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REMOVING LOW-FREQUENCY ARTEFACTS FROM TOPOLOGY-OPTIMIZED SURFACES

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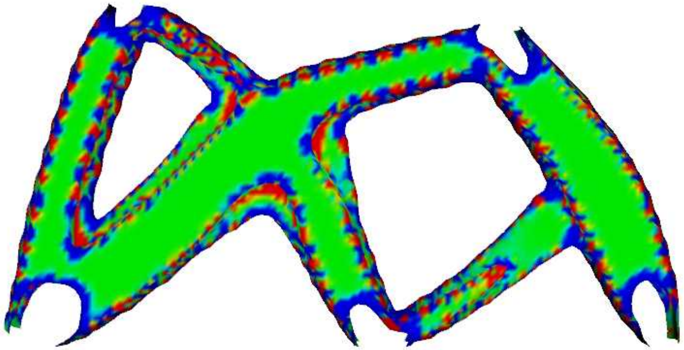

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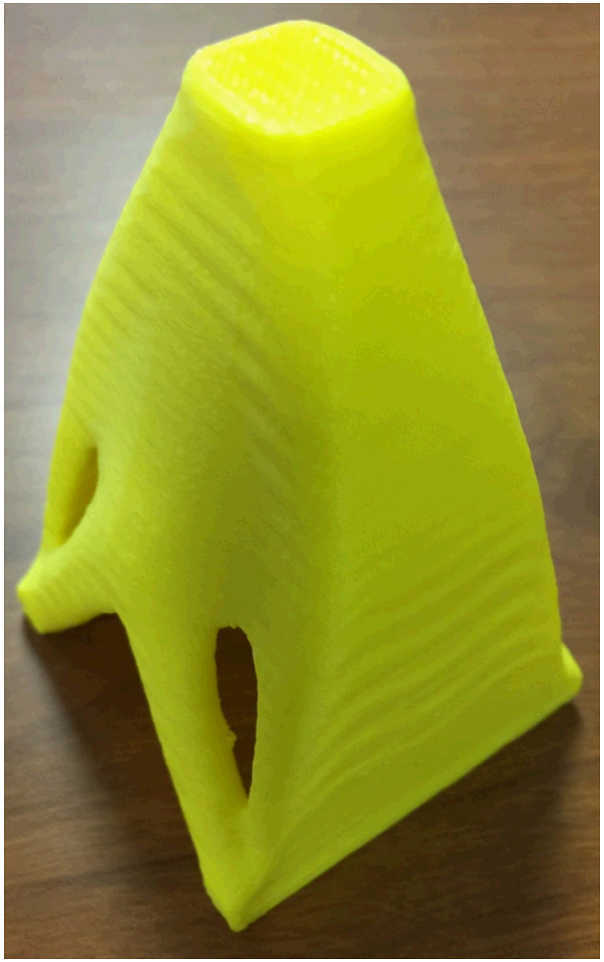
Topology-optimized parts

