



Nonlinear Dynamics of a Spring-Supported Piston in a Vibrated Liquid-Filled Housing:

II. Experiments

