Finite-Difference Algorithm for 3D Orthorhombic Elastic Wave Propagation

Richard P. Jensen, Leiph A. Preston and David F. Aldridge
Geophysics Department, Sandia National Laboratories, Albuquerque, New Mexico

Summary

Many geophysicists concur that an ***orthorhombic*** elastic medium, characterized by three mutually orthogonal symmetry planes, constitutes a realistic representation of seismic anisotropy in shallow crustal rocks. This symmetry condition typically arises via a dense system of vertically-aligned microfractures superimposed on a finely-layered horizontal geology:

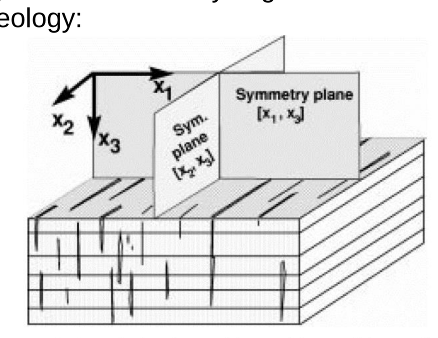


Fig. 1. An orthorhombic model caused by parallel vertical cracks embedded in a medium composed of thin horizontal layers. Orthorhombic media have three mutually orthogonal planes of mirror symmetry.

From Tsvankin, 1997, *Geophysics*.

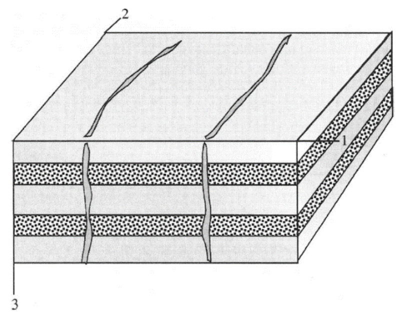
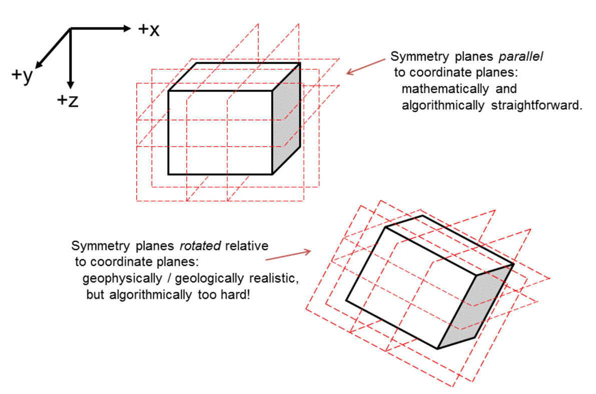


Fig. 1. Schematic diagram of long vertical fractures aligned in the 2,3-plane embedded in a TI medium with a vertical symmetry axis.

From Schoenberg and Helbig, 1997, *Geophysics*.

However, various geological deformation processes will rotate the symmetry planes away from alignment with the global XYZ coordinate planes:

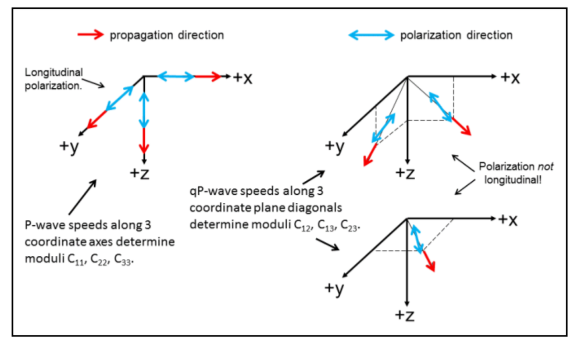


Present algorithmic assumption: 3 principal axes of orthorhombic elastic modulus tensor aligned with global XYZ coordinate axes.

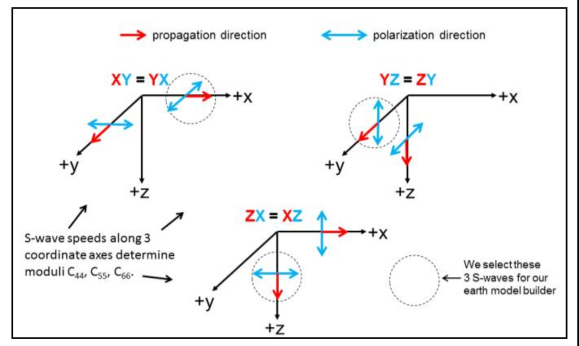
Rotated principal axes lead to significant algorithmic complications! Subject of future R&D.