Thermal

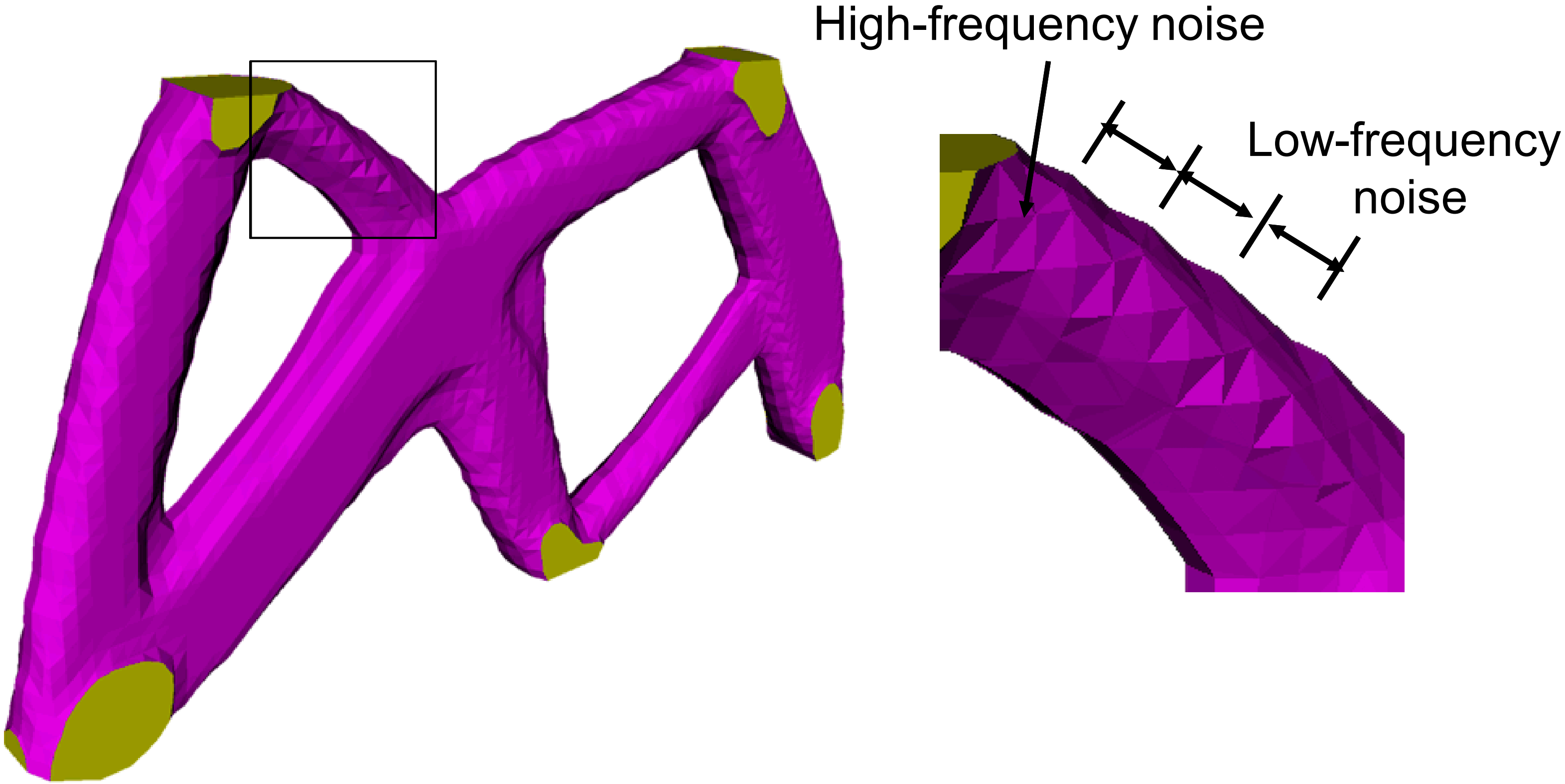


Thermal + Structural

Structural

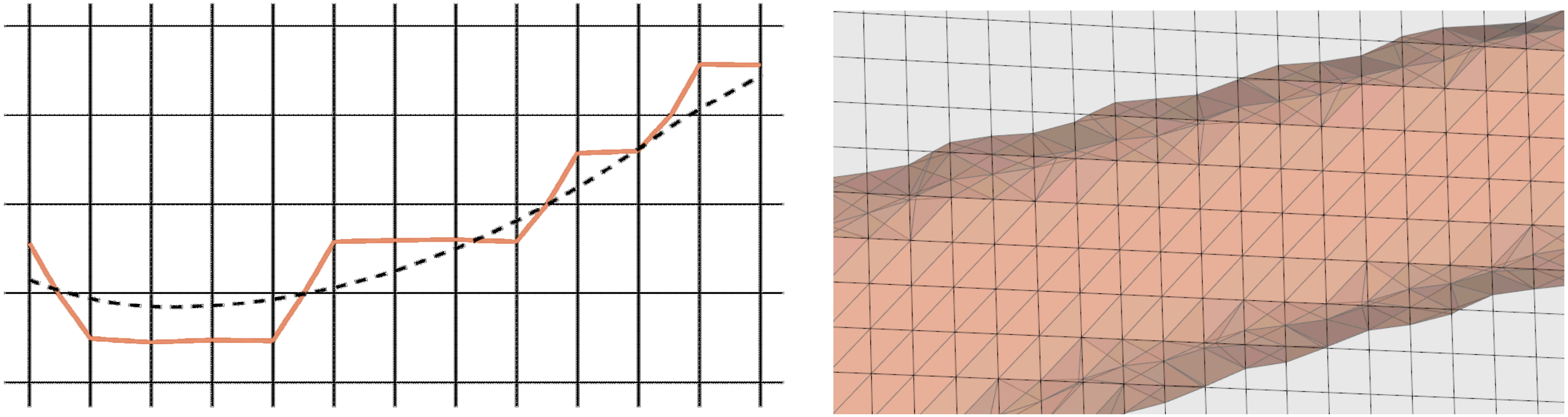
Error categorization by frequency

- High-frequency: “random noise”
- Low-frequency: “ripples”



