



Multiphase Effects in Spring-Mass-Damper Systems

**Timothy J. O'Hern, John R. Torczynski,
and Jonathan R. Clausen**

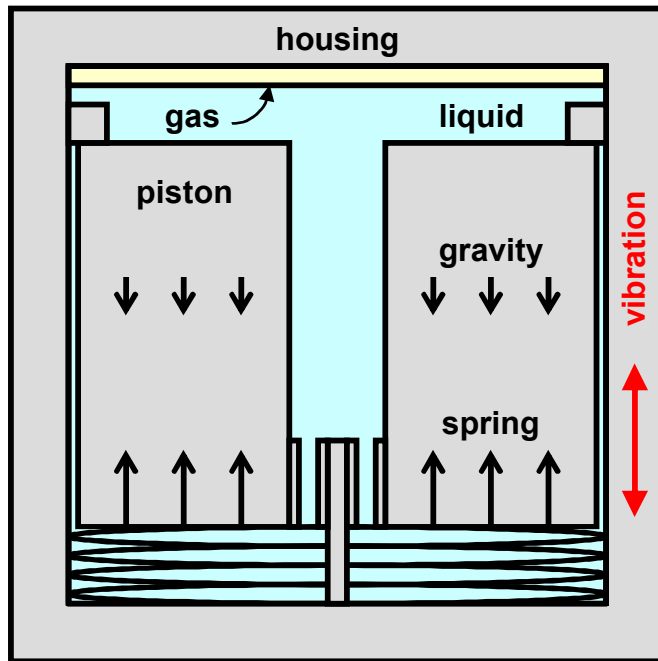
**Engineering Sciences Center
Sandia National Laboratories
Albuquerque, New Mexico, USA**

***ASME IMECE
Phoenix, AZ, November 11-17, 2016***

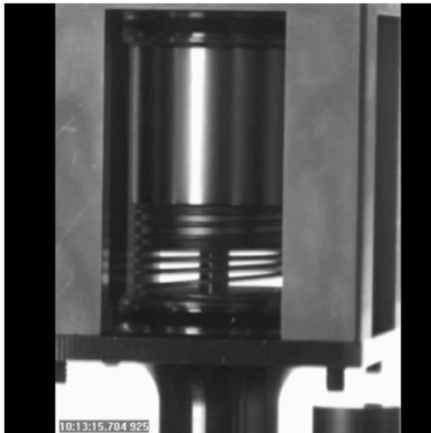
**The authors wish to thank Louis A. Romero and Gilbert L. Benavides
of Sandia National Laboratories for many helpful interactions.**

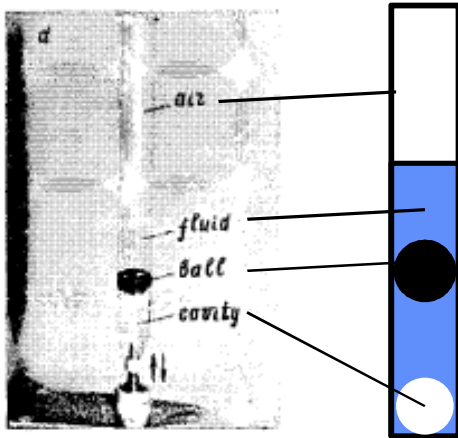


Outline



1. Background/Motivation/Define
2. Experiments
3. Theory
4. Simulations
5. Conclusions and Future Work





Chelomey (1984)¹

- Heavy ball resting at bottom of liquid-filled tube
- Vibrating the tube causes the heavy ball to rise vertically in the tube
- Ball reaches a stable state with air cavity visible at the base of the tube

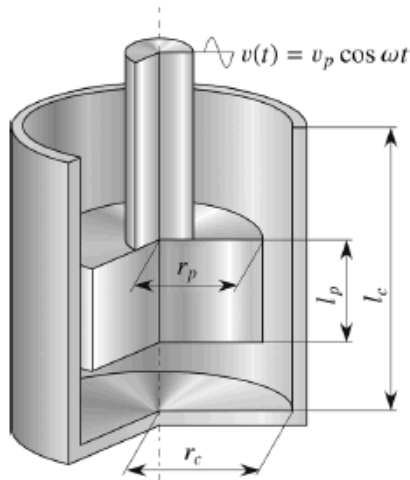


Fig. 3 Cross-sectional view of an oil damper

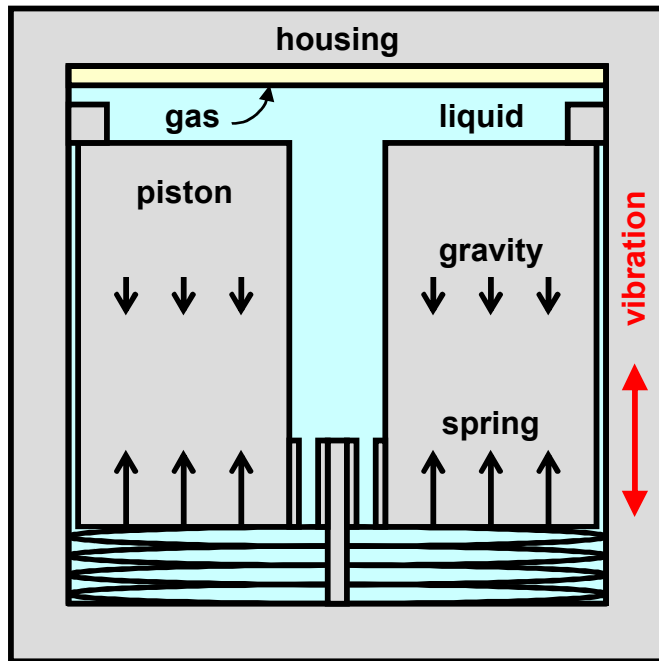
Asami et al. (2014)²

- Numerically and experimentally studied flow field around moving piston in damper
- Measured flow field above and below driven, oscillating piston
- Not multiphase, but dynamic system under vibration

1. Chelomey, V., (1985), "Paradoxes in Mechanics Caused by Vibrations," Meccanica., 20(4), pp. 314-316.

2. Asami, T., Honda, I., and Ueyama, A. (2014), "Numerical Analysis of the Internal Flow in an Annular Flow Channel Type Oil Damper," J. Fluids Engineering, 136.

Strange Vibration-Induced Dynamics

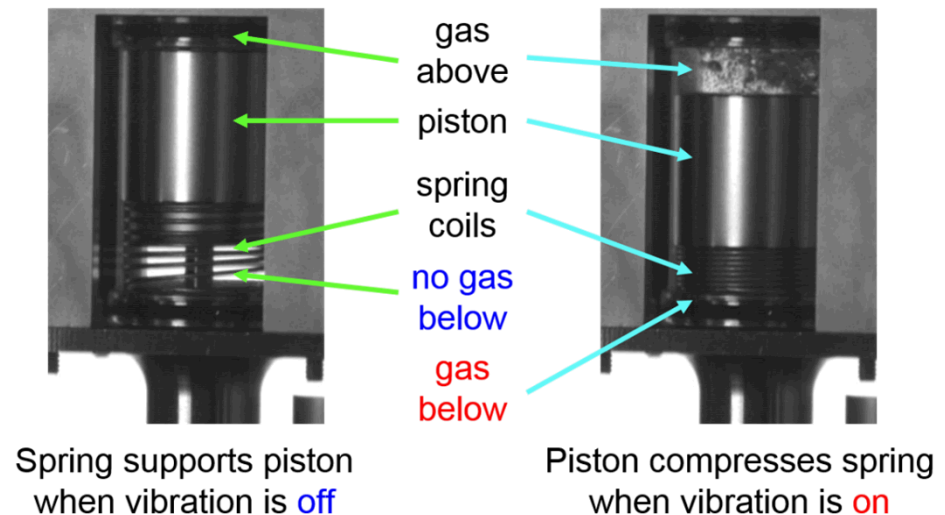
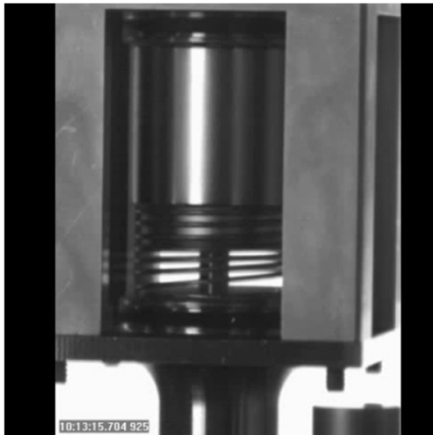


Spring-mass-damper system

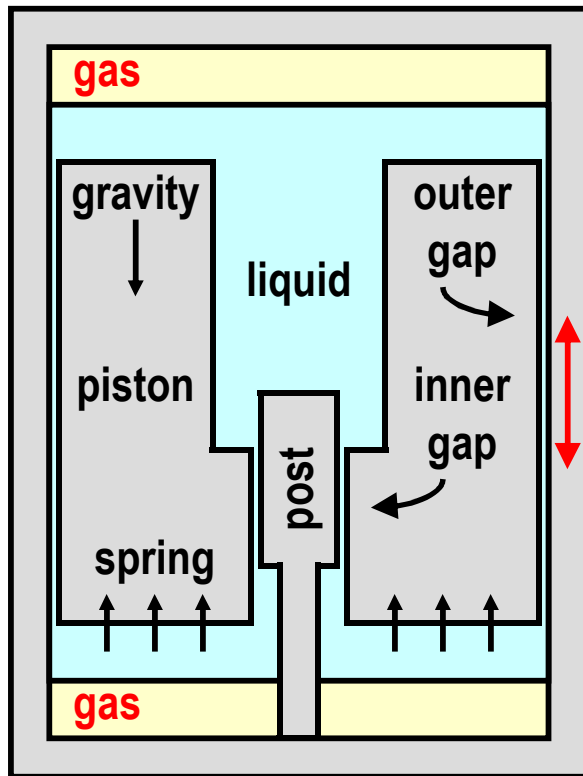
- Piston moves vertically in housing
- Spring supports it against gravity
- Viscous liquid provides damping
- Small amount of gas is present

Housing is vibrated vertically

- Gas moves down below piston
- Piston moves down against spring



How do bubbles get to floor?



Bubbles under vibration move in unexpected ways!

vertical vibration

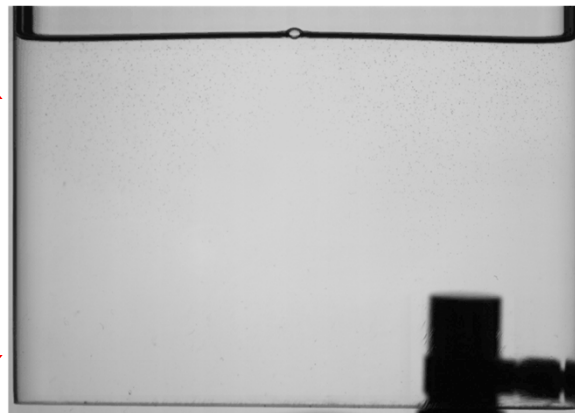
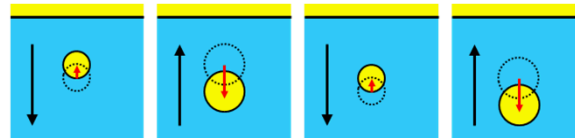
$$g = g_0 + g_1 \cos \omega t$$

Some gas moves down below piston

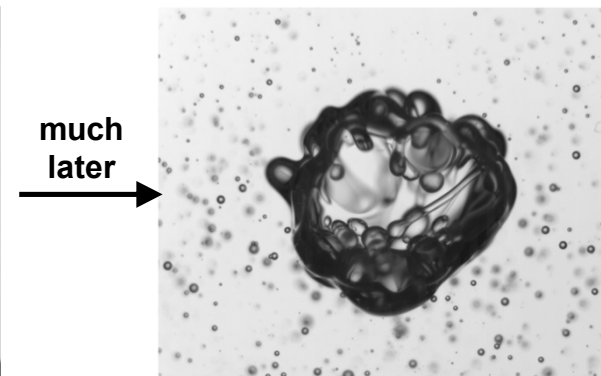
- Bjerknes forces push bubbles down
- Create & stabilize a lower gas region