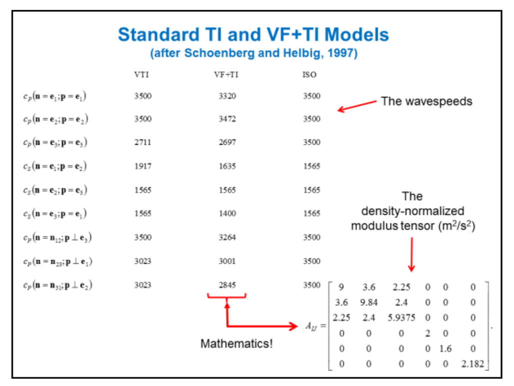
Mathematically, the elastic stress-strain constitutive relations for an orthorhombic body contain nine independent moduli. In turn, these moduli can be determined by observing (or prescribing) nine independent P-wave and S-wave phase speeds along different directions (Brown, 1989):

6 P-Wave Speeds / Directions:



6 S-Wave Speeds / Directions:





Our initial test modeling utilizes the "standard model" of a VF+TI (vertical fractures + transverse isotropic) elastic model of Schoenberg and Helbig (1997), plus its TI and isotropic counterparts.

The anisotropic elastic ***velocity-stress system***, a set of 9 coupled, first-order, linear, inhomogeneous PDEs forms the mathematical basis for our explicit time-domain finite-difference (FD) numerical algorithm. All partial derivatives are discretized with centered and staggered FD operators that are 2nd-order in time and 4th-order in space:

Governing PDE System: Anisotropic Elastic Velocity-Stress System

$$\rho \frac{\partial v_i}{\partial t} - \frac{\partial \sigma_{ij}}{\partial x_j} = f_i + \frac{\partial m^a_{ij}}{\partial x_j}$$ 3 equations of motion

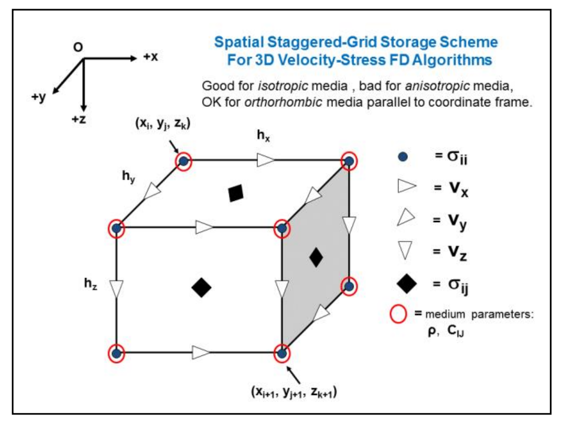
$$\frac{\partial \sigma_{ij}}{\partial t} - c_{ijkl} \frac{\partial v_k}{\partial x_l} = \frac{\partial m^s_{ij}}{\partial t}$$ 6 stress/strain constitutive relations

Nine, coupled, first-order, linear, non-homogeneous partial differential equations.

Wavefield variables: $v_i(\mathbf{x},t)$ - velocity vector; $\sigma_{ij}(\mathbf{x},t)$ - stress tensor

Earth model parameters: $\rho(\mathbf{x})$ - mass density; $c_{ijkl}(\mathbf{x})$ – modulus tensor ($c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}$)

Body sources: $f_i(\mathbf{x},t)$ - force vector; $m_{ij}(\mathbf{x},t)$ - moment tensor



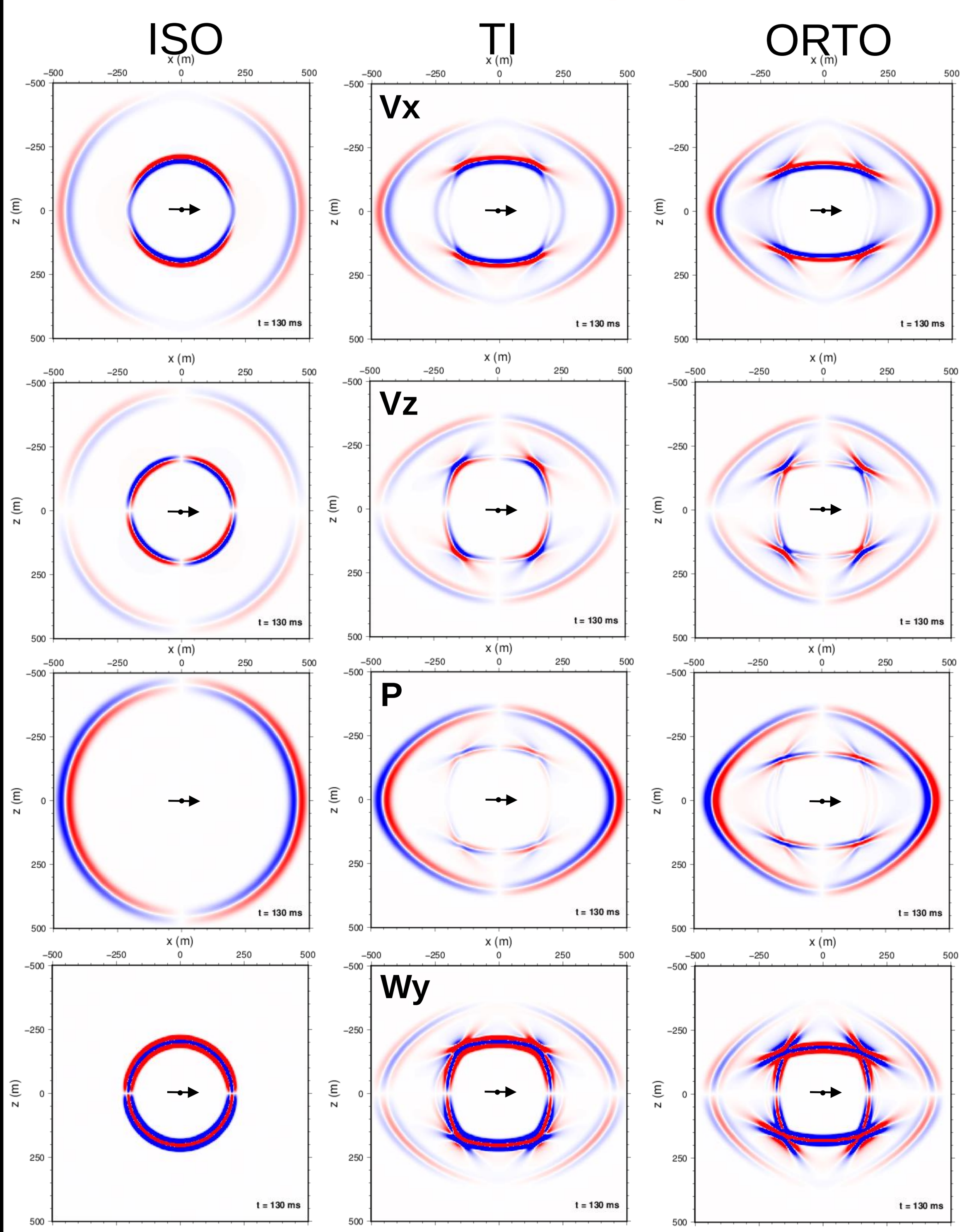
Wavefront Timeslice Plots for Different Seismic Energy Source Types

Columns: media types (ISO = isotropic, TI = transverse isotopic, ORTO = orthorhombic)

Rows: **wavefield variables (Vx, Vy, Vz = particle velocities, P = pressure, Wx, Wy, Wz = particle rotation rates)**

Source wavelet: Gaussian pulse, derived via double integration of a 50 Hz peak frequency Ricker wavelet

X-Direction Force (Fx) Source



Outer wavefront: P-wave

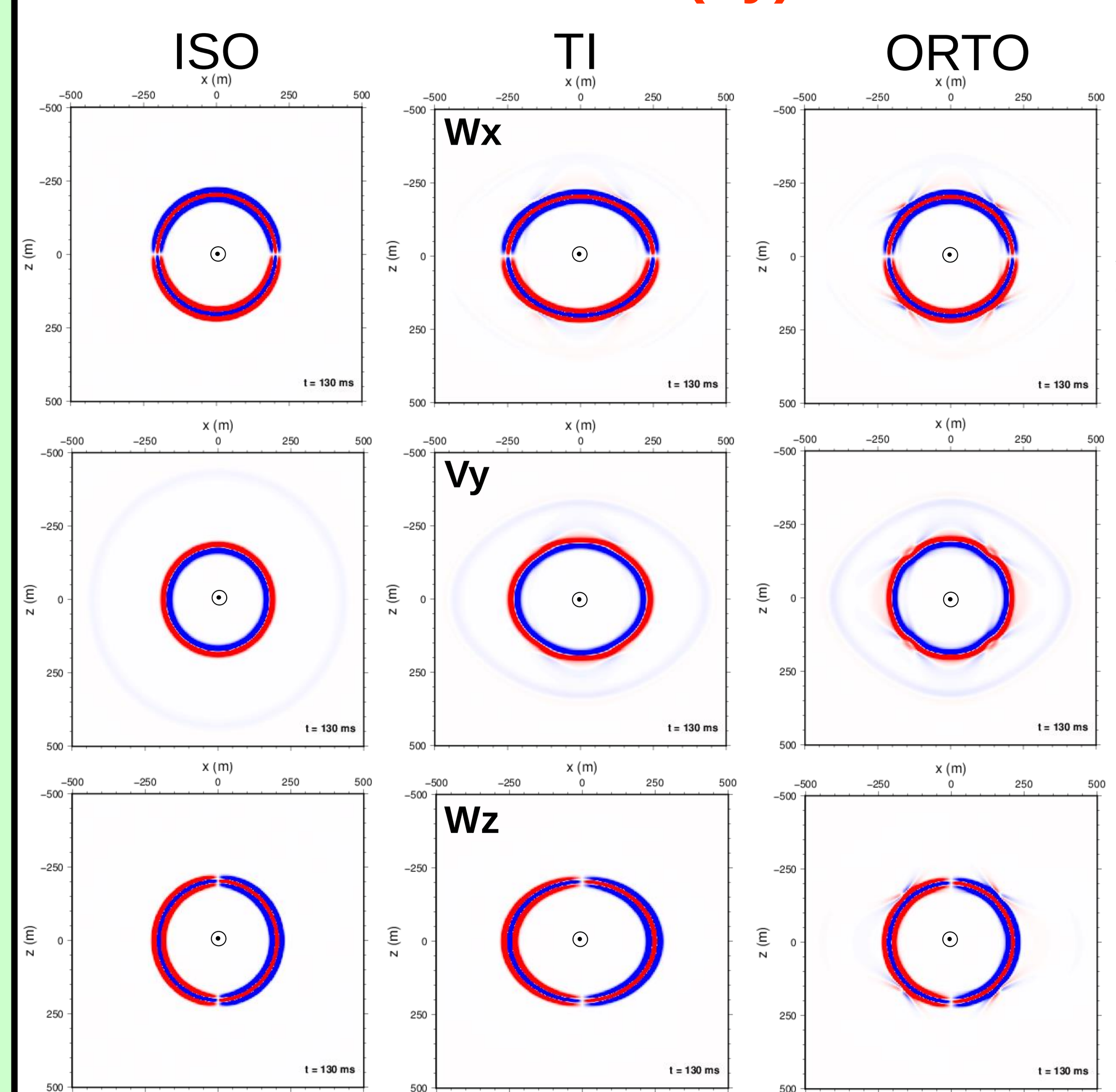
Inner wavefront: SV wave

Triplications in TI and ORTO SV wavefronts

Pressure wavefronts propagate with both P and S speeds in TI and ORTO media

Weak rotation wavefronts propagate with P speed in TI and ORTO media

Y-Direction Force (Fy) Source



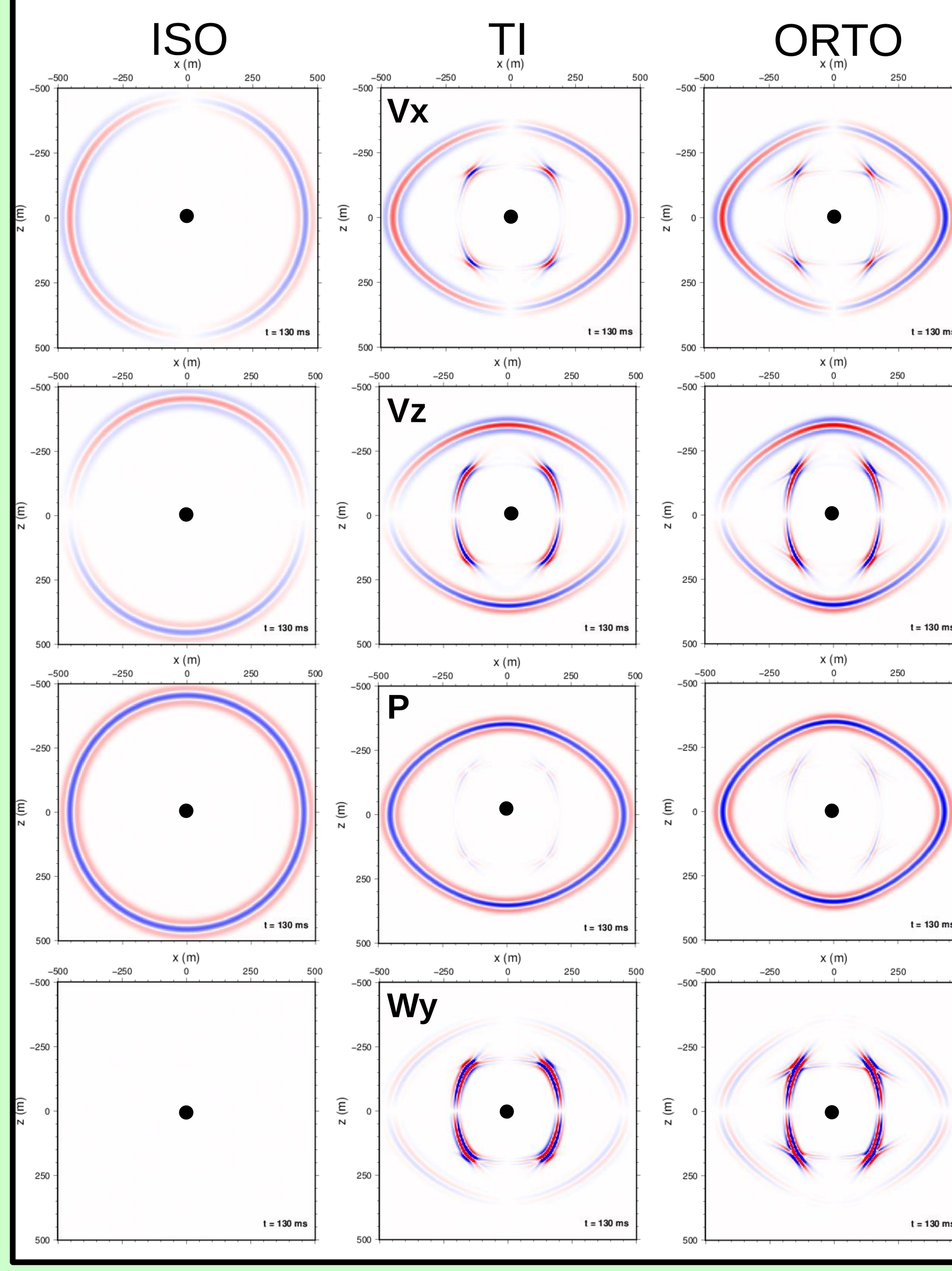
Non-circular rotation wavefronts in TI and ORTO media.

Fy source generates SH motion in all 3 media types

Note weak transverse polarized P wave in all 3 media

Wx and Wz nodal planes rotated 90° rel to each other

Isotropic Explosion Source



Explosion source generates SV waves in TI and ORTO

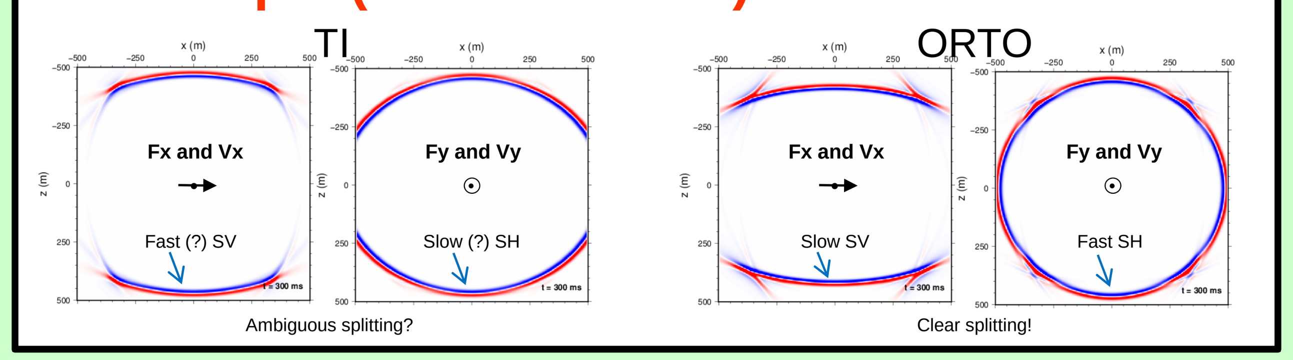
Note triplications

Strong shear from explosion source in TI and ORTO

Non-circular pressure wavefronts, and weak S-wave pressure signal, in TI and ORTO

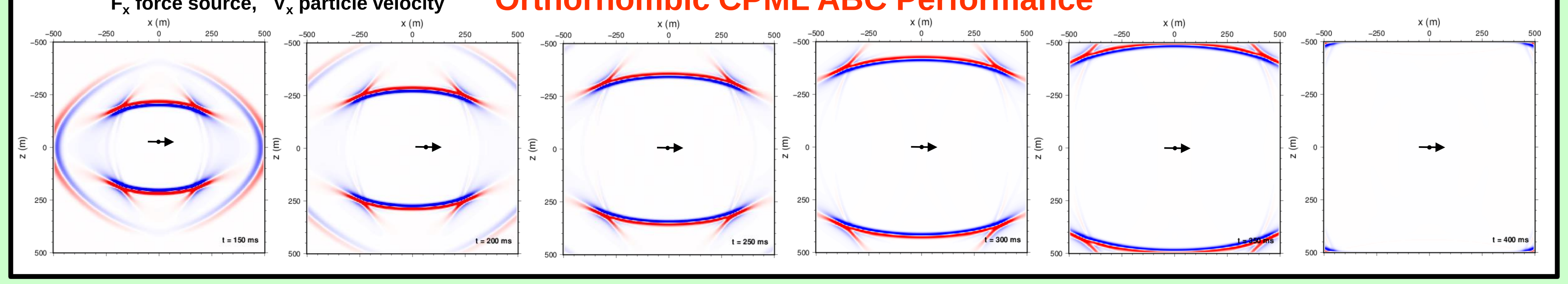
Weak P and strong SV rotation wavefronts in TI and ORTO

Split (Fast and Slow) Shear Waves



Orthorhombic CPML ABC Performance

F_x force source, V_x particle velocity



P- and SV-wavefronts cleanly exit edges and corners of computational grid *without* generating any visible reflected or diffracted energy!

Conclusions

Explicit time-domain finite-difference numerical algorithm demonstrates known anisotropic seismic phenomena of:

1) Complex wavefront shapes, 2) Pressure / rotation propagating with *both* P / S speeds,

3) Split (fast and slow) shear waves, 4) *Shear* waves from isotropic explosion.

Future algorithmic work includes implementation of:

1) Orthorhombic stress-free surface, 2) Rotated modulus tensor principle axes.

Acknowledgements

Sandia National Laboratories is a multi-mission laboratory managed and operated by Sandia Corporation, a wholly-owned subsidiary of Lockheed Martin Corporation, for the US Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. The present work is conducted under the auspices of the Source Physics Experiment (SPE) funded by the Office of Defence Nuclear Nonproliferation Research and Development (DNN R&D) of the NNSA.

References

Brown, R.J., 1989, Relationships between the velocities and the elastic constants of an anisotropic solid possessing orthorhombic symmetry: CREWES consortium report.
Schoenberg, M., and Helbig, K., 1997, Orthorhombic media: modeling of elastic wave behavior in a vertically fractured earth: Geophysics, **62**, 1954-1974.
Tsvankin, I., 1997, Anisotropic parameters and P-wave velocity for orthorhombic media: Geophysics, **62**, 1292-1309.