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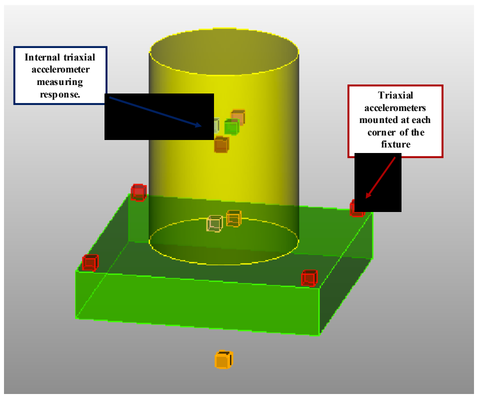

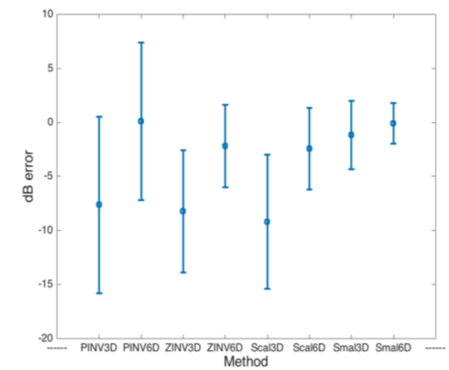

Input Test Specification for 6 DOF

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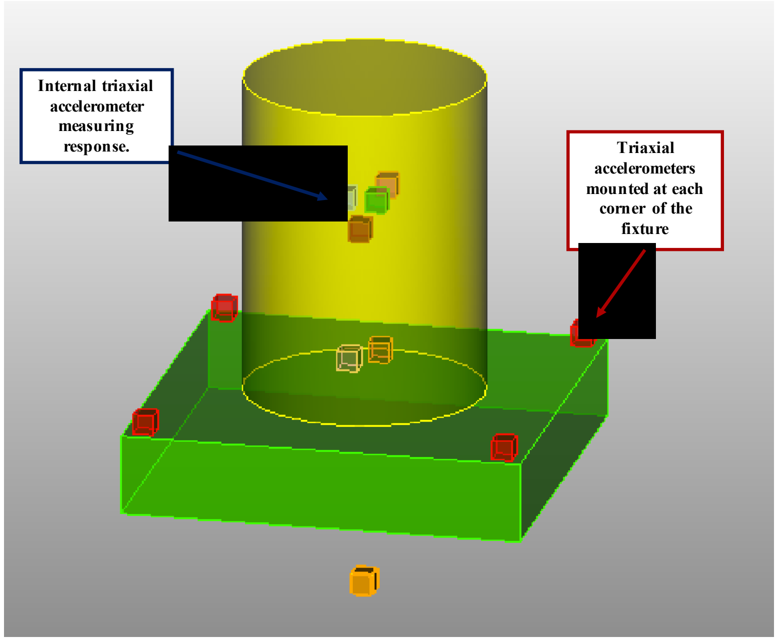
Abstract

- Recent studies have found that 6-DOF testing better replicates the stress environment during actual field tests.
- Unfortunately, it is a rare occasion where a field test can be sufficiently instrumented such that the subsystem/component 6-DOF inputs can be directly derived.
- However, a recent flight test of a Sandia system was instrumented sufficiently such that the input could be directly derived for a particular subsystem.
- This is compared to methods for deriving 6-DOF test inputs from field data with limited instrumentation.
  - There are four methods used for deriving 6-DOF input with limited instrumentation.
- In addition to input comparisons, actual response measurement during the flight are compared to the various 6-DOF tests as method for comparing input 6-DOF derivation.
- This work focuses on best methods to replicate an actual field test with a laboratory test.
  - Derivations of environmental specifications will be the focus of future work. This would include straight-line specifications and appropriate probability and statistics (P99/C90 or 1:500)

Big Picture Steps

- On 6DOF shaker table, build transmissibility function $[H(\omega)]$ from base input of fixture to internal gages.
- Given flight data with internal responses, use $[H(\omega)]$ to develop a set of inputs, $[S_{xx}(\omega)]$, that will as best as possible replicate internal response from flight if put on 6 DOF shaker table.
- Compare to internal responses of 6 DOF to Flight from various methods of generating 6 DOF input.

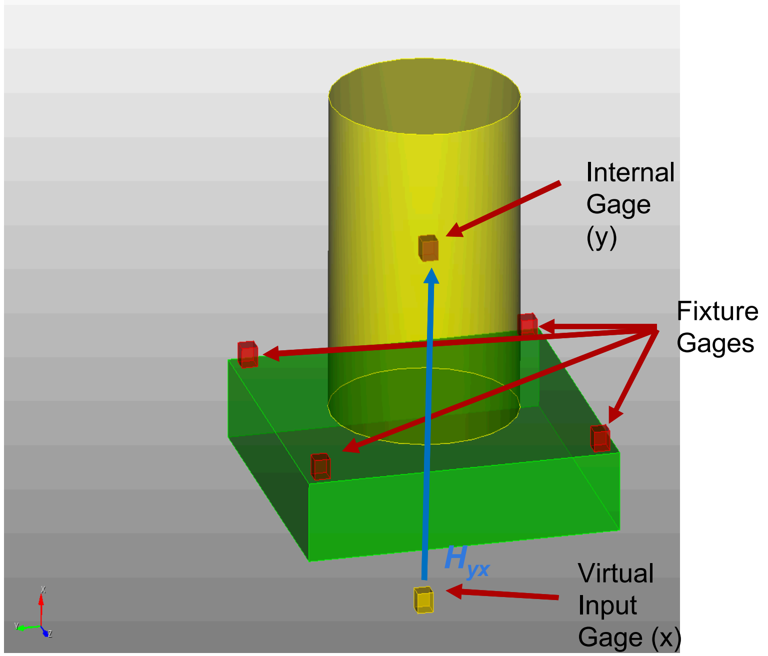


Standard Math Background

- Transmissibility function is derived from input to output locations of interest internal to the Subsystem.
  - $[H_{yx}] = [G_{xx}]^{-\dagger}\,[G_{xy}]$
  - $[G_{xy}]$ is the input/output cross-spectral density matrix, $[G_{xx}]$ is the input auto spectral density matrix, and $-\dagger$ is the typical Moore-Penrose generalized inverse.
  - $[G_{xx}]$ derived from 4 gages on fixture (discussed on next few slides)
- Positive Semidefinite
  - Determinant > 0
  - Eigenvalues > 0
  - Check at each frequency
- Condition Number
  - Using Singular Value Decomposition [X] = [U]*[S]*[V]'.
    - the ratio of the largest singular value of [S] to the smallest
  - *What values are acceptable*
- All Matrices are 3 dimensional and each concept applies at a particular frequency
  - True Throughout the Presentation

Developing Transmissibility Function

- Transmissibility function is derived from input to output locations of interest internal to the Subsystem.
  - $[H_{yx}] = [G_{xx}]^{-\dagger} [G_{xy}]$
- We want to build this from a 6 DOF input into the base of the fixture.
  - 3 Translational Accelerations
  - 3 Rotational Accelerations
- Assume the fixture remains rigid.
- If we knew 6 DOF input at virtual location we could calculate fixture gage response



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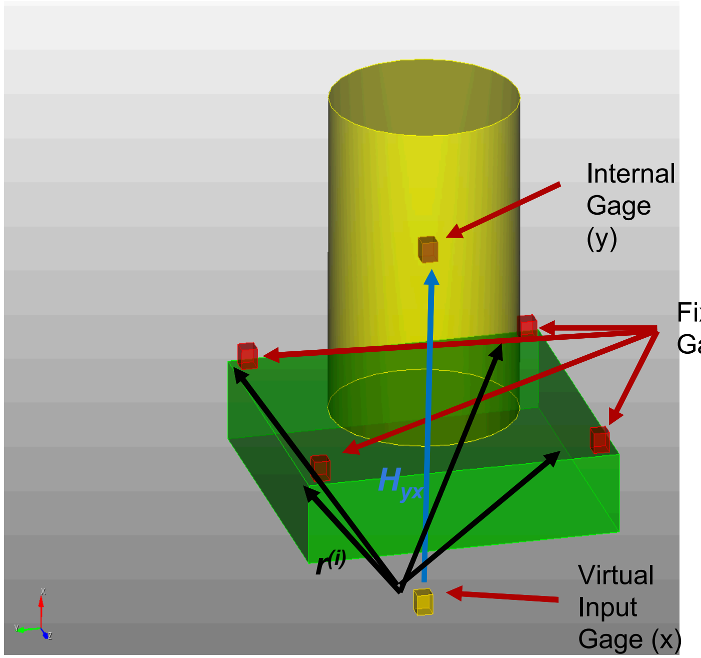


