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Diffeomorphism Group Representations in Relativistic Quantum Field Theory

Gerald A. Goldin and David H. Sharp

It has been an honor to present this talk in memory of my friend and colleague S. Twareque Ali, who passed away in January 2016. – G. A. Goldin

Abstract. We explore the role played by the diffeomorphism group and its unitary representations in relativistic quantum field theory. From the quantum kinematics of particles described by representations of the diffeomorphism group of a space-like surface in an inertial reference frame, we reconstruct the local relativistic neutral scalar field in the Fock representation. An explicit expression for the free Hamiltonian is obtained in terms of the Lie algebra generators (mass and momentum densities). We suggest that this approach can be generalized to fields whose quanta are spatially extended objects.

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1. Introduction

The focus of this article is on the role played by diffeomorphism groups and their representations in relativistic quantum field theory (QFT).

We highlight the important fact that diffeomorphism group representations arise naturally if one starts with the well-known Fock representation of the free, neutral relativistic scalar field, describing noninteracting bosons [6, 7]; and we show how this occurs. The mass density and the momentum flux density obtained in this way are reference frame-dependent constructs; they are not local in spacetime, and not Lorentz covariant. However, they are essential to the description of what one actually *measures* – particle locations at particular times, and/or trajectories. Indeed, such measurements are always taking place in a specific inertial reference frame.

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Our main idea is to reverse the direction of this construction. That is, we propose to start with a fixed frame of reference, and with respect to that frame, to obtain the quantum kinematics described by unitary representations of the group of diffeomorphisms of space and its semidirect product with the group of scalar functions. Then we identify hierarchies of such representations, and introduce intertwining creation and annihilation fields. Only at that point do we introduce the spacetime symmetry group (the Poincaré group in this case) which provides the information needed to construct local relativistic fields out of the intertwining fields.

In Sec. 2 of this article, we briefly review results on diffeomorphism group representations in nonrelativistic quantum theory. Sec. 3 highlights our earlier characterization of hierarchies of representations. In Sec. 4 is about obtaining current algebra and the corresponding diffeomorphism group representations from the field theory of relativistic neutral scalar bosons. In Sec. 5, we discuss more generally how one can begin with "nonrelativistic" representations of the diffeomorphism group and the current algebra, and obtain a relativistic quantum field from them. We also express the free relativistic Hamiltonian explicitly in terms of the original particle density and current operators. Sec. 6 mentions some possible directions opened up by our approach.

2. Diffeomorphism groups in Galilean quantum theory

Diffeomorphism groups and their unitary representations play a fundamental role in nonrelativistic (Galilean) quantum theory [1, 2]. To set the stage, we review this briefly.

Let Σ be the manifold of physical space. Of course Σ can be regarded as a submanifold of spacetime, with (for example) $t = 0$. In Galilean theory, Σ is independent of the velocity of the observer. Let $\mathcal{D} = C_0^\infty(\Sigma)$ be the group of compactly-supported smooth real-valued functions on Σ, under pointwise addition, and let $\mathcal{K} = \mathit{Diff}_0(\Sigma)$ be the group of compactly supported C^∞ diffeomorphisms of Σ, under composition. Let $G = \mathcal{D} \times \mathcal{K}$ be the natural semidirect product of these groups. Then the irreducible, continuous unitary representations of G describe the quantum kinematics of a wide variety of physical systems.

For $(f, \phi) \in \mathcal{D} \times \mathcal{K}$, let us write a continuous unitary representation (CUR) of G as $U(f)V(\phi)$. Under very general conditions, one may realize the representation in a Hilbert space $\mathcal{H} = L^2_\mu(\Gamma, \mathcal{M})$, with [2]

$$[U(f)\Psi](\gamma) = \exp i\langle\gamma, f\rangle\Psi(\gamma),$$

$$[V(\phi)\Psi](\gamma) = \chi_\phi(\gamma)\Psi(\phi\gamma)\sqrt{\frac{d\mu_\phi}{d\mu}(\gamma)}\,; \qquad (2.1)$$

where: Γ is a configuration space whose elements (denoted γ) are continuous linear functionals on $\mathcal{D}$; $\langle\gamma, f\rangle$ denotes the value of γ at f; $\mathcal{M}$ is a complex inner product space (accommodating vector-valued wave functions), and $\Psi \in$

