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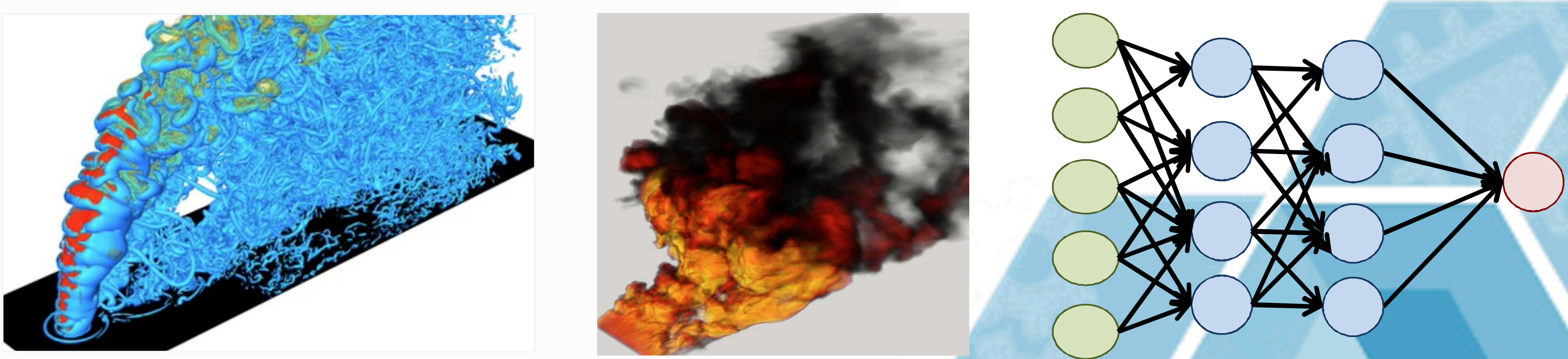

Using Tensor Theory to Embed Invariances: A Case Study from Turbulence Modeling

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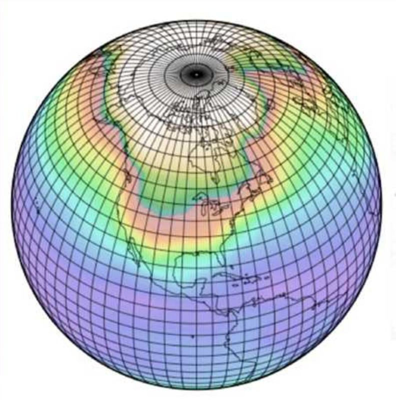
Objective