Two gas regions: upper and lower

- Both are quasi-stable (stationary)



create lower gas region



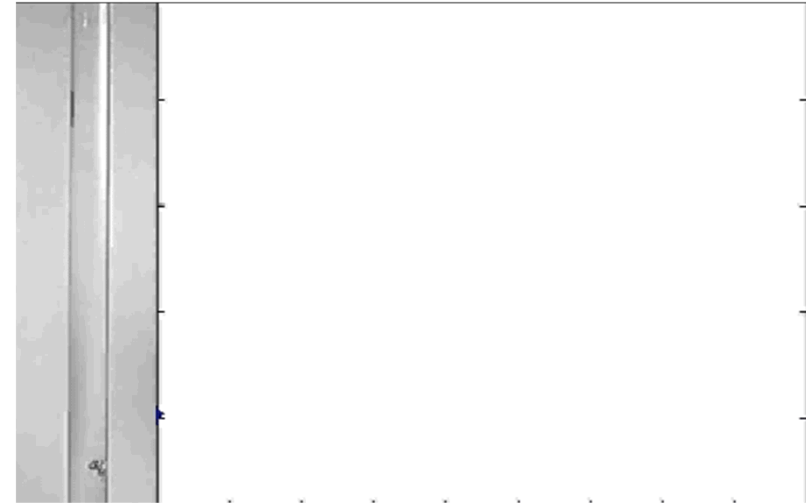
stabilize it

Related Work on Bubble Vibration Effects

Bubbles and bubble clusters stabilized by vibration have been seen in other contexts

- Crum (1975) "Bjerknes forces on bubbles in a stationary sound field," *J. Acoust. Soc. Am.*, 57(6), 1363-1370.
- Ellenberger, van Baten, and Krishna (2005) "Exploiting the Bjerknes force in bubble column reactors," *Chem. Eng. Sci.*, 60, 5962-5970.
- Jameson and Davidson (1966) "The motion of a bubble in a vertically oscillating liquid: theory for an inviscid liquid, and experimental results," *Chem. Eng. Sci.*, 21, 29-34.
- Waghmare, Rice, and Knopf (2008) "Mass Transfer in a Viscous Bubble Column with Forced Oscillations," *Int. Eng. Chem. Res.*, 47, 5386-5394.
- Waghmare, Knopf, and Rice (2007) "The Bjerknes Effect: Explaining Pulsed-Flow Behavior in Bubble Columns," *AIChE J.*, 53(7), 1678-1686
- Brian Ebling and the Oklahoma State University group

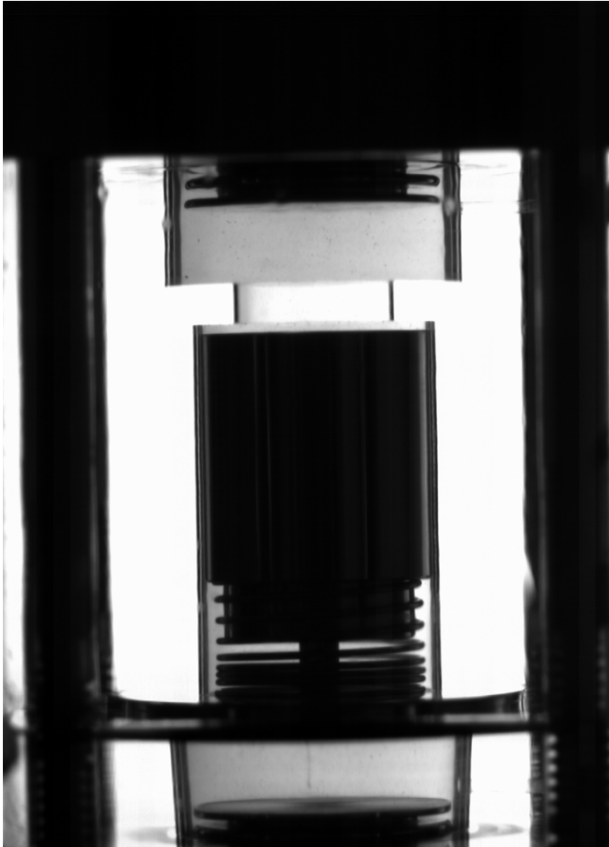
Ellenberger et al. move a bubble cluster up & down in a tube by sweeping vibration frequency



**Ellenberger, Vandu, and Krishna;
air bubble in water, 0125 mm p-p,
85-100 Hz, 2 Hz/sec sweep**

**[http://www.science.uva.nl/research/cr/
BubbleMotionVibration/](http://www.science.uva.nl/research/cr/BubbleMotionVibration/)**

Cavitation is Not the Usual Source of Bubbles and Problems



500 fps



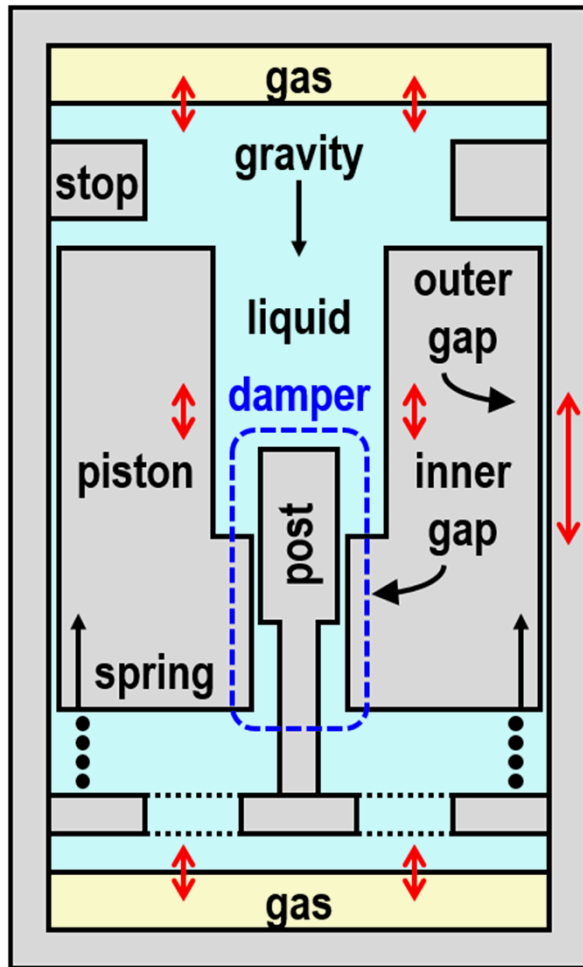
5100 fps

20-cSt PDMS,
40 Hz, 28 g peak
acceleration, 4.34 mm
displacement

Much harder shaking
than the rest of the
cases shown today

Unusual conditions here – this does not normally occur

How Vibration Makes Piston Go Down



Gas regions form **pneumatic spring**

- Bjerknes forces push some gas down
- Stiffness is $\sim 100\times$ piston's spring

Enables new mode with **low damping**

- Piston and liquid can move together
- Little liquid is forced through gaps

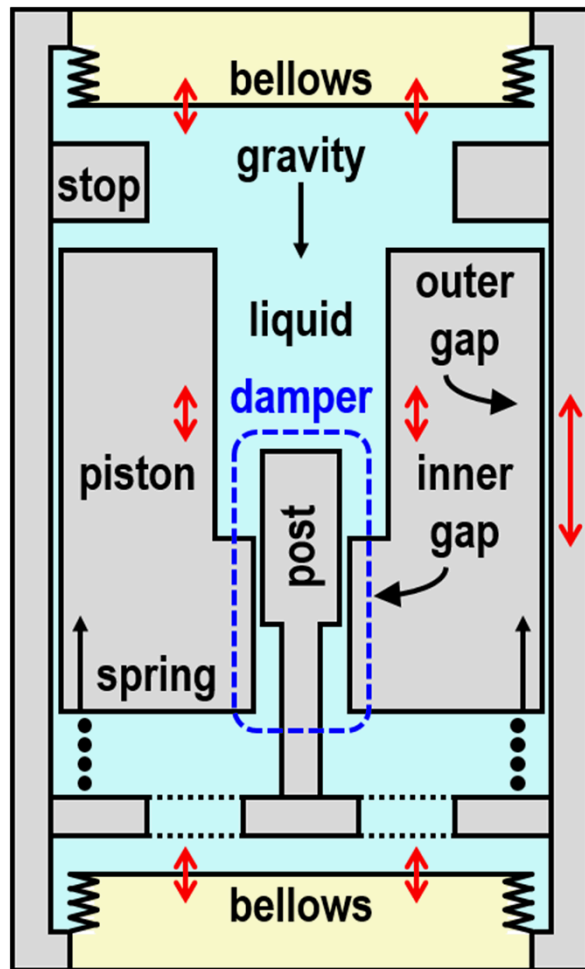
Low damping gives **strong resonance**

- Piston+liquid mass with gas spring
- Frequency $f \approx (K_{\text{gas}}/M_{\text{total}})^{1/2}/(2\pi)$

Gap nonlinearity produces **net force**

- Damping depends on piston position
- Piston moves down to shorten gap

Analyze Analogous Bellows System



Gas regions are problematic to treat

- Upper/lower split of gas is not known
- Motion is transient and complicated

So replace gas regions with bellows

- Compressibility is well characterized
- Choose to be similar to gas regions

System suited to theory & simulation

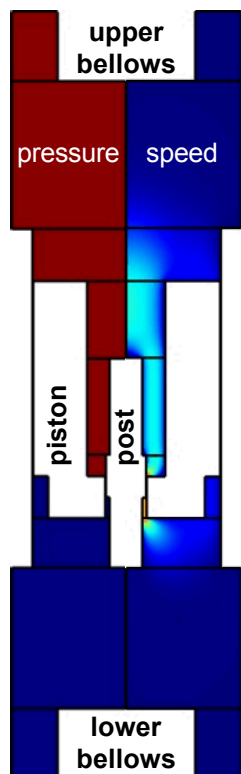
- Liquid: incompressible Navier-Stokes equations with moving boundaries
- Objects: Newton's 2nd Law (" $F = ma$ ")

Theory and Simulation Agree

Theory gives 2-DOF nonlinear damped harmonic oscillator

- Liquid quasi-steady-Stokes, solids Newton's 2nd Law
- Liquid damping & added mass depend on piston position

Oscillation amplitude & drift agree with Navier-Stokes ALE



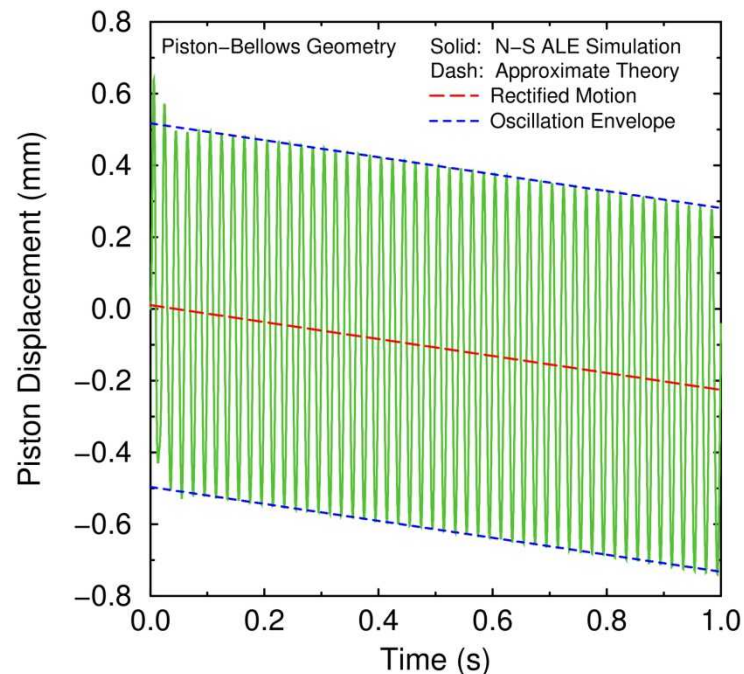
Navier-Stokes Eqns.
Newton's 2nd Law

$$\lambda = \frac{A_B}{A_p + (A_G/2)}$$

$$F_{\text{rect}} = \lambda \frac{d\beta_{11}}{dz_1} \langle z_1 \dot{z}_2 \rangle$$

$$U_{\text{rect}} = \frac{F_{\text{rect}}}{\beta_{11}} \text{ (drift)}$$

liquid added masses		liquid damping coefficients		gravity-buoyancy
$(\tilde{\mathbf{M}} + \mathbf{M}) \ddot{\mathbf{Z}} + (\tilde{\mathbf{B}} + \mathbf{B}) \dot{\mathbf{Z}} + \tilde{\mathbf{K}} \mathbf{Z} = \mathbf{f}$				
object masses	object damping coefficients	object spring constants		
$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}_i = 0, \quad \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma}_i = 0$ $\mathbf{u}_i = \begin{cases} U \hat{\mathbf{e}}_z & \text{on } S_i \\ 0 & \text{on other walls} \end{cases}$ $S_i = \frac{1}{2} \left(\frac{\partial \mathbf{u}_i}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}_i^T}{\partial \mathbf{x}} \right)$	$\mathbf{Z} = \begin{pmatrix} Z_p \\ Z_B \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}$ $\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$	$\mathbf{f} = - \begin{pmatrix} M_{pG} \\ M_{BG} \end{pmatrix} g_r \cos \omega t$ $\beta_{ij} = \frac{2\mu}{U^2} \int_V \mathbf{S}_i : \mathbf{S}_j dV$ $m_{ij} = \frac{\rho}{U^2} \int_V \mathbf{u}_i \cdot \mathbf{u}_j dV$		$\tilde{\mathbf{K}} = \begin{pmatrix} K_p & 0 \\ 0 & K_B \end{pmatrix}$ $\tilde{\mathbf{B}} = \begin{pmatrix} B_p & 0 \\ 0 & B_B \end{pmatrix}$ $\tilde{\mathbf{M}} = \begin{pmatrix} M_p & 0 \\ 0 & M_B \end{pmatrix}$



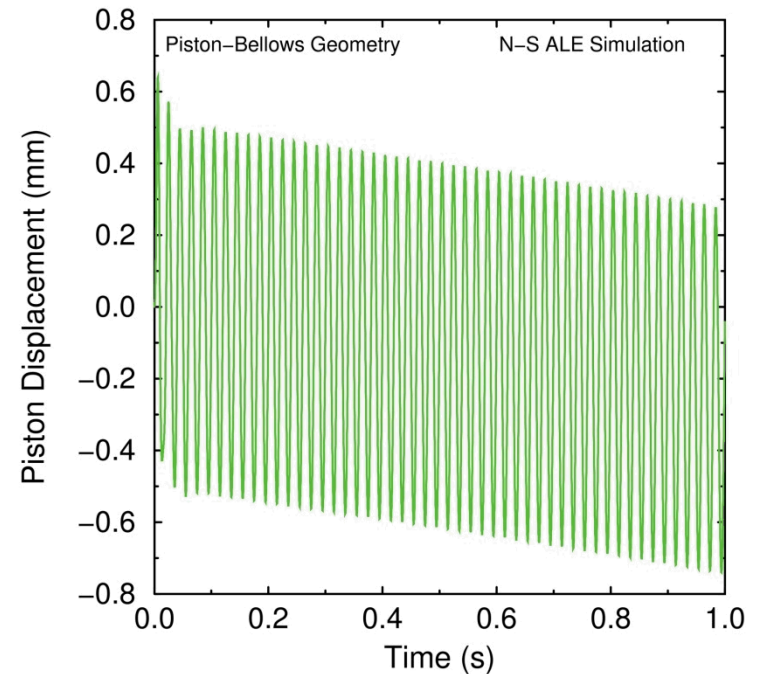
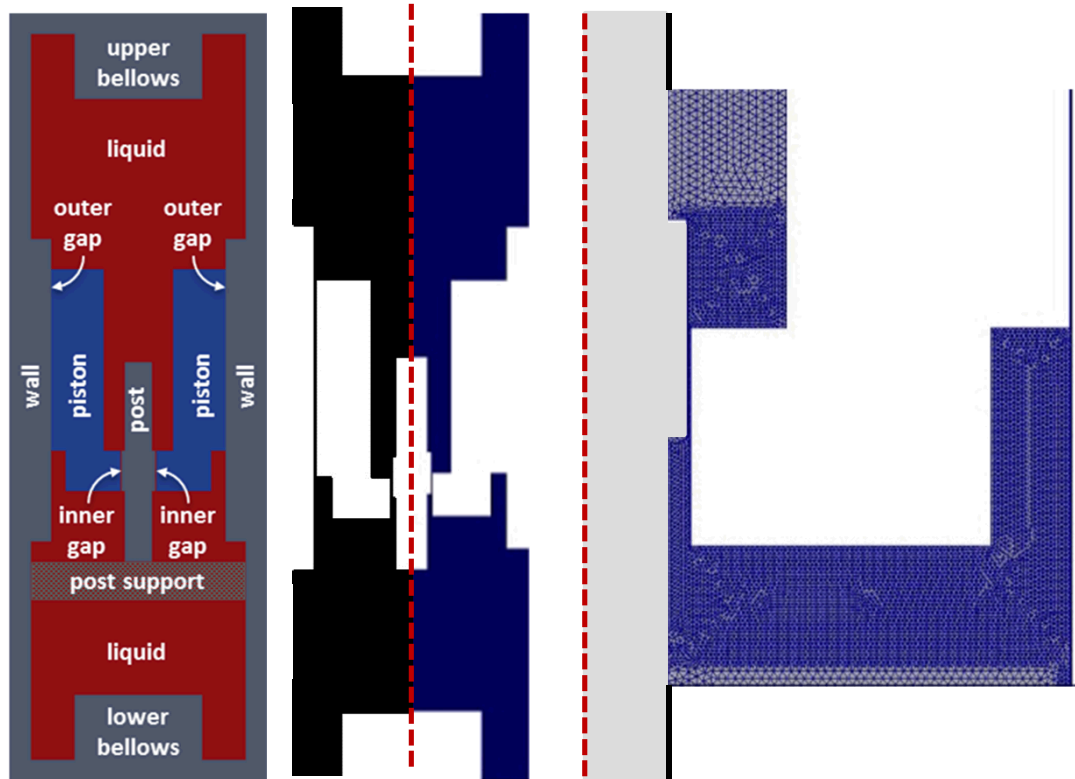
Theory and Simulation

Numerical Simulation

Coupled ALE simulation of piston, bellows, & liquid motion

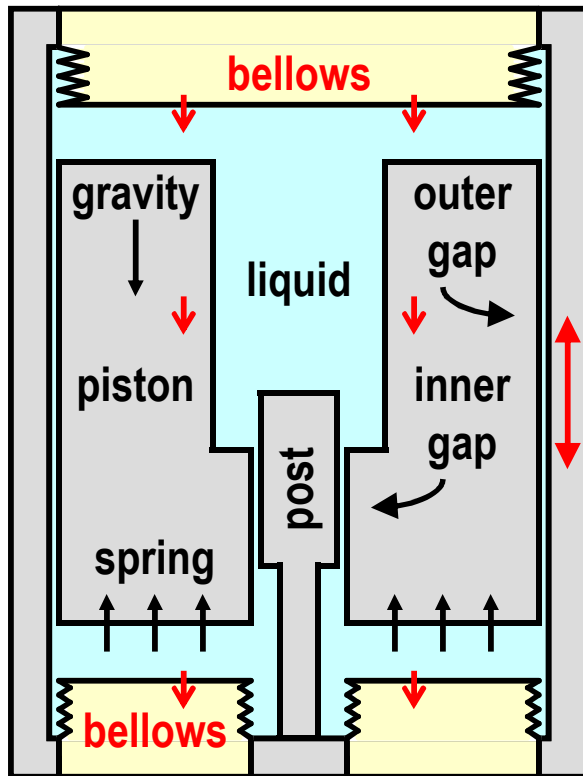
- Moving-boundary incompressible Navier-Stokes eqns
- Sliding-mesh scheme mitigates sheared elements
- Newton's 2nd Law (" $F = ma$ ") for piston and bellows

Piston oscillates strongly and drifts downward slowly



Sierra ARIA Finite Element Simulation

Analyze Analogous Bellows System



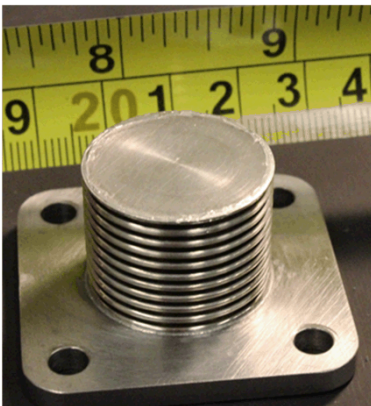
Gas regions are hard to analyze

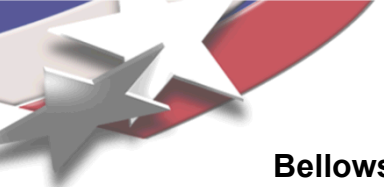
- Upper/lower split of gas is not known
- Motion is transient and complicated

So replace gas regions with bellows

- Compressibility is well characterized
- Choose pressure-volume properties to be similar to gas regions

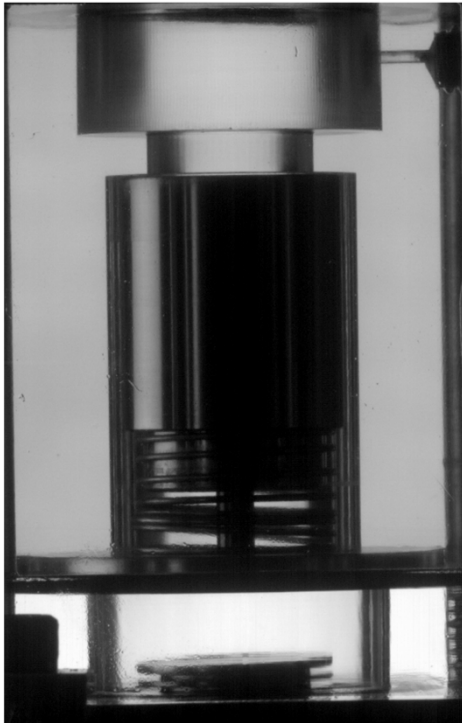
System suited to experiments, theory, and simulation





Experiments with Bellows

Bellows



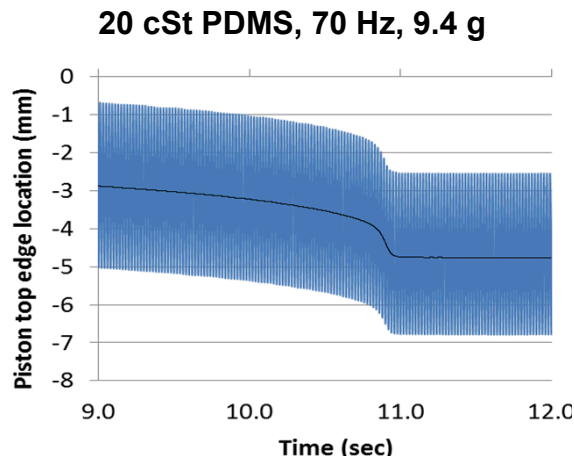
Bellows

Different bellows sizes chosen to act as bubbles of typical sizes

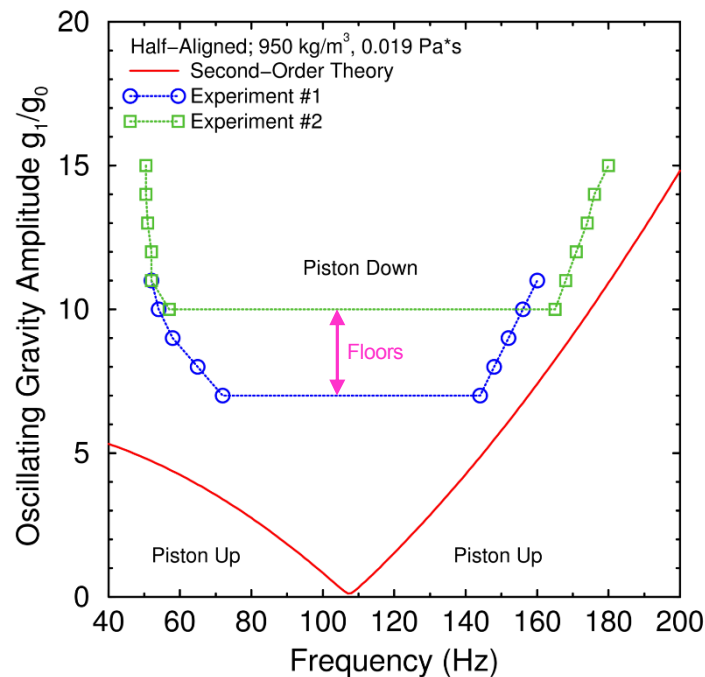
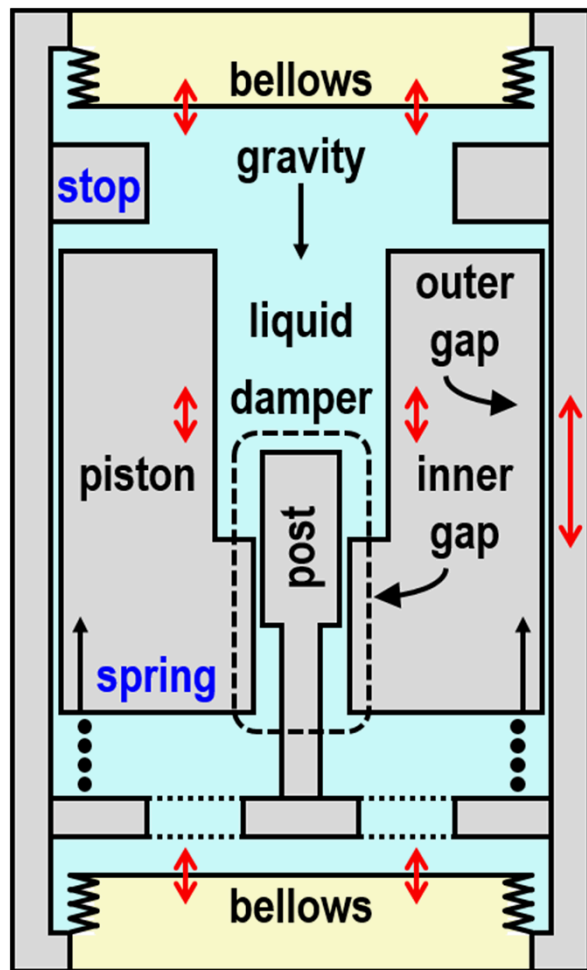
- Commercial Servometer bellows
 - $d = 2.5$ cm and $d = 1.9$ cm bellows span expected compressibility range
 - Control experiments without bellows
 - No bellows
 - Single bellows
- } no piston motion

Vibration conditions

- Commercial Labworks shaker
- Frequency: 40-200 Hz
- Acceleration: up to 30 g (30×9.81 m/s²)
- Peak-to-peak displacement: 0-2 mm



Experiments Differ Significantly



Regime map: amplitude vs. frequency

- Rectified force equals spring preload

Experiments have amplitude floors

- Not reproducible from day to day

Stop may be causing the difference

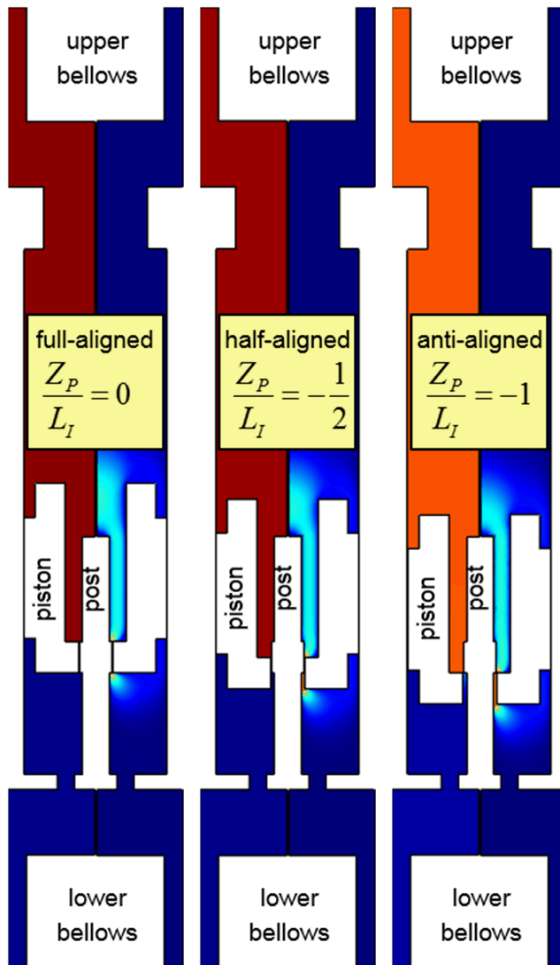


- ## So replace stop with a second spring

- ## Investigate two-spring system

- **Focus on rectification nonlinearity**
- **Study stop interaction subsequently**

Two-Spring System



Piston positions of significance

- Full-aligned: maximum damping
- Half-aligned: zero spring force
- Anti-aligned: small damping

Analyses: from complex to simplest

- Navier-Stokes and Newton's 2nd Law
- Full ODE (quasi-steady-Stokes)
- Oscillation + drift model (from ODE)
- Quasi-steady equilibrium (from ODE)

$$\rho \frac{D\mathbf{u}}{Dt} = -\frac{\partial p}{\partial \mathbf{x}} + \mu \nabla^2 \mathbf{u} + \rho(\mathbf{g} - \mathbf{a}), \quad \nabla \cdot \mathbf{u} = 0, \quad \mathbf{u} = \mathbf{u}_{\text{wall}};$$

$$\tilde{\mathbf{M}}\ddot{\mathbf{Z}} = -\tilde{\mathbf{B}}\dot{\mathbf{Z}} - \tilde{\mathbf{K}}\mathbf{Z} + \tilde{\mathbf{M}}(\mathbf{g} - \mathbf{a}) + \mathbf{F}_{\text{liquid}}, \quad \mathbf{u}_{\text{wall}} = \mathbf{u}_{\text{wall}}[\mathbf{Z}, \dot{\mathbf{Z}}]$$

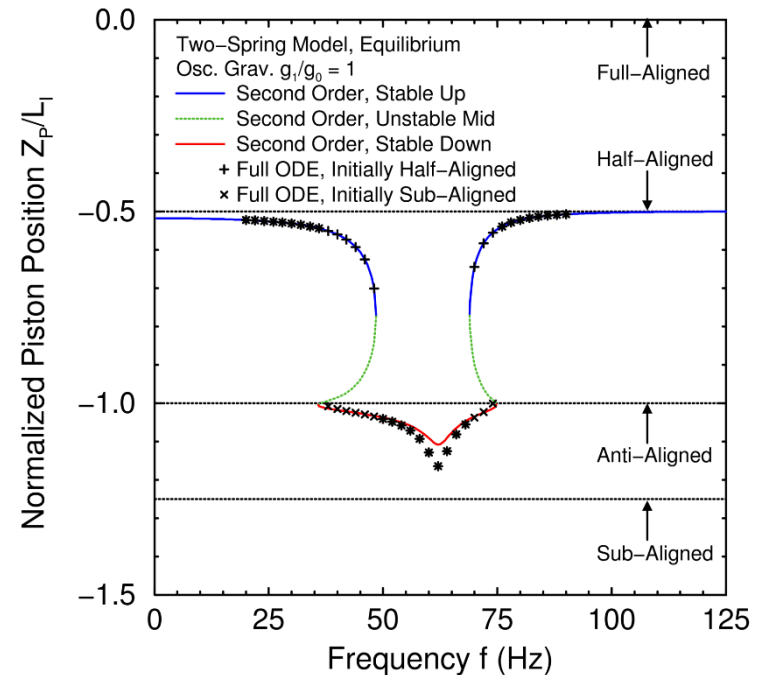
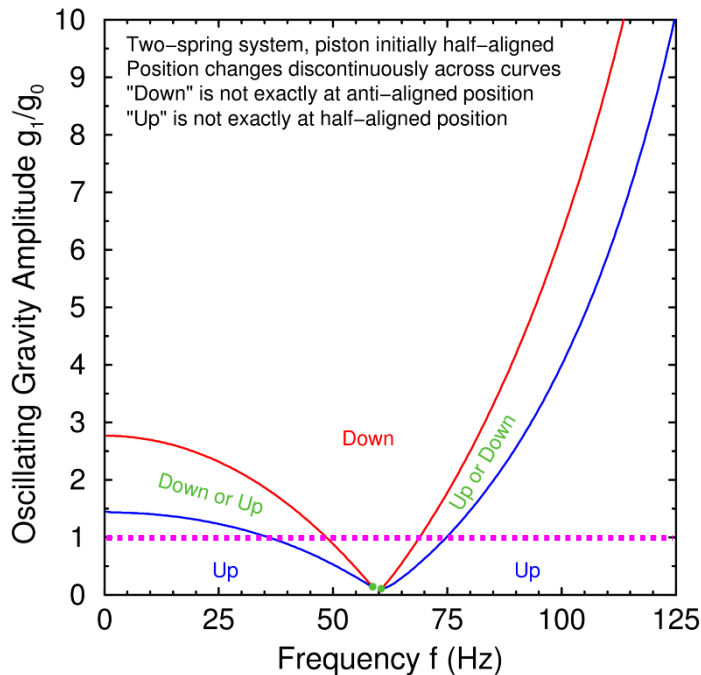
$$(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}])\ddot{\mathbf{Z}} + (\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}])\dot{\mathbf{Z}} + \tilde{\mathbf{K}}\mathbf{Z} = \mathbf{F}, \quad \mathbf{F} = \mathbf{F}_0 \sin[\omega t]$$

$$(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}_{\text{drift}}])\ddot{\mathbf{Z}}_{\text{oscil}} + (\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}_{\text{drift}}])\dot{\mathbf{Z}}_{\text{oscil}} + \tilde{\mathbf{K}}\mathbf{Z}_{\text{oscil}} = \mathbf{F}_{\text{oscil}}$$

$$(\tilde{\mathbf{M}} + \mathbf{M}[\mathbf{Z}_{\text{drift}}])\ddot{\mathbf{Z}}_{\text{drift}} + (\tilde{\mathbf{B}} + \mathbf{B}[\mathbf{Z}_{\text{drift}}])\dot{\mathbf{Z}}_{\text{drift}} + \tilde{\mathbf{K}}\mathbf{Z}_{\text{drift}} = \mathbf{F}_{\text{drift}}$$

$$\mathbf{F}_{\text{oscil}} = \mathbf{F}_0 \sin[\omega t], \quad \mathbf{F}_{\text{drift}} = -\left\langle \mathbf{Z}_{\text{oscil}} \frac{\partial \mathbf{B}}{\partial \mathbf{Z}} \dot{\mathbf{Z}}_{\text{oscil}} \right\rangle$$

Multiple Equilibrium Piston Positions



Equilibrium piston position versus amplitude & frequency

- Two stable states: up, down (unstable state between: mid)
- Up & down regions separated by multi-state regions

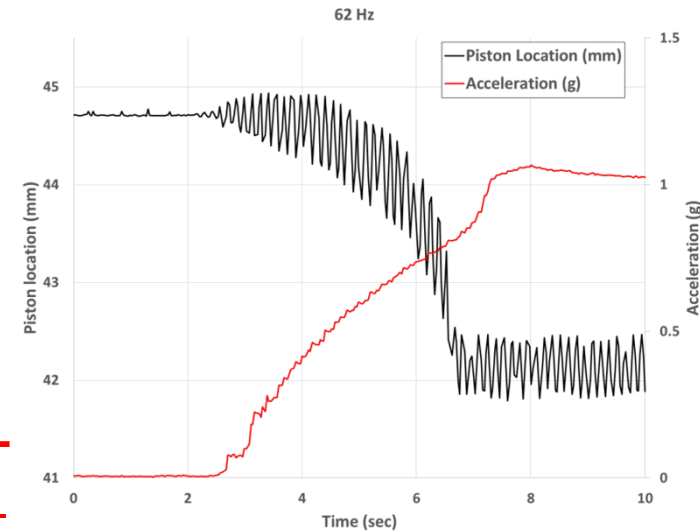
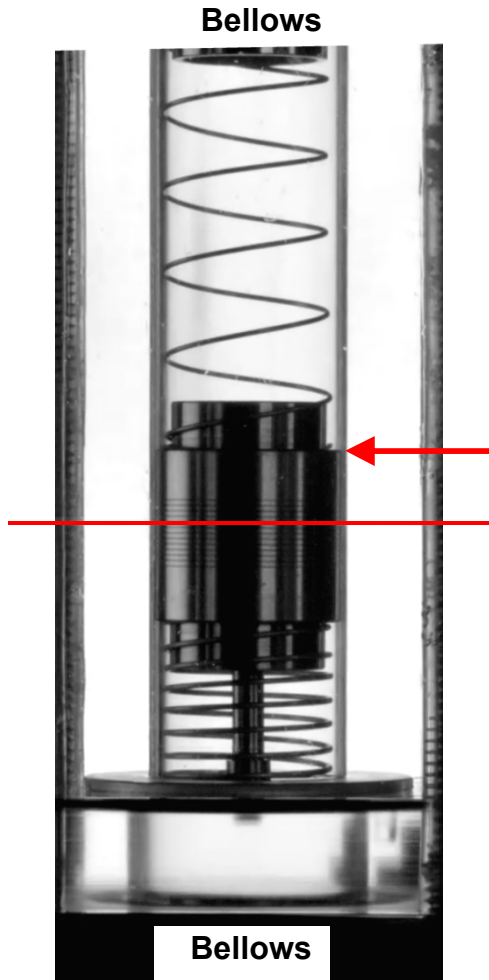
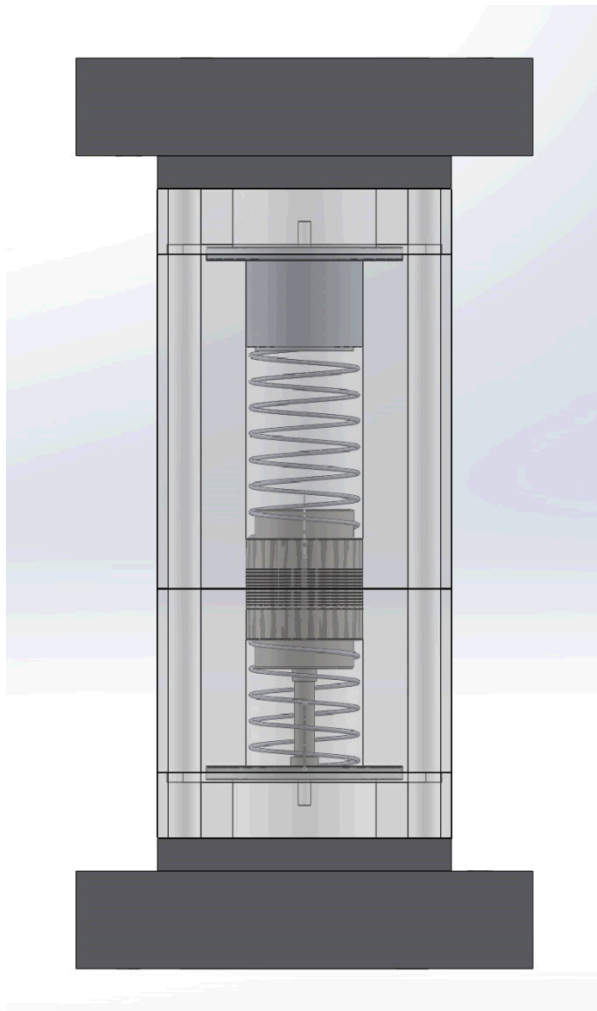
Position is multi-valued versus frequency at fixed amplitude

- Quasi-steady equilibrium agrees well with full ODE

Two-Spring Experiment

Replace upper piston stop with a second spring

- Piston does not have to pull away from stop to start motion

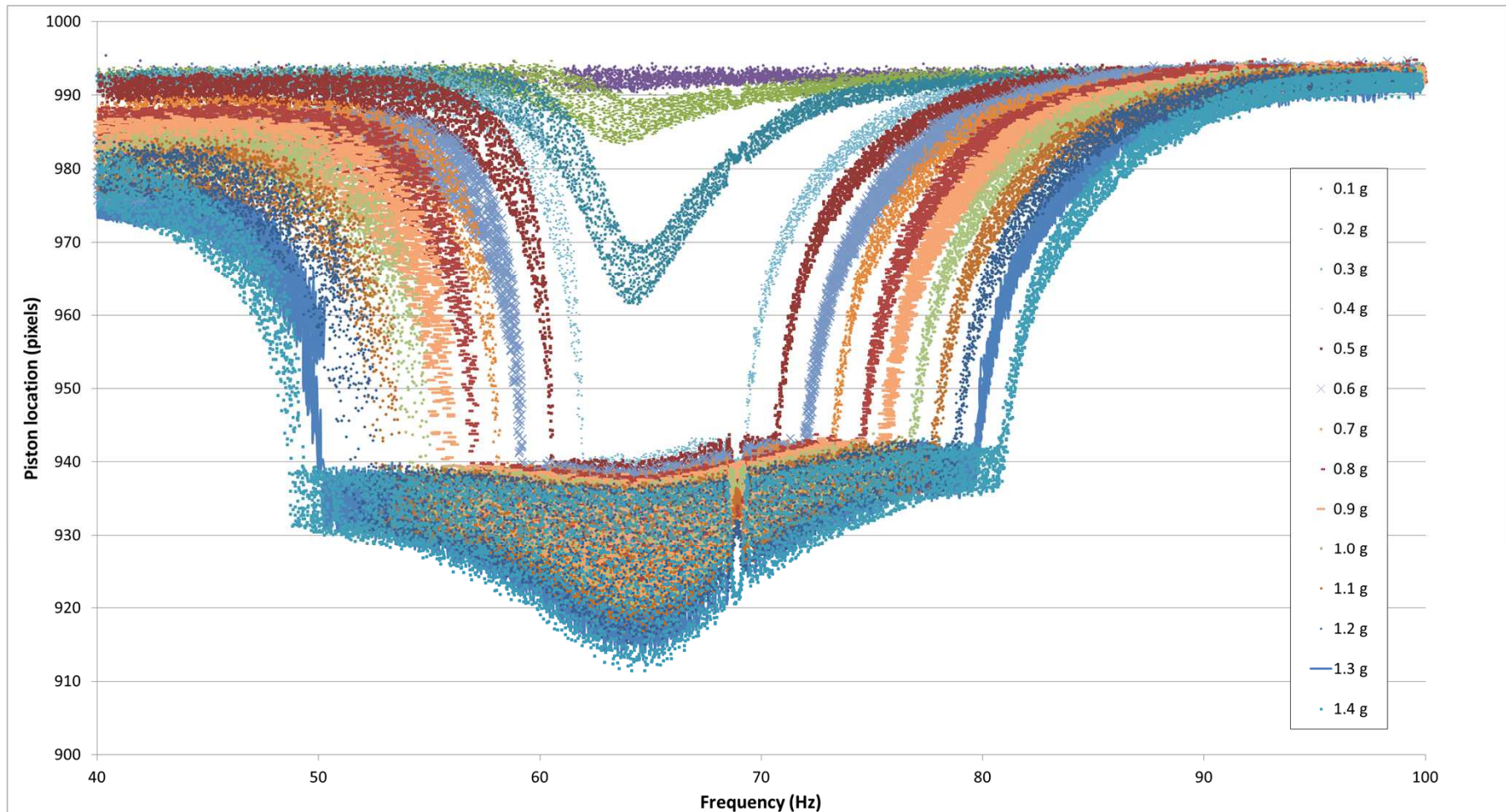


- Piston motion requires lower acceleration
 - (e.g., 1 g vs. 20 g)
- Motion much more repeatable

62 Hz, 1.0 g, 20 cSt PDMS, 1" bellows

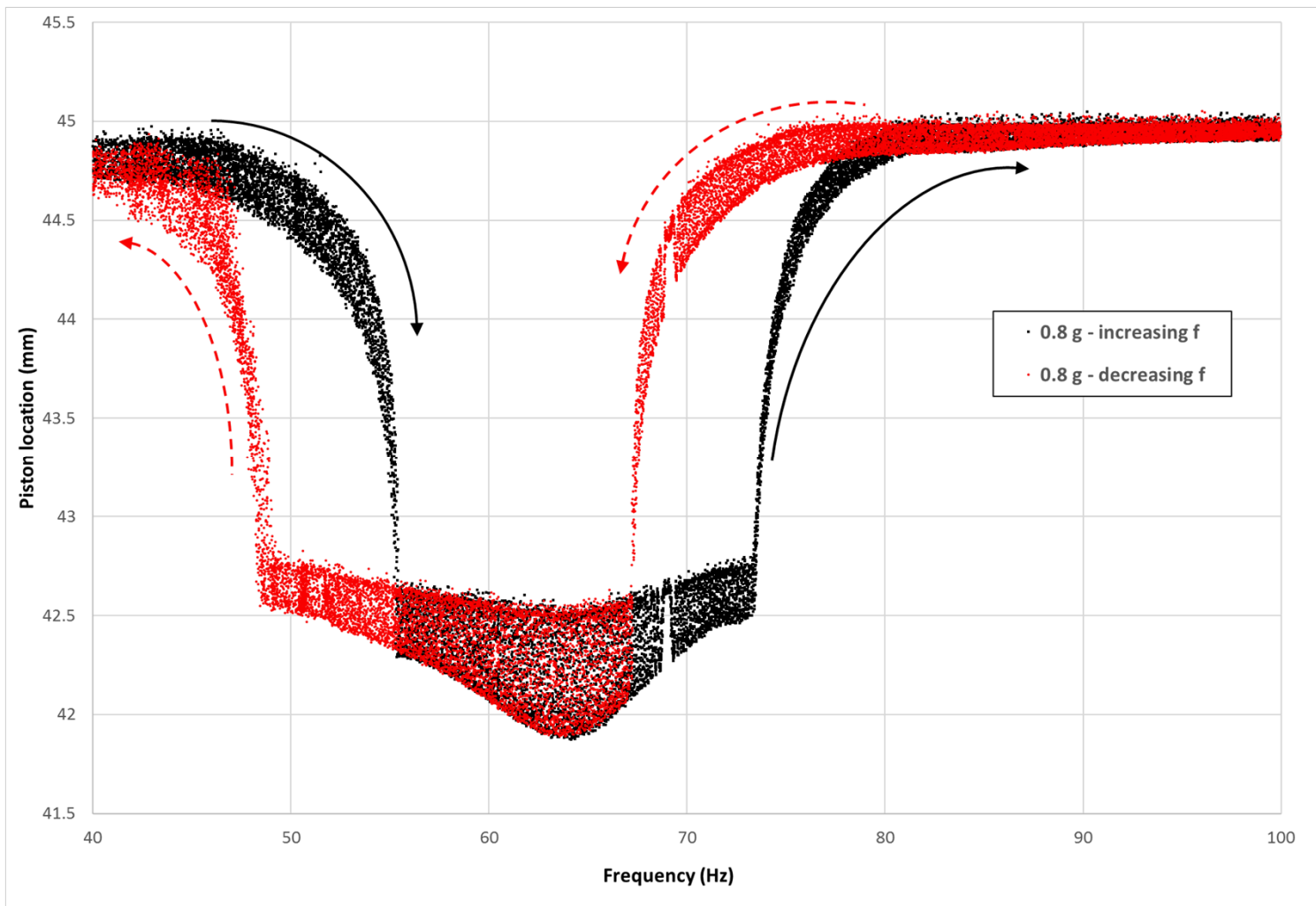
Two-Spring Experiment

40-100 Hz sine sweep, 5 minutes long, 0.1 g steps



Piston starts moving downward at 0.3 g, completely down at 0.4 g. With stops transition was at 10 g.

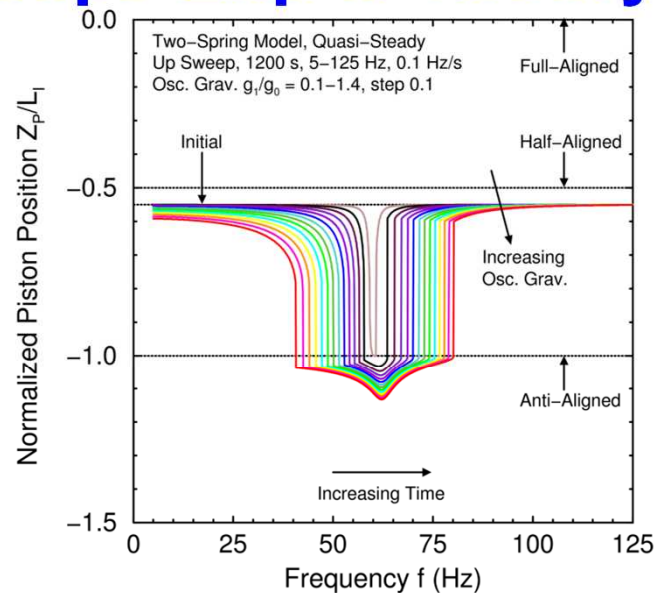
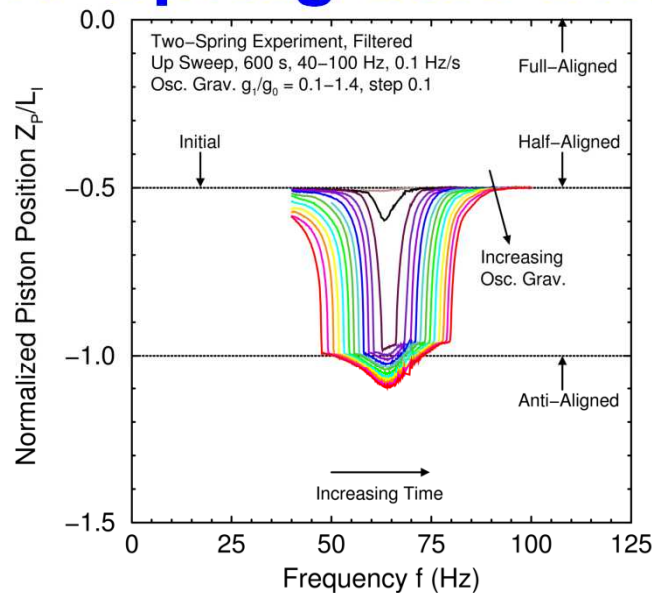
Experimental Data



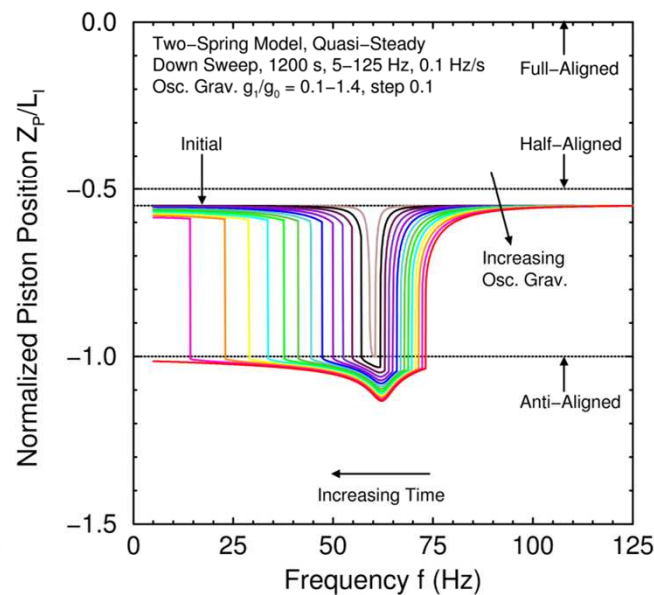
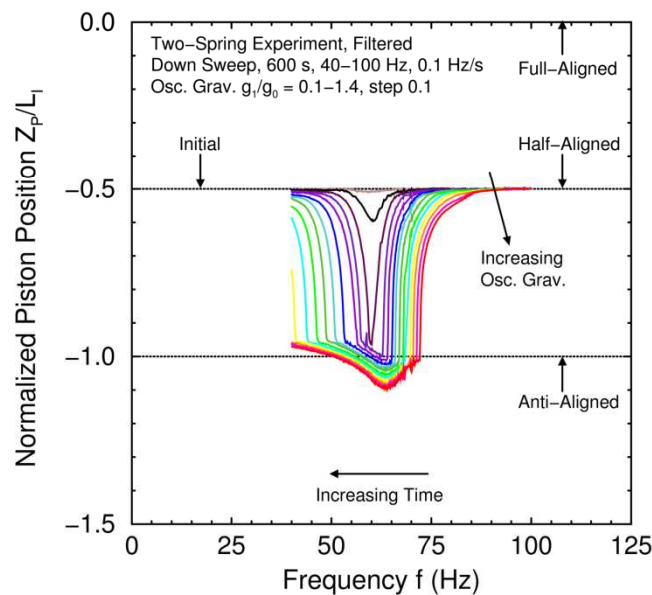
Hysteresis between increasing and decreasing f cases

Two-Spring Sine Sweeps Exp & Theory

Increasing f
sine sweep



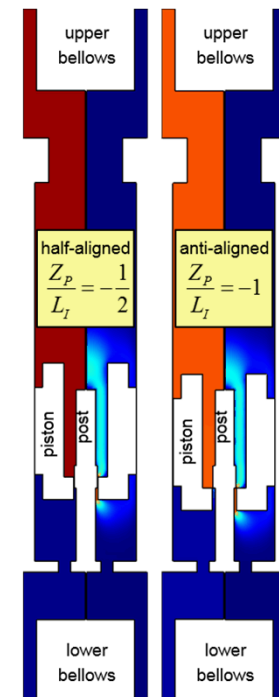
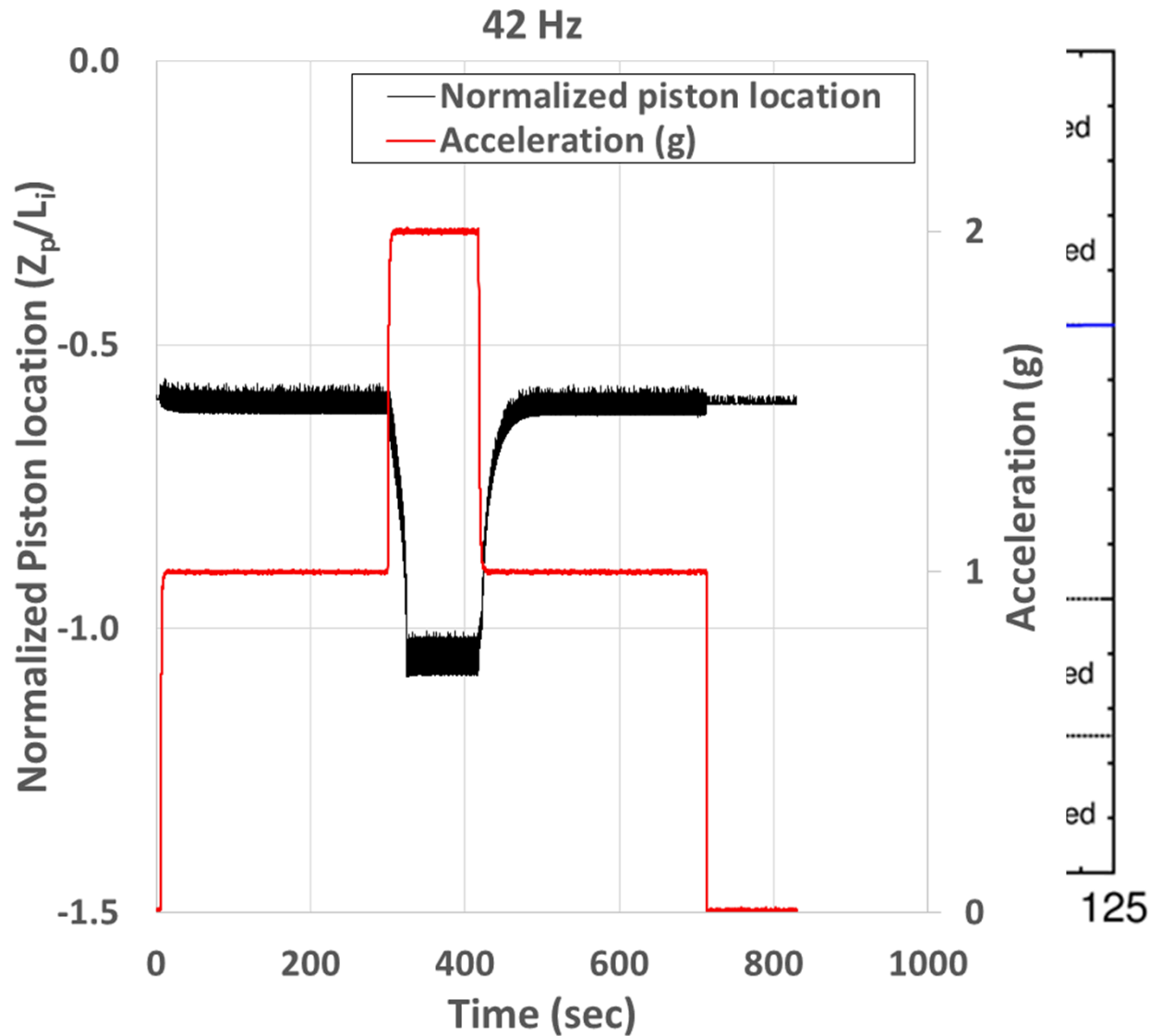
Decreasing f
sine sweep



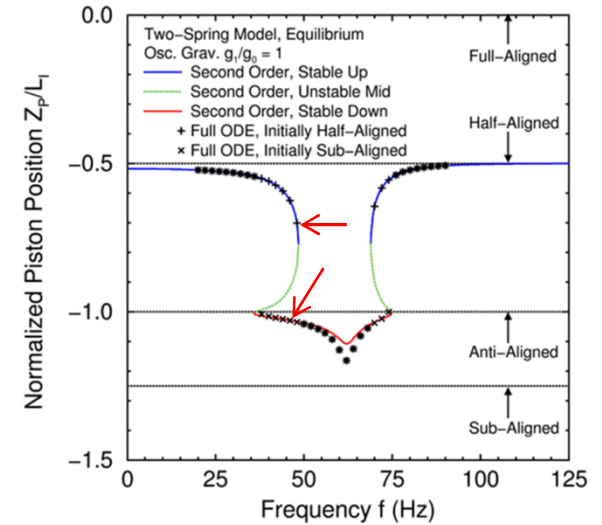
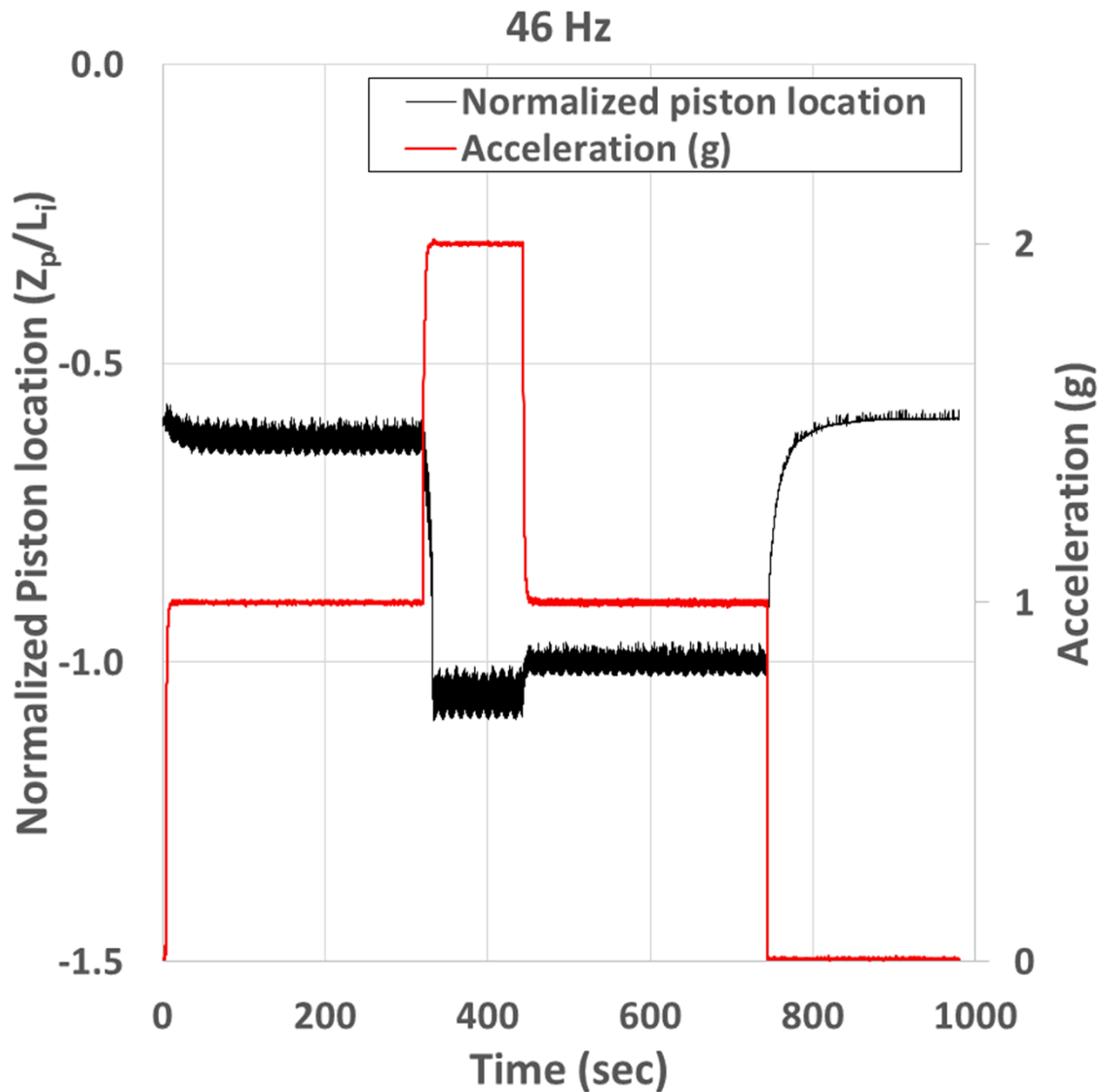
Experiments

Quasi-Steady Theory

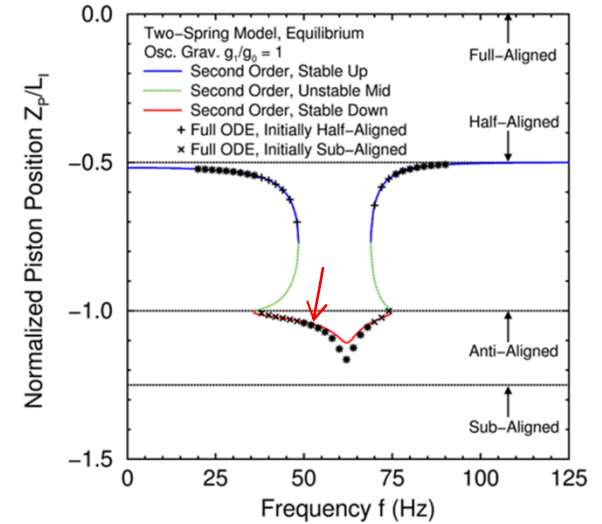
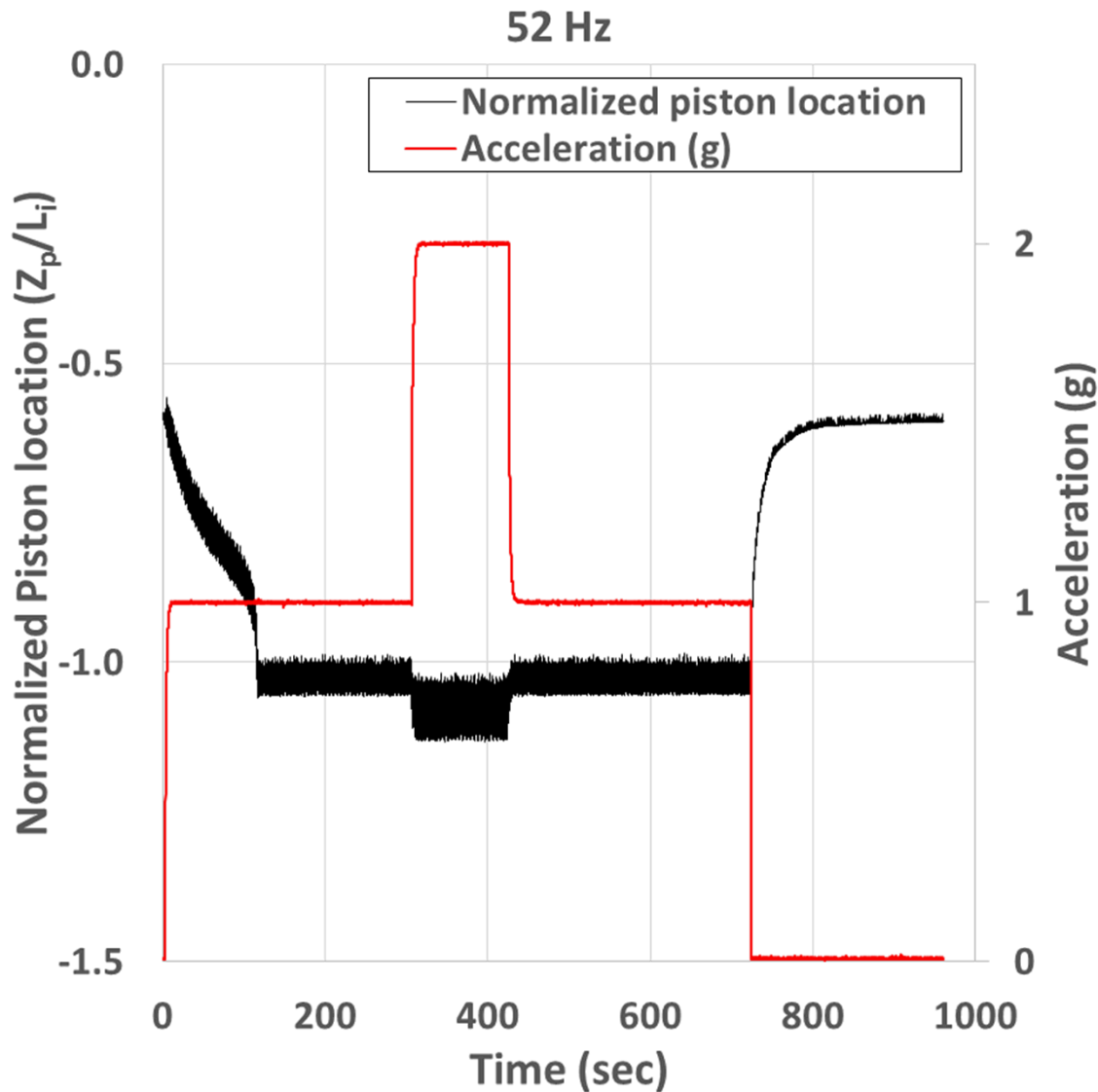
Two-Spring Experiment – Validation Data



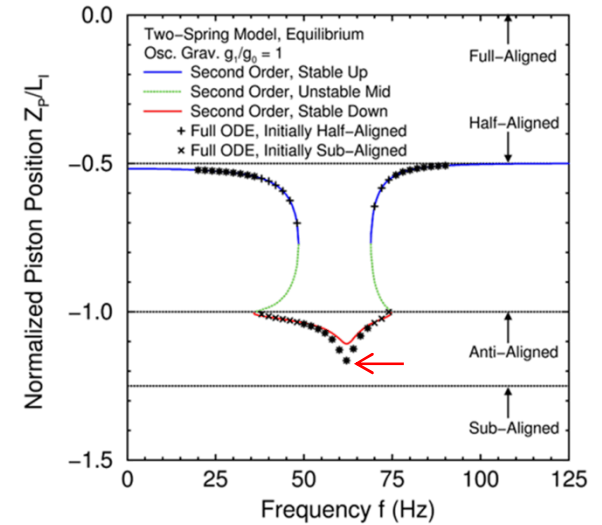
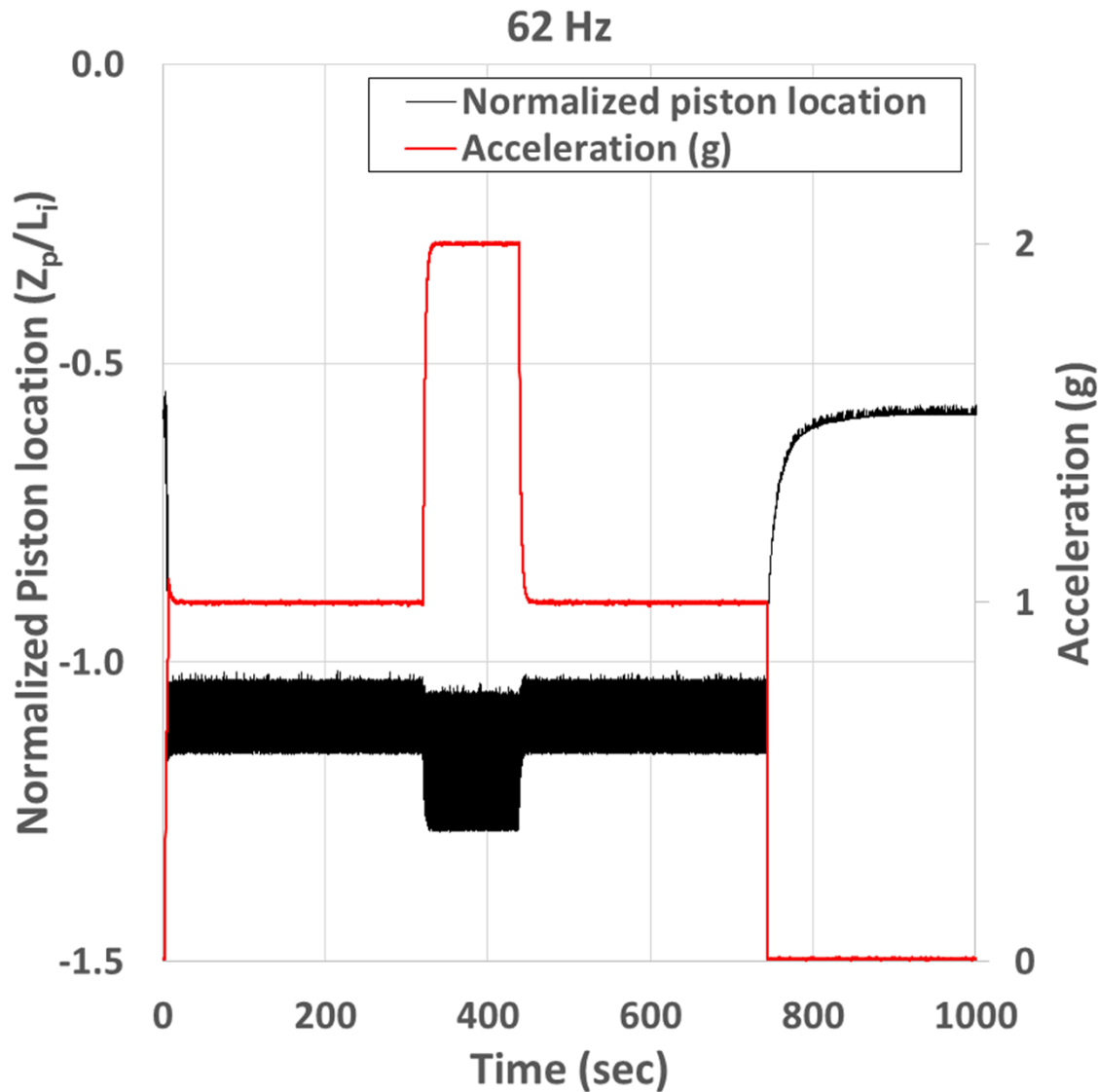
Two-Spring Experiment – Validation Data



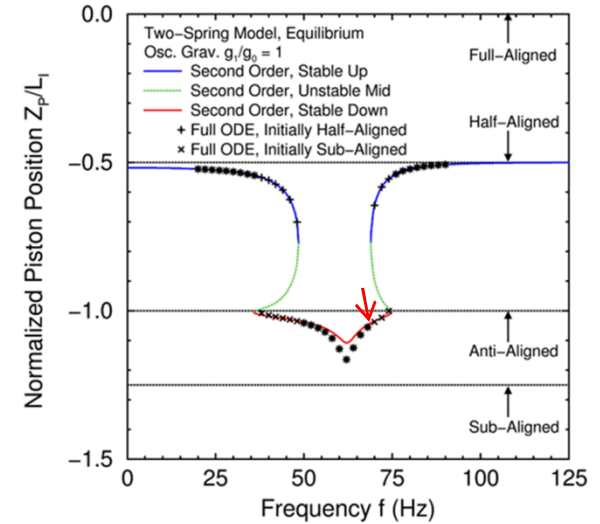
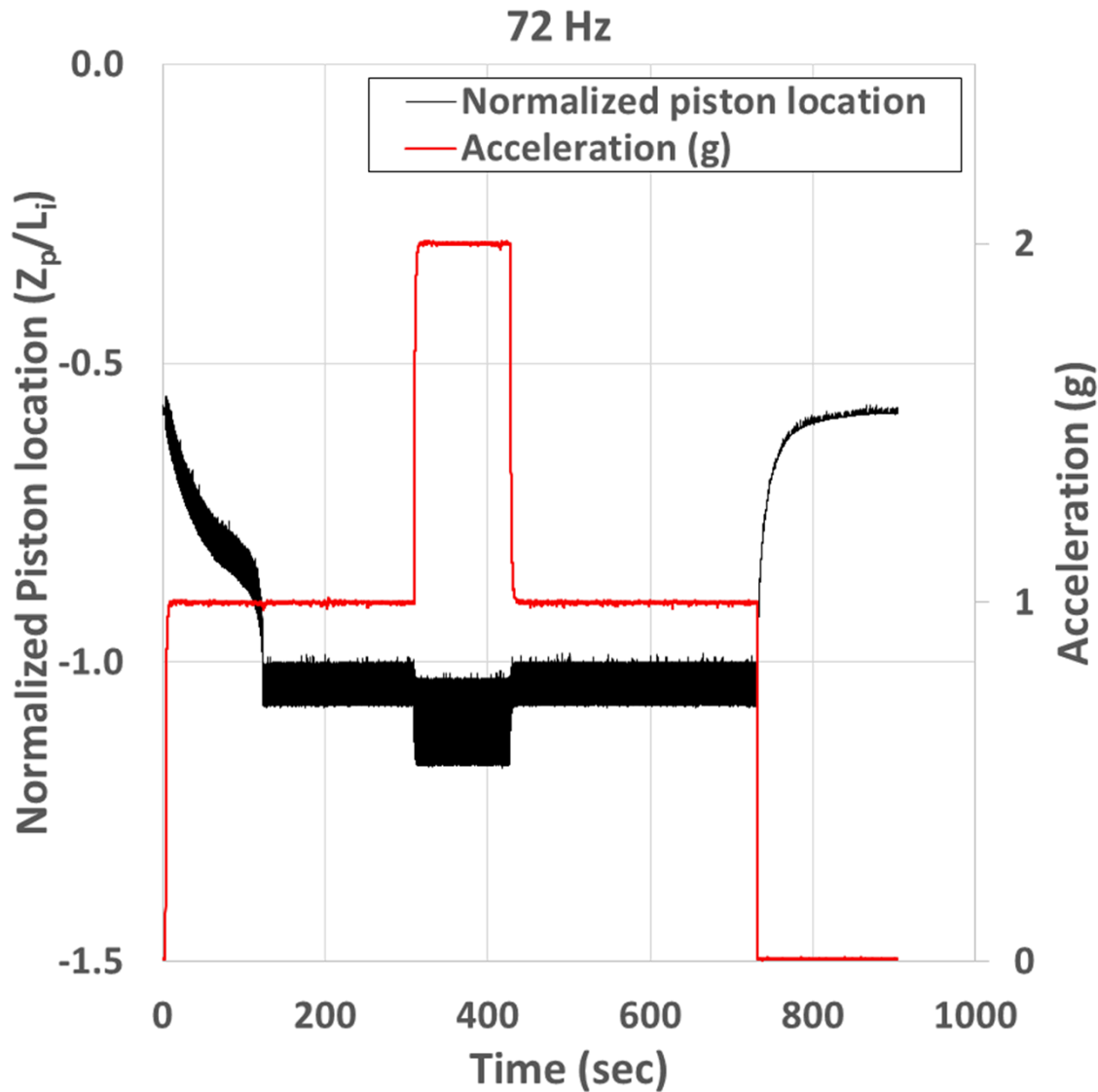
Two-Spring Experiment – Validation Data