$\mathcal{H}$ takes values in $\mathcal{M}$; $\phi\gamma$ denotes the natural group action of a diffeomorphism $\phi \in \mathcal{K}$ on Γ; μ is a measure on Γ quasiinvariant under diffeomorphisms; $d\mu_\phi/d\mu$ is the Radon-Nikodym derivative of the transformed measure with respect to the original one; and $\chi_\phi(\gamma)$ is a unitary 1-cocycle acting in $\mathcal{M}$. Each of these has a fairly direct physical interpretation in quantum mechanics.

The Lie algebra of self-adjoint local current operators is defined in a continuous unitary representation $U(f)V(\phi)$ of $\mathcal{G}$ by:

$$\rho(f) = m \lim_{s\to 0} \frac{U(sf) - I}{is}, \quad J(\mathbf{g}) = \hbar \lim_{s\to 0} \frac{V(\phi_s^{\mathbf{g}}) - I}{is} \tag{2.2}$$

where: $f \in \mathcal{D}$, $\mathbf{g} \in \mathrm{vect}_0^\infty(\Sigma)$, and $\phi_s^{\mathbf{g}}$ is the flow generated by the vector field $\mathbf{g}$ under the real parameter s; m is a unit mass, and $\hbar$ is Planck's constant (over 2π). Here ρ describes the (space-averaged) mass density, and J the (space-averaged) momentum flux density. Then:

$$[\rho(f_1), \rho(f_2)] = 0, \qquad [\rho(f), J(\mathbf{g})] = i\hbar\rho(L_{\mathbf{g}}f)$$
$$[J(\mathbf{g_1}), J(\mathbf{g_2})] = -i\hbar J([\mathbf{g_1}, \mathbf{g_2}])\,, \tag{2.3}$$

where $[\mathbf{g_1}, \mathbf{g_2}]$ is the Lie bracket of the vector fields.

This framework unifies descriptions of the quantum kinematics of a wide variety of systems. Particular families of representations describe configuration spaces and exchange statistics for N-particle systems, including systems of indistinguishable particles satisfying bosonic statistics, fermionic statistics, and parastatistics. In $(2+1)$-dimensional spacetime, one obtains anyonic (braid group) statistics and non-abelian braid statistics. Tightly-bound composite particles (quantum dipoles, quadrupoles, etc.) are also described.

One further obtains configuration spaces of infinite but locally finite particle systems, as in statistical mechanics, as well as infinite systems with accumulation points. Systems of extended configurations, such as vortex patches, filaments, and tubes, arise also obtained in this framework [2, 3, **?**].

3. Hierarchies of representations

The irreducible unitary diffeomorphism group representations fall naturally into *hierarchies*, whose intertwining operators (satisfying a natural commutator bracket with the densities and currents) create and annihilate objects of the same kind. These intertwining operators have an interpretation as "second-quantized" fields, which are general enough to describe not only point particles but extended objects [4, 5].

For example, consider a family of N-particle representations, where $N = 0, 1, 2, \ldots$. We have the Hilbert spaces $\mathcal{H}_N$, and the unitary representations $U_N(f)V_N(\phi)$. We are entitled to call the family a *hierarchy* if for each N there exists an operator-valued distribution $\psi_N^* : \mathcal{H}_N \to \mathcal{H}_{N+1}$ (the creation field), such that for all $h \in \mathcal{D}$, $f \in \mathcal{D}$, and $\phi \in \mathcal{K}$,