- Constitutive models are used to model unresolved physical processes

Turbulence

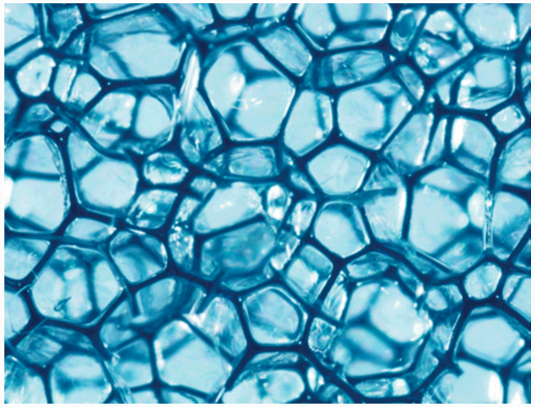
Van Dyke

Climate Modeling



Henderson-Sellers

Materials Science



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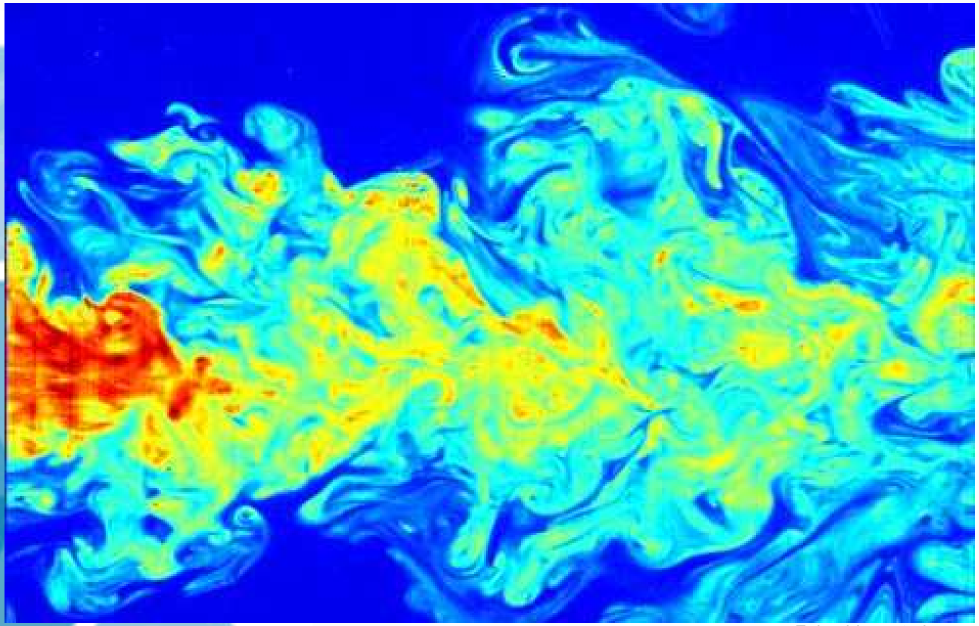
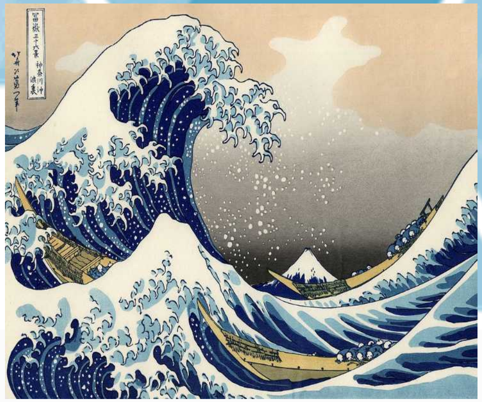
- Growing interest in applying machine learning to constitutive modeling
- We present a method of embedding known invariance constraints directly into these data-driven constitutive models
- Case study from turbulence modeling

Turbulence

Chaotic 3-D fluid motion at a continuum of scales



Fukushima et al.

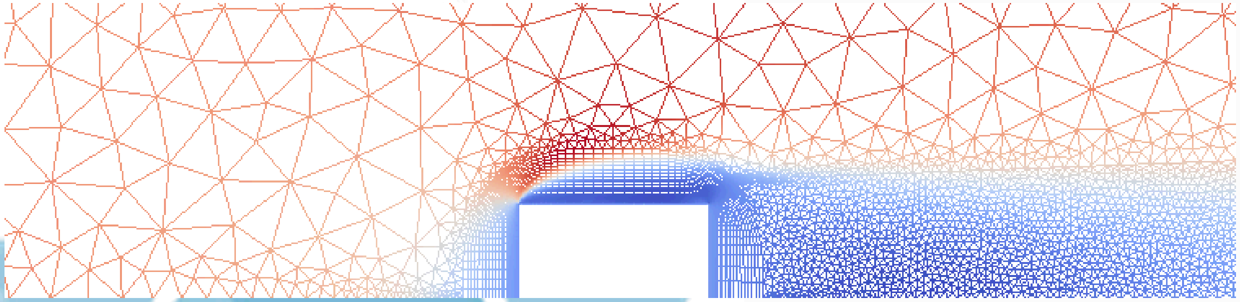


Hokusai (c 1830)

http://www.windturbinesyndrome.com/2011/wind-turbine-turbulence-what-are-the-micro-climate-effects/

https://brilliant.org/wiki/rocket-physics/

Turbulence Simulations



Direct Numerical Simulation (DNS)

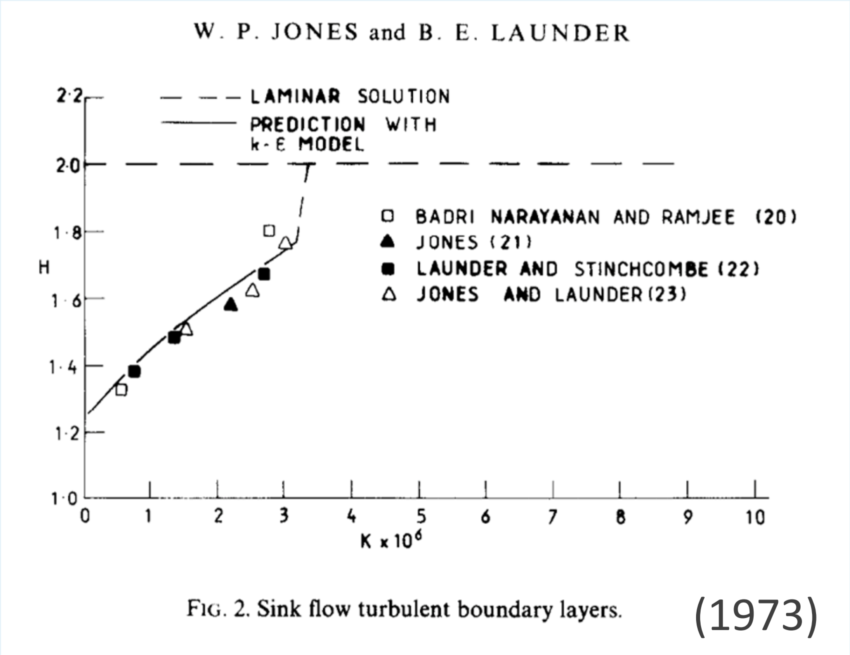
- Hundreds of millions of core hours for even simple flows
- Exact

Reynolds Averaged Navier Stokes (RANS)

- Orders of magnitude less computationally expensive
- Approximate

Deep Learning for Turbulence Modeling

- In RANS, use simplifying assumptions to get computational efficiency
    - Need model for unknown term: the Reynolds stress anisotropy tensor **A**
- Default model: Linear Eddy Viscosity Model
    - Based on theory + sparse experimental data
- Our approach: Deep neural network
- Inputs: Mean strain rate tensor **S**, mean rotation rate tensor **R**
- Outputs: Reynolds stress anisotropy **A**
- **Tensors are the natural language of scientific computing**



W. P. JONES and B. E. LAUNDER

FIG. 2. Sink flow turbulent boundary layers.

(1973)

Deep Learning for Turbulence Modeling

- Inputs: Tensors **S**, **R**
- Output: Tensor **A**

- Would like to enforce Galilean invariance
  - Invariance to inertial coordinate frame transformations
  - Borrow some ideas from group theory, representation theory
  - All Galilean invariant tensors that are a function of **S** and **R** lie on a tensor basis: the *integrity basis* of **S** and **R** for the orthogonal group

$$\mathbf{A}(\mathbf{Q}\mathbf{S}\mathbf{Q}^T, \mathbf{Q}\mathbf{R}\mathbf{Q}^T) = \mathbf{Q}\mathbf{A}(\mathbf{S}, \mathbf{R})\mathbf{Q}^T$$

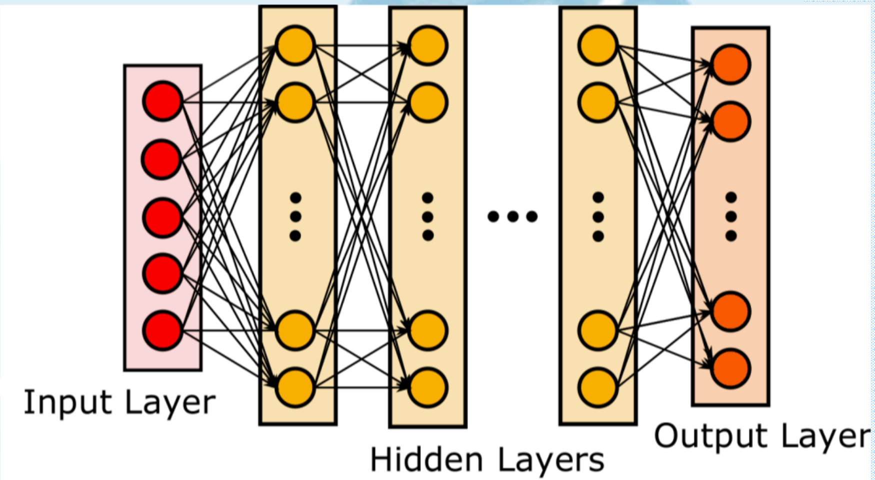
$$\mathbf{A} = \sum_{n=1}^{10} f^{(n)} \mathbf{B}^{(n)}$$