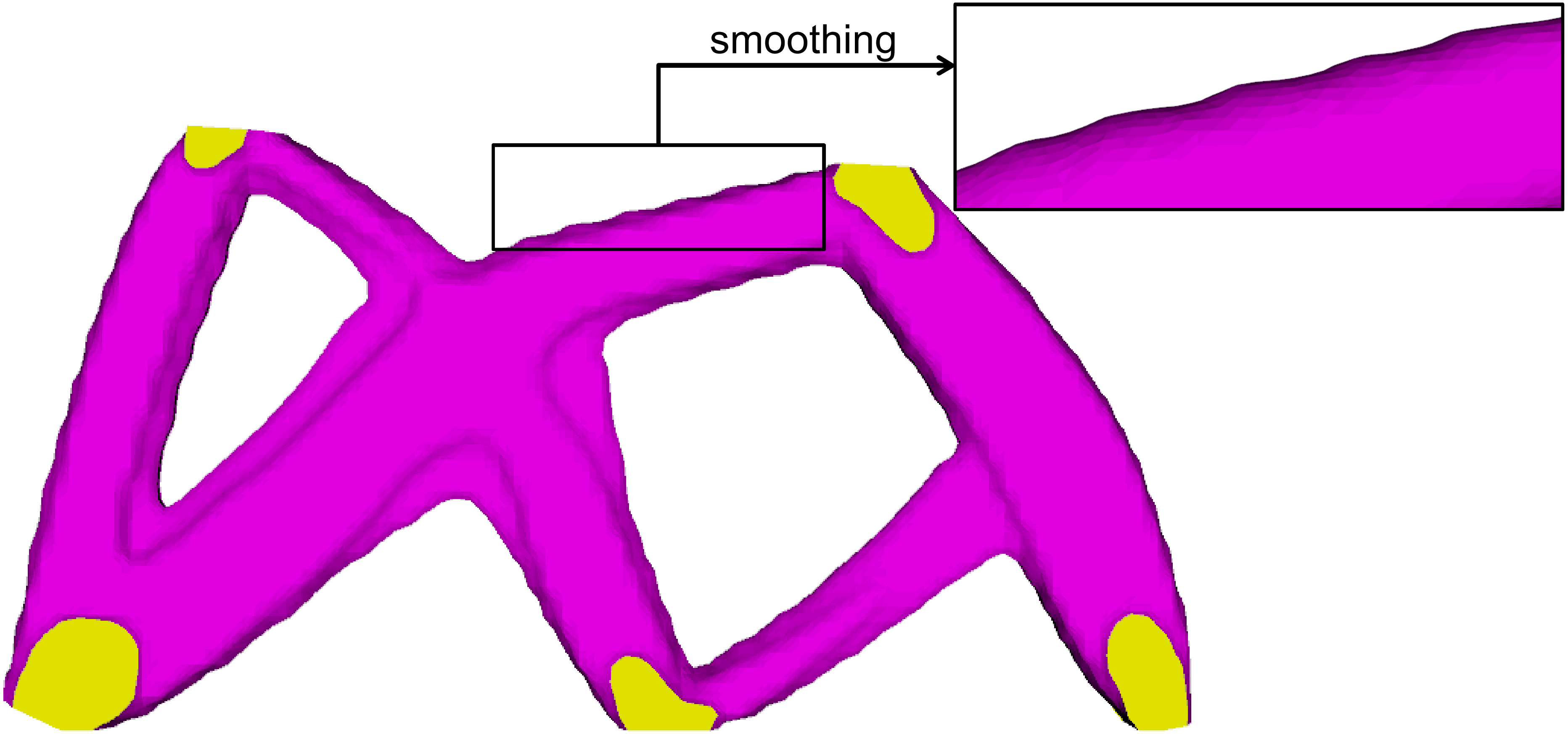
Density penalization

- Topology optimization is computed on a density field
- Nonbinary densities are penalized, causing “snapping” near the surface



Persistence of low frequencies

- Local "averaging" methods often preserve ripples



How to define frequency?