$$U_{N+1}(f)\psi_N^*(h) = \psi_N^*(U_{N=1}(f)h)U_N(f)\,, \tag{3.1}$$

$$V_{N+1}(\phi)\psi_N^*(h) = \psi_N^*(V_{N=1}(\phi)h)V_N(\phi)\,. \tag{3.2}$$

Physically, if we begin with an N-particle state, create a new particle in state h and then transform the resulting $(N+1)$-particle state by the unitary group representation U_{N+1} (or respectively, V_{N+1}), the result is the same as if we first transform the N-particle state by U_N (resp. V_N), and then create the new particle in the state obtained by transforming the 1-particle state h by U_1 (resp. V_1). The construction is nontrivial because creation and annihilation within a hierarchy must respect the particle statistics (bosonic, fermionic, or anyonic). When $\mathcal{H}_1 = L^2(\mathbb{R}^3, \mathbb{C})$, as in the 1-particle representation of $\mathcal{D} \times \mathcal{K}$, we write $\psi^*(h) = \int \psi^*(\mathbf{x})h(\mathbf{x})d^3x$. We call $\psi^*(\mathbf{x})$ an *intertwining field* for the hierarchy. In effect, $\psi^*(\mathbf{x})$ creates a particle at $\mathbf{x} \in \Sigma$, and its adjoint $\psi(\mathbf{x})$ annihilates a particle at $\mathbf{x}$. These ideas were used to obtain q-commutation relations for anyons. [**?**, **?**].

Consequently ψ^* and ψ also obey natural brackets with the local current algebra generating the group representations. Furthermore they obey a bracket with each other: canonical commutation relations, when they intertwine N-particle Bose representations; anticommutation relations when they intertwine N-particle Fermi representations; and q-commutation relations when they intertwine N-anyon representations (in 2-space), where q is the anyonic phase. They are non-relativistic fields.

In terms of ψ^* and ψ, we may write the current algebra generators (formally) as operator-valued distributions:

$$\rho(\mathbf{x}) = \psi^*(\mathbf{x})\psi(\mathbf{x}), \quad \mathbf{J}(\mathbf{x}) = (1/2i)\{\psi^*(\mathbf{x})\nabla\psi(\mathbf{x}) - [\nabla\psi^*(\mathbf{x})]\psi(\mathbf{x})\}\,. \tag{3.3}$$

We thus have a rather beautiful unifying, current-algebraic description of a wide variety of distinct nonrelativistic quantum systems as representations of a local symmetry group. It is then natural to ask if there a role to be played by the diffeomorphism group and its representations in *relativistic* quantum field theories.

4. The diffeomorphism group and the free relativistic neutral scalar Bose field

Suppose we begin with the free neutral scalar relativistic field in the Fock representation. We use the following notational conventions to write the main equations of interest.

In Minkowski spacetime, $x^\mu = (x^0, \mathbf{x})$, with $\mu = 0,1,2,3$; and where $x^0 = ct$; the metric tensor $g_{\mu\nu} = \mathrm{diag}\,[1,-1,-1,-1]$. The covariant momentum 4-vector is $p_\mu = (p_0, \mathbf{p})$, where $p_0 = E/c$, E being the energy. The wave number 4-vector is $k = (k_0, \mathbf{k})$, where $E = \hbar\omega$ and $k_0 = \omega/c = E/\hbar c$, and $\mathbf{p} = \hbar\mathbf{k}$. Since $E^2 = p^2c^2 + m^2c^4$, we have $\omega^2 = \mathbf{k}^2c^2 + (m^2c^4/\hbar^2)$. For a given value of $\mathbf{k}$, we thus also write $k_0 = \omega_{\mathbf{k}}/c$, where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2c^2 + (m^2c^4/\hbar^2)}$.