- $$\begin{Bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{z}_1 \\ \vdots \\ \ddot{x}_4 \end{Bmatrix}_{12x1} = \begin{bmatrix} 1 & 0 & 0 & 0 & r_z^1 & -r_y^1 \\ 0 & 1 & 0 & -r_z^1 & 0 & r_x^1 \\ 0 & 0 & 1 & r_y^1 & -r_x^1 & 0 \\ 1 & 0 & 0 & 0 & r_z^2 & -r_y^2 \\ 0 & 1 & 0 & -r_z^2 & 0 & r_x^2 \\ 0 & 0 & 1 & r_y^2 & -r_x^2 & 0 \\ 1 & 0 & 0 & 0 & r_z^3 & -r_y^3 \\ 0 & 1 & 0 & -r_z^3 & 0 & r_x^3 \\ 0 & 0 & 1 & r_y^3 & -r_x^3 & 0 \\ 1 & 0 & 0 & 0 & r_z^4 & -r_y^4 \\ 0 & 1 & 0 & -r_z^4 & 0 & r_x^4 \\ 0 & 0 & 1 & r_y^4 & -r_x^4 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ r\ddot{x} \\ r\ddot{y} \\ r\ddot{z} \end{Bmatrix}_{6x1}$$
- $\{a_f\} = [R]\{a_x\}$
- $\{a_x\} = [R]\backslash\{a_f\}$ -> least-squares solution (A*x =b)

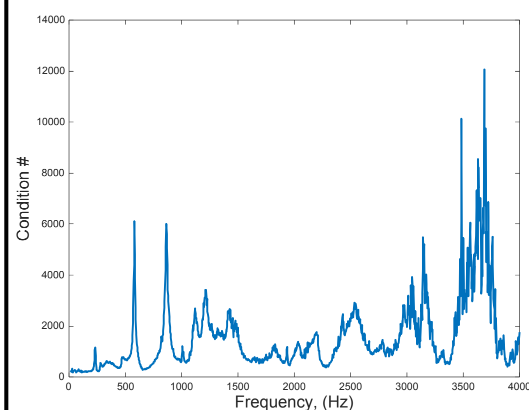
Numerical Set-up

- We have 10 responses and we need 6 inputs at so many frequency lines.
  - H is 10x6x#of frequency spacing.
  - In this presentation, we write the matrices as if they are 2D and drop the # of frequency spacing. But realize each matrix function is solved at each frequency.
  - X = pinv(H) Y. The solution is the one which norm(X) is the smallest.
- Future work may include selection of best gages to get ideal input.

PINV Method (1)

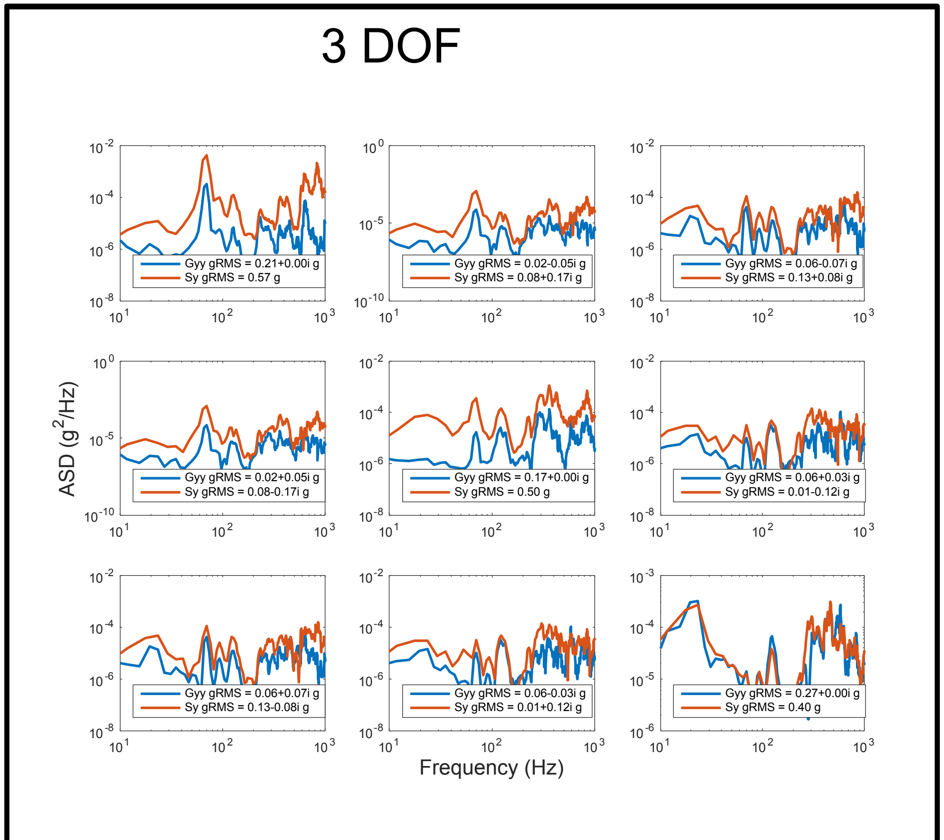
- The concept is relatively simple
  - Typical input-output relation for linear systems
  - $\{Y(\omega)\} = [H(\omega)]\{X(\omega)\}$.
  - We want to know the input $\{X(\omega)\} = [H(\omega)]^{-\dagger}\{Y(\omega)\}$.
- Implementation is a little complex
  - The output $y(t)$ is provided in the time domain from the flight data.
    - Use Fourier Transform to get $Y(\omega)$.
  - Interpolate $[H(\omega)]$ to same frequency spacing and band as $Y(\omega)$.
    - $[H(\omega)]$ had a coarser frequency spacing.
  - Perform Operation $\{X(\omega)\} = [H(\omega)]^{-\dagger}\{Y(\omega)\}$.
  - Use $\{X(\omega)\}$ to calculate $[S_{xx}]$.
- Checks
  - Conditioning of $[H(\omega)]$
  - Positive Semidefinite $[S_{xx}]$
    - Determinant > 0
    - Eigenvalues > 0
    - Check at each frequency

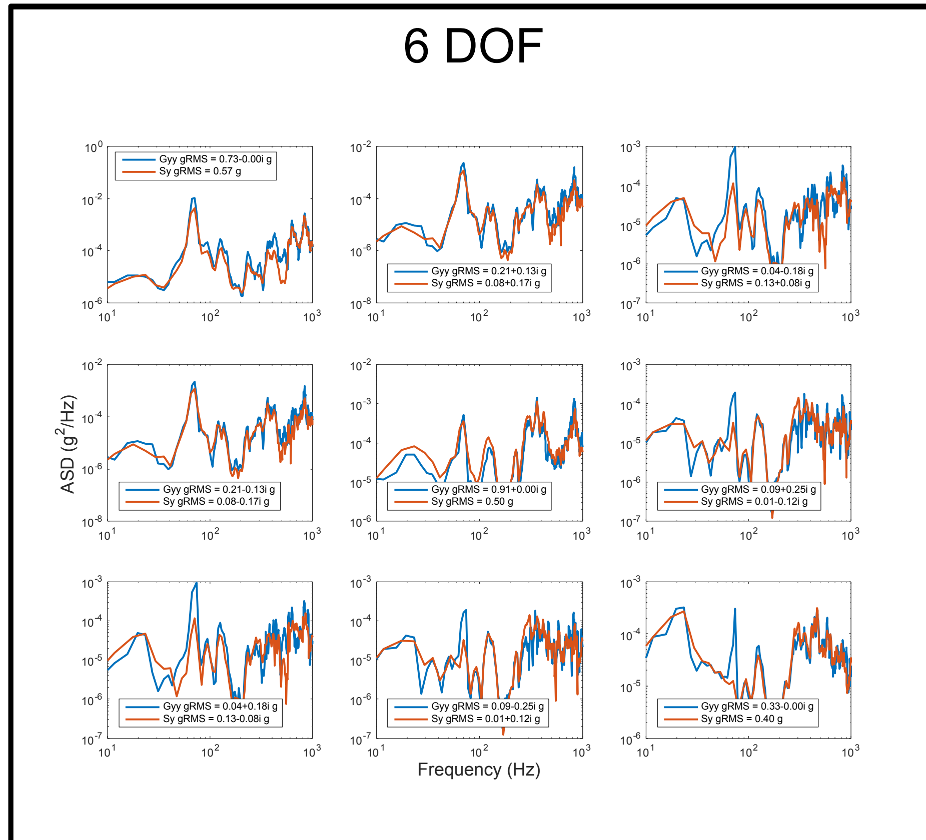
Check condition of $[H(\omega)]^{-\dagger}$.



Method (1) Forward Problem

- Run the Forward Problem as a Check $[S_y] = [H][S_x][H]'$
- There are going to be some errors





S_{xx}=Z*S_{yy}*Z' Method (2)

- Concept
  - $[Z(\omega)] = [H(\omega)]^{-\dagger}$
  - Input Spectral Density Matrix: $[S_{xx}(\omega)] = \; [Z(\omega)] * \left[S_{yy}(\omega)\right] * [Z(\omega)]'$
  - $[S_{yy}(\omega)]$ is the response spectral density matric from the flight data.
- Implementation
  - Interpolate $[S_{yy}(\omega)]$ to match frequency spacing of $[H(\omega)]$
- Checks
  - Conditioning of $[H(\omega)]$
  - Positive Semidefinite $[S_{xx}]$
    - Determinant > 0
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Method (2) Forward Problem

- Run the Forward Problem as a Check $[S_y] = [H][S_x][H]'$
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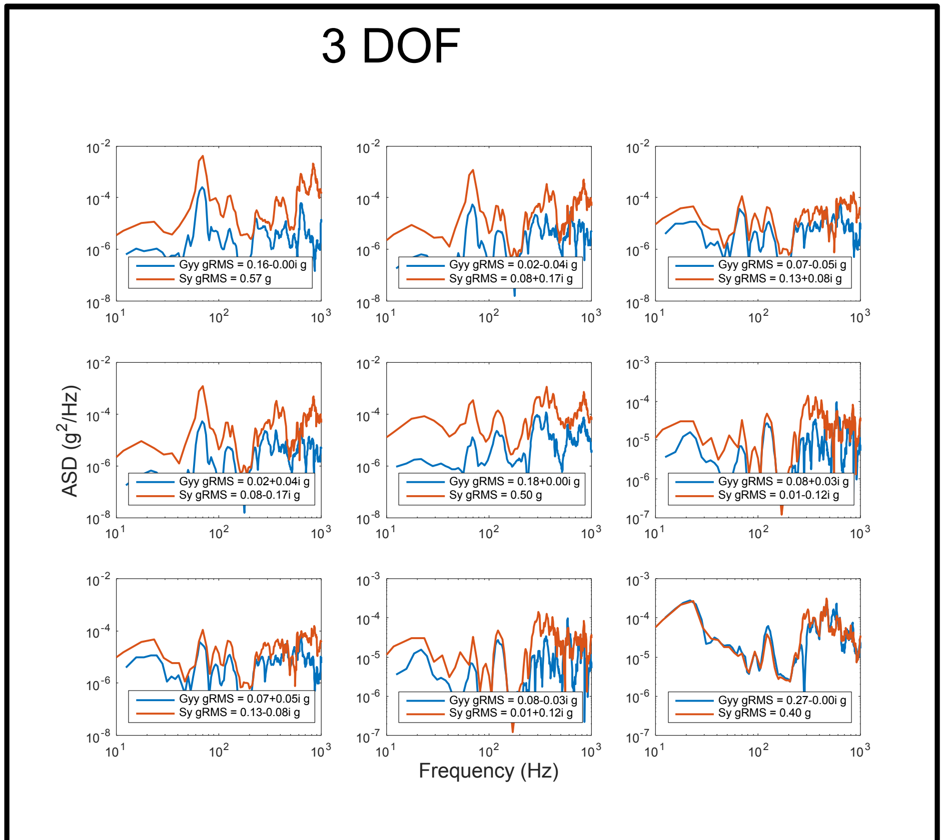
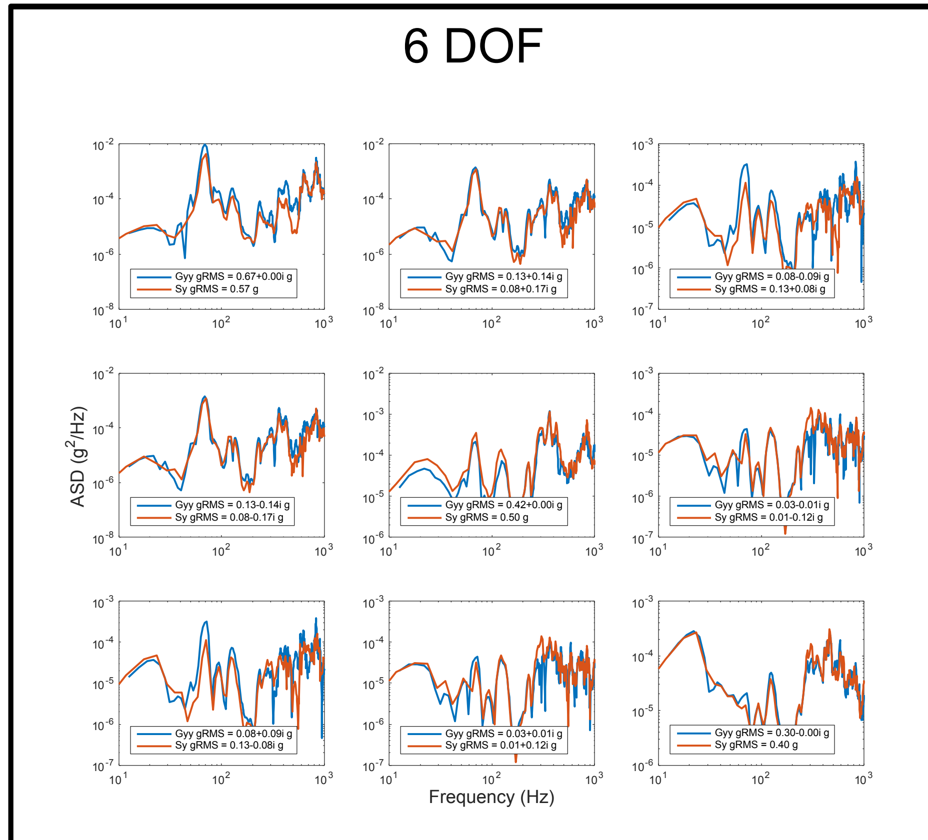
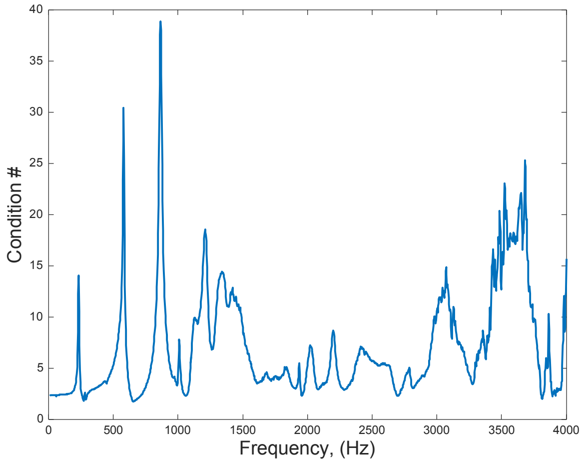




S_x=Z*S_y*Z' with Scaling Method (3)

- Smallwood proposed a scaling method to assure that the input SDM is positive semidefinite
  - This is more relevant when we start talking about drawing straight line specifications and coming up with test specifications.
  - Also useful if you just derive the diagonal of the input SDM for specifications
  - It obviously comes at a cost and I wanted to see if I could figure out the cost with this scaling.
- Scaling Concept
  - $[S_{yy}]_{new} = [S_s][S_{yy}]_{old}[S_s]$
  - $S_{s,ii} = \sqrt{\frac{1}{S_{yy,old,ii}}}$

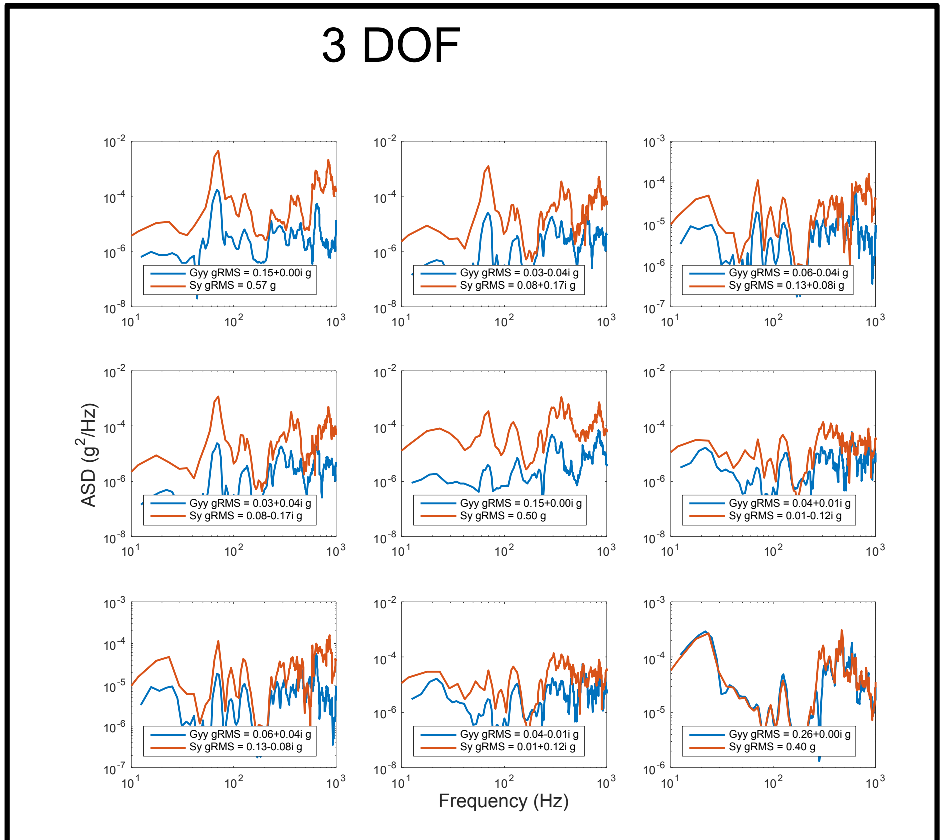
S_x=Z*S_y*Z' with Scaling Method (3)

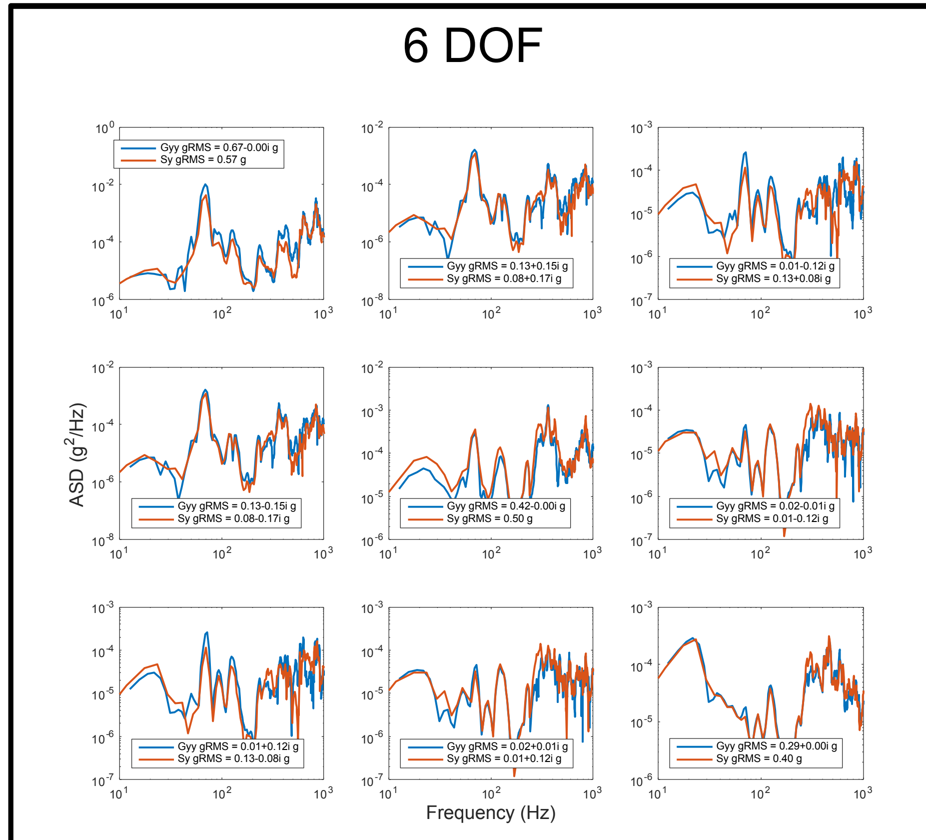


- Scaling Concept Continued
  - $[S_{yy}]_{new} = [S_s][S_{yy}]_{old}[S_s]$
  - $S_{s,ii} = \sqrt{\frac{1}{S_{yy,old,ii}}}$
  - $[S_{xx}(\omega)]_{scaled} = [Z(\omega)] * [S_{yy}(\omega)]_{new} * [Z(\omega)]'$
  - Scale back the results
    - $[S_{xx}]_{actual} = [Z] * [S_s]^{-1} * [H] * [S_{xx}]_{scaled} * [H]^T * [S_s]^{-1} * [Z(\omega)]'$
    - These are a function of frequency.
- Implementation
  - Interpolate $[S_{yy}(\omega)]$ to match frequency spacing of $[H(\omega)]$
- Same Checks as previous (conditioning, positive semidefinite)

Method (3) Forward Problem

- Run the Forward Problem as a Check $[S_y] = [H][S_x][H]'$
- There are going to be some errors





S_x=Z*S_y*Z' with Tikhonov Regularization Method (4)

- Tikhonov for solving $Ax = b$
  - $\hat{x} = [A^T A - \lambda^2 I]^{-1} A^T b$
- We want to solve $[H][S_x][H]' = [S_y]$; Remember these are matrices, but I am going to drop the [].
  - $A = H, X = S_x H', B = S_y$
  - $S_x H' = [H'H \ - \lambda^2 I]^{-1} H' S_y$
  - $S_x H' = C; where\ C = \ [H'H \ - \lambda^2 I]^{-1} H' S_y$
  - Do another Tikhonov Regularization, but we want to solve for S_x
  - $[S_x H']' = C' \ \rightarrow HS_x' = C'; Tikhonov\ (A = H, X = S_x', B = C')$
  - $\widehat{S_x'} = [H'H \ - \lambda^2 I]^{-1} H' C'$
  - Iterate with setting $S_y = H\widehat{S_x}H'$

Method (4) also includes

- We actually don't give it the true S_y.
- We give it a $\widehat{S_y}$ that is
  - We provide the autospectral density of the response (diagonal of S_y) into $\widehat{S_y}$. $\widehat{S_{yii}} = S_{yii}, for\ i = 1, \dots, n\ (size\ S_y)$.
  - We make sure the phase and coherence of $\widehat{S_y}$ is compatible with H and S_x (Though before first Tikhonov iteration this is a guess of $diag(1e-6)$).
    - $C = HS_xH'$
    - Normalize off diagonal terms (see Cap 2009)
      - $C_{ijN} = \frac{c_{ij}}{\sqrt{c_{ii}c_{jj}}}$
    - $\widehat{S_{y_{ijN}}} = C_{ijN};\ \ when\ i \neq j$ (Off diagonal Terms)
    - Scale Back Off diagonal terms $\widehat{S_{y_{ij}}} = \widehat{S_{y_{ijN}}}\sqrt{S_{yii}S_{yjj}}, when\ i \neq j$
  - We calculate the S_x using Tikhonov and $\widehat{S_y}$ with iterations.

J.S. Cap, D.G. Tipton, D.O. Smallwood, ***"The Derivation of Random Vibration Specifications From Field Test Data for Use with a Six Degree-of-Freedom Shaker Test,"*** *80th Shock and Vibration Symposium, San Diego, CA, October 25-29, 2009.*

Method (4) Forward Problem

- Run the Forward Problem as a Check $[S_y] = [H][S_x][H]'$
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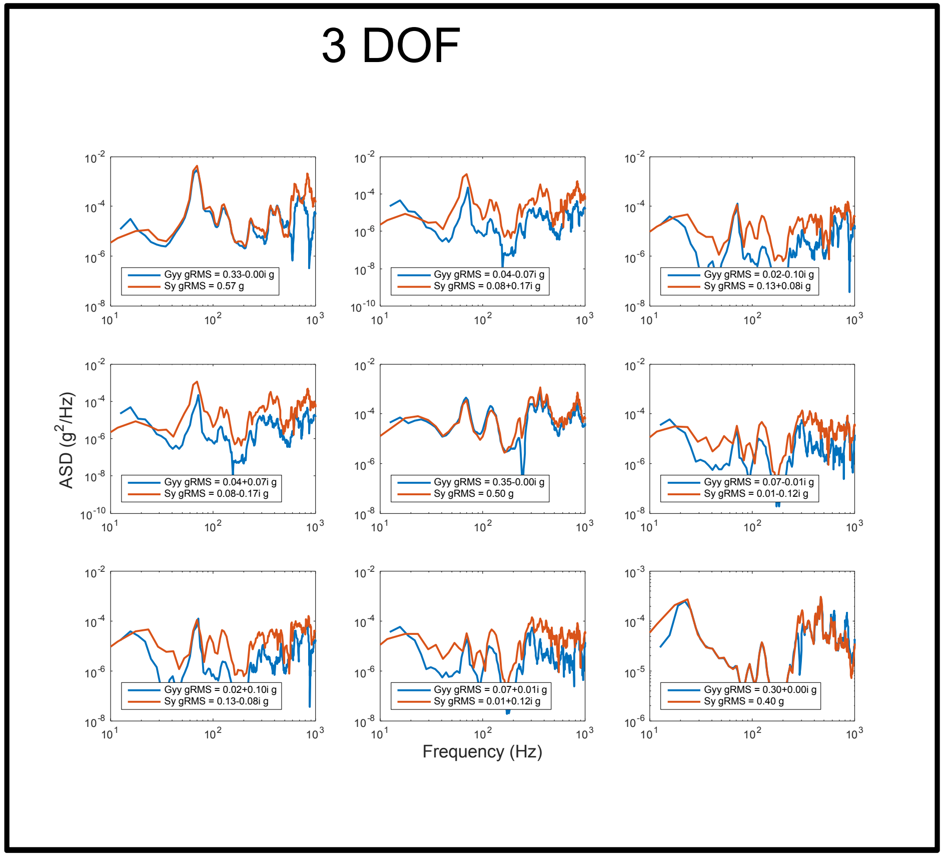
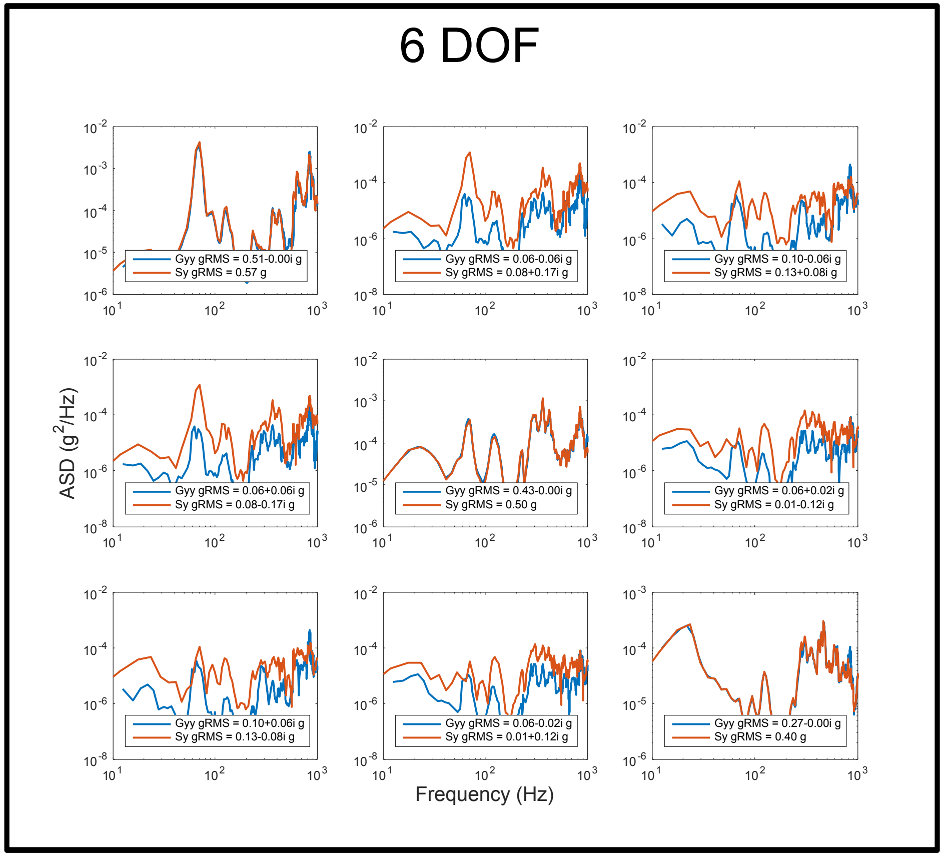
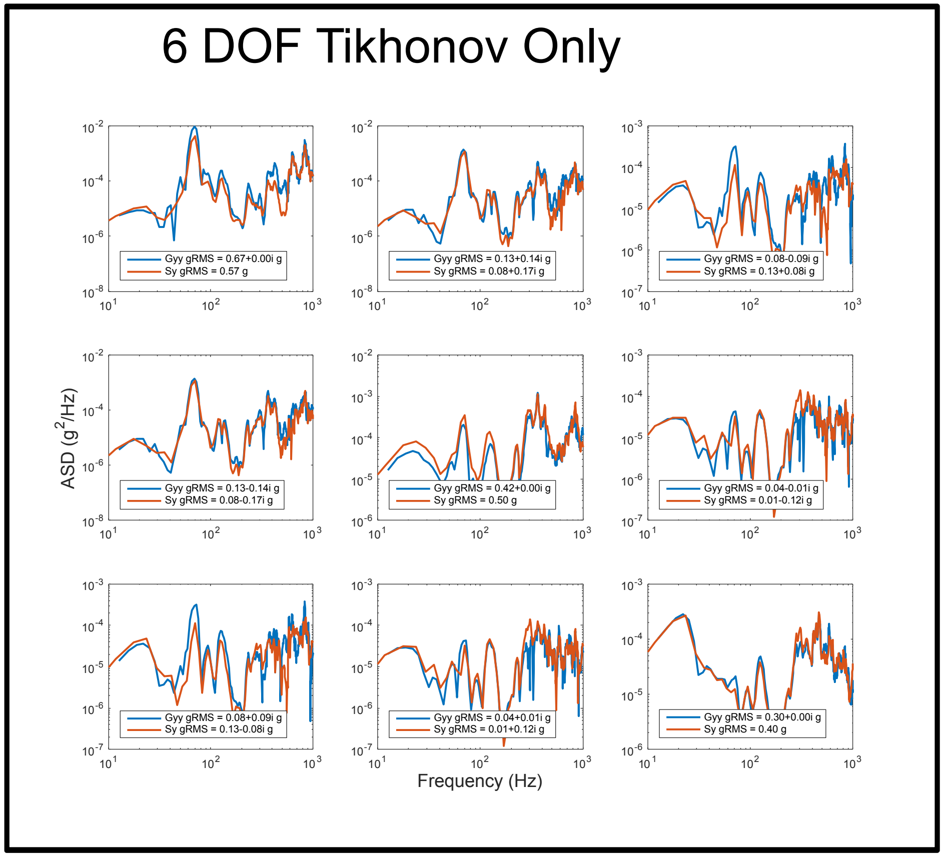
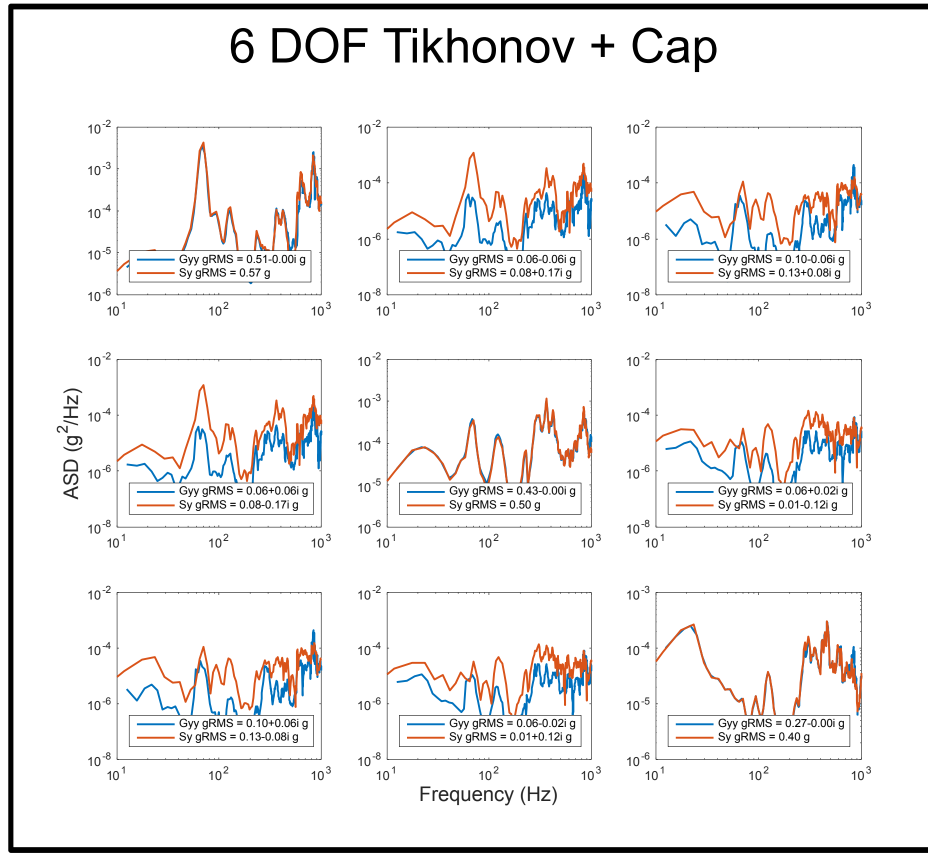




Method (4) Forward Problem

- Compare Only Tikhonov and Tikhonov + Cap’s (modified Off diagonals)





Results – Deep Thoughts

- We know it is not going to be perfect.
  - We have two different units (one that flew and one that we tested)
  - Numerical issues
- The standard now becomes how did the methods compare to if we just derived straight from the flight test instrumentation with no inverse
  - There has to be some relative comparison to try and account for unit-to-unit at the bare minimum.
  - Recall generally we will have to do some type of inverse, for rarely are subsystems & components instrumented enough.
- Trying to distill all information down is difficult
  - I will walk you through one method.
  - Laura will provide another

Response Comparisons

- Let's put the PSD into 6th Octaves and form a dB error between method and actual flight

$$dB_{error} = 10 * \log_{10} \frac{G_{method}(\omega_{6th})}{G_{flight}(\omega_{6th})}$$

$$G(\omega_{6th}) = ASD\ at\ 6th\ octave\ centers$$

- At each similar instrumented gage (between flight and experiment) there is a *dB* error at each center 6th Octave frequency from 10 to 4,000 *Hz* (53 points).
  - There were 10 similar gages thus each method has 530 *dB* error points
  - We can simply take the mean and standard deviation and plot these to try and give an overall feel for how well each method does compared to deriving straight from flight.

Comparisons of all Methods Forward Problem

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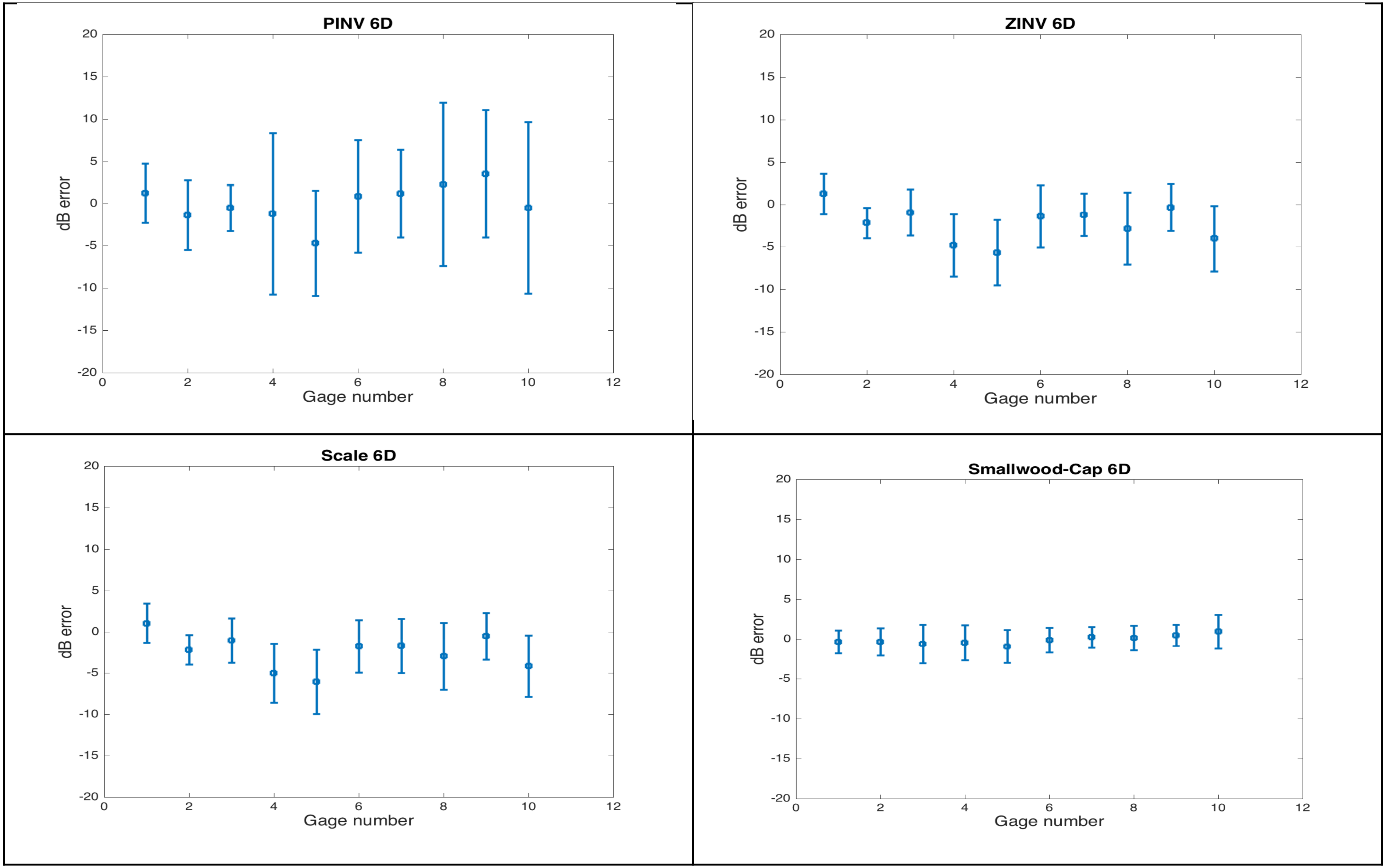

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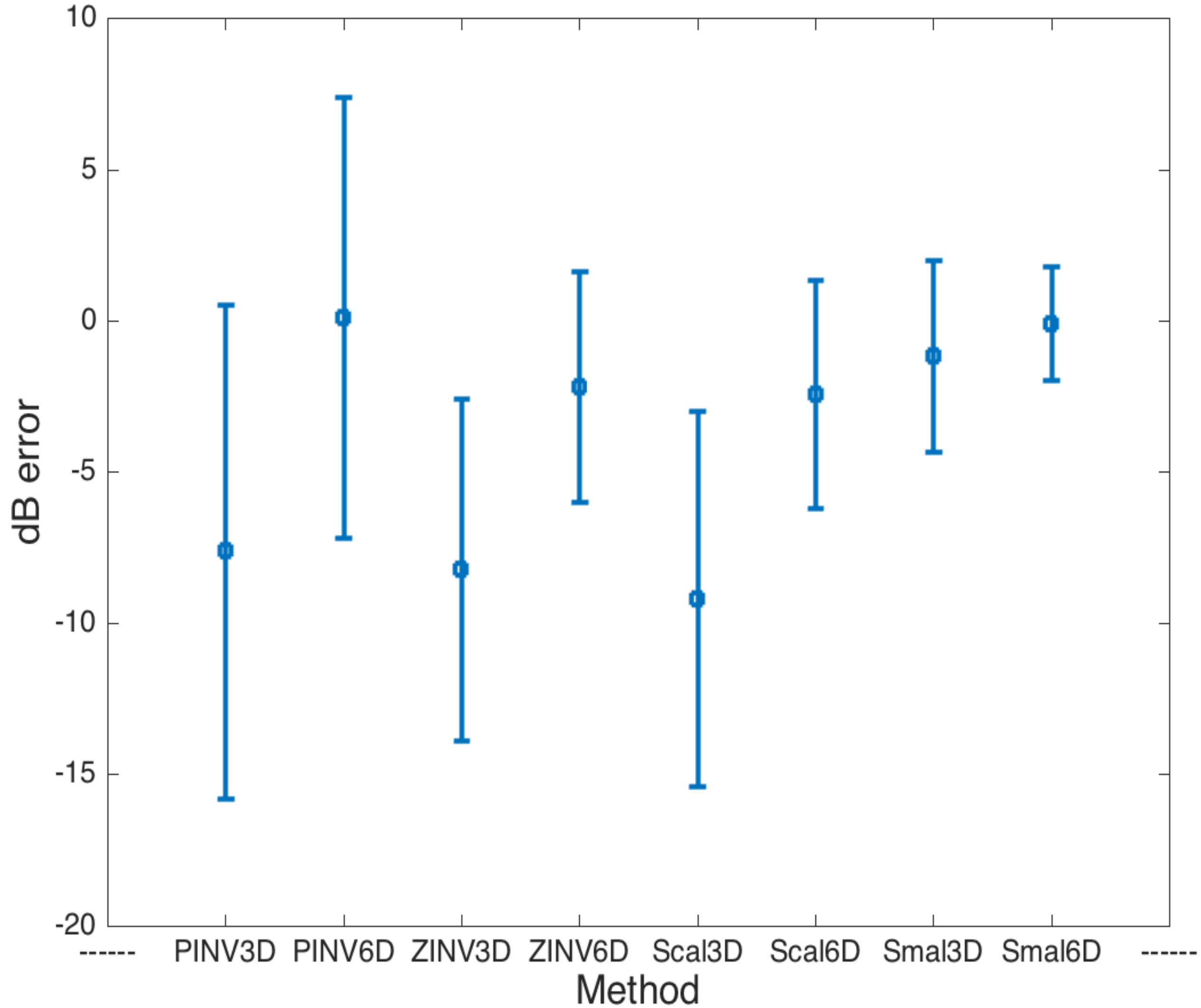


Conclusions

- Before we can run (well Mike is just trying to up to speed with most in the 6 DOF world), we walked.
  - Eventually we want to derive Environmental Specifications at the 6 DOF level.
- This work looked at how to come up with 6 DOF inputs given responses from an actual flight test.
- Explored 4 different mathematical methods
- It appears that method 4 works the best
  - Though, this is very metric dependent and consultation with PRT & Experimentalist is key.
- It appears that we would prefer 6 DOF over 3 DOF.

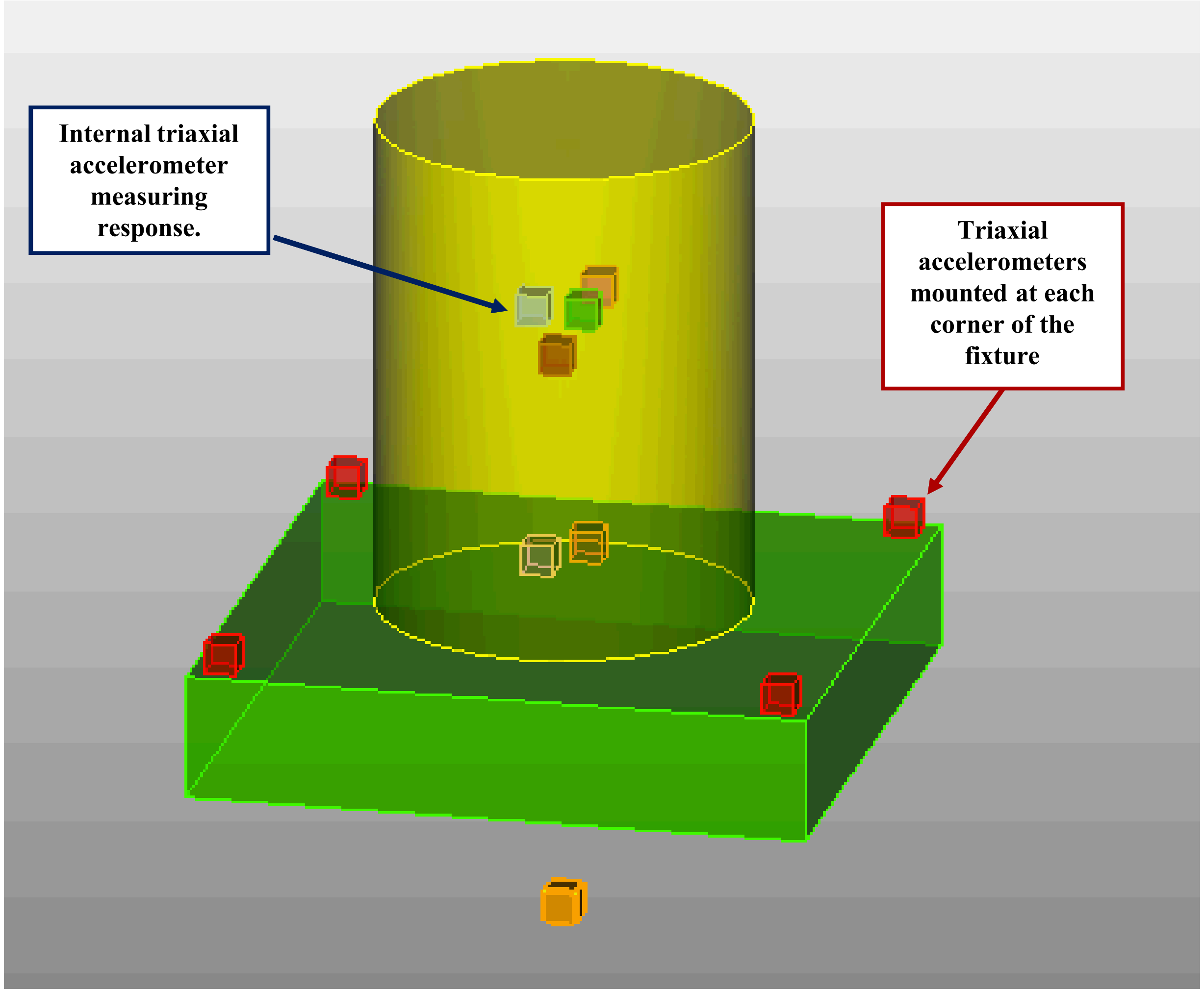
Future Work

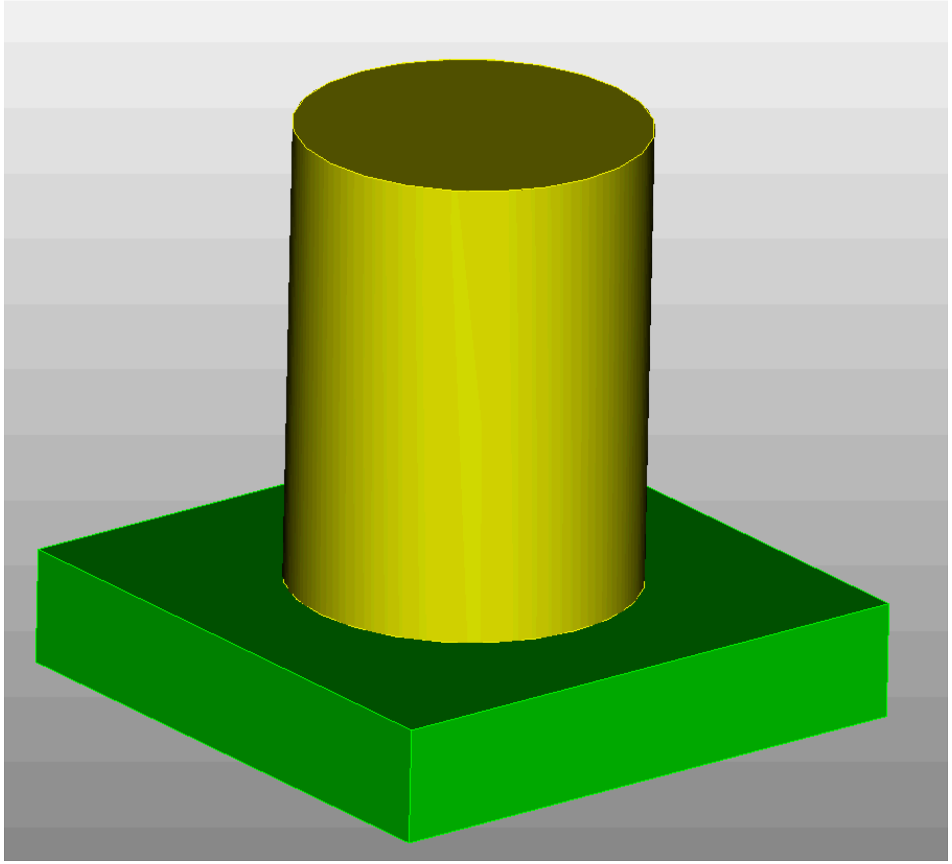
- Develop specifications (straight line, 1/500 or P99/90)
  - Look at given just diagonal of input SDM
    - Experimentalist will figure coherence and phase
  - Maybe just give coherence
  - Positive Semidefinite
- Look at scaling a particular location during input generation such that internal widgets are more important than the legs as an example – Implementing Optimization Algorithm
- What to do when there is only one gage on a component
- Is there an ideal set of gages for inverse
- Look at what happens when we do inverse with certain gages and make predictions at other gages.

BACK UPS

Big Picture Next Is Envi. Spec Derivation

- In regards to Envi. Specifications
  - I think one could first derive appropriate 6 DOF inputs, then apply appropriate statistics (P99/50 or 1/500), and then envelop
  - Alternatively one could do what we do in the past and at each instrumented flight channel develop appropriate statistics (P99/50), envelop, and then generate 6 DOF input from these.
  - Both methods could suffer from input SDM not being positive semidefinite.
- When do we start trying to match stress states instead of accelerations?
- Also comes the question of how to derive specifications with limited to no instrumentation on say a component A, but component B, C, and D do have instrumentation.


Internal triaxial accelerometer measuring response.
Triaxial accelerometers mounted at each corner of the fixture



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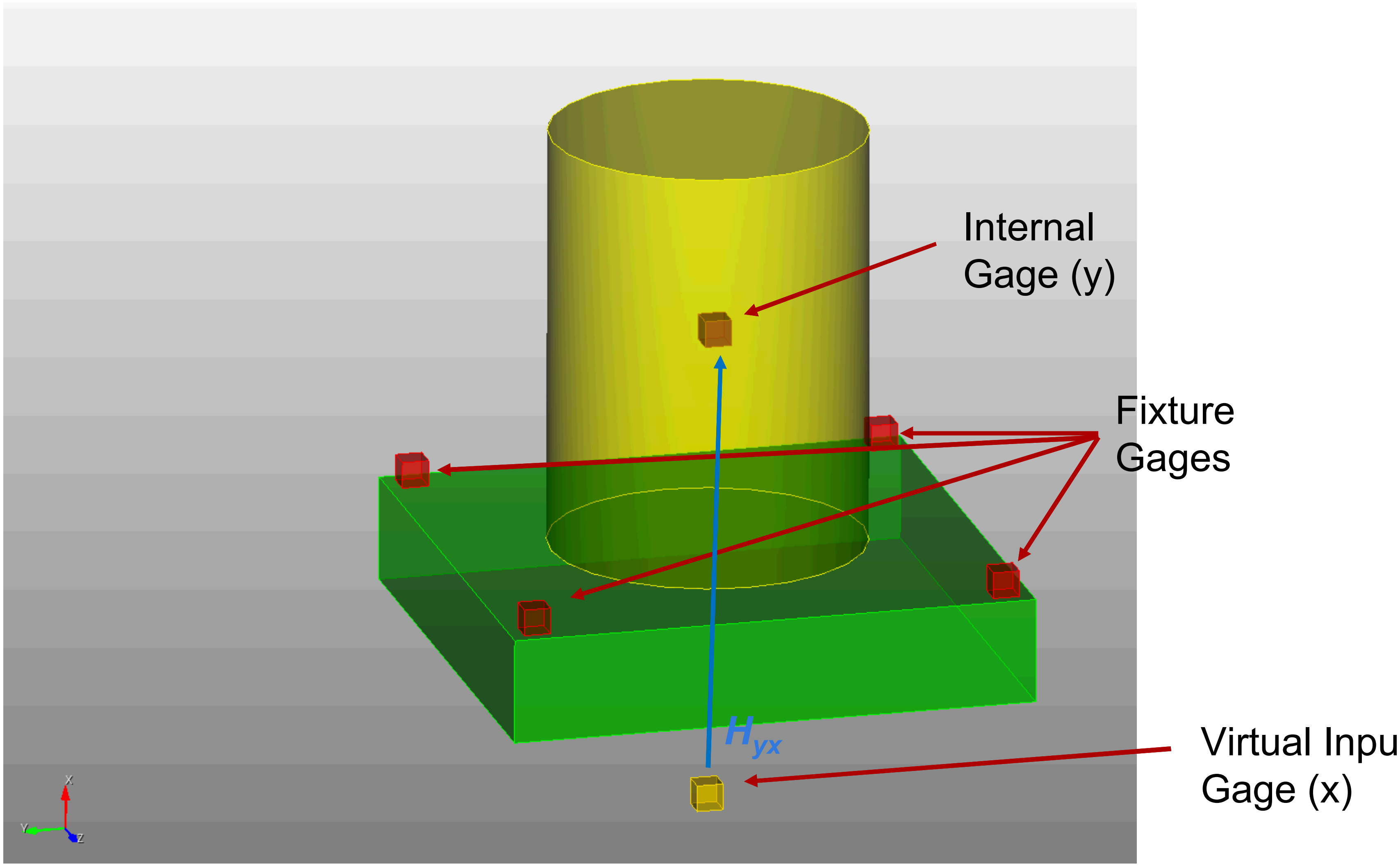

PINV Method (1)

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  - $\{Y(\omega)\} = [H(\omega)]\{X(\omega)\}$.
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- Implementation is a little complex
  - The output $y(t)$ is provided in the time domain from the flight data.
    - Use Fourier Transform to get $Y(\omega)$.
  - Interpolate $[H(\omega)]$to same frequency spacing and band as $Y(\omega)$.
    - $[H(\omega)]$ had a coarser frequency spacing.
  - Perform Operation $\{\mathrm{X}(\omega)\} = [H(\omega)]^{-\dagger}\{Y(\omega)\}$.
  - Convert $\mathrm{X}(\omega)$ into time domain with inverse Fourier Transform
  - Compute input spectral density matrix $[S_{xx}]$, given translation and rotation input accelerations: $x(t), y(t), z(t), rx(t), ry(t), rz(t)$.

Comparison of Inputs

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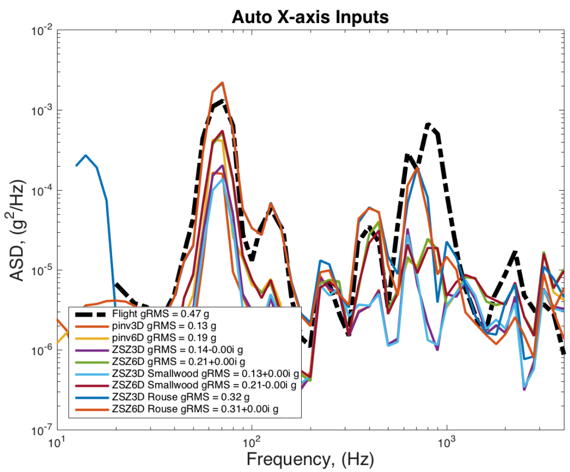
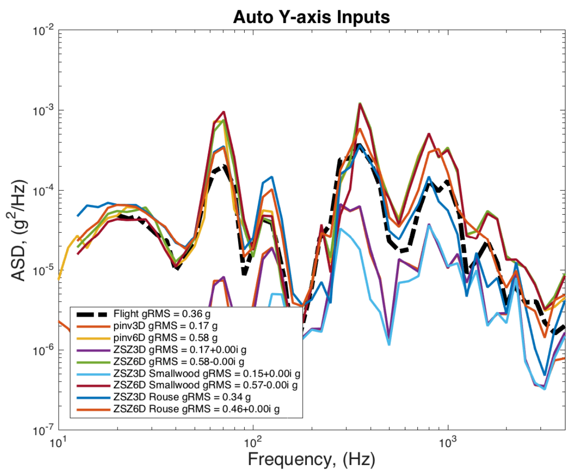
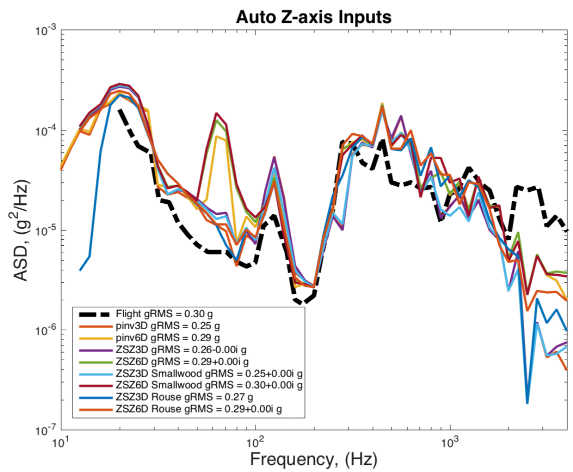
- Translation Inputs have 8 comparisons to Flight Test derivation.
  - The 4 methods
  - Each method includes derivation with translations (X, Y, and Z)- 3D
  - Each method includes derivation with translation and rotation - 6D







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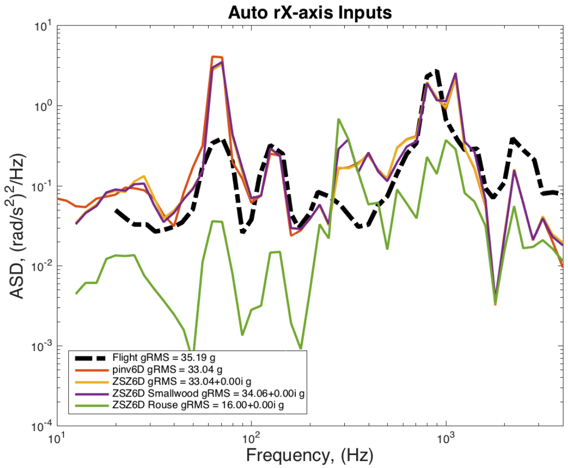

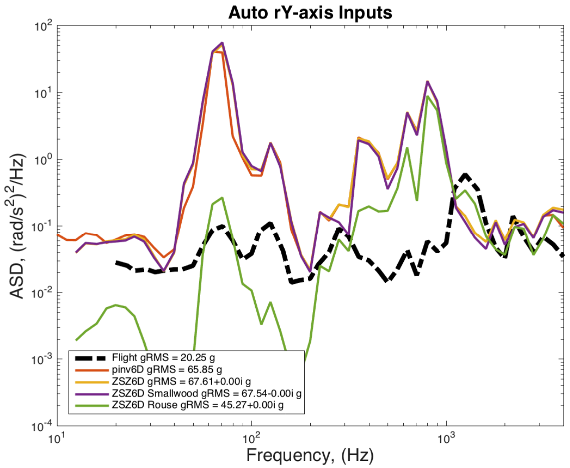

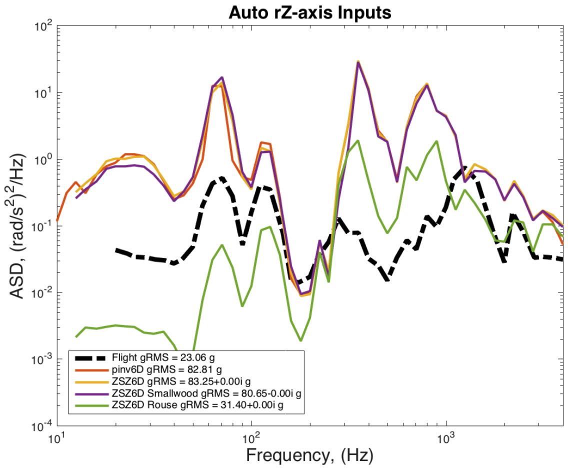

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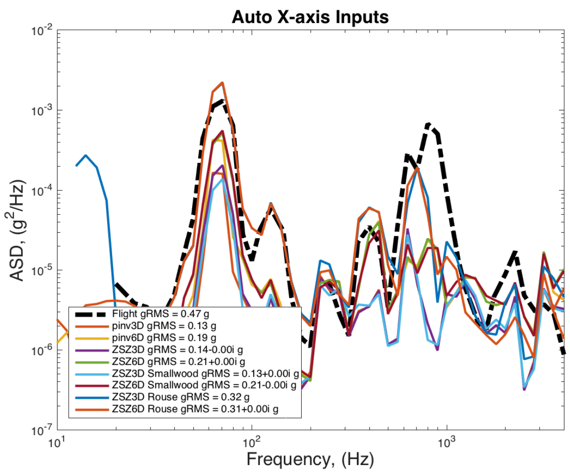

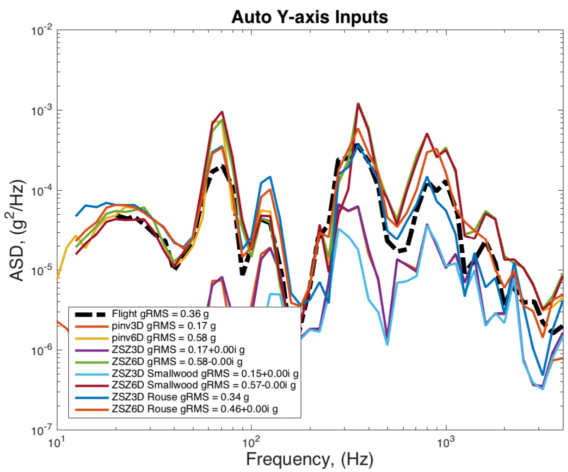

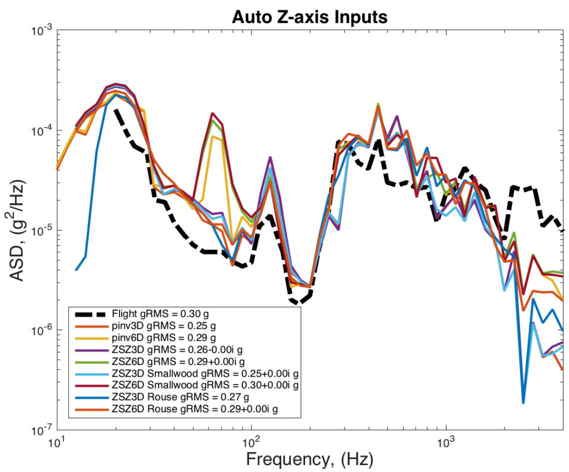

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