+DCOPF
(LOAD MISMATCH)





IEEE-118^[10]

118 nodes

54 generators

91 loads

186 network elements/lines

	Cost (\$)	AC Feasible?
UC	811,658 (base)	NO
UC+DCOPF	814,715 (+0.4%)	NO
Local UC+ACOPF	843,591 (+3.9%)	YES
UC+DCOPF+RUC	844,922 (+4.1%)	YES
Global UC+ACOPF	835,926 (+3.0%)	YES

- Key Takeaway: Results indicate considerable divergence between the market settlements and stability/reliability requirements

Computational Results (Local Method)

10 piecewise linear segments, relative MIP gap tolerance 0.1%

	UC MILP	UC+DCOPF MILP	UC+ACOPF MILP SLP		UC+DCOPF+RUC MILP SLP	
Solution Time (s)						
6-Bus	0.13	0.21	0.88(3)	0.07(50)	1.02(1, 1)	0.06(33)
RTS-79	1.86	6.76	88.71(3)	0.75(36)	10.37(1, 2)	0.45(26)
IEEE-118	5.04	21.42	110.17(2)	5.06(46)	57.2(1, 1)	3.71(33)
Cost (\$)						
6-Bus	101, 270	106, 987	101, 763		102, 523	
RTS-79	823, 145	823, 894	895, 281		896, 169	
IEEE-118	811, 658	814, 715	843, 591		844, 922	

- Most of the OA algorithm time spent in the MILP
- UC+ACOPF: 5x-15x slower than the UC+DCOPF
- UC+DCOPF+RUC: 1.5x-5x slower than the UC+DCOPF

Local v. Global UC+ACOPF Method

Case	Problem Formulation	Upper Bound	Lower Bound	Relative Gap (%)	CPU Time (s)
6-Bus	Global	101,763	101,655	0.11%	3.6
	Local	101,763	-	0.11%	0.95
RTS-79	Global	895,096	893,967	0.13%	266.4
	Local	895,281	-	0.15%	89.46
IEEE-118	Global	835,926	833,057	0.34%	8480
	Local	843,591	-	1.25%	115.23

- On the largest test case, the approximation method is over 70x faster, at the cost of 0.91% in relative optimality gap change
- Why should we care about a 0.91% gap? What tolerance is solved in practice?
- Note: Thermal limits different in global solution method (apparent power thermal limit) and local solution method (current thermal limit) so a direct comparison (above) is inexact

ONGOING CHALLENGES

- Hybrid strategy that leverages current MILP solvers that industry and ISOs use and balances between quality and speed
- Applying decomposition techniques for parallel, distributed optimization (e.g., ADMM for the UC problem in Feizollahi et al. [11] and ADMM for the ACOPF problem in Sun et al. [12])
- Improving the performance of the master problem of the OA algorithm
- Comparing the fidelity and computational performance to current market practices on larger scale networks

- A. Castillo, C. Laird, C. A. Silva-Monroy, J.-P. Watson, R.P. O'Neill. "The Unit Commitment Problem with AC Optimal Power Flow Constraints." Transactions on Power Systems (2016) DOI: [10.1109/TPWRS.2015.2511010](https://doi.org/10.1109/TPWRS.2015.2511010)
- A. Castillo, P. Lipka, J.-P. Watson, S.S. Oren, R.P. O'Neill. "A Successive Linear Programming Approach to Solving the IV-ACOPF." Transactions on Power Systems (2015) DOI: [10.1109/TPWRS.2015.2487042](https://doi.org/10.1109/TPWRS.2015.2487042)
- J. Liu, A. Castillo, J.-P. Watson, C. Laird. "Global Solution Strategies for the Network-Constrained Unit Commitment (NCUC) Problem with Nonlinear AC Transmission Models." submitted to Mathematical Programming Computation (2016)