Let $a^*_{\mathbf{k}}$ and $a_{\mathbf{k}}$ be (respectively) creation and annihilation operators for the free *relativistic* neutral scalar field in the Fock representation, for a particle with energy $E = \hbar\omega_{\mathbf{k}}$ and 3-momentum $\mathbf{p} = \hbar\mathbf{k}$. Then we have,

$$[a_{\mathbf{k}_1}, a^*_{\mathbf{k}_2}] = \omega_{\mathbf{k}_1}\delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2). \tag{4.1}$$

From these (relativistic) operators, one constructs the (nonrelativistic) fields from which particle measurement operators are obtained [?]:

$$\phi_1(x) = \int_{k_0>0} \frac{d^3k}{k_0} \frac{1}{(2\pi)^{3/2}} (k_0)^{1/2} e^{-ikx} a_{\mathbf{k}} \,, \tag{4.2}$$

and setting $t = 0$ and $k_0 = \omega_{\mathbf{k}}$, we have

$$\phi_1(\mathbf{x}, 0) = \int \frac{d^3k}{\omega_{\mathbf{k}}^{1/2}} \frac{1}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} \,. \tag{4.3}$$

Then ϕ_1 and ϕ_1^* satisfy the equal-time canonical commutation relations of the intertwining fields ψ and ψ^* discussed above; i.e., without any coefficient in the δ-function:

$$[\phi_1(x^0, \mathbf{x}), \phi_1^*(y^0, \mathbf{y})]_{x^0=y^0} = \delta^{(3)}(\mathbf{x} - \mathbf{y}). \tag{4.4}$$

Next ρ and $\mathbf{J}$ can be defined in terms of these field operators on each N-particle subspace of the Fock representation, using Eq. (3.3). They in turn satisfy the local current algebra (2.3). When exponentiated, we obtain the N-boson representations of the semidirect product group G – i.e., "nonrelativistic" quantum mechanics existing in within relativistic quantum field theory. We also highlight the remarkable fact that ρ and $\mathbf{J}$ (unlike the relativistic fields) are actually defined as distributions over space at a fixed time.

This construction is the one whose direction we want to reverse. That is, having obtained ψ and ψ^* by intertwining diffeomorphism group representations at $t = 0$ is a specific inertial reference frame, we write relativistic creation and annihilation operators $a_{\mathbf{k}}^*$ and $a_{\mathbf{k}}$ using the inverse transform of the equations above. From these, we construct the relativistic quantum field.

5. Relativistic QFT from diffeomorphism group representations: general approach

5.1. Motivation

The spacetime symmetry group informs us how to relate one inertial frame of reference to another. Traditionally, relativistic QFT begins with the introduction of fields assumed to be covariant with respect to the Poincaré group, encoding the physics of special relativity into the theory right from the start.

But the group of diffeomorphisms of spacetime is, in a sense, incompatible with the Poincaré symmetry. With the exception of $(1+1)$-dimensional spacetime, general diffeomorphisms disrupt the Minkowskian causal structure. More specifically, call a diffeomorphism of Minkowski space M *causal* if for any pair of points $x, y \in M$, it preserves the sign $(+, 0, -)$ of $(x-y)^2 = (x-y)_\mu(x-y)^\mu$ (summation convention, with the Minkowski metric). In $(1+1)$-dimensional spacetime, an infinite-dimensional group of causal diffeomorphisms exists, defined by independent actions on each of the two light cone coordinates. But in Minklowski spacetimes having more than one space dimension, the causal diffeomorphisms are limited to the finite-dimensional group of

Poincaré transformations together with uniform dilations. In Galileian spacetime, on the other hand, diffeomorphisms that act on spatial coordinates only (possibly in a time-dependent way) are among those respecting the causal structure. Thus in the Galilean case we have an infinite-dimensional group.

This incompatibility of general spacetime diffeomorphisms with special relativity, together with the value of the diffeomorphism group approach in describing general quantum kinematics, leads us to the idea of describing measurements of particles or other entities (field quanta) in a *fixed* inertial frame – the frame in which the actual measurements occur. Then we need not worry about covariance; the measurement operators can be noncovariant and nonlocal in the spacetime (although local in space at a fixed time). Of course, the corresponding operators for measurements taken in a different inertial reference frame will be different, and not obtainable directly from the first set of operators until after covariant fields (or some other way of encoding the spacetime symmetry) have been introduced. We thus defer the construction of fields covariant under the spacetime symmetry until later.

Our idea is natural if we focus on describing *spatial configurations* in a relativistic theory (with Poincaré symmetry). One must grapple with the fact that the shapes of spacetime regions, the particular choices of spacelike surfaces such as hyperplanes in Minkowskian spacetime, and the shapes of regions within those surfaces, change with the reference frame of the observer. Therefore, since we are *forced* at some stage to deal with noncovariant objects, it makes sense to begin with them. This is why we specify a frame of reference *before* beginning the constructions that lead to particle configuration spaces, and *without* having yet identified the spacetime symmetry.

5.2. Anticipated steps

Thus we envision the following steps in the program.

1. Choose an inertial frame of reference F (the frame of the observer). We have not yet built in how observations in one inertial frame are related to those in another.

2. Call the spacetime as observed from F by the name M_F. Introduce a coordinate system for M_F in which the coordinate $\mathbf{x}$ refers to space, and t to time. A "spacelike" surface Σ_F, coordinatized by $\mathbf{x}$, is obtained by setting $t = 0$ in M_F. We postulate a Euclidean metric on Σ_F, which is to play the role of the manifold Σ in the earlier discussion. Evidently this approach is general enough to include spatial manifolds Σ_F having nontrivial topology, as well as higher-dimensional spacetimes (e.g., 10- or 26-dimensional) with some of the spatial dimensions compactified.

It is natural to regard the different spacetimes M_F as fibers in a bundle over a base space of inertial frames. Each fiber carries a copy of the theory. The spacetime symmmetry should eventually establish the isomorphism of the theories (diffeomorphism group representations, etc.) in different fibers.

3. Define the group $G = \mathcal{D} \times \mathcal{K}$ with respect to Σ_F and consider its continuous unitary representations as discussed above. We remark that even in relativistic QFT, we need to describe (spatial) configurations based on

observable locations and motions of entities (particles, excitations). Unitary representations of $G(\Sigma_F)$ are natural because one-parameter groups of diffeomorphisms (i.e., flows) describe possible smooth motions of configurations in physical space. The infinitesimal generators of such flows are local currents.

4. Identify one or more *hierarchies* of representations, describing configurations consisting of entities of the same type (to be interpreted as quanta of the same field). Introduce creation and annihilation fields as operators intertwining the unitary representations in the hierarchy.

At this stage in the general development, we have a full description of the field quanta, but we do not yet have the field – nor do we have a description of the dynamics, which is to be provided as usual by a Hamiltonian operator.

5. The next step is to introduce a (covariant) relativistic field, defined making use of the creation and annihilation fields intertwining the hierarchy of diffeomorphism group representations in inertial frame F. This is where the particular choice of spacetime symmetry (relating different reference frames) is introduced. The configuration-space entities in the hierarchy of diffeomorphism group representations are interpreted as quanta of this field as observable in the reference frame F.

6. Finally, write the Hamiltonian H describing the (relativistic) dynamics. While H may be expressed in terms of the relativistic quantum field, the preceding construction means that it may also be expressed explicitly in terms of the local currents (the infinitesimal generators in the diffeomorphism group representations with which we started), together with the (nonrelativistic) intertwining fields. At the end of the construction, the physics described by the relativistic field and Hamiltonian should not depend on the particular reference frame F with which we began.

5.3. Constructing the relativistic free neutral scalar field

Carrying out first four steps in $(3+1)$-dimensioanl Minkowski space, for the hierarchy of N-particle Bose representations of $G = \mathcal{D} \times \mathcal{K}$, we obtain the creation and annihilation fields ϕ_1^*, ϕ in Fock space satisfying Eq. (4.4).

To construct the relativistic field $\phi(x)$, we use the three-dimensional inverse transforms of $\phi_1(\mathbf{x},0)$ and $\phi_1^*(\mathbf{x},0)$, corresponding to Eq. (4.3); thus,

$$\frac{1}{(2\pi)^{3/2}} \int d^3x\, \phi_1(\mathbf{x},0) e^{i\mathbf{k}\cdot\mathbf{x}} \;=\; a_{\mathbf{k}}/\omega_{\mathbf{k}}^{1/2}\,, \tag{5.1}$$

and correspondingly for the adjoint. This is precisely the point where relativistic invariance has been put in "by hand" – the definitions of $a_{\mathbf{k}}^*$ and $a_{\mathbf{k}}$ from the intertwining fields ϕ_1^* and ϕ_1 are such as to satisfy a relativistic bracket.

Then the relativistic field is, as usual,

$$\phi(x) = \frac{1}{(2\pi)^{3/2}\sqrt{2}} \int_{k_0>0} \frac{d^3k}{k_0} (a_{\mathbf{k}} e^{-ikx} + a_{\mathbf{k}}^* e^{ikx})\,. \tag{5.2}$$

We stress that both ϕ_1 and ϕ are operator-valued distributions in the same Hilbert space, for any value of c — including the Galilean limit $c \to \infty$.

5.4. The free relativistic Hamiltonian in terms of local currents

We conclude by expressing the free relativistic Hamiltonian in terms of diffeomorphism group generators (i.e., local currents), in this representation. In a calculation beginning with

$$H = \int \frac{d^3k}{k_0} \hbar\, \omega_{\mathbf{k}}\, a^*_{\mathbf{k}} a_{\mathbf{k}} = \frac{1}{2} \int d^3x : \pi(x)^2 + \nabla\phi(x)\cdot\nabla\phi(x) + m^2\phi(x)^2 : \quad (5.3)$$

where the colons :: denote normal ordering, we obtain the singular-looking expression,

$$H = \int\int d^3x\, d^3y\, F(\mathbf{x}-\mathbf{y})\, \rho(\mathbf{x})\, \exp \int_{\mathbf{x}}^{\mathbf{y}} \frac{1}{2\rho(\mathbf{x}')} \mathbf{K}(\mathbf{x}') d\mathbf{x}' , \quad (5.4)$$

where $\mathbf{K}(\mathbf{x}) = \nabla\rho(\mathbf{x}) + 2i\mathbf{J}(\mathbf{x})$, and $F(\mathbf{x}-\mathbf{y}) = \int d^3k\, \omega_{\mathbf{k}} \exp[i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})]$. One can obtain the Galilean free Hamiltonian explicitly as a limiting case of the above relativistic Hamiltonian.

We remark that there are ways to make mathematical sense of the apparently singular expressions in Eq. (5.4) involving products and quotients of the (spatially) local density and current operators.

6. Discussion and concluding remarks

The example of the free relativistic boson field serves as a model for more general quantum field theories built up from hierarchies of diffeomorphism group representations.

The relativistic fermion case is not quite as straightforward as the boson field. Had we used the hierarchy of fermionic N-particle representations of the group, we would have obtained fields satisfying equal-time canonical anticommutation relations acting in the Fock space of antisymmetric wave functions. But we must take account of particle spin; in the spinless case, after introducing the Hamiltonian, one cannot satisfy the important propoerty of local causality.

Note, however, that ruling out spin 0 fermions does not mean that antisymmetric N-particle representations of diffeomorphism groups are irrelevant. If we consider spin 1 bosons, for example, we need to include both symmetric and antisymmetric spatial wave functions in order to allow for all of the possible spin symmetries under particle exchange.

One natural direction in which to extend these ideas is to non-Fock representations of G describing infinitely many particles. A second direction is toward representations of G describing configurations of extended objects. Intertwining such representations are fields that create vortex loops, strings, or more general embedded manifolds. Fock-like representations of relativistic fields whose quanta have spatial extent would be of great interest from this point of view.

We also anticipate the value of studying interacting relativistic quantum fields constructed from diffeomorphism group representations.

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