Modeling Near-Surface Metallic Clutter Without the Excruciating Pain

Christine Downs[1,2], Chester J Weiss[1,3], Jeffrey Bach[1], and Jeffery T Williams[1]

Exceptional service in the national interest

Sandia National Laboratories

Abstract

An ongoing problem in modeling electromagnetic (EM) interations with the near-surface and related anthropogenic metal clutter is the large difference in length scale between the clutter dimensions and their resulting EM response. For example, observations evidence shows that cables, pipes, and rail lines can have a strong influence far from where they are located, even in situations where these artefacts are volumetrically insignificant over the scale of the model. This poses a significant modeling problem for understanding geohazards in urban environments because of the very fine numerical discretization required for accurate representation of an artefact embedded in a larger computational domain. We adopt a sub-grid approximation and impose a boundary condition along grid edges to capture the vanishing fields of a perfect electric conductor.

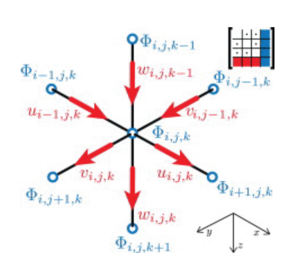
We work in a Cartesian system where the EM fields are solved via finite volumes in the frequency domain in terms of the Lorenz gauged magnetic vector (A) and electric scalar (φ) potentials. The electric field is given simply by A-grad(φ), and set identically to zero along edges of the mesh that coincide with the center of long, slender, metallic conductors. A simple extension to bulky artefacts, like blocks or slabs, involves endowing all such edges in their interior with the same "internal" boundary condition. In essence, we apply the "perfect electric conductor" boundary condition to select edges interior to the modeling domain. We note a few minor numerical consequences of this approach, namely: the zero-E field internal boundary condition destroys the symmetry of the finite volume coefficient matrix; and, the accuracy of representation of the conducting artefact is restricted by the relatively coarse discretization mesh. The former is overcome with the use of preconditioned bi-conjugate gradient methods instead of the quasi-minimal-residual methods (QMR). Both are matrix-free iterative solvers— thus avoiding unnecessary storage— and both exhibit generally good convergence for well-posed problems. The latter is more difficult to overcome without either modifying the mesh (potentially degrading the condition number of the conefficient matrix) or with novel mesh sub-gridding. Initial results show qualitative agreement with the expected physics.

Problem Statement & Approach

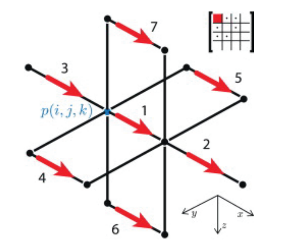
Discretization on staggered grid for finite difference computations

Following Weiss (Computers & Geosciences, 2013) the governing Maxwell equations were cast in the frequency ω domain and further decomposed in terms of their magnetic vector **A** and electric scalar φ potentials, mimetically distributed on a staggered Cartesian Yee (1966) grid. Choosing the Lorenz gauge $\nabla\cdot\mathbf{A} = \mathrm{i}\omega\mu_0\hat{\sigma}\Phi = k^2\Phi$ and explicitly enforcing charge conservation results in a coupled set of equations:

$$\begin{pmatrix} -\nabla^2 + k^2 & \nabla k^2 - k^2\nabla \\ \nabla\cdot k^2 - k^2\nabla\cdot & -\nabla\cdot k^2\nabla + k^4 \end{pmatrix} \begin{pmatrix} \mathbf{A} \\ \Phi \end{pmatrix} = \begin{pmatrix} \mu_0 \mathbf{J}_s \\ \mu_0\nabla\cdot\mathbf{J}_s \end{pmatrix}$$



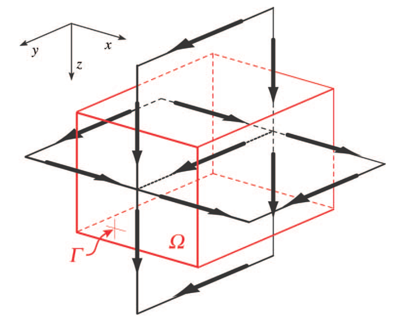
Finite difference stencil for discrete grad and div operators. Electric scalar potential (blue dots) is defined on cell vertices.



Finite difference stencil for discrete Laplacian Operator. Magnetic vector potential (red arrows) is defined on cell edges.

whose discrete version is solved iteratively using the stabilized bi-conjugate gradient method (van der Vorst, 1992) for complex-valued, non-symmetric matrices. Observe that the matrix operator in the continuous case (shown above) is symmetric, which would allow for use of a linear solver that exploits this property (e.g. QMR). However, imposing the "thin wire approximation" ad hoc, as we show-below, results in non-symmetric system of equations.

Discretization of the continous coupled system of equations shown above is done in two steps. For the three coupled equations associated with top row of the system, we assume locally linear polynomial order on the potentials and integrate each equation over a region Ω centered on the corresponding x, y and z edges (for the x, y and z equations). If a cell (or row of cells) was assigned a high conductivity in an attempt to represent a slender "wire", the effect would be propagated to the multiple finite difference stencils (in all three coordinate directions!) neighboring the high con-ductivity cell(s). Hence, the thinness of the wire in the numerical simulation would be severley compomised. The second step is to integrate the 4th equation, again assuming local linearity over a staggered node centered volume a step which further accentuates the "bigness" of the thin wire.

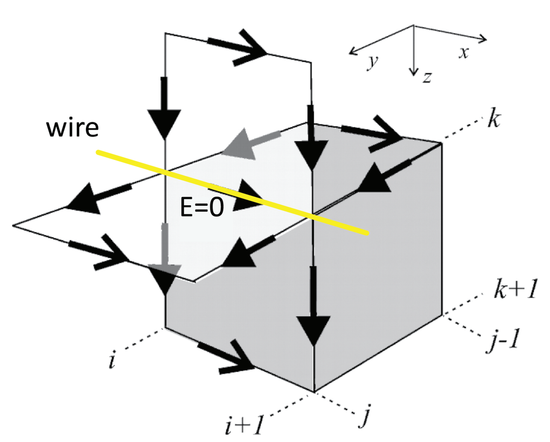


Defining perfect conductor in staggered grid

To minimize the footprint of a thin, filamentary conductor (a "wire") on the finite difference discretization template, we start with the observatation that due to "skin effects", there is no electric field inside a perfect conductor. Hence, on edges of the Cartesian grid where the wire resides, we replace the equations in x, y or z resulting from volume integration with the auxiliarly condition:

$$\hat{\xi}\cdot\mathbf{E} = -\mathrm{i}\omega\,\hat{\xi}\cdot(\mathbf{A}-\nabla\Phi) = 0$$

where $\hat{\xi}$ is taken to be a unit vector in the direction along which the wire is oriented. Additional accuracy could be achieved by further prescription of the ln(r) fall-off in electric field orthogonal to the wire, however we find the simple expression above to be sufficient. Notice that replacement of the usual finite difference equations by the one just described breaks the symmetry of the linear system, thus necessitating use of solver tolerant of non-symmetric systems, such as BiCGstab (van der Vorst, 1992).



Correspondng Author: Christine Downs, cdowns@sandia.gov

[1] Geophysics Department, Sandia National Laboratories, Albuquerque NM

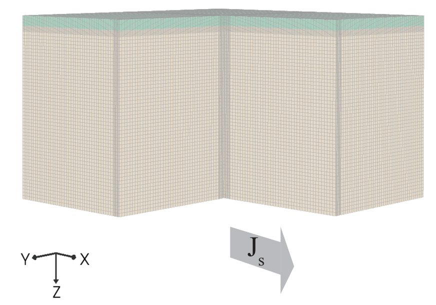
[2] University of South Florida, Tampa, FL

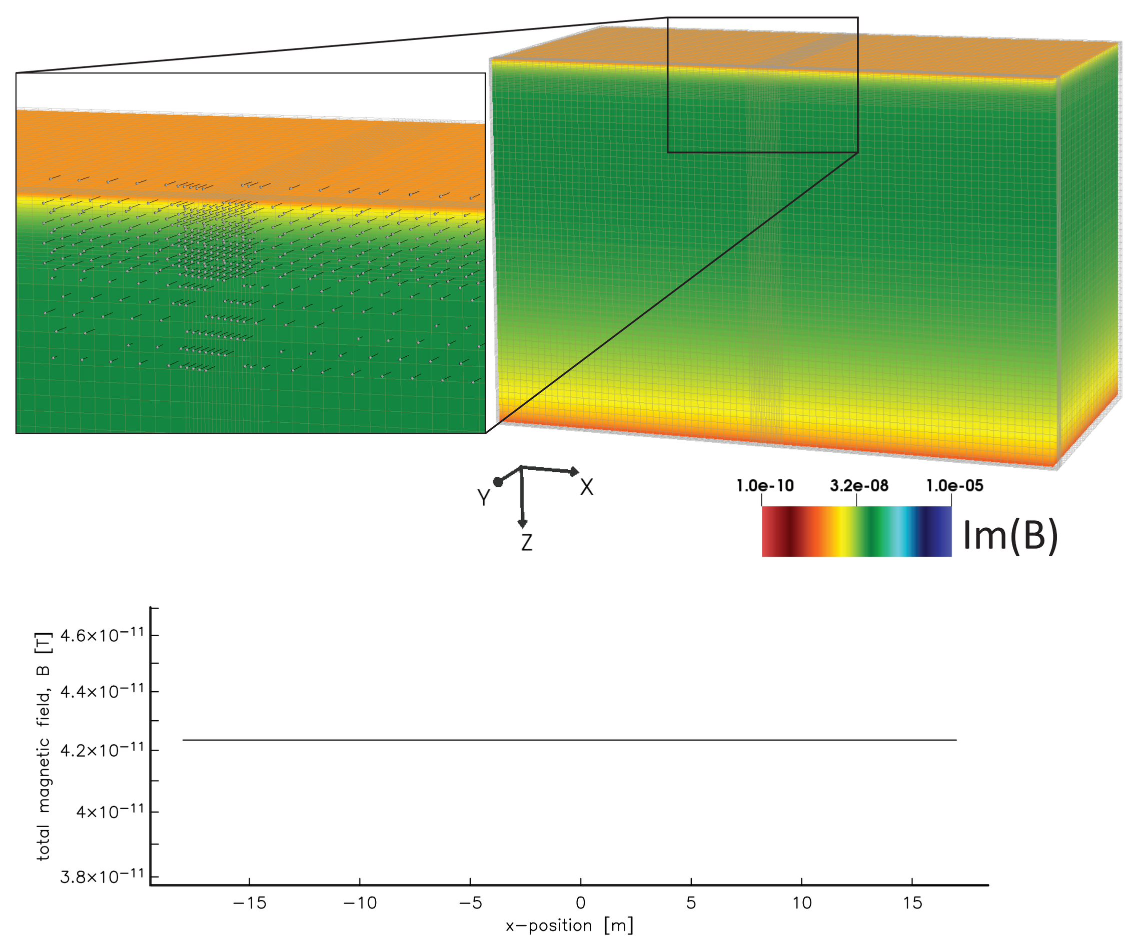
[3] University of New Mexico, Alburquerque, NM

Intersection of Geology and Infrastructure: B-field response

Horizontal layers without metallic clutter.

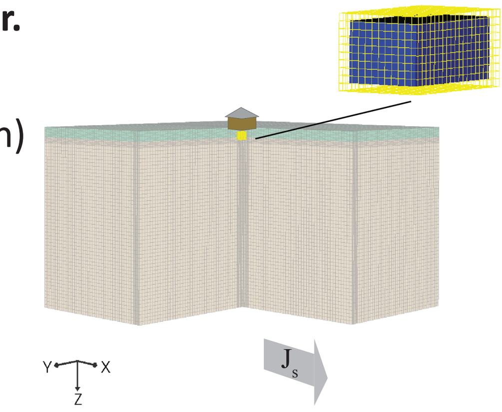
- 100 m x 100 m x 50 m
- Relative conductor (0.01 S/m, green) over a resistor (0.001 S/m, tan)
- X-directed, 25 kHz plane wave source

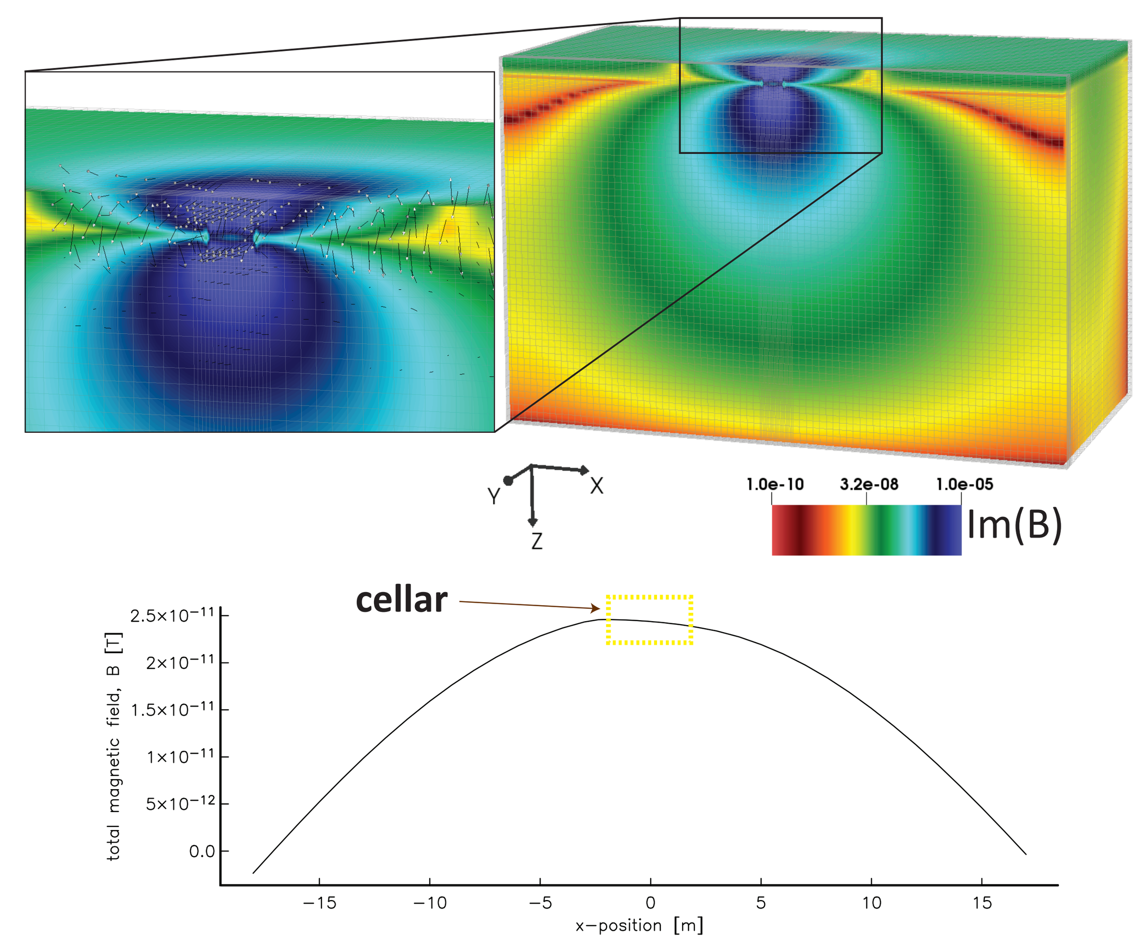




Horizontal layers with metallic clutter.

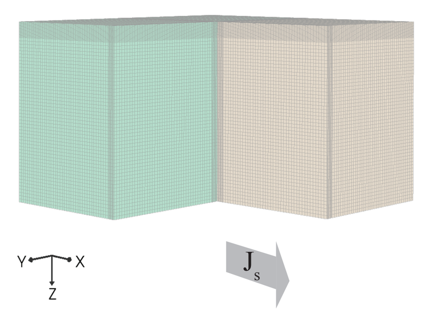
- 100 m x 100 m x 50 m
- Relative conductor (0.01 S/m, green) over a resistor (0.001 S/m, tan)
- Metallic clutter is an air-filled, rectangular wire mesh (cellar)
- cellar is 3 m x 5 m x 1.5 m
- X-directed, 25 kHz plane wave

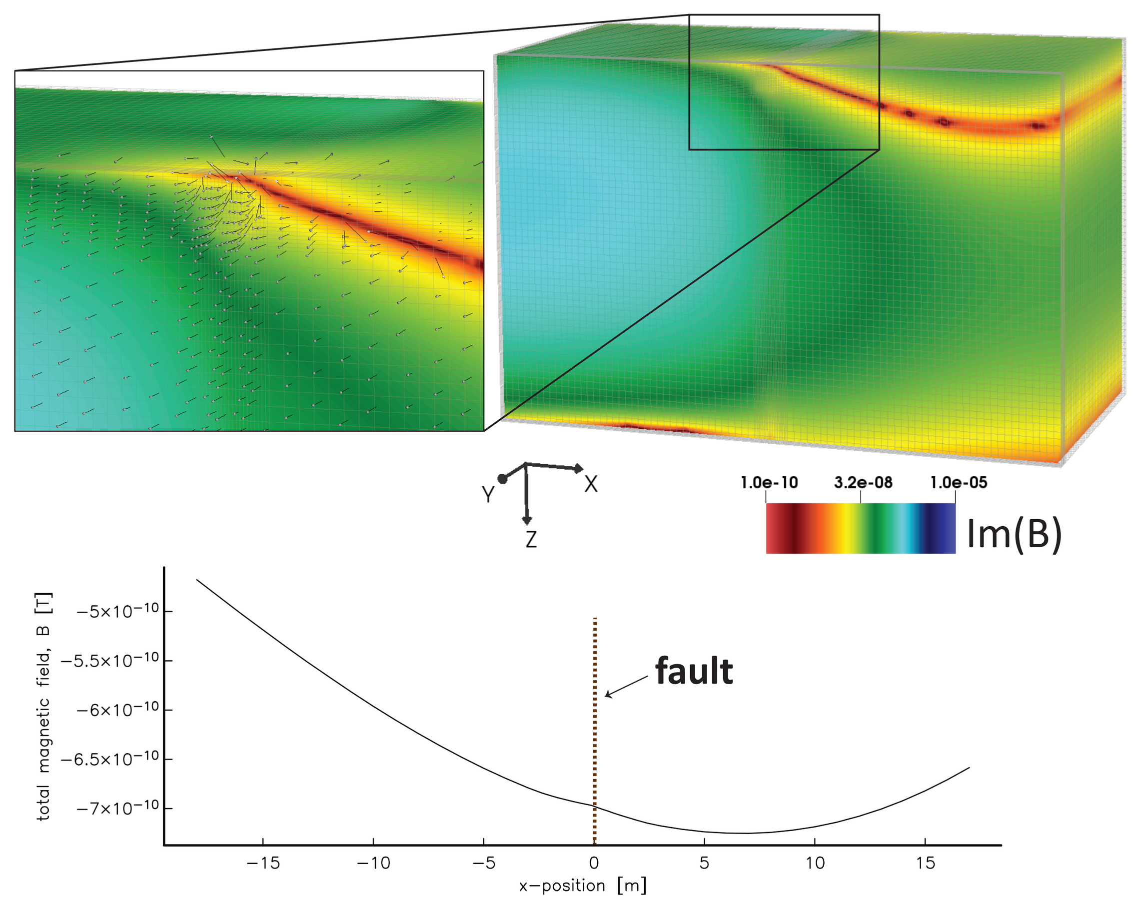




Vertical fault without metallic clutter.

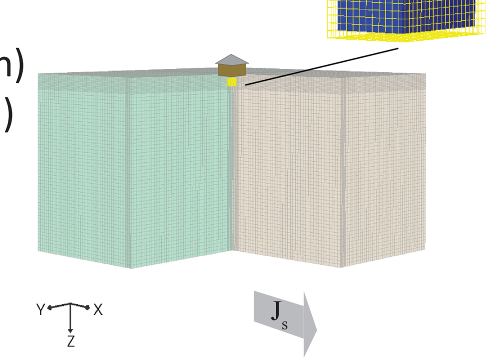
- 100 m x 100 m x 50 m
- Relative conductor (0.01 S/m, green) adjacent to a resistor (0.001 S/m, tan)
- X-directed, 25 kHz plane wave source (perpendicular to fault strike)

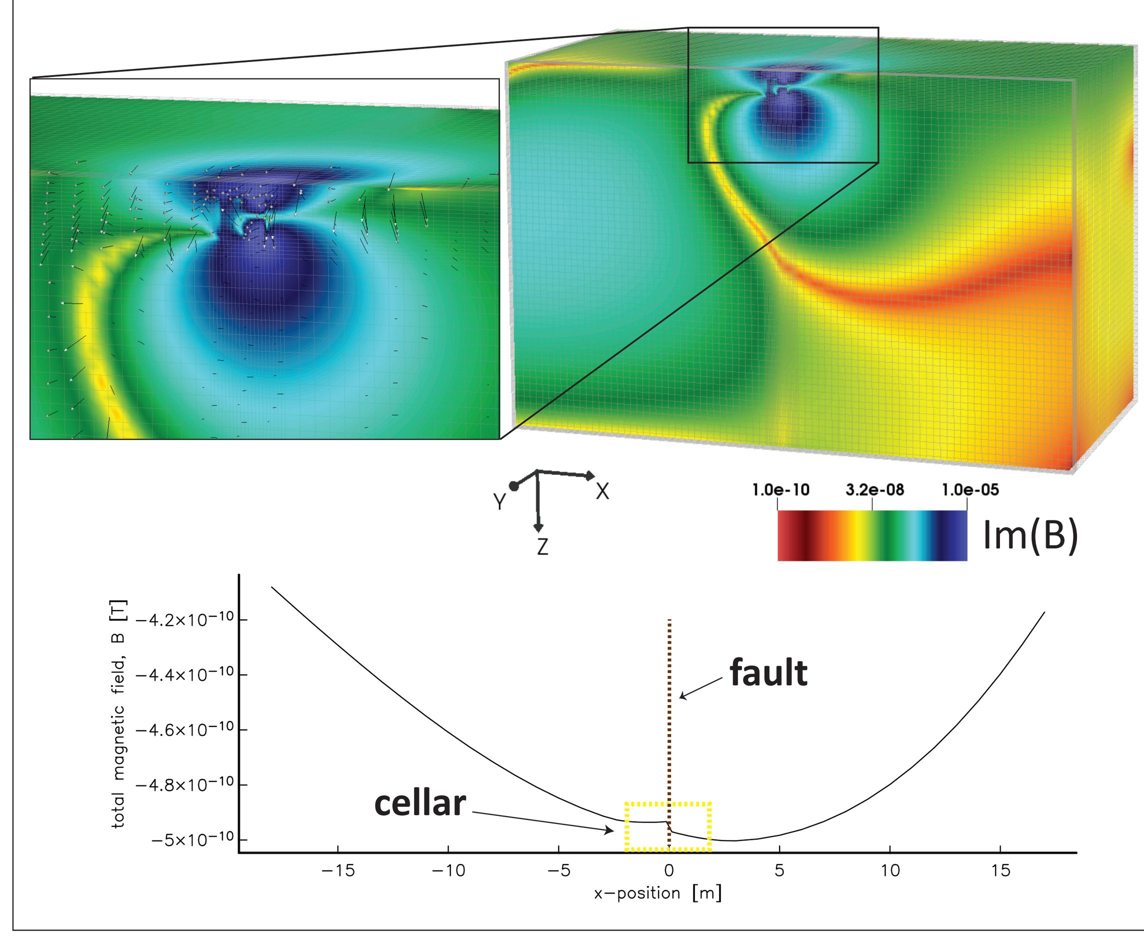




Vertical fault with metallic clutter

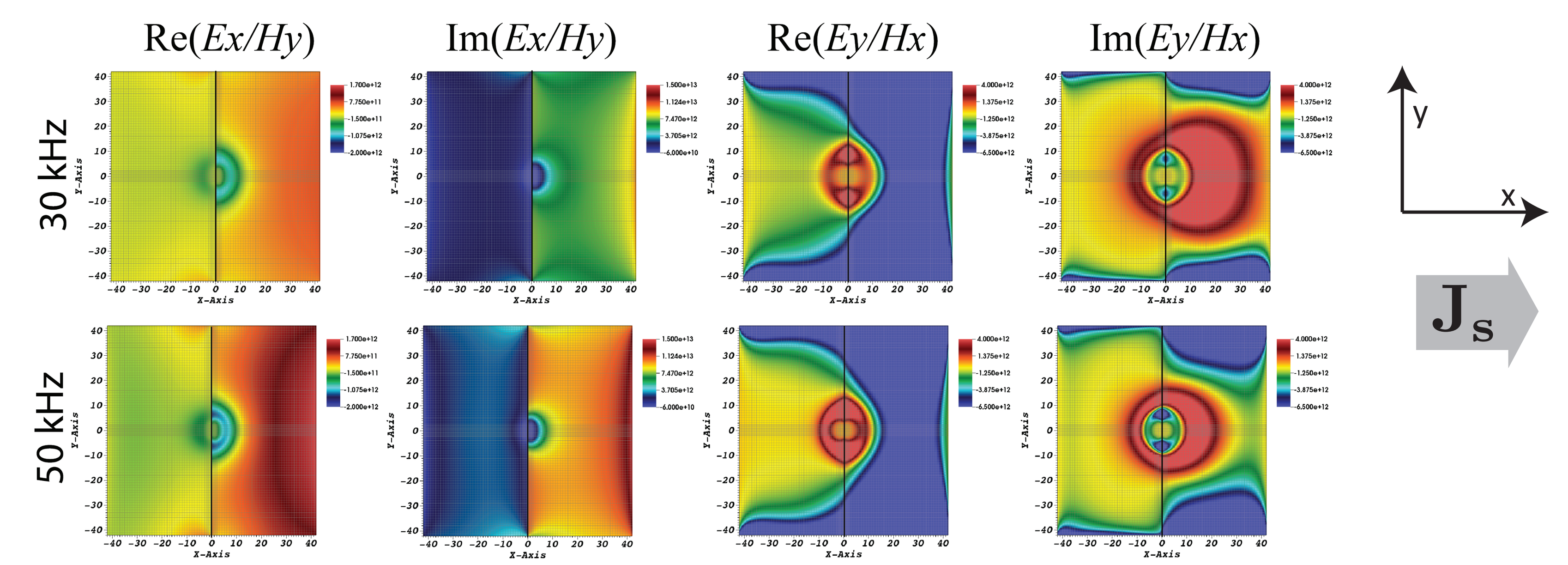
- 100 m x 100 m x 50 m
- Relative conductor (0.01 S/m, green) adjacent to a resistor (0.001 S/m, tan)
- Metallic clutter is an air-filled, rectangular wire mesh (cellar)
- Cellar is 3 m x 5 m x 1.5 m
- X-directed, 25 kHz plane wave (perpendicular to fault strike)





Digging Deeper: Impedance Tensors

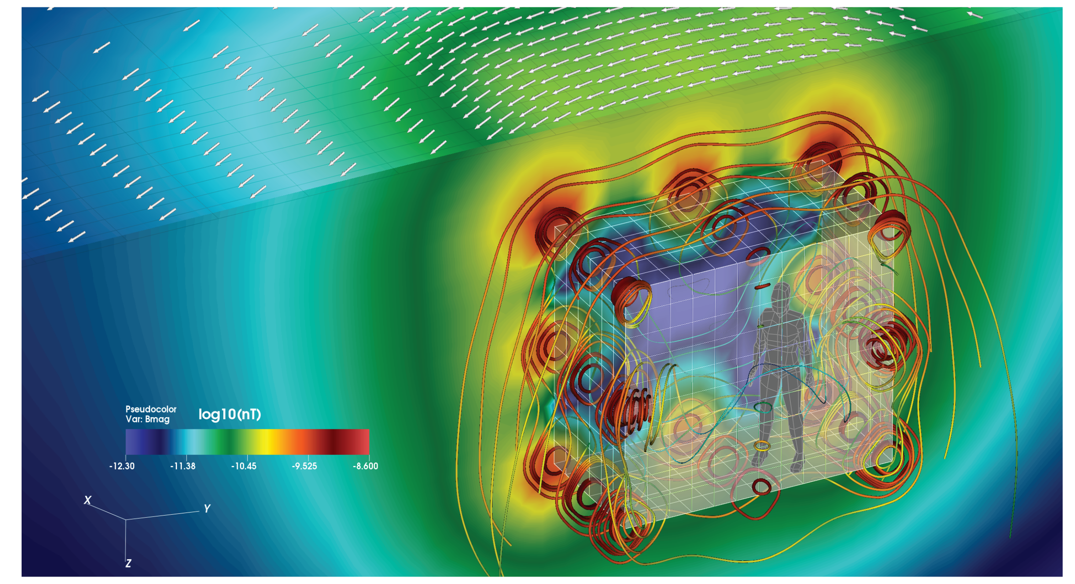
To further understand the expression of metallic clutter in the presence of geologic structure we also consider the magentotelluric impedance tensor, a proxy for apparent conductivity (and phase) which is especially useful in when the plane wave source strength is unknown. Observe the strong Ey/Hx response which, in the absence of the metal clutter, would be identically zero.



Visualizing the Physics

Streamlines and magnitude of the out-of-phase com-ponent of magnetic induction B for a 25 kHz plane wave vertically incident upon an undergound void coarsely stabilized by rebar. Direction of Im(B) on the air-earth interface is shown by the white arrows. Figure is shown for scale.

Rebar is located along the outer edges of the void and along interecting midlines across all six of its faces. Simulation results using the thin wire approxi-mation show that rebar effectively channels electric current and generates a strong magnetic field circu-lating around each of the rebar segments parallel to the source polarization. Some leakage of currents into connected rebar segments orthogonal to the source polarization is also evident.



Summary and Conclusions

The simple "set E to zero" thin wire approximation is a simple and effective means for capturing the electromagnetic response of strong filamentary conductors embedded in a 3D conducting medium.

Outstanding questions:

1. Cost/benefit analysis of including the higher order ln(r) terms for problems of geo-significance.
2. What is the effective "size" of the wire when discretized as we've done? One half of a mesh cell? Something else?
3. What is the best way to represent wires that are non-conformal to the FD mesh? (e.g. skewed or in the middle of a cell)

The EM response of a geologic structure, particularly where the conductivity contrast is relatively low, is easily overprinted by the reponse of anthropogenic metal clutter, especially when the sourcing fields (either natural or manmade) are polar-ized in the direction along which the clutter is oriented.

However, geologic structure can be distinguished from metal clutter when the clutter is small in comparison. That is, the mesoscale electromagnetic response of "large" geologic features (e.g. faults) is only locally disrupted by the response of the clutter.

The thin wire approximation is expandable to thin "sheets" by applying E=0 to neighboring edges over a given surface.

Sandia National Laboratories · The University of New Mexico · USF University of South Florida · Built on LDRD Laboratory Directed Research and Development

Sandia National Laboratories is a multi-mission laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.