Unknown coefficients

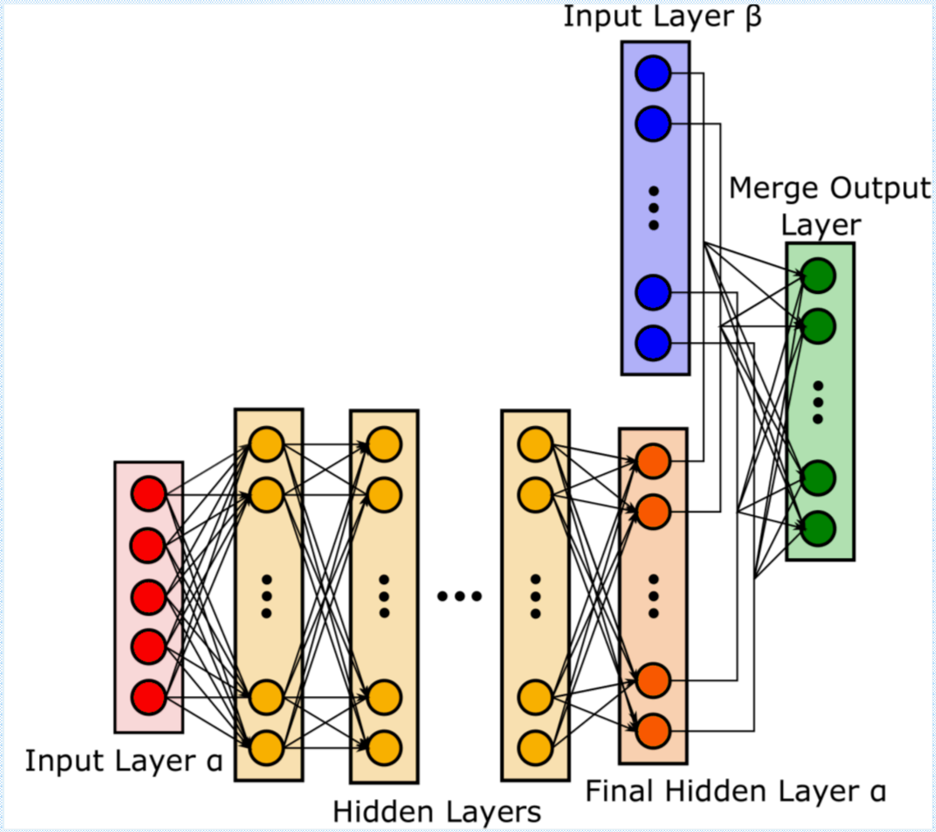
Known Tensor Basis

Embedding Galilean Invariance

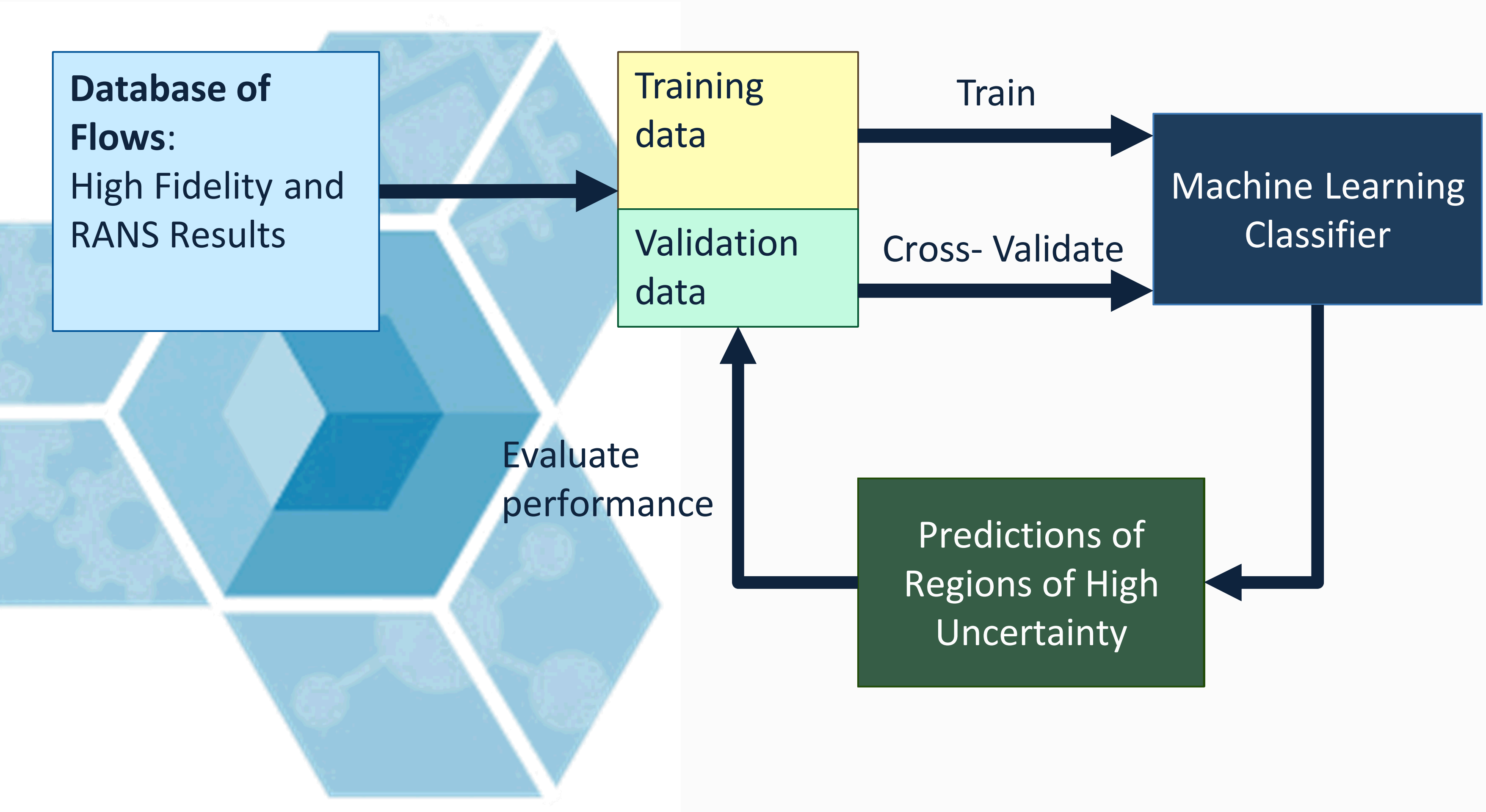
Multi-Layer Perceptron (MLP)



Tensor Basis Neural Network (TBNN)



Classifier Development



Database of Flows:
High Fidelity and RANS Results

Training data

Validation data

Train

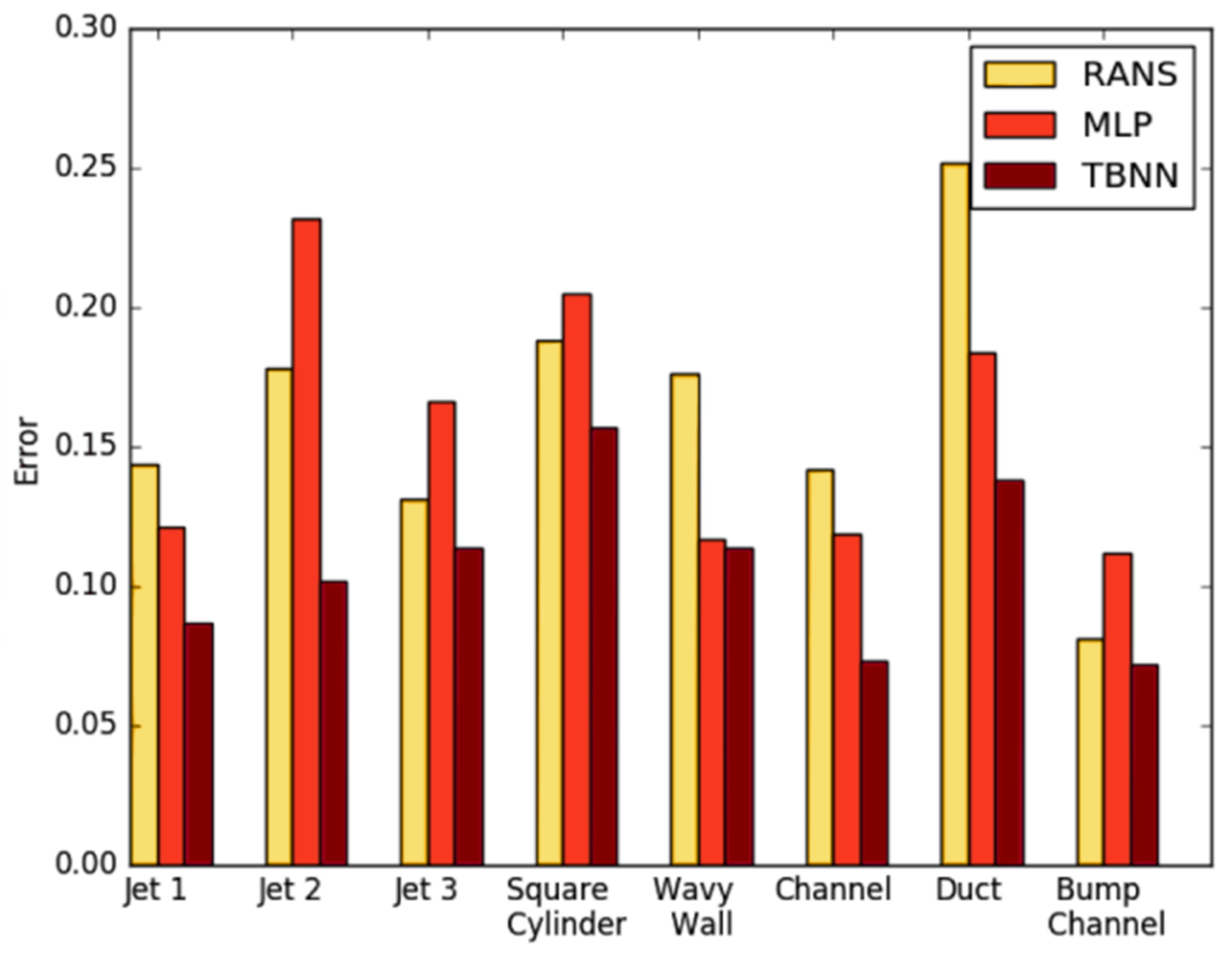
Cross- Validate

Machine Learning Classifier

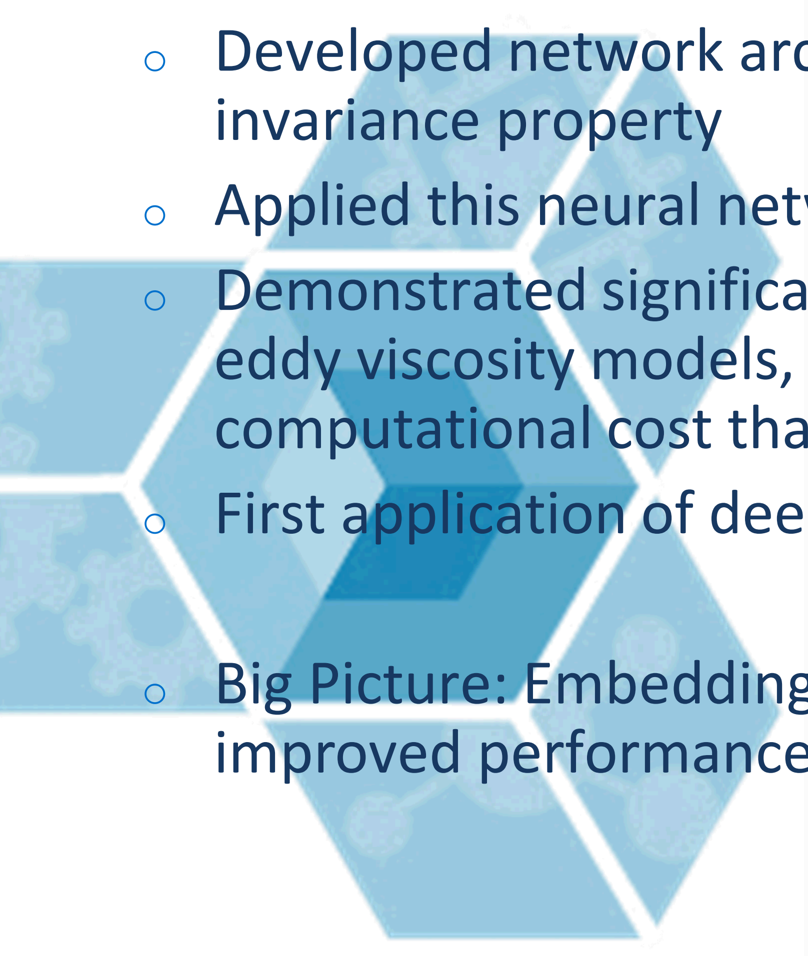
Predictions of Regions of High Uncertainty

Evaluate performance

Deep Learning for Turbulence Modeling



Conclusions



- Developed network architecture to embed known tensor invariance property
- Applied this neural network to turbulence modeling
- Demonstrated significant improvement over conventional eddy viscosity models, at orders of magnitude lower computational cost than DNS
- First application of deep learning to turbulence modeling

- Big Picture: Embedding domain knowledge can give improved performance, especially in data-limited scenarios

References

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Questions