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Removing Low-frequency Artefacts from Topology-Optimized Surface Meshes

Salvatore S. Elder[1] and William Roshan Quadros[2]

[1]School of Applied and Engineering Physics, Cornell University, Ithaca, NY 14853, USA. [2]Sandia National Laboratories, PO Box 5800, MS 0897, Albuquerque, NM 87185, USA.

Introduction

- Topology optimization is a scheme in which physics codes are run to design parts automatically, subject to specified boundary conditions and minimization objectives.
- We have observed low-frequency surface ripples present in the output, however, which are difficult to remove using traditional smoothing methods.

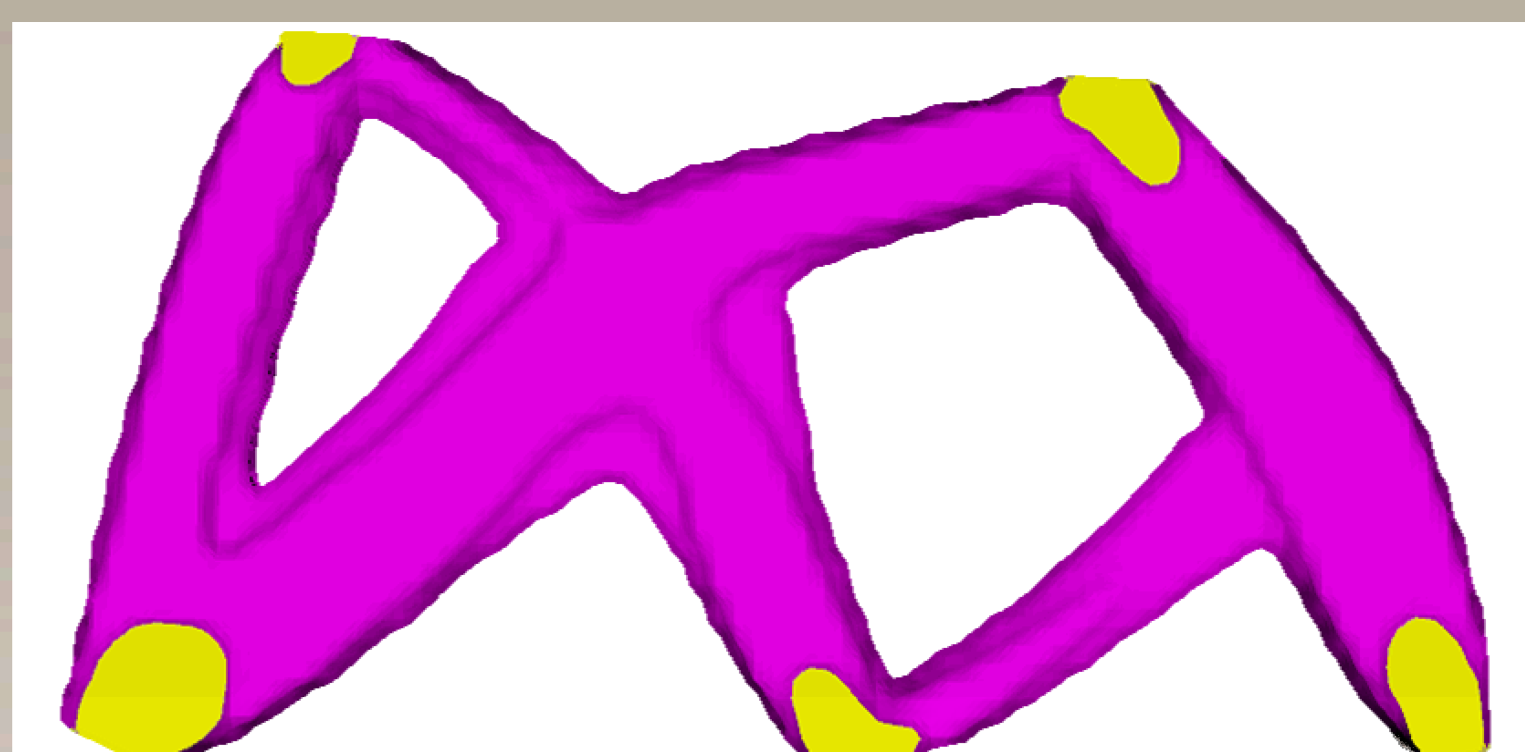


Fig. 1: A topology optimized shape with quasi-periodic bumps along the surface.

- Topology optimization output is formulated as a discretely sampled material density field $0 \leq \rho \leq 100\%$.
- Intermediate values of ρ are penalized, so that the grid nodes tend to snap toward 0 or 100%.
- The final surface is taken to be the $\rho = 50\%$ isosurface defined by the nodal grid values.

Ripple origin

- The possible effect of such penalization is illustrated in the artificial example of Fig. 2. Compare to the actual topology optimization output of Fig. 3.

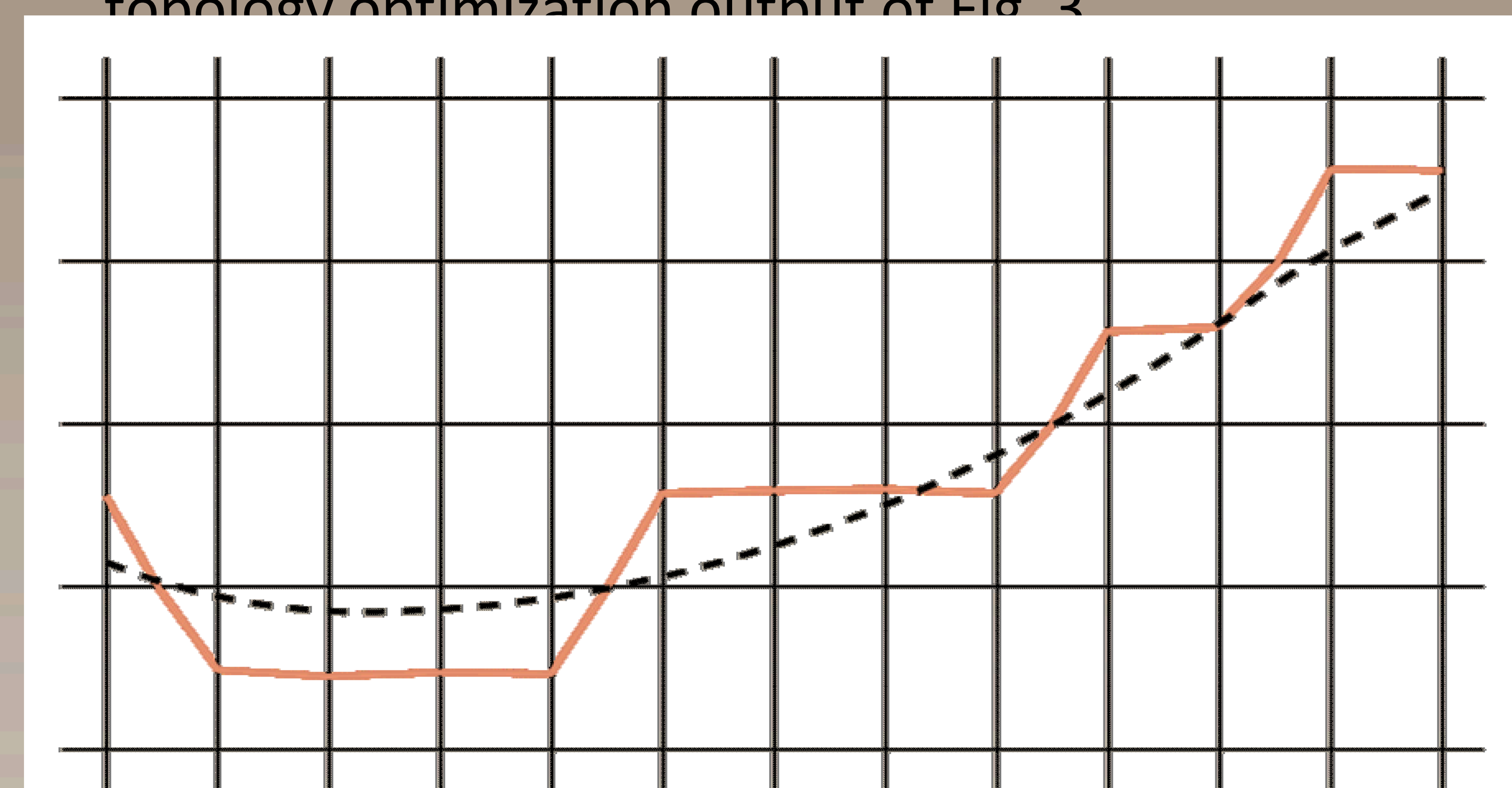


Fig. 2: Suppose the dashed line represents the true surface optimum. The solid orange line shows the result of snapping all $\rho < 50\%$ to 0 and all $\rho > 50\%$ to 100%.

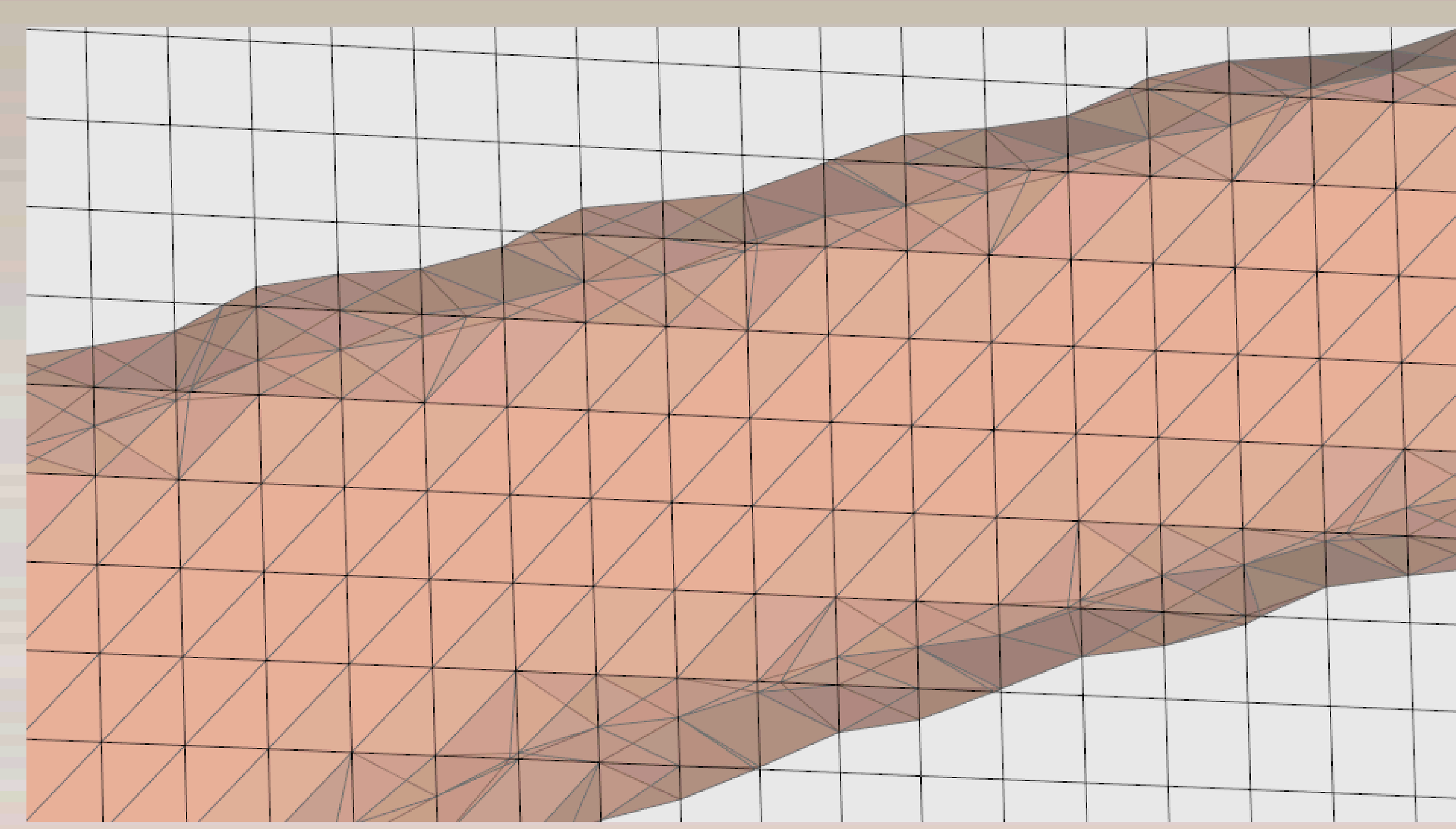


Fig. 3: A close-up of a part of the shape of Fig. 1, with the discrete grid used by the topology optimization code overlaid. Note the low-frequency ripples and their similarity to the pattern seen in Fig. 2.

Frequency formulation

- Consider graph signals $\mathbf{x}$, which are column vectors containing a value for each of the N nodes. Any linear Laplacian has a matrix representation $\mathbf{L}$.
- Suppose

$$\mathbf{L}\,\mathbf{v}_i = \lambda_i\,\mathbf{v}_i, \qquad i = 1, 2, \ldots, N.$$

- By analogy to Fourier series, the N eigenvalues can be thought of as *frequencies* and the N eigenvectors can be thought of as *oscillatory modes*.
- For any $\mathbf{x}$, we can write

$$\mathbf{x} = \sum_{i=1}^{N} c_i\,\mathbf{v}_i.$$

- If we had c_i and $\mathbf{v}_i$, then we could filter $\mathbf{x}$ according to any transfer function $f(\lambda)$ we wanted:

$$\mathbf{x}' = \sum_{i=1}^{N} f(\lambda_i)\,c_i\,\mathbf{v}_i.$$

- If $\mathbf{L} = \mathbf{L}^T$ (i.e., the Laplacian is symmetric), then

$$\mathbf{v}_m^T(\mathbf{L}\,\mathbf{v}_n) = (\mathbf{v}_m^T\mathbf{L}^T)\mathbf{v}_n = (\mathbf{L}\,\mathbf{v}_m)^T\mathbf{v}_n\,.$$

Rewrite the LHS and RHS using the fact that $\mathbf{v}_m$ and $\mathbf{v}_n$ are eigenvectors of $\mathbf{L}$:

$$\lambda_n(\mathbf{v}_m^T\mathbf{v}_n) = \lambda_m(\mathbf{v}_m^T\mathbf{v}_n).$$

If the λ_i are distinct, then

$$(\mathbf{v}_m^T\mathbf{v}_n) = \delta_{mn}.$$

Therefore, once some $\mathbf{v}_j$ is known, we can calculate the corresponding coefficient c_i by dotting $\mathbf{x}$ with $\mathbf{v}_j$:

$$\mathbf{v}_j^T\mathbf{x} = \sum_{i=1}^{N} c_i\,\mathbf{v}_j^T\mathbf{v}_i = \sum_{i=1}^{N} c_i\,\delta_{ji} = c_j.$$

- A method of Vallet and Lévy allows for the explicit calculation and filtering of c_i and $\mathbf{v}_i$ even for fairly large meshes.

Proposed method

- In Vallet and Lévy's approach, the vertex coordinates $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are filtered to change a mesh's shape.
- It is not clear how to select f to remove the ripples, however.
- We have observed that the Gaussian curvature does an excellent job of highlighting the low-frequency ripples in topology optimized surfaces. See Fig. 5.

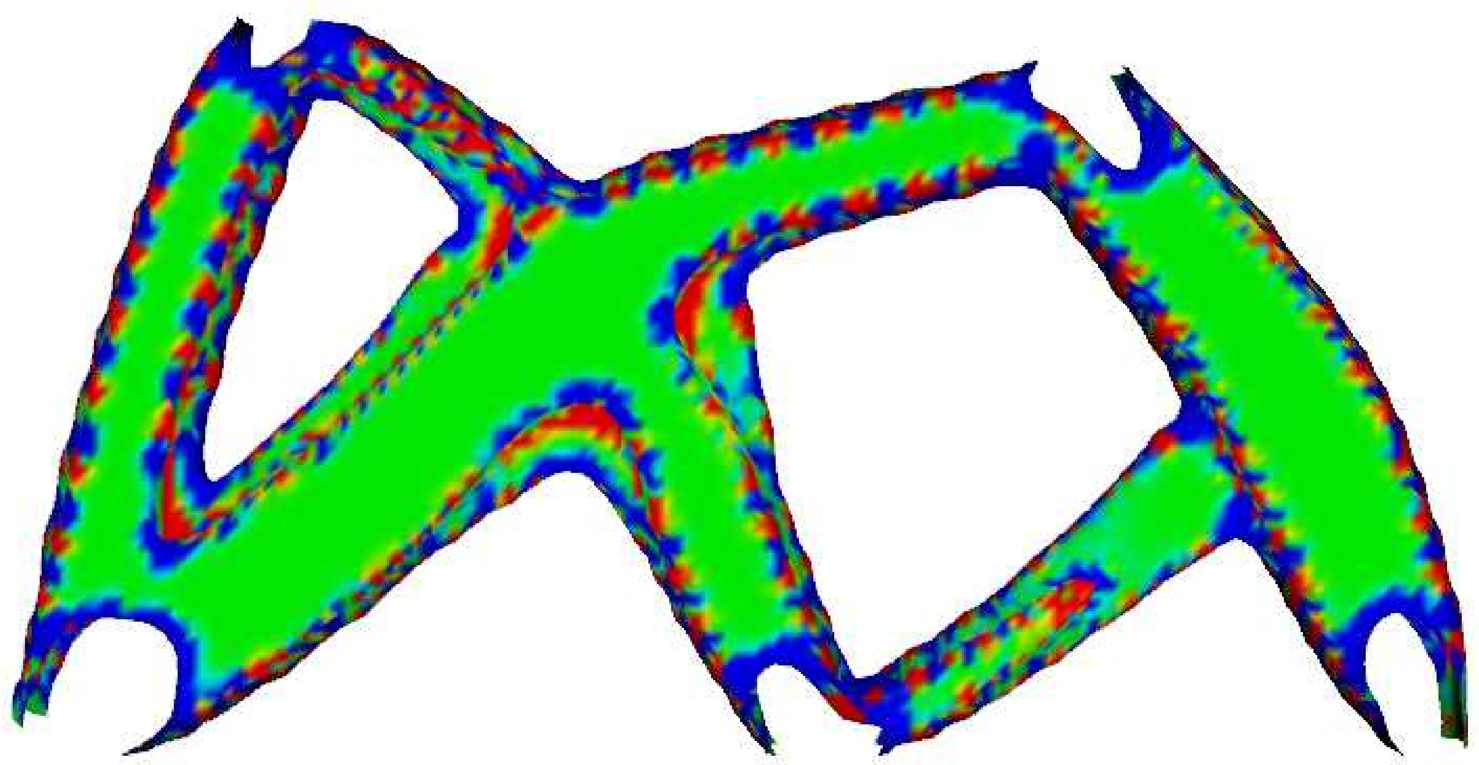


Fig. 5: The Gaussian curvature of a topology optimized surface. Red regions correspond to positive curvature (hills and valleys), blue regions to negative curvature (saddle points), and green regions to relatively flat areas (curvature ≈ 0). In this case, the positive curvature appears to highlight the low-frequency ripples.

- We suggest the following procedure to select $f(\lambda)$:

1. Construct a signal, $\boldsymbol{\gamma}$, defined on the mesh vertices. Set $\boldsymbol{\gamma} = 1$ wherever the curvature is positive and above a certain threshold, and $\boldsymbol{\gamma} = 0$ everywhere else.
2. Project this signal into frequency space, obtaining a coefficient a_i for each frequency mode:
$$\boldsymbol{\gamma} = \sum_{i=1}^{N} a_i \, \mathbf{v}_i$$
3. Select a band-stop filter $f(\lambda)$ for which $f \to 0$ near any λ_m for which a_m is large.

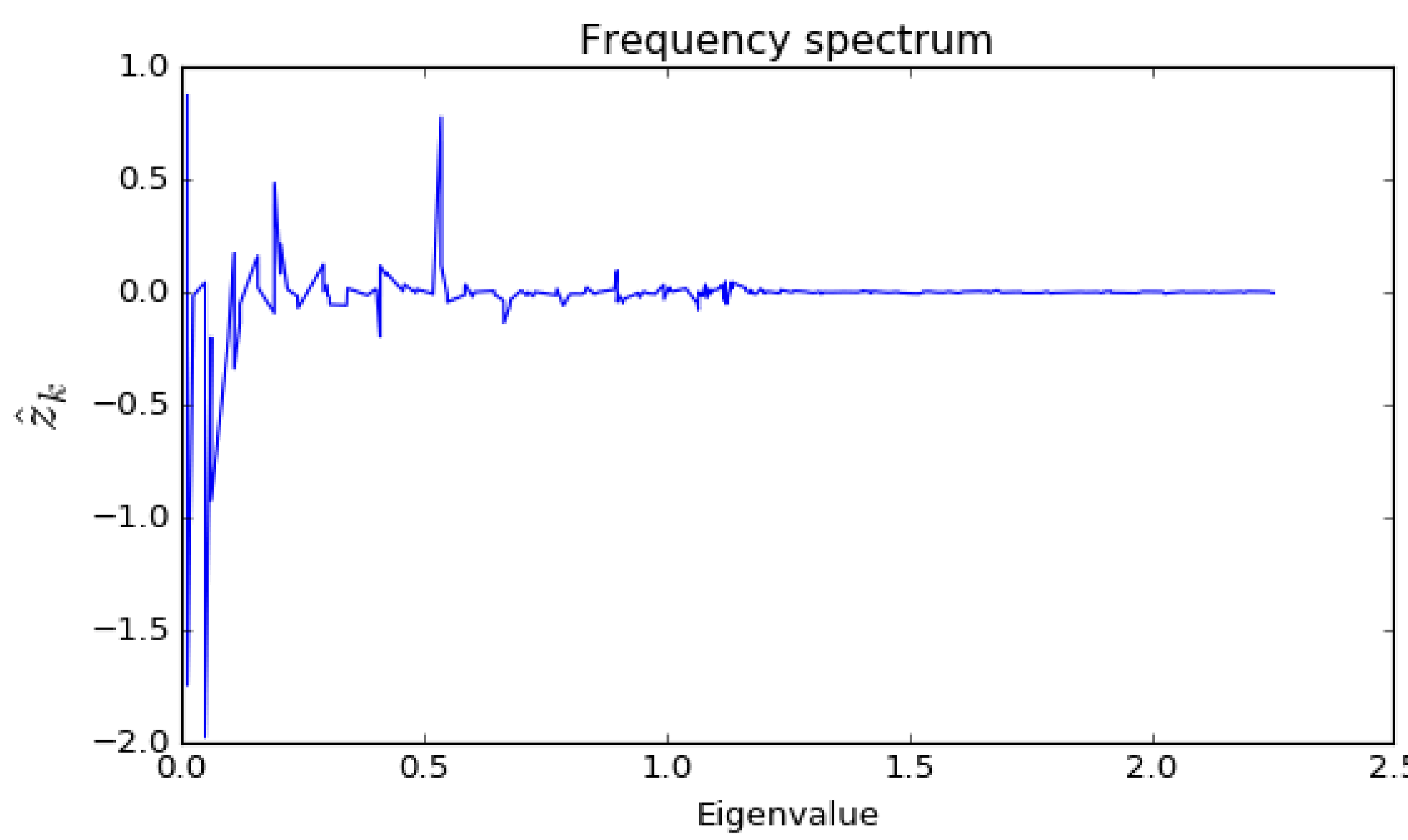
This makes sense because mesh spectra are linear. Thus

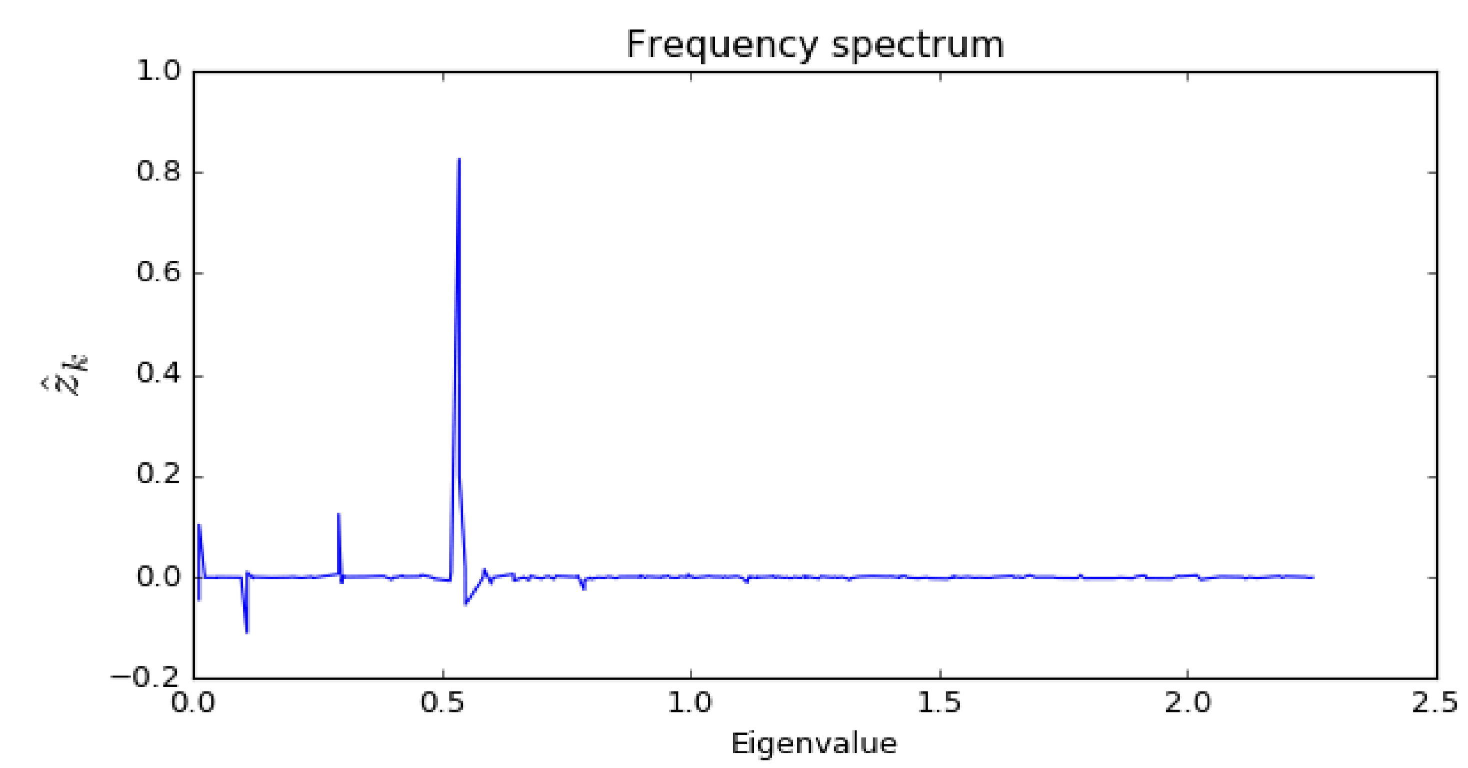
$$\mathbf{x}^{\text{smooth}} + \mathbf{x}^{\text{ripples}} = \sum_{i=1}^{N} (c_i^{\text{smooth}} + c_i^{\text{ripples}}) \mathbf{v}_i$$

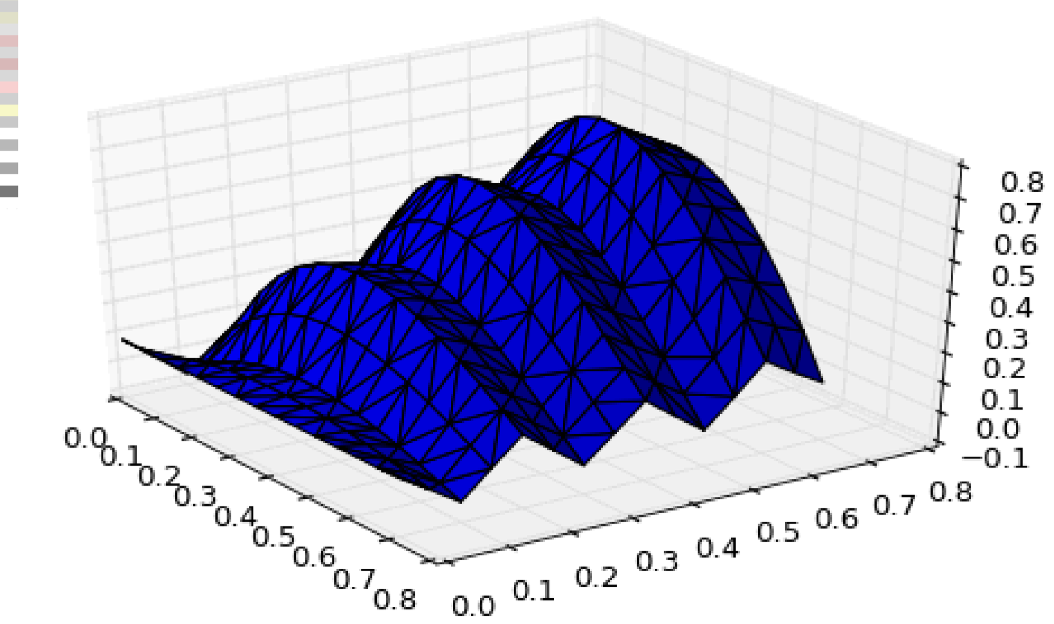
- The procedure should identify the largest of c_i^{ripples}.
- Even if the amplitude and overtones of $\boldsymbol{\gamma}$ differ from those of the ripples, they will almost certainly have the same principal modal components. Analogously, a violin and a tuning fork sounding an A4 share a fundamental frequency of 440 Hz.

Results

- We created a cubic Bézier patch to represent a portion of a topology optimized surface, and superimposed a sinusoid on top.
- Fig. 6 shows the result of removing all eigenmode components for which $0.5 \leq \lambda \leq 0.57$. Also shown is the spectrum of the ripples alone. (We used Python and SciPy.)
- Both spectra have a peak near $\lambda = 0.52$, due to the ripples. This clear correspondence helps to justify the curvature approach.







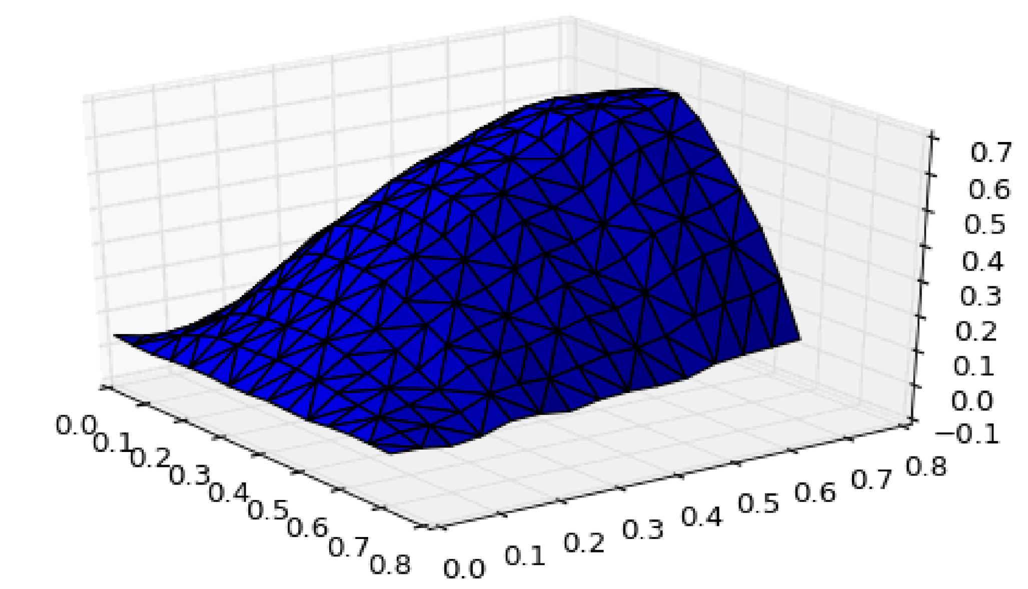


Fig. 6: Top: The mesh to be smoothed. We added the prominent ripples artificially. **Top, previous column:** The spectrum of the mesh. **Bottom, previous column:** The spectrum of the ripples we added. Note the prominent peak shared by both spectra. **Bottom:** The result of removing the aforementioned peak from the spectrum of the mesh, then inverting the frequency transform. The ripples have been removed, leaving us with a smooth mesh.

Conclusions

We have identified systematic errors present in topology optimized surfaces, and explored a processing pipeline for removing them. We suggest the use of curvature as a means of locating the errors in frequency space, and we remove undesired ripples from an example mesh. Future work would bring the ideas explored in this poster to full maturity, with frequency detection and smoothing implemented for topology-optimized surfaces.

This poster is adapted from the 2016 IMR research note "Removing low-frequency artefacts from topology-optimized surfaces" (Elder and Quadros). See references at the end of the note, especially "Optimal surface smoothing as filter design" (Taubin et al.) and "Spectral geometry processing with manifold harmonics" (Vallet and Lévy).

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