- Signals $\mathbf{x}$ are column vectors containing a value for each of the *N* nodes. Laplacian = matrix $\mathbf{L}$.
- Eigenvalue problem:
  - $\mathbf{L}\,\mathbf{v}_i = \lambda_i\,\mathbf{v}_i, \qquad i = 1, 2, \ldots, N$

Frequencies

Normal Modes

- Differential Equations 101: $\sin \omega x$ are eigenfunctions of

$$L = -\nabla^2 = -\frac{\mathrm{d}^2}{\mathrm{d}x^2}$$

Properties

- **Completeness:**

$$\mathbf{x} = \sum_{i=1}^{N} c_i \, \mathbf{v}_i$$

Fourier series

- **Orthogonality:**

If $\mathbf{L} = \mathbf{L}^T$ then

$$\mathbf{v}_m^T \left(\mathbf{L} \, \mathbf{v}_n \right) = (\mathbf{v}_m^T \mathbf{L}^T) \mathbf{v}_n = (\mathbf{L} \, \mathbf{v}_m)^T \mathbf{v}_n$$

$$\lambda_n (\mathbf{v}_m^T \mathbf{v}_n) = \lambda_m (\mathbf{v}_m^T \mathbf{v}_n)$$

If the λ_i are distinct, then

$$\left(\mathbf{v}_m^T \mathbf{v}_n \right) = \delta_{mn}$$

Explicit frequency method

- Once $\mathbf{v}_i$ are known, find c_j with inner product:

$$\mathbf{x} = \sum_{i=1}^{N} c_i\, \mathbf{v}_i$$

$$\mathbf{v}_j^T \mathbf{x} = \sum_{i=1}^{N} c_i\, \mathbf{v}_j^T \mathbf{v}_i = \sum_{i=1}^{N} c_i\, \delta_{ji} = c_j$$

- Reconstruct **x** applying transfer function *f*:

  - $\mathbf{x}' = \sum_{i=1}^{N} f(\lambda_i)\, c_i\, \mathbf{v}_i$

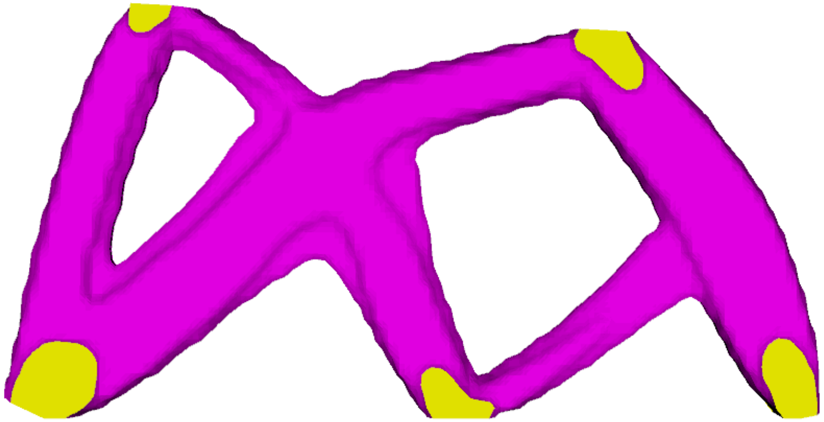
Polynomial filters

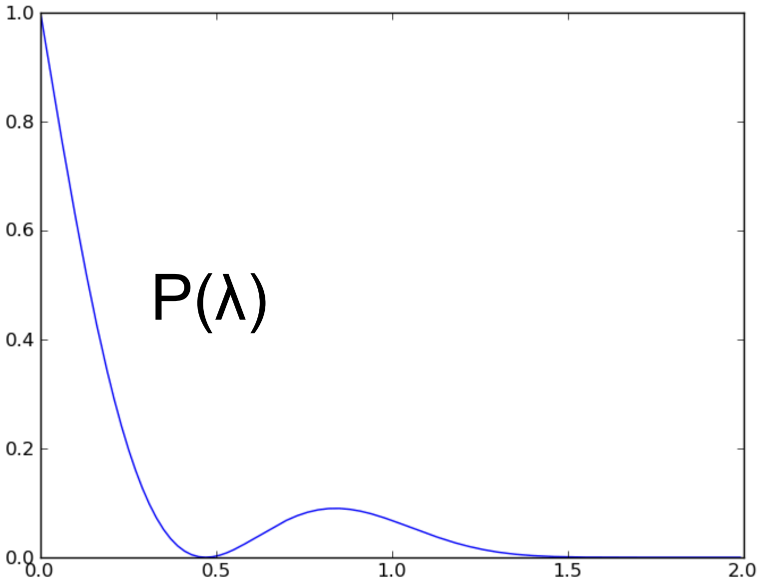
- $P(\mathbf{L})$ has eigenvalues $P(\lambda_i)$.

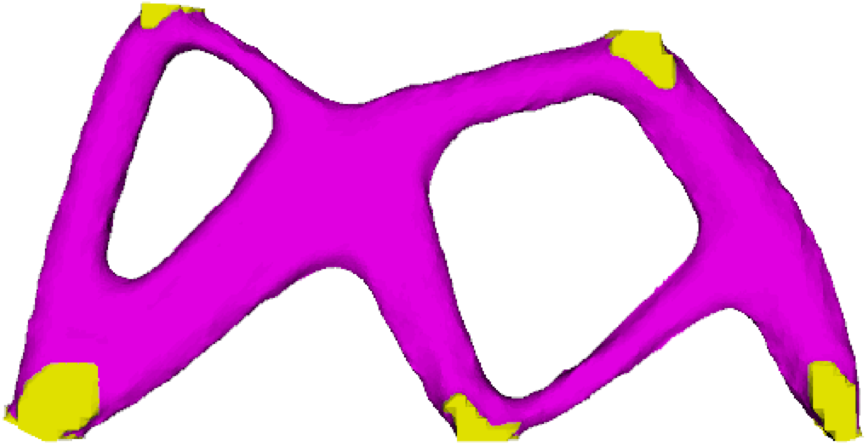
$$\mathbf{x} = \textstyle\sum_{i=1}^{N} c_i \, \mathbf{v}_i$$

$$P(\mathbf{L})\,\mathbf{x} = \sum_{i=1}^{N} c_i \, P(\mathbf{L})\,\mathbf{v}_i = \sum_{i=1}^{N} P(\lambda_i)\, c_i \, \mathbf{v}_i$$

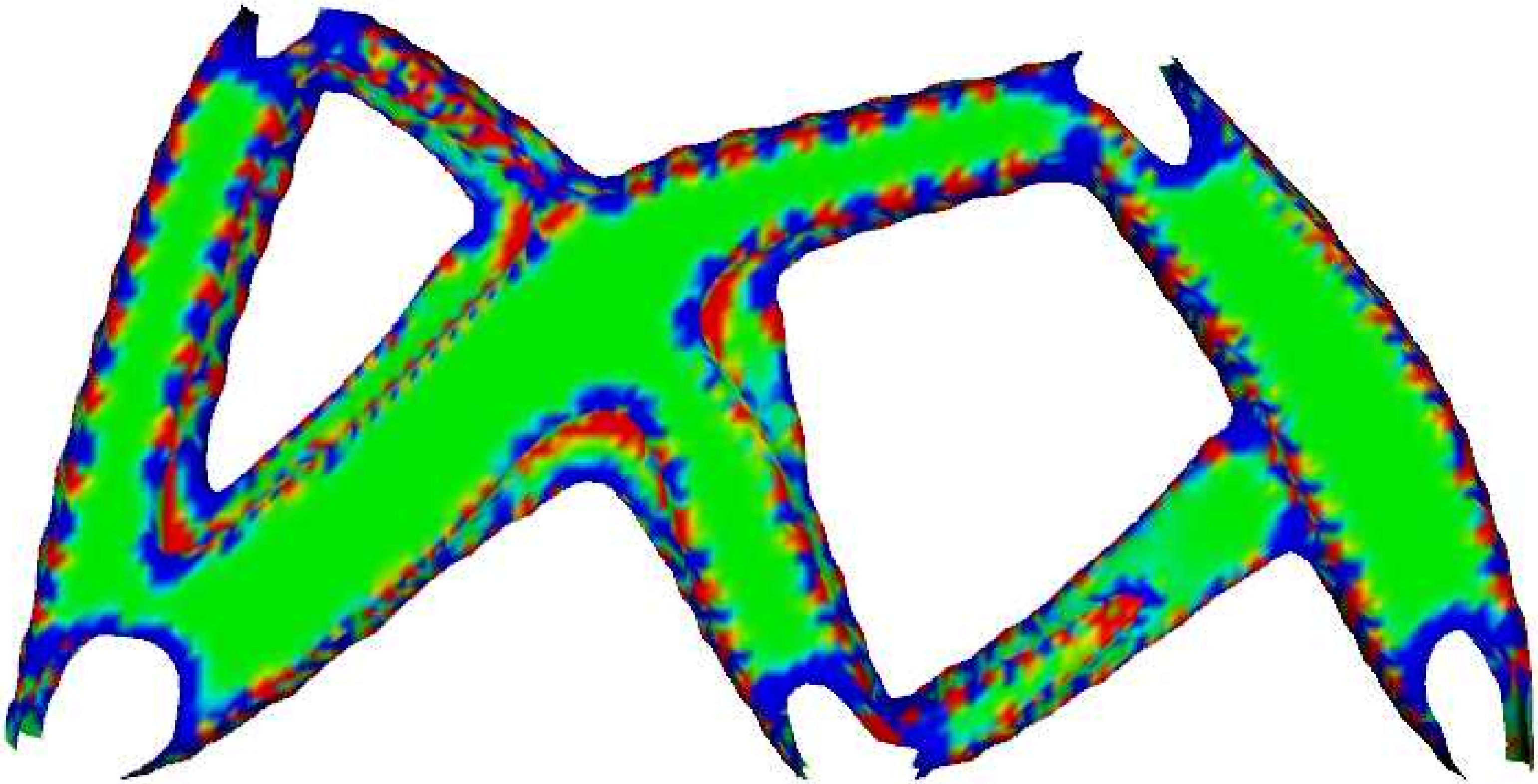
Polynomials are hard to control!







Curvature highlights ripples



positive **zero** **negative**

Proposed method

1. Construct signal $\boldsymbol{\gamma}$. Set $\boldsymbol{\gamma} = 1$ for red nodes and $\boldsymbol{\gamma} = 0$ elsewhere.

2. Project into frequency space:

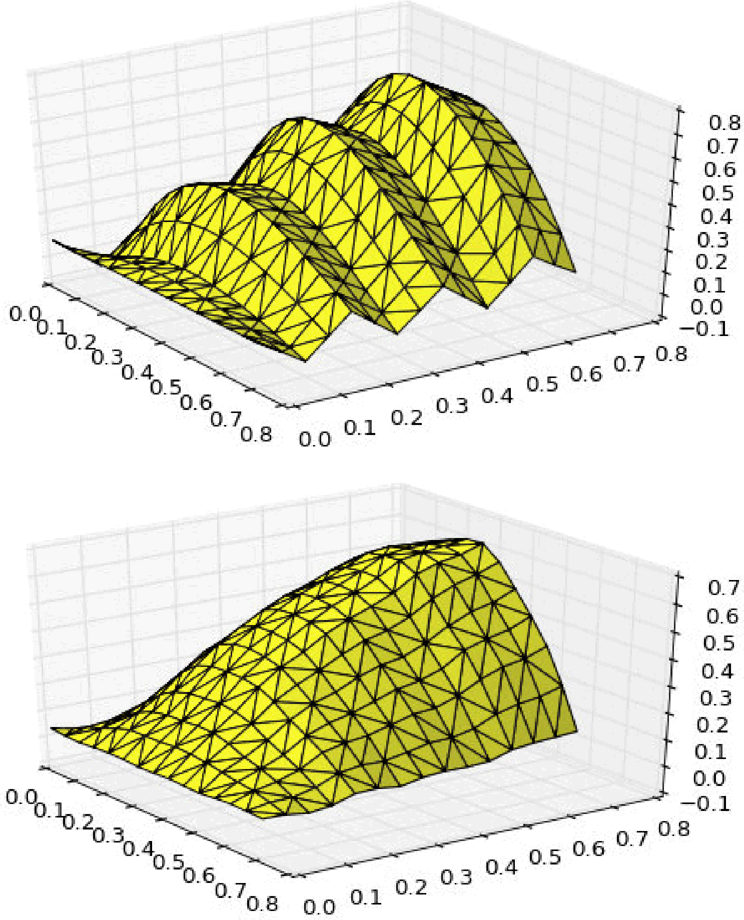
$$\boldsymbol{\gamma} = \sum_{i=1}^{N} a_i \, \mathbf{v}_i$$

