Plane Wave Electromagnetic Transmission and Reflection from a Thin Geologic Layer

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Introduction and Summary

In seismic reflection exploration, a *thin* geologic bed in defined as one with thickness less than about one-fourth wavelength of an incident plane wave (Widess, 1973). In this case, distinct reflection arrivals from top and bottom bed boundaries are difficult to recognize. Information regarding layer thickness and material properties is encoded in the amplitude of the composite (reflected and/or transmitted) response. Due to the significantly larger wavelength of a diffusing electromagnetic (EM) wave, virtually all geologic layers are considered extremely thin via this definition. This is particularly true for a single hydraulic fracture (~1 cm width) or even a fracture zone (~ meters width). Seismic and (low-frequency) EM wavelengths are given by

$$\lambda(f)\big|_{\mathrm{SEIS}} = \frac{c}{f}, \qquad \lambda(f)\big|_{\mathrm{EM}} = \frac{c(f)}{f} \approx \sqrt{\frac{4\pi}{\sigma\mu f}},$$

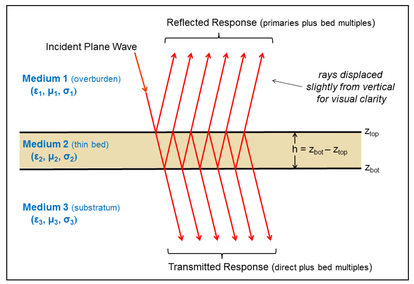
where c is phase speed, and σ and μ are current conductivity and magnetic permeability. For example, the wavelength of a 30 Hz sinusoid propagating with seismic velocity of 3000 m/s in an elastic medium is 100 m, whereas the analogous EM signal diffusing through a conductive body of 0.1 S/m (clayey shale) has wavelength 1826 m. This larger wavelength has implications for the resolution capabilities of the EM prospecting method.

In this investigation, we restrict consideration to a normally-incident plane EM wave, and calculate reflection and transmission responses of a uniform-thickness layer embedded between two (possibly dissimilar) homogeneous and isotropic halfspaces. Exact frequency-domain expressions for the electric vector **E** and magnetic vector **B** are obtained via two equivalent methodologies: 1) summing primaries and all intrabed multiples (the SEIS way), and 2) solving a boundary value problem by imposing wavefield continuity at the interfaces (the EM way). Time-domain responses are obtained by inverse numerical Fourier transformation.

We predict observable **E** and **B** responses, in both reflection and transmission, from layers that are several orders of magnitude smaller than the incident dominant wavelength. Large conductivity or permeability contrast, as with a hydraulic fracture injected with a suitable EM detection agent, enhances response amplitude. In contrast with LaBrecque et al. (2016), we find that permittivity contrast exerts negligible influence at the low frequencies typically used in EM geophysical exploration. However, responses appear to depend on the fixed (parameter x thickness) product, a result that agrees with First Born Approximation scattering theory. An obvious extension of this investigation involves a non-normal incident plane wave, which offers the intriguing possibility of EM Amplitude *vs.* Offset (EMAVO) analysis.

Normal Incidence Reflection and Transmission

The Geophysical Basis



The Mathematical Basis

Sum the infinite series of individual arrivals (with appropriate phase delays and amplitude attenuations) to obtain

Electric Vector *Reflection and Transmission Response Filters*

$$\mathrm{REF}(\omega) = \frac{R_{top}(\omega) + R_{bot}(\omega)e^{+iK_2(\omega)2h}}{1 + R_{top}(\omega)R_{bot}(\omega)e^{+iK_2(\omega)2h}}, \quad \mathrm{TRN}(\omega) = \frac{\left[1 + R_{top}(\omega)\right]\left[1 + R_{bot}(\omega)\right]e^{+iK_2(\omega)h}}{1 + R_{top}(\omega)R_{bot}(\omega)e^{+iK_2(\omega)2h}}.$$

where the normal incidence ***reflection coefficient*** and ***complex wavenumber*** are:

$$R_{top}(\omega) = \frac{K_1(\omega)/\mu_1 - K_2(\omega)/\mu_2}{K_1(\omega)/\mu_1 + K_2(\omega)/\mu_2}, \qquad K(\omega) = \sqrt{i\omega\mu(\sigma - i\omega\varepsilon)}.$$

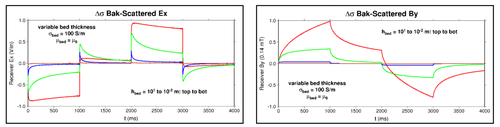
Magnetic (**B**) vector responses obtained by multiplying by $K(\omega)/\omega$.

Time-Domain EM Responses

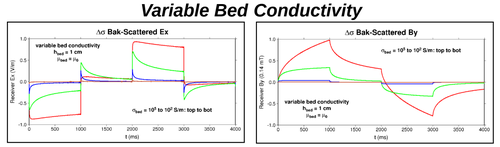
Obtained by inverse discrete Fourier transforming frequency-domain spectra. Source waveform is alternating polarity square pulse sequence (1 s on+, 1 s off, 1 s on-).

Backward-Scattered (= Reflected) EM Responses

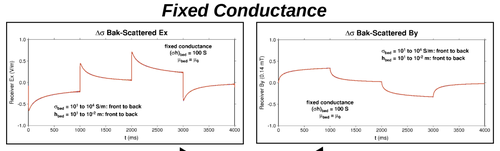
Variable Bed Thickness



Variable Bed Conductivity

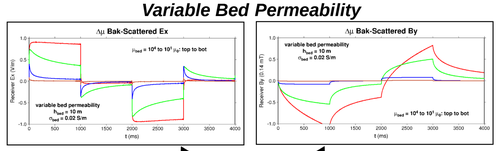


Fixed Conductance



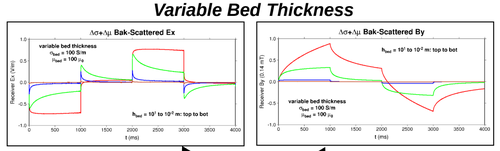
Fixed conductivity x thickness product yields fixed responses

Variable Bed Permeability



Permeability responses *opposite sign* as conductivity responses

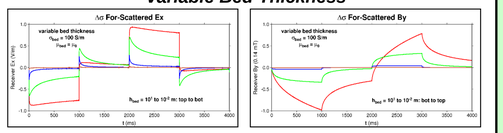
Variable Bed Thickness



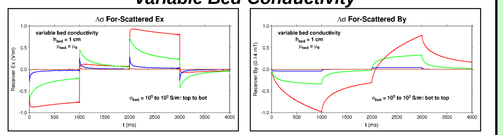
Joint conductivity + permeability contrast *suppresses* back-scattering

Forward-Scattered (= Transmitted – Direct) EM Responses

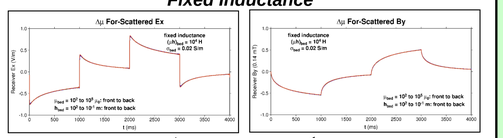
Variable Bed Thickness



Variable Bed Conductivity

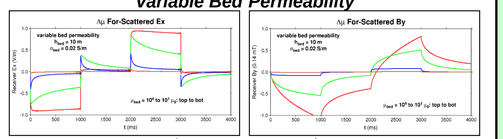


Fixed Inductance



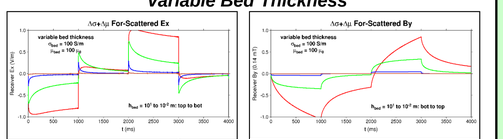
Fixed permeability x thickness product yields fixed responses

Variable Bed Permeability



Permeability responses *same sign* as conductivity responses

Variable Bed Thickness

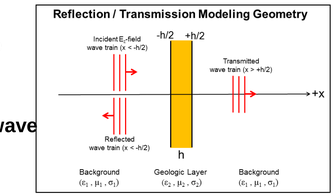


Joint conductivity + permeability contrast *enhances* fore-scattering

Backward-Scattered / Forward-Scattered EM Signal Amplitude Investigation

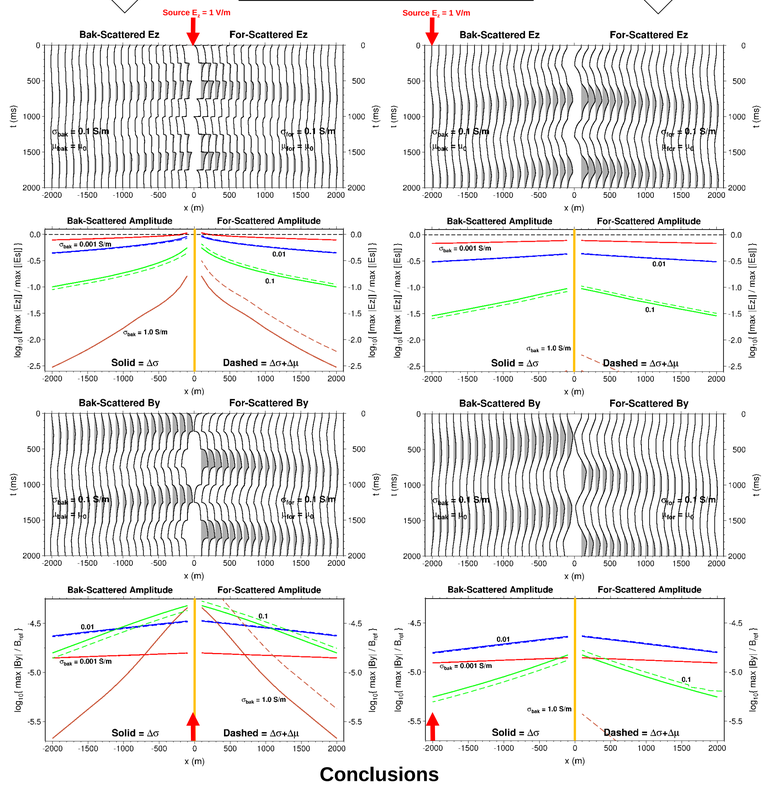
Source waveform:
alt. polarity square pulse sequence; period = 1 s.

Proximal plane wave Ez source 1 V/m



h_{bed} = 5 m
σ_{bed} = 10 S/m
$\mu_{bed} = \mu_0 , 10\mu_0$

Distal plane wave Ez source 1 V/m



Conclusions

1) Backward / forward-scattered signal levels decay with distance from embedded layer.
2) Signal levels for distant source lower than for near source.
3) Layer conductivity contrast generates little difference in back- and fore-scattered signal amplitudes.
4) Joint conductivity / permeability contrast enhances fore-scattering and suppresses back-scattering.
5) All E and B signal levels appear potentially measurable in a field experiment context.

Acknowledgements

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References

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Widess, M.B., 1973, How thin is a thin bed?: Geophysics, **38**, 1176-1180.