Two-Spring Experiment – Validation Data

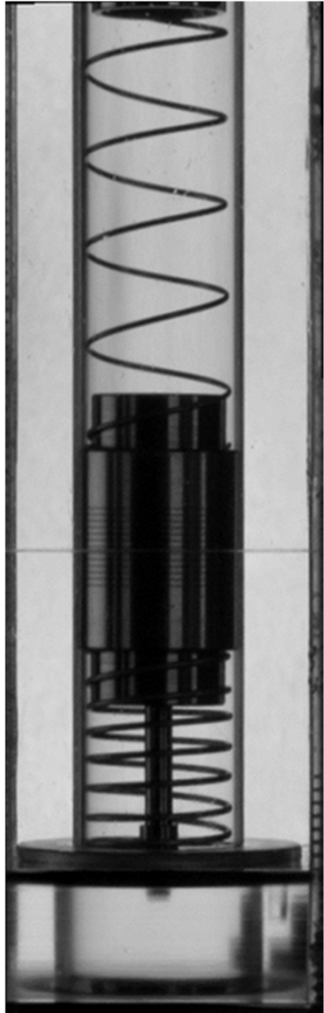


Two-Spring Experiment – Validation Data

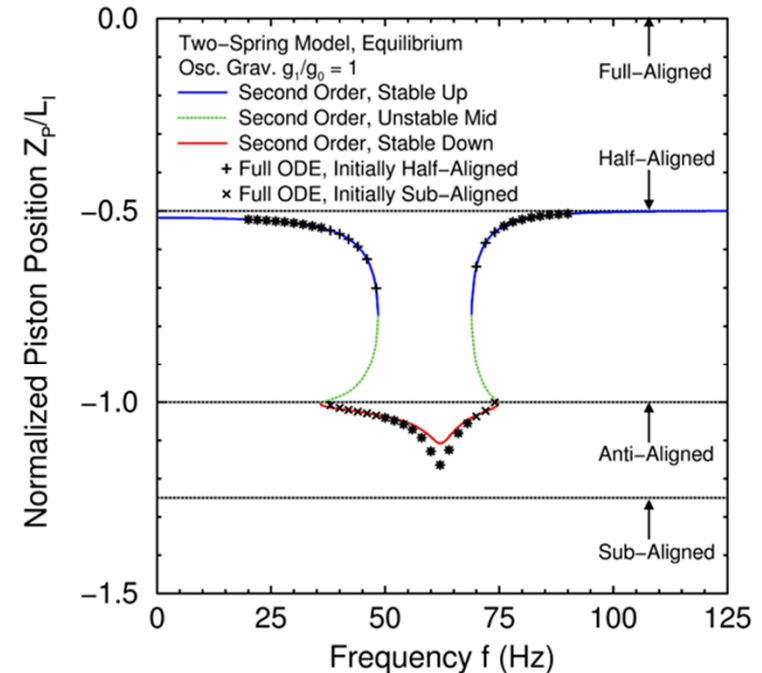
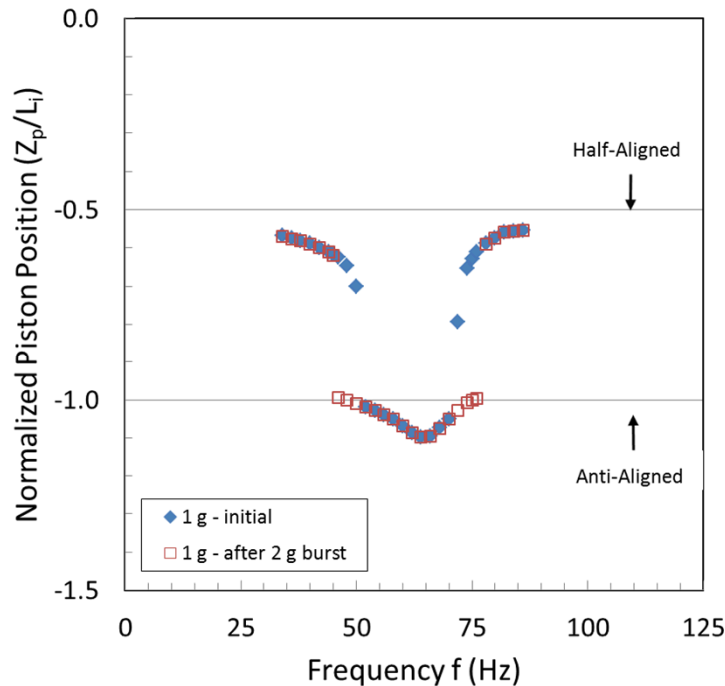


Two-Spring Experiment

Bellows



Bellows



Experiments and Models in good agreement

- Still need to run with real spring constants, bellows characteristics for direct comparison
- Much better experimental repeatability and agreement with theory without the stops

Conclusions and Future Work

gas above + vibration = gas below



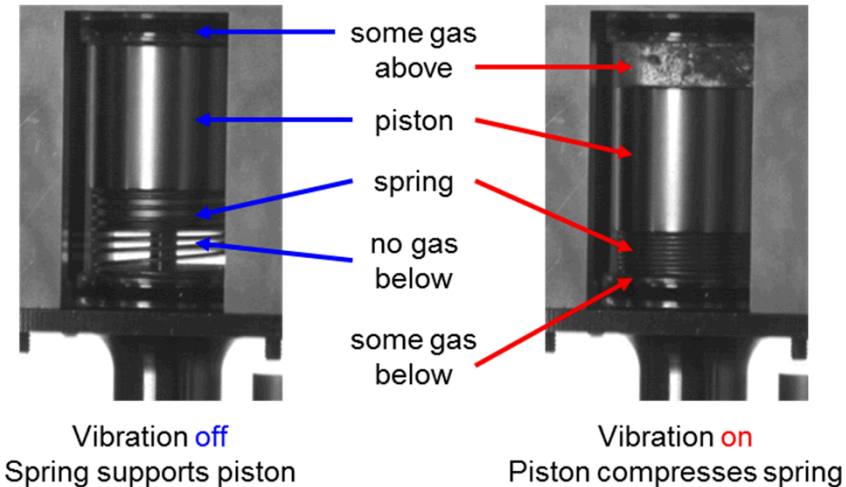
gas above + gas below = gas spring



gas spring + piston mass = resonance



resonance + gap nonlinearity = net motion



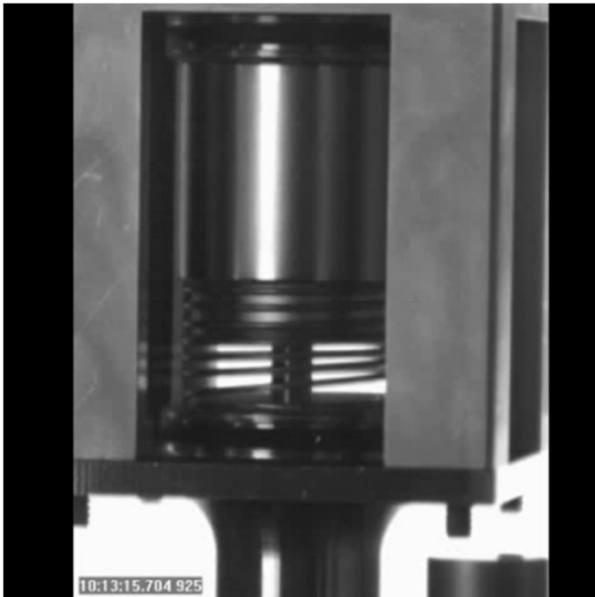
Gas changes how an immersed body responds to vibration

- Clear physical picture of route to net motion (rectification)
- Couette mode dominates Poiseuille mode in gap-controlled case of interest
- Good agreement between theory & simulation (bellows)

Much work remains to be done for complete understanding

- Compare theory, simulation, and experiments in detail
- Investigate effects of friction and contact forces (at stops in experiment)
- Study how gas divides between upper & lower regions

“Gas-Enabled Resonance and Rectified Motion of a Piston in a Vibrated Housing Filled with a Viscous Liquid,” (2016), Romero, L. A., Torczynski, J. R., Clausen, J. R., O’Hern, T. J., Benavides, G. L., *ASME Journal of Fluids Engineering*, 138(6), 554-573.



Questions?

- Experiments, analysis, and simulations are performed in order to predict the behavior of a multiphase spring-mass-damper system subjected to vibration. The geometry of interest is a spring-supported piston that fits closely within a vibrated liquid-filled cylinder, where the damping caused by forcing liquid through narrow gaps depends almost linearly on the piston position. Work to date has demonstrated that the addition of a little gas to this otherwise liquid-filled system completely changes its dynamics. When no gas is present, the piston's vibrational response is highly overdamped due to the viscous liquid being forced through the narrow gaps. When a small amount of gas is added, Bjerknes forces cause some of the gas to migrate below the piston. The resulting pneumatic spring enables the liquid to move with the piston so that little liquid is forced through the gaps. This "Couette mode" thus has low damping and a strong resonance near the frequency given by the pneumatic spring constant and the piston mass. Near this frequency, the piston response is large, and the nonlinearity from the varying gap length produces a net force on the piston. This "rectified" force can be many times the piston's weight and can cause the piston to move downward, compressing its supporting spring.
- A surrogate system in which the gas regions are replaced by upper and lower bellows with similar compressibility is studied. A theory for the piston and bellows motions is compared to finite element simulations. The liquid obeys the unsteady incompressible Navier-Stokes equations, and the piston and the bellows obey Newton's 2nd Law. Due to the large piston displacements near resonance, an Arbitrary Lagrangian Eulerian (ALE) technique with a sliding-mesh scheme is used to limit mesh distortion. Theory and simulation results for the piston motion are in good agreement.
- Experiments are performed with upper and lower bellows of appropriate characteristics replacing the gas. Liquid viscosity, bellows compressibility, vibration amplitude, and gap geometry are varied to determine their effects on the frequency at which the rectified force is sufficient to move the piston downward. This critical frequency is found to depend on, and whether the experiment is run by varying vibration acceleration amplitude at fixed frequency, by varying frequency at fixed acceleration amplitude, and whether the frequency is increased or decreased with time.
- While the theory and simulations are in good agreement, the experiments show systematic differences due to additional damping and end effects on the piston.