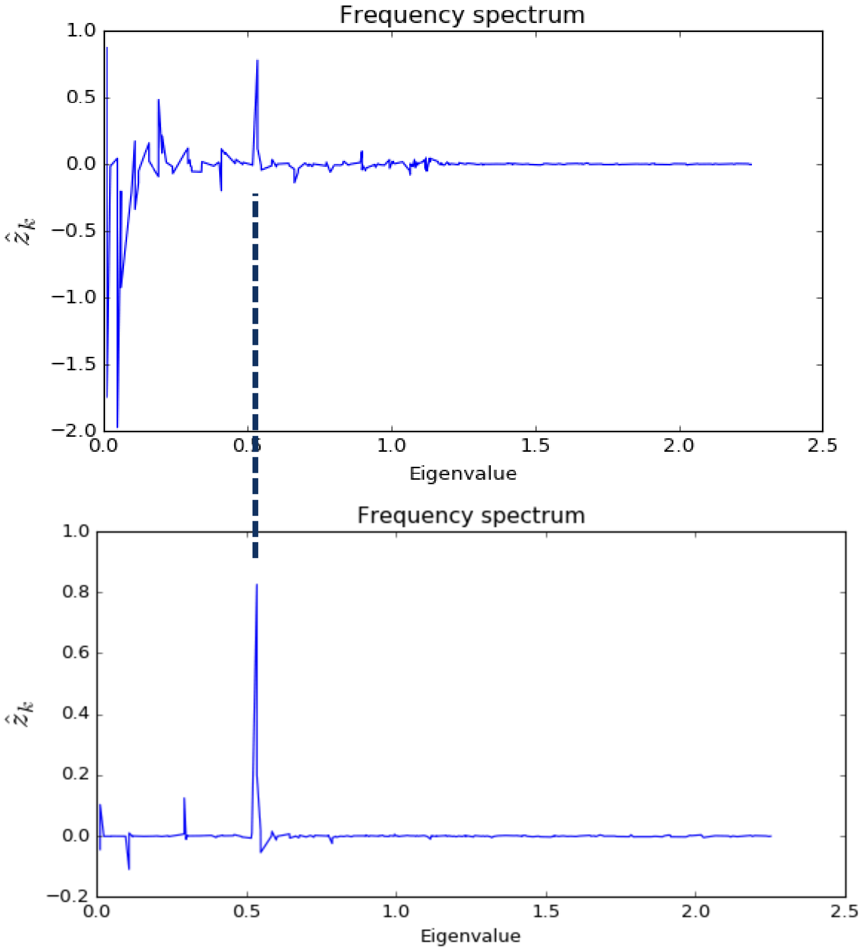
3. Choose $f \to 0$ near any λ_m for which a_m is large.

4. Filter shape with f.

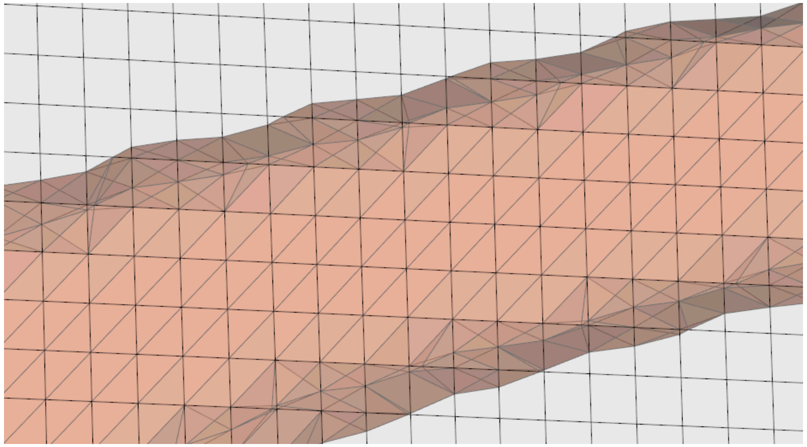
Mesh spectra are linear

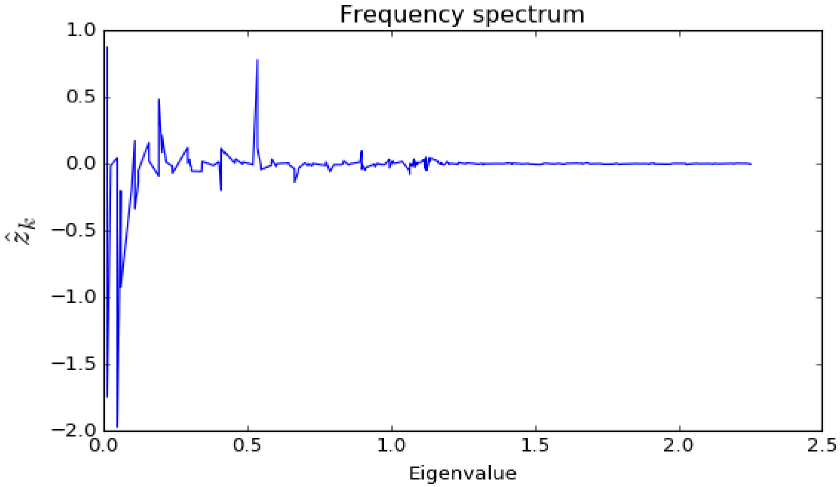
$$\mathbf{x}^{\text{smooth}} + \mathbf{x}^{\text{ripples}} = \sum_{i=1}^{N} (c_i^{\text{smooth}} + c_i^{\text{ripples}})\mathbf{v}_i$$

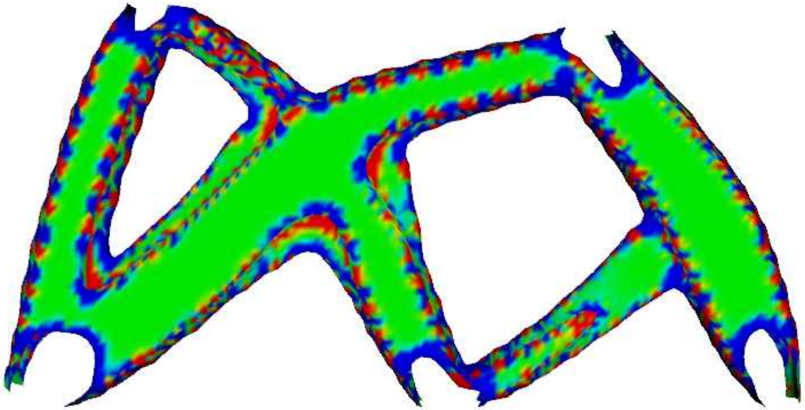


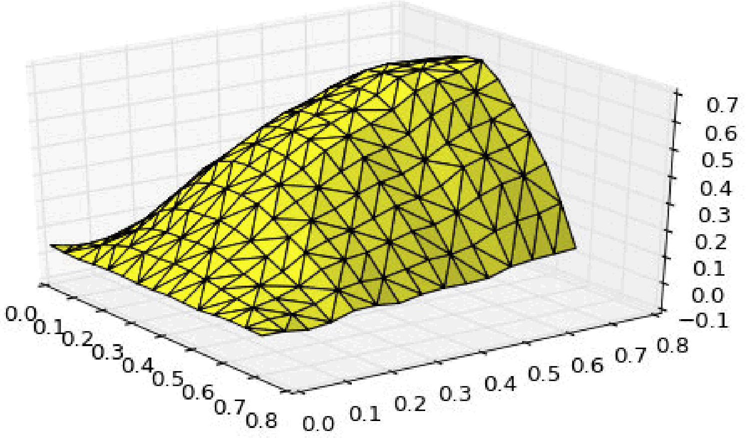


Summary




Frequency spectrum
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
$\hat{z}_k$
0.0
0.5
1.0
1.5
2.0
2.5
Eigenvalue




0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Thank You!