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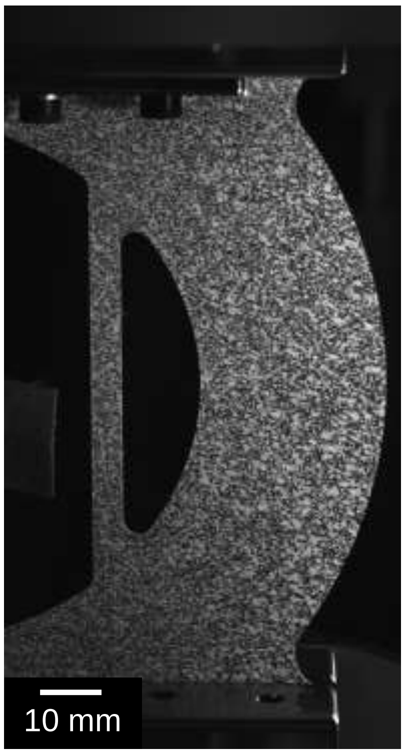

Identifying and Validating Material Models with Non-Unique Parameter Sets

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U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

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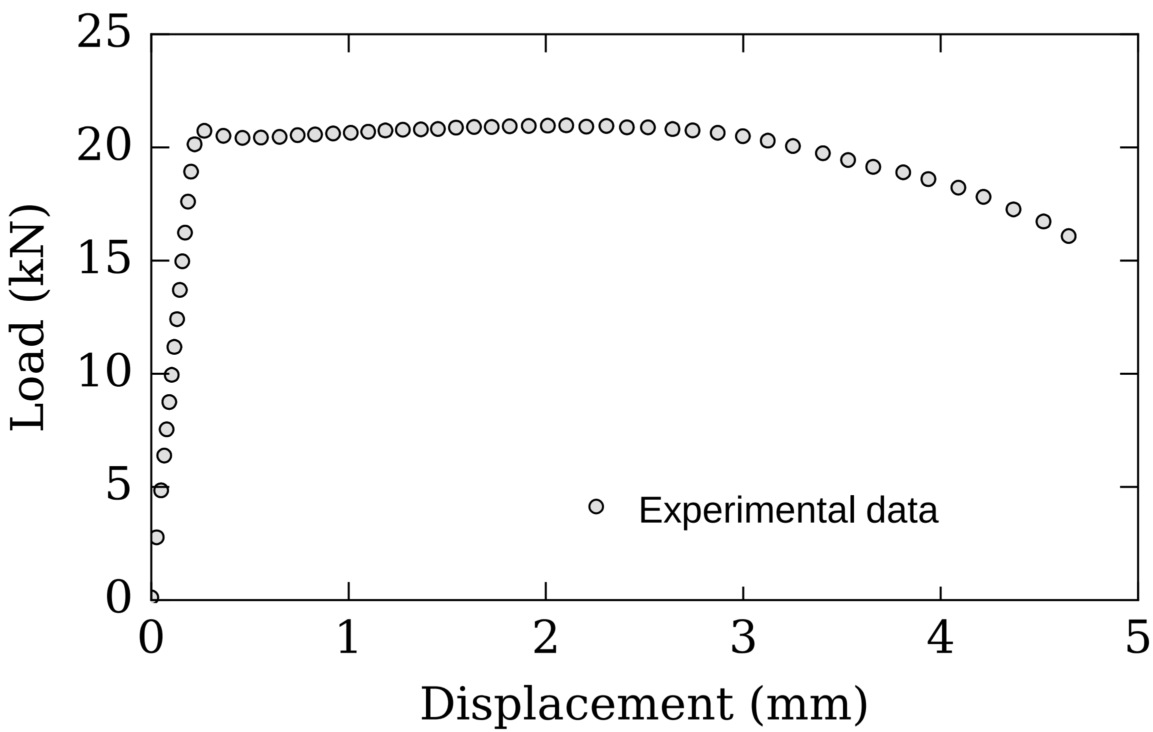
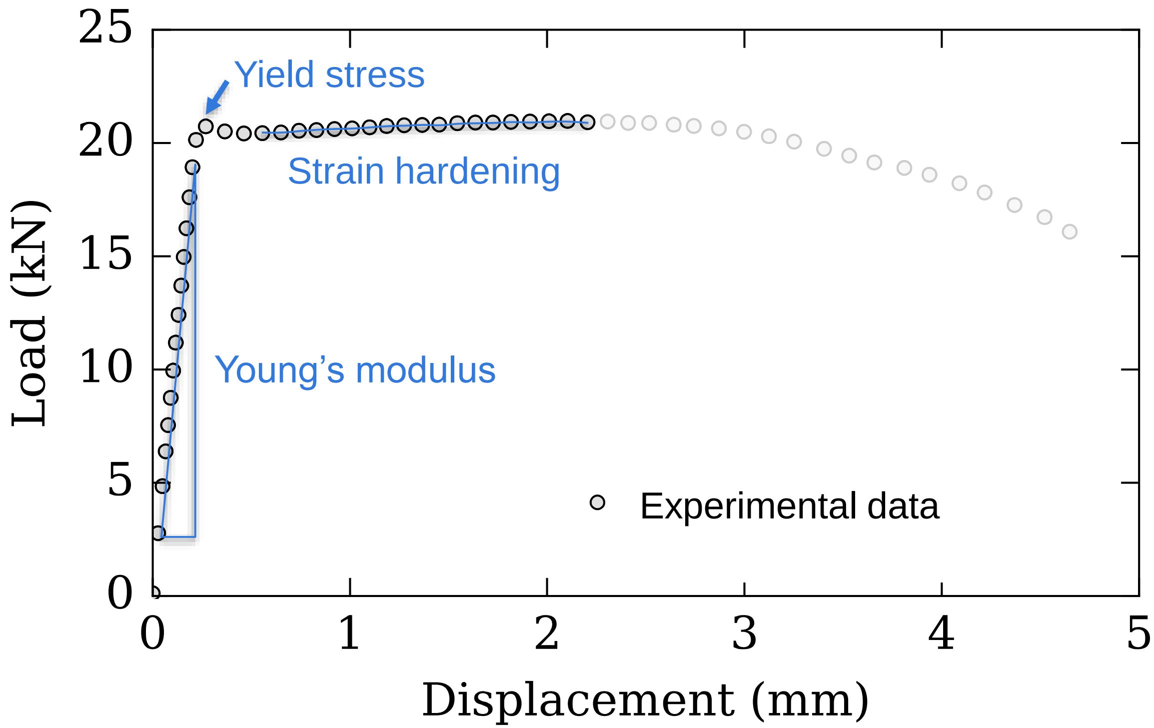
Modeling material and component behavior is critical for modern engineering.

$$\sigma = \mathrm{f}(\zeta\underbrace{(\varepsilon, p, \dot{p}, T)}_{\text{loading conditions}}, \xi\underbrace{(E, \upsilon, \sigma_o, H)}_{\text{material properties}})$$



Modeling material and component behavior is critical for modern engineering.

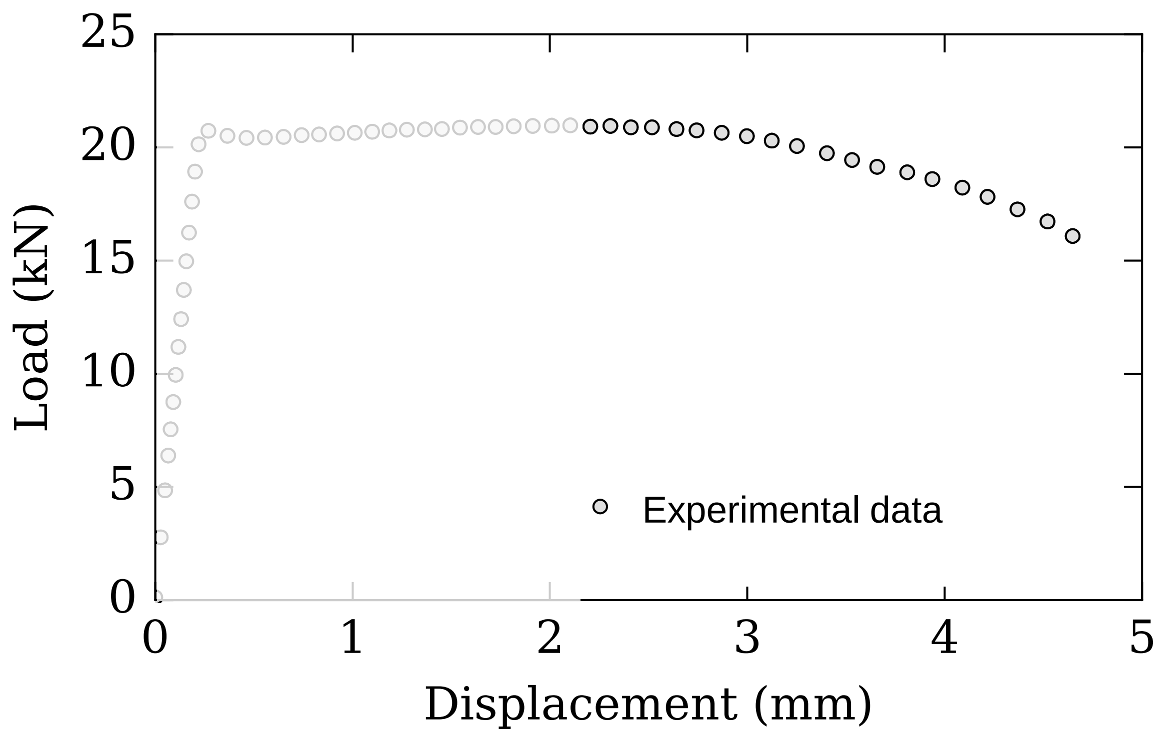
Low strain regime:
- Uniaxial tension
- Identification of model parameters accomplished analytically

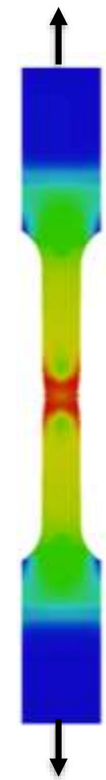


Modeling material and component behavior is critical for modern engineering.

High strain regime:

- Multiaxial stress state
- Identification of model parameters requires inverse problem





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High strain regime:

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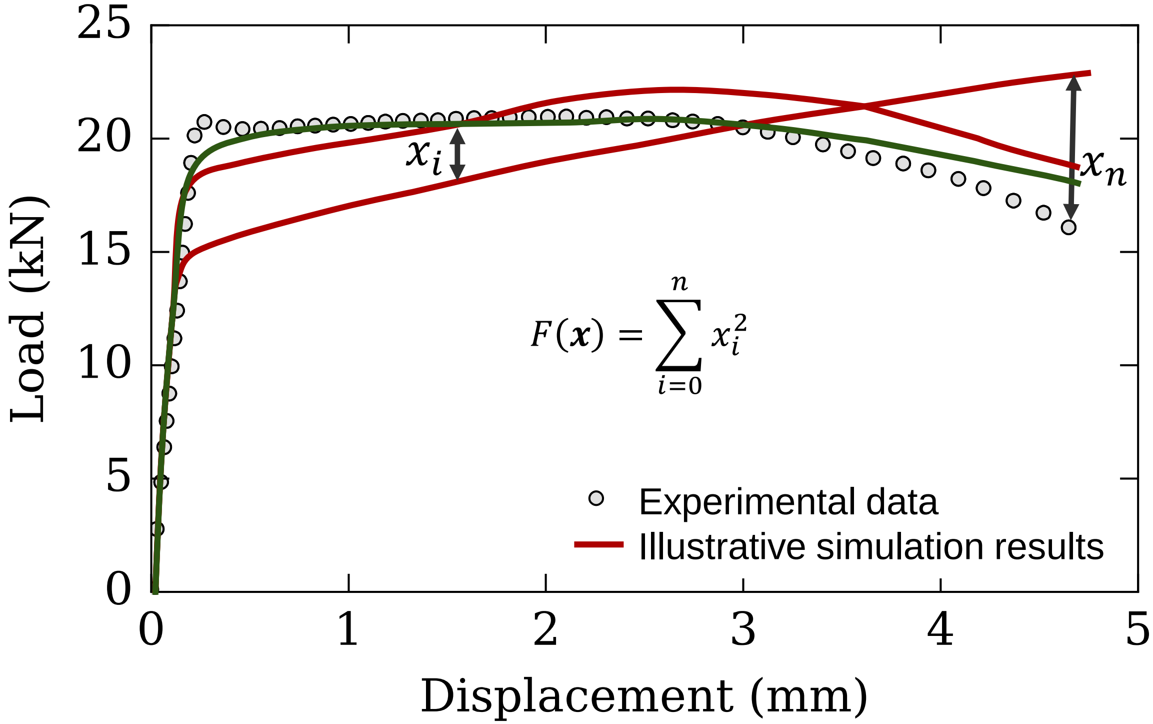

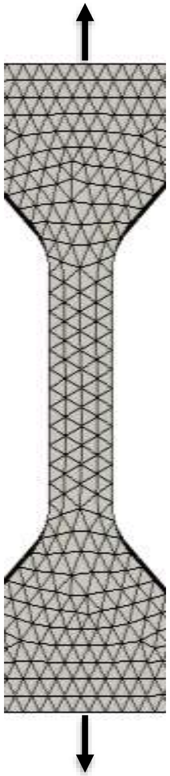

Traditional material identification is inadequate.

Limitations

- Global information misses local deformation
- Many tests required to calibrate complex models
- Simple stress state does not reflect complex, real-world loading conditions

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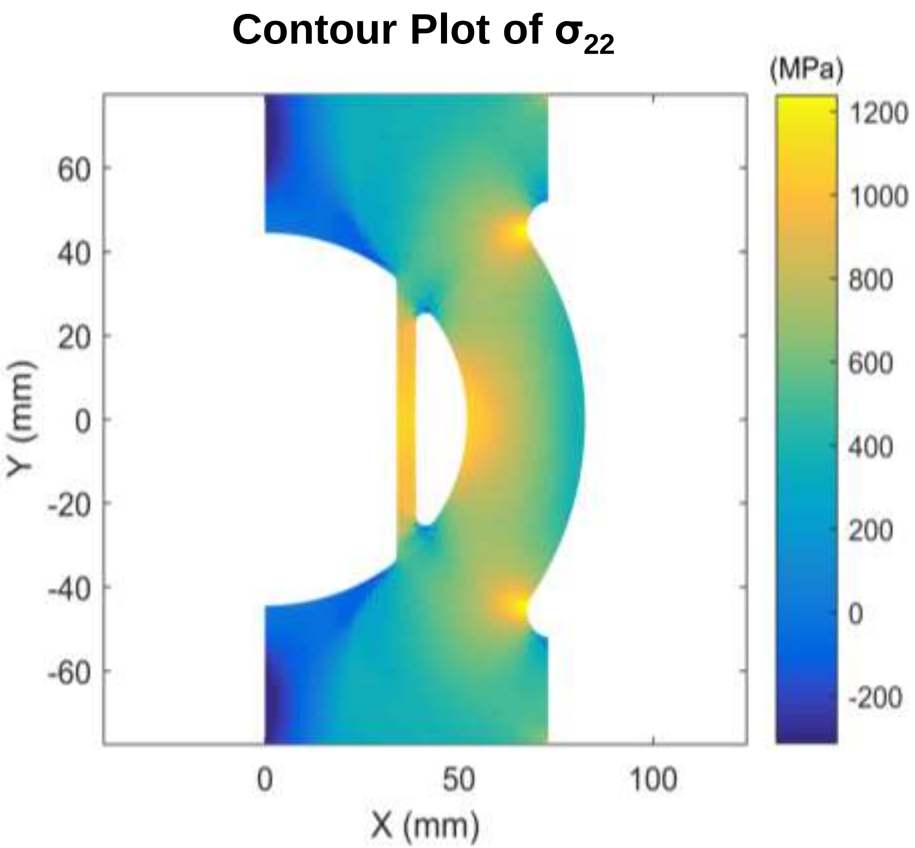
http://money.cnn.com/2014/01/22/autos/small-car-crash-test/
Accessed 29 Aug 2016

High-throughput, high-quality material identification addresses limitations of traditional material ID.



<u>Capitalize on:</u>

- Full-field deformation measurements: Digital Image Correlation (DIC)
- Inverse techniques: Virtual Fields Method (VFM)

Virtual Fields Method is a powerful inverse technique.

Principle of Virtual Power

$$\underbrace{\int_{V_o} \left((\det F) \sigma \cdot F^{-T} \right) : \delta \dot{F} \, dV}_{\text{Internal Power, } P_{int}} = \underbrace{f \cdot \overline{\delta v}}_{\text{External Power, } P_{ext}}$$

σ	Cauchy Stress
F	Deformation Gradient
f	Resultant Load
V	Sample Volume
∂v	Virtual Velocity
$\partial \dot{F}$	Virtual Velocity Gradient

Material Identification Procedure

1. Select material model and specimen geometry
2. Measure specimen deformation during loading
3. Calculate stress with initial guess of model parameters
4. Compute cost function
   - $\Psi = \sum_{\text{time}} [P_{int} - P_{ext}]^2$
5. Iterate on model parameters until cost function is minimized

Pierron and Grédiac (2012) *The Virtual Fields Method*. Springer
Kramer and Scherzinger (2014) SAND2014-17871

304L stainless steel is a model material.

Viscoplastic BCJ-MEM material model

- Isotropic; Von Mises equivalent stress
- Neglecting temperature effects

p = equivalent plastic strain
$\dot{p}$ = equivalent plastic strain rate

$$\sigma_f(p,\dot{p},\xi) = \sigma_o\left\{1+\operatorname{asinh}\left[\left(\frac{\dot{p}}{b_\sigma}\right)^{1/m_\sigma}\right]\right\} + \frac{H}{R_d}\left[1-\exp(-R_d p)\right]\left\{1+\operatorname{asinh}\left[\left(\frac{\dot{p}}{b_\kappa}\right)^{1/m_\kappa}\right]\right\}$$

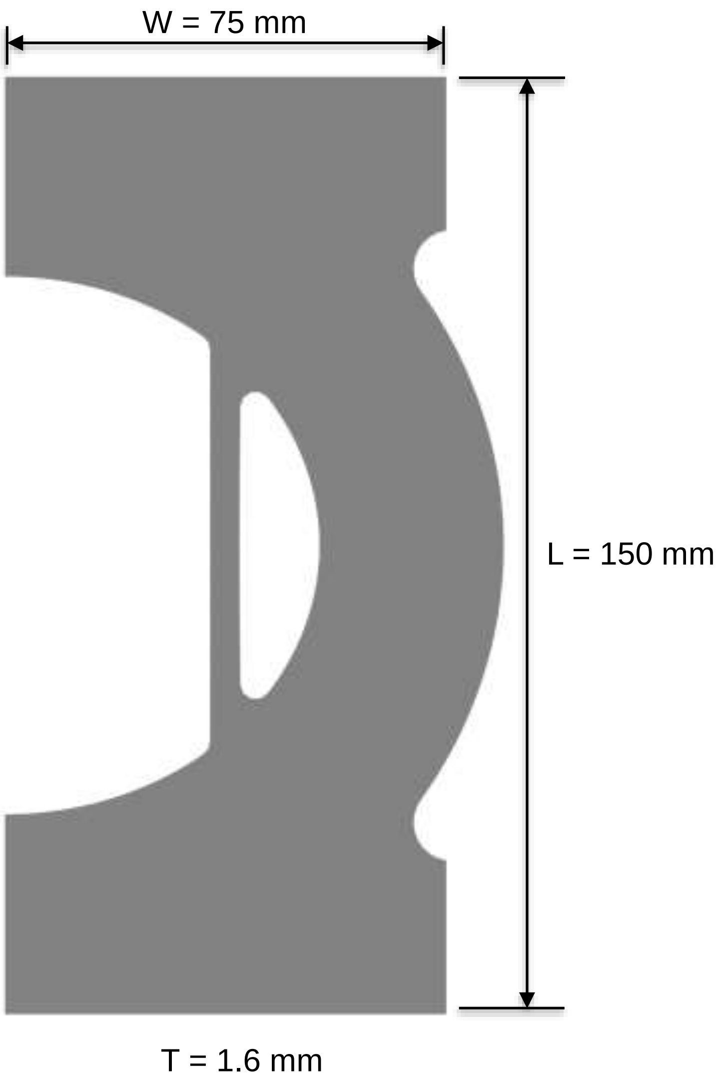
Reference Model Parameter Values from Traditional Material Identification

Parameter	Symbol	Value	Units
Quasi-static Yield Stress	σ_o	307.9	MPa
Hardening Variable	H	2.692	GPa
Dynamic Recovery	R_d	2.601	--
Rate-Dependent Exponent (Yield)	m_σ	3.169	--
Rate-Dependent Exponent (Hardening)	m_κ	3.169	--
Rate-Dependent Coefficient (Yield)	b_σ	16.25	s^{-1}
Rate-Dependent Coefficient (Hardening)	b_κ	16.25	s^{-1}

Novel specimen geometry provides better material identification.

Design Criteria

- Maximize strain/stress heterogeneity
- Maximize range of strain rates
- Minimize large gradients near sample edges
- Planar sample with complex 2D geometry



Complex specimen geometry induces stress and strain rate heterogeneity in sample.

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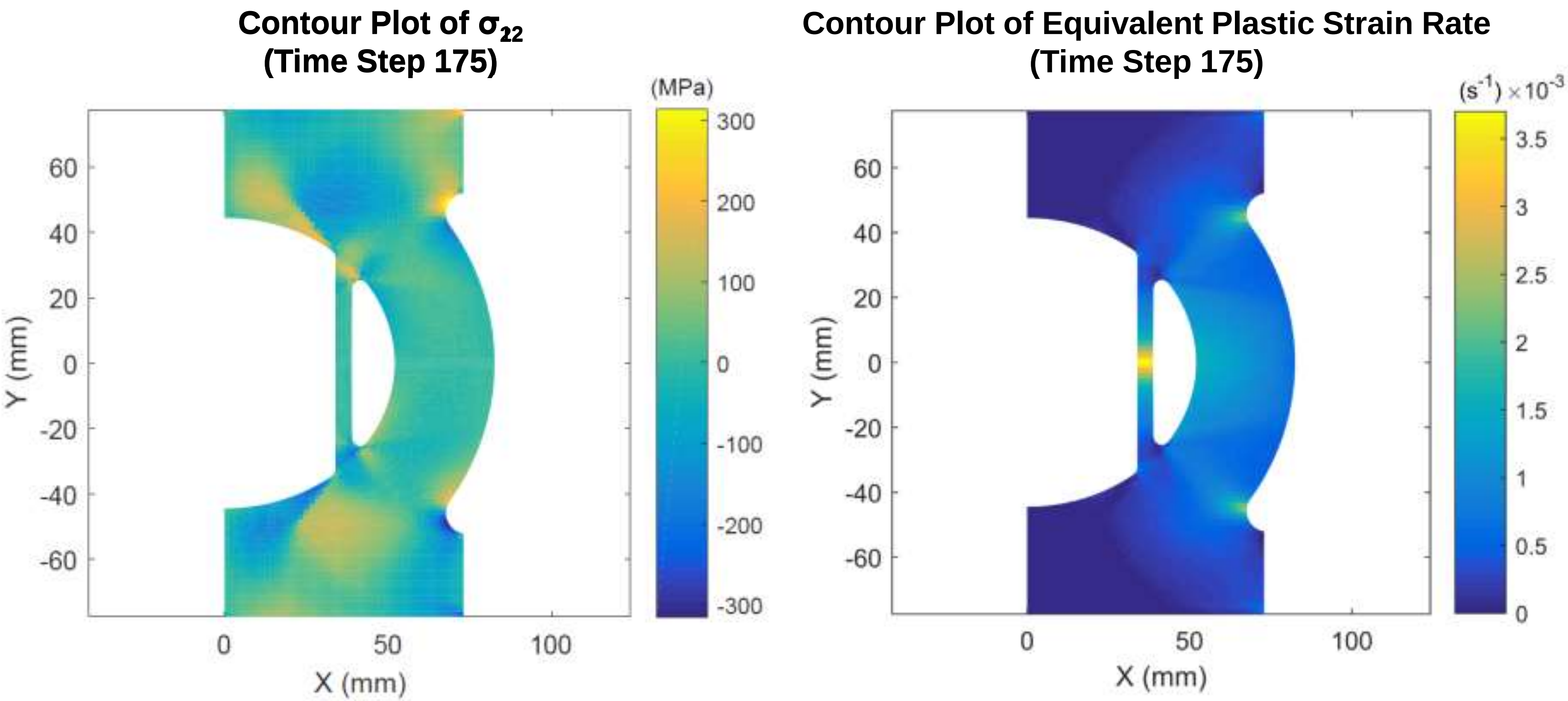
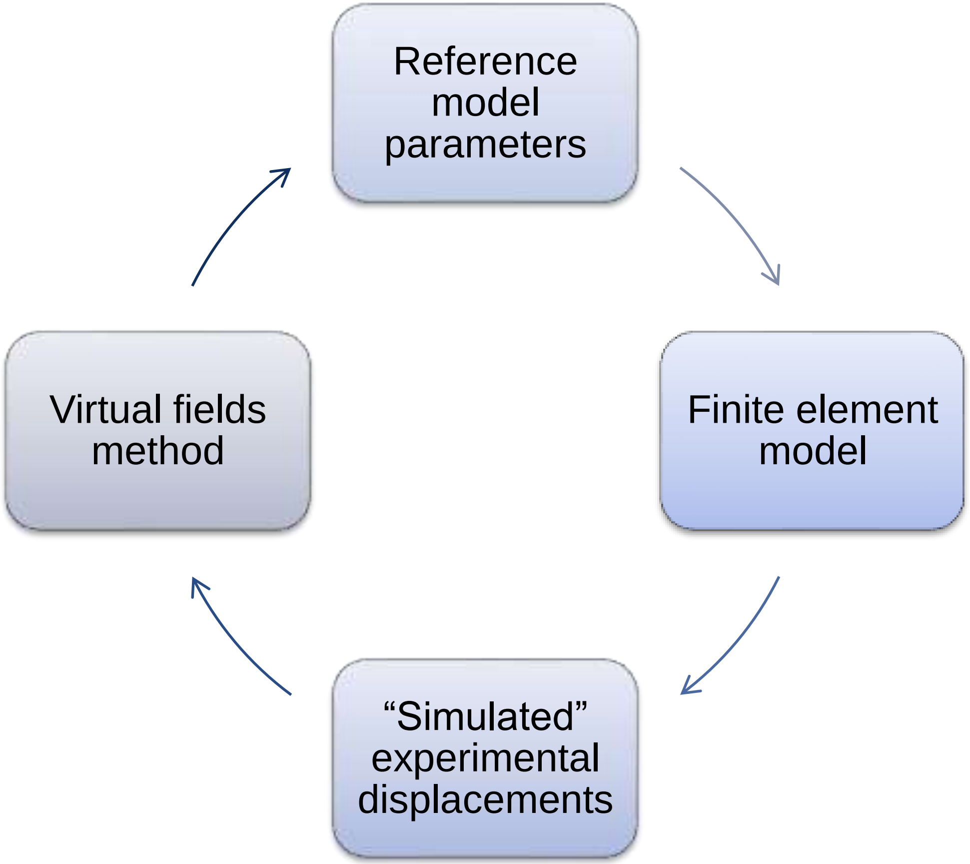
Predicted Results from FEM Simulation



Algorithm is verified with simulated experiments.



Three displacement fields

1. Case 1: "Exact" displacements from FEM simulation

2. Case 2: 0.1 µm random noise added to FEM displacements
   - ~ 1/500 pixel DIC noise floor

3. Case 3: 1.0 µm random noise added to FEM displacements
   - ~ 1/50 pixel DIC noise floor

VFM Algorithm Inputs

- Initial Guess: +20 % from exact parameter value
- Bounds: +/- 90 % from exact parameter value

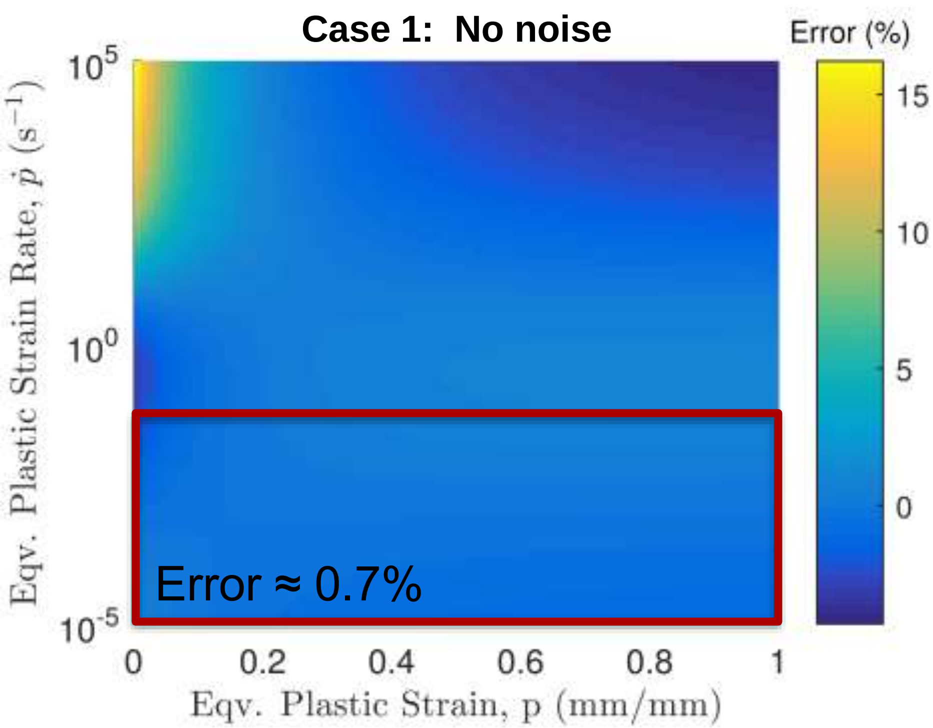
Material identification results have larger than expected error.

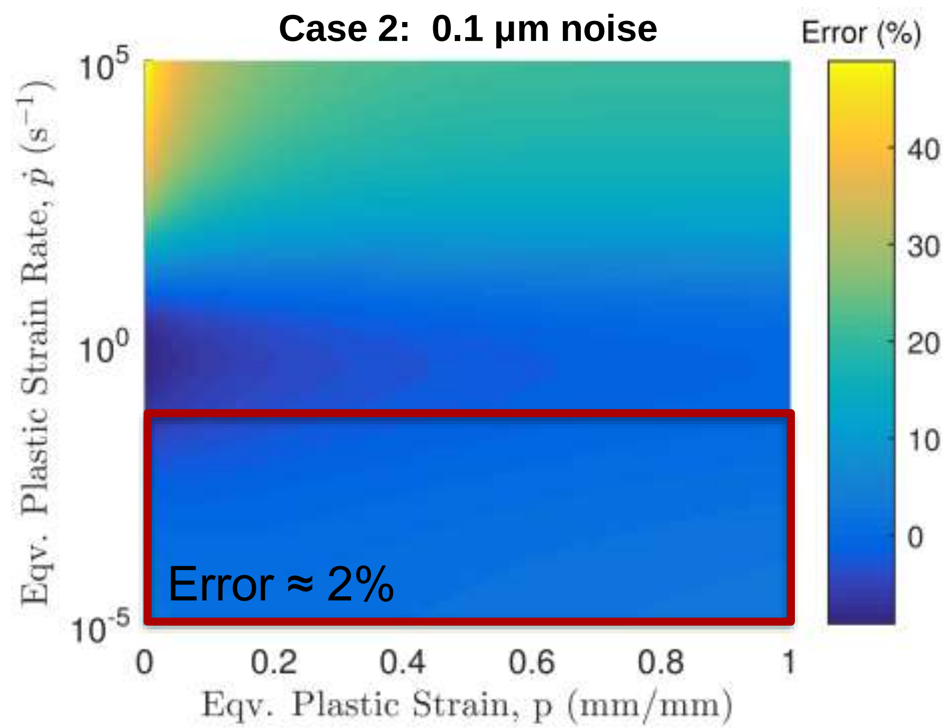
			Case 1: No noise	Case 2: 0.1 μm noise	Case 3: 1.0 μm noise
Parameter	Reference Value	Units	Percent Error of Identified Parameters		
σ_o	307.9	MPa	-1.5	-3.5	-6.3
H	2.692	GPa	2.3	0.6	20.4
R_d	2.601	--	-0.5	5.4	90.0
m_σ	3.169	--	17.9	40.8	9.0
m_κ	3.169	--	-17.4	9.9	11.8
b_σ	16.25	s^{-1}	10.8	10.3	90.0
b_κ	16.25	s^{-1}	10.9	10.4	90.0
Cost Function Residual			2.1e-7	3.2e-7	1.1e-4

Parameter set is not unique.

$$\sigma_f(p,\dot{p},\xi) = \sigma_o\left\{1+\operatorname{asinh}\left[\left(\frac{\dot{p}}{b_\sigma}\right)^{1/m_\sigma}\right]\right\} + \frac{H}{R_d}\left[1-\exp(-R_d p)\right]\left\{1+\operatorname{asinh}\left[\left(\frac{\dot{p}}{b_\kappa}\right)^{1/m_\kappa}\right]\right\}$$

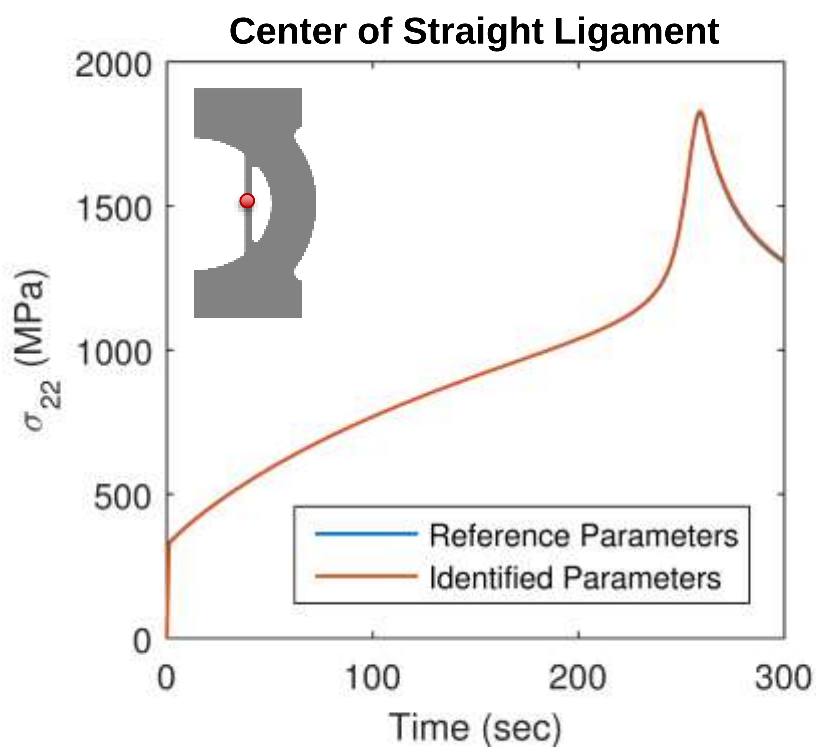
$$\text{Flow Stress Error} = \frac{\sigma_f(p,\dot{p},\xi_{\text{identified}}) - \sigma_f(p,\dot{p},\xi_{\text{reference}})}{\sigma_f(p,\dot{p},\xi_{\text{reference}})}$$

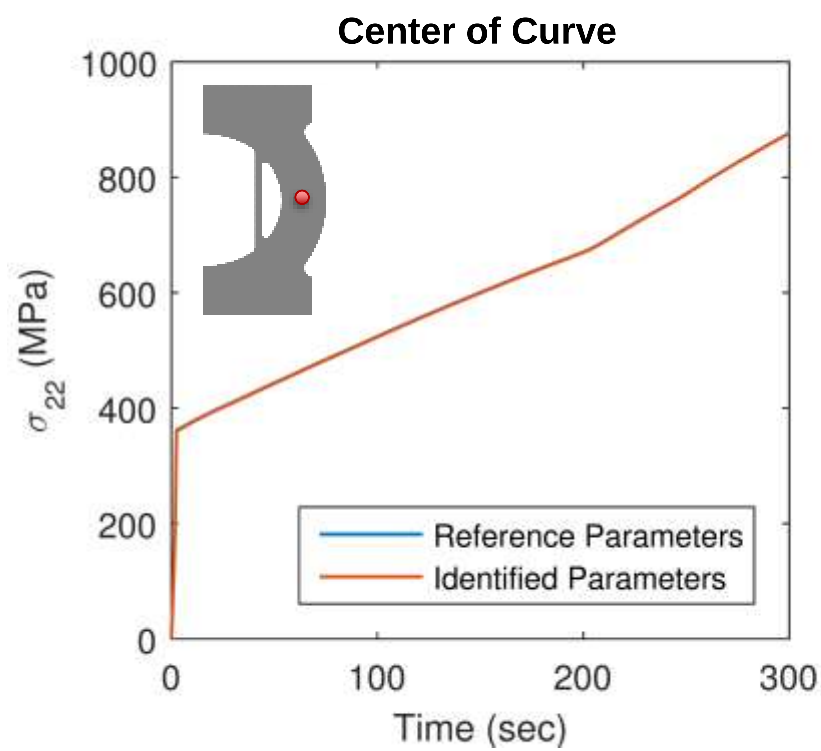




Reconstructed stress fields are invariant for different parameter sets.

Case 1: No noise

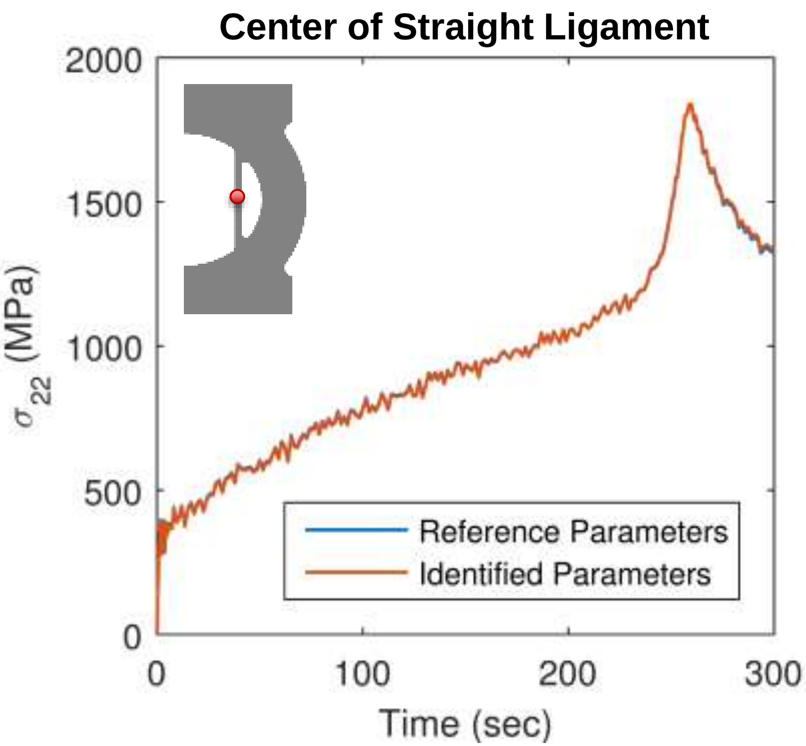


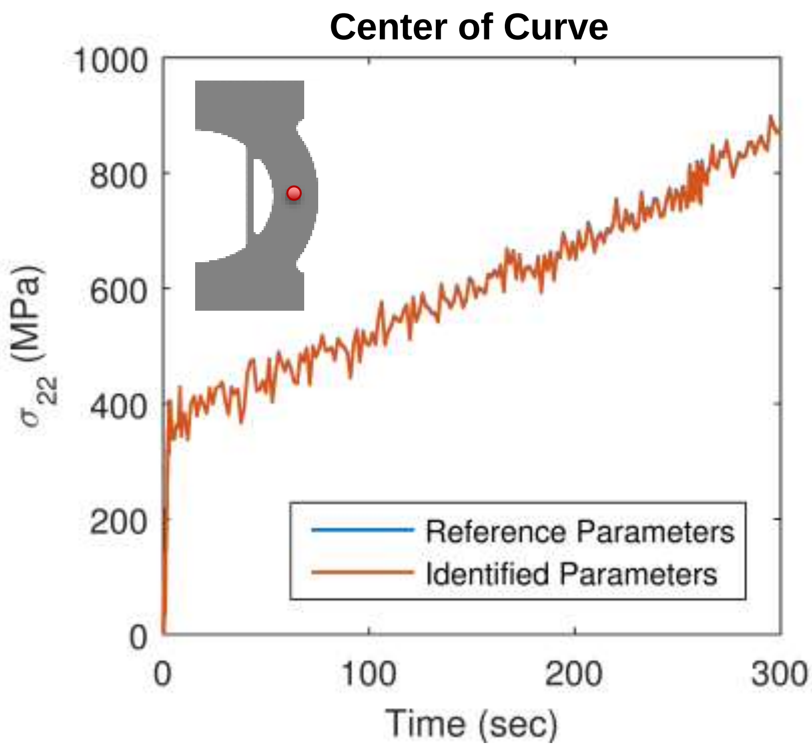


< 2% Error in Von Mises Stress

Reconstructed stress fields are invariant for different parameter sets.

Case 2: 0.1 μm noise

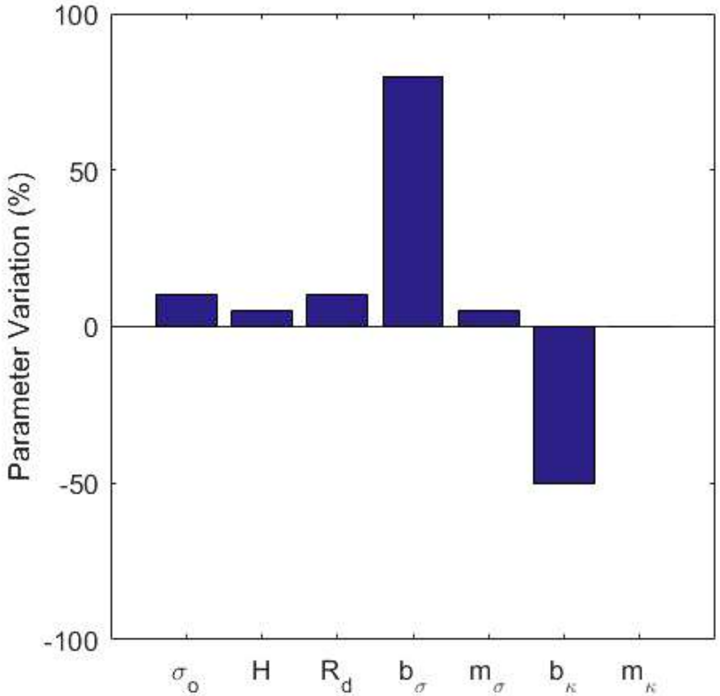


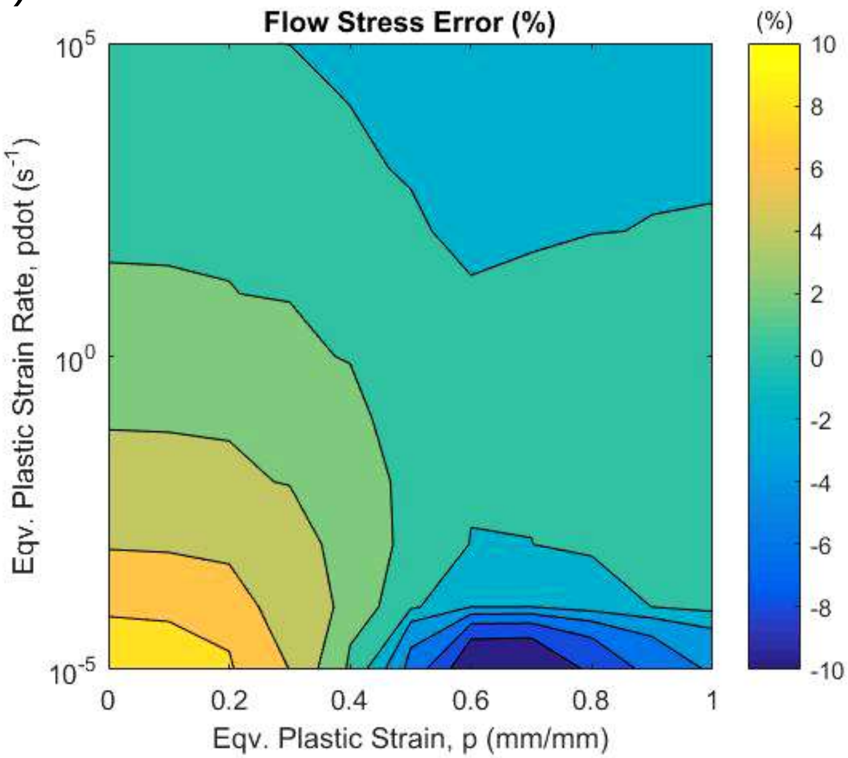


< 5% Error in Von Mises Stress

Infinite solutions exist as a hypersurface in 7D space.

- Varied each parameter from -90% to +90% reference value, in 5% increments
  - ➢ 95 billion combinations
- Retained parameter sets with flow stress error:
  - Median < 1%
  - Maximum < 10%
  - 20,423 parameter sets ($2 \cdot 10^{-5}$ %)





How do we know if a parameter set is correct?

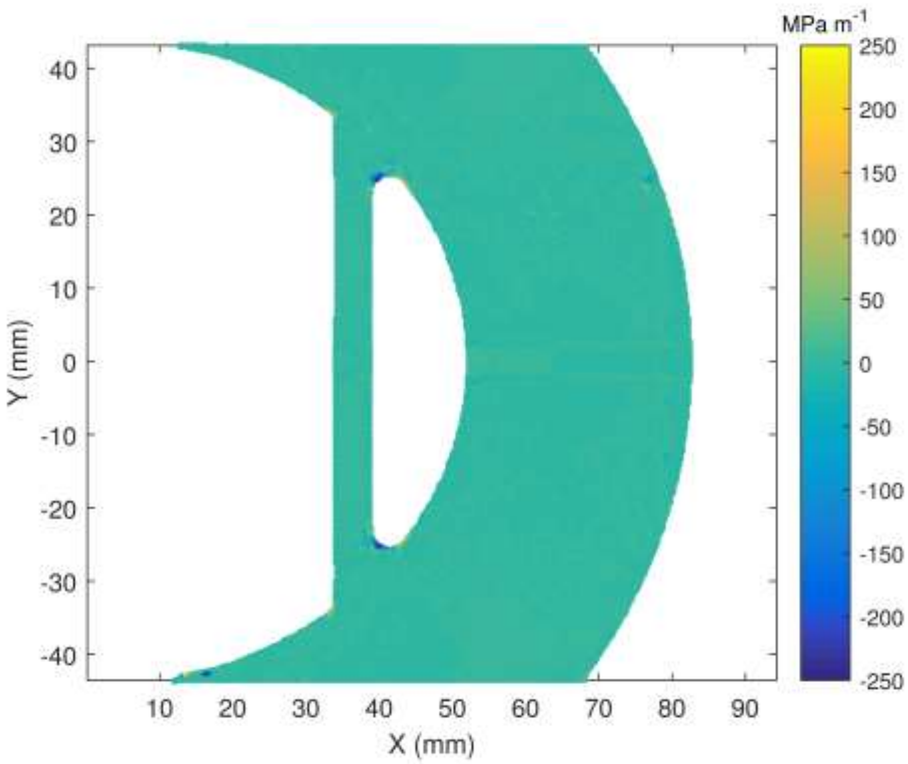
Local Equilibrium

- Parameters identified from global equilibrium:
  - $\int_{V_o} \left((\det F)\sigma \cdot F^{-T}\right) : \delta\dot{F}\; dV = f \cdot \overline{\delta v}$
- Calculate local equilibrium from reconstructed stresses:
  - $\dfrac{\partial\left((\det F)\sigma \cdot F^{-T}\right)_{ij}}{\partial X_i} = 0$
- Violations of local equilibrium highlight regions of the sample where material model (with identified parameters) fails to be accurate
  - e.g. high strains, high strain rates, shear-dominated, etc…

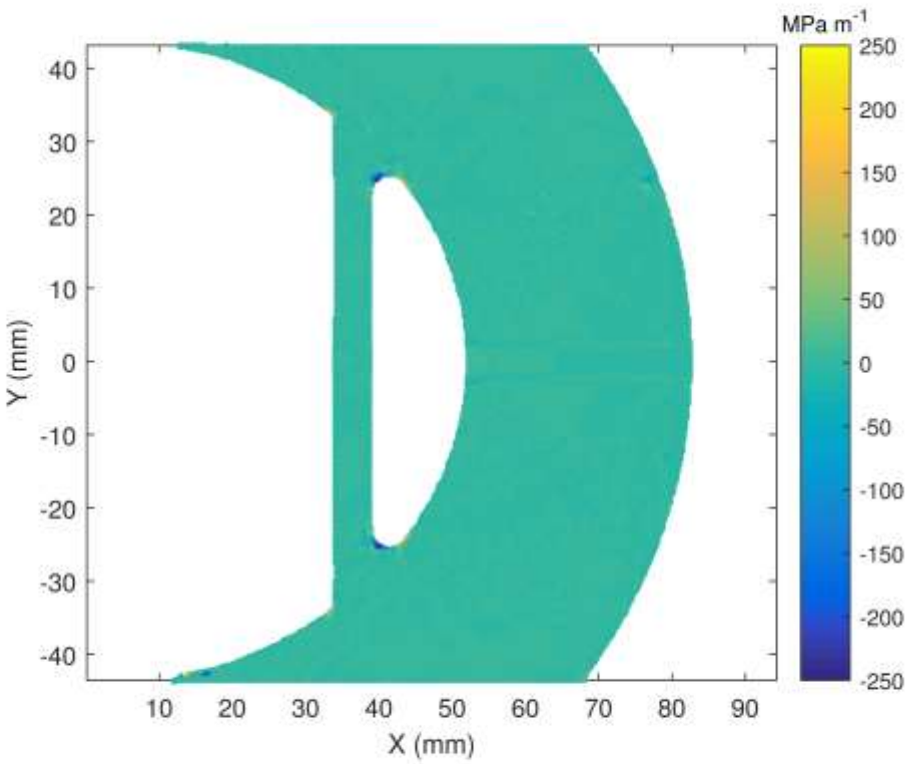
Equivalent model parameters cause equilibrium to be obeyed.

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} = 0$$

Reference Model Parameters



Identified Parameters
Case 1: No Noise



Equivalent model parameters cause equilibrium to be obeyed.

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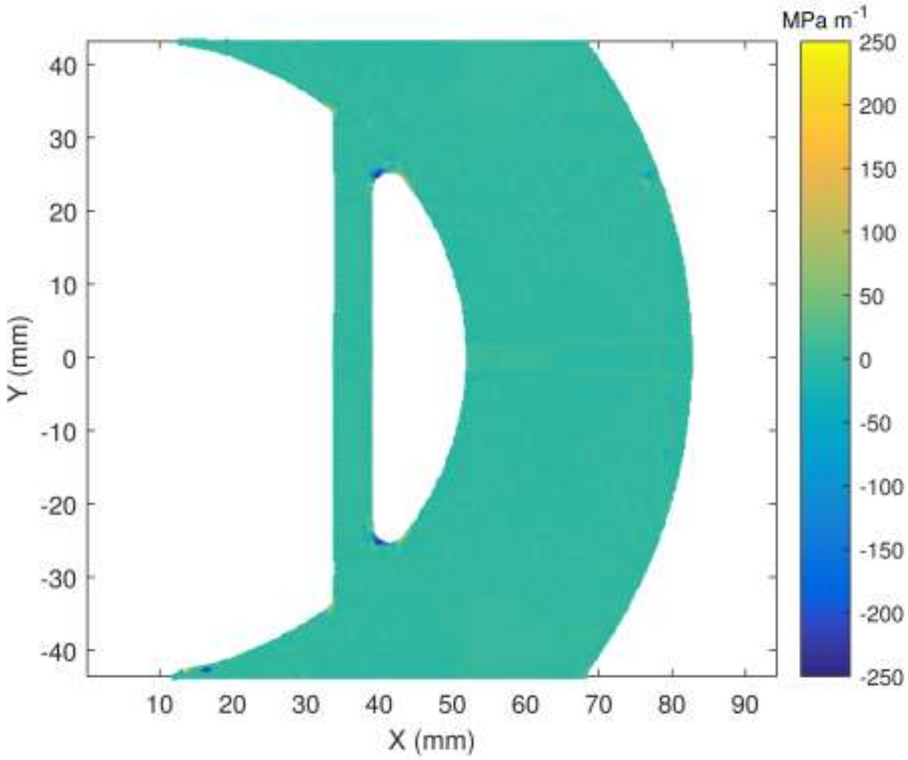
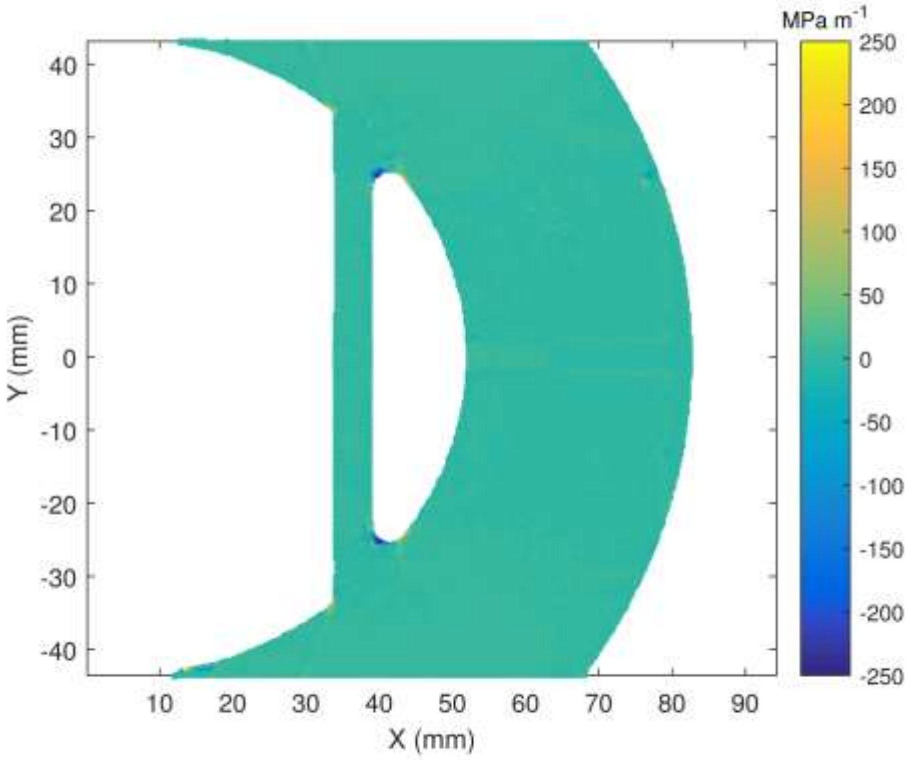
Reference Model Parameters



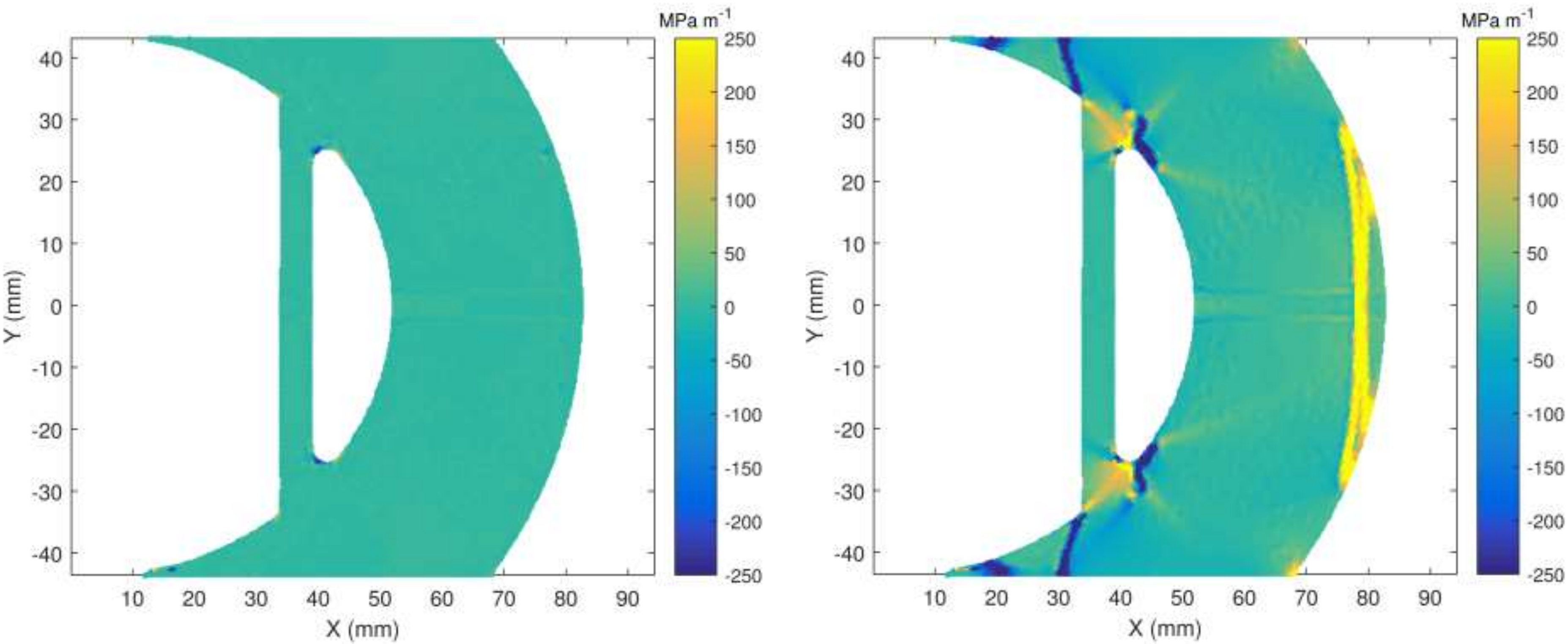
Identified Parameters
Case 2: 0.1 μm



Incorrect model parameters cause equilibrium to be violated.

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} = 0$$

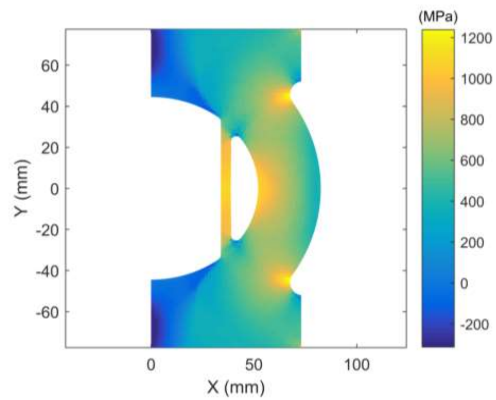
Reference Model Parameters **Arbitrary (Incorrect) Model Parameters**

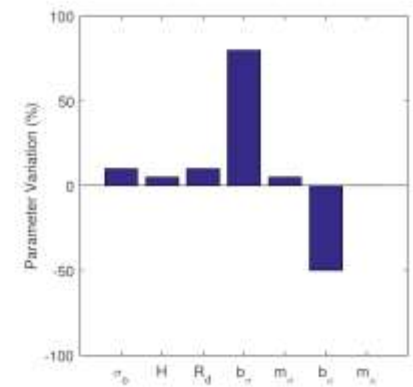


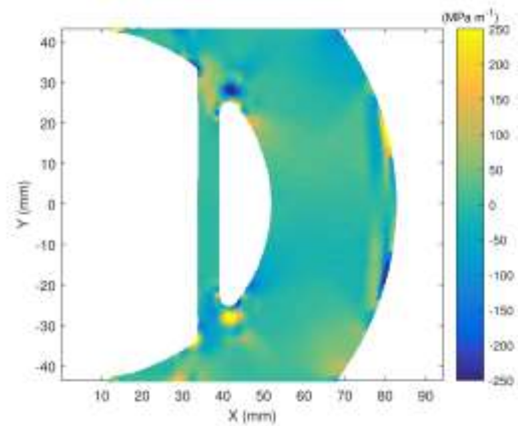
Goal: Perform better material characterization by utilizing full-field data instead of only global measurements

- Combined digital image correlation and the virtual fields method
- Validated implementation using simulated experimental data
- Discovered that model parameters are not unique
- Developing equilibrium method for material identification validation

$$\underbrace{\int_{V_o} \left((\det F)\sigma \cdot F^{-T}\right) : \delta\dot{F}\, dV}_{\textit{Internal Power}} = \underbrace{f \cdot \overline{\delta v}}_{\textit{External Power}}$$







$$\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = 0$$

Sneak peak at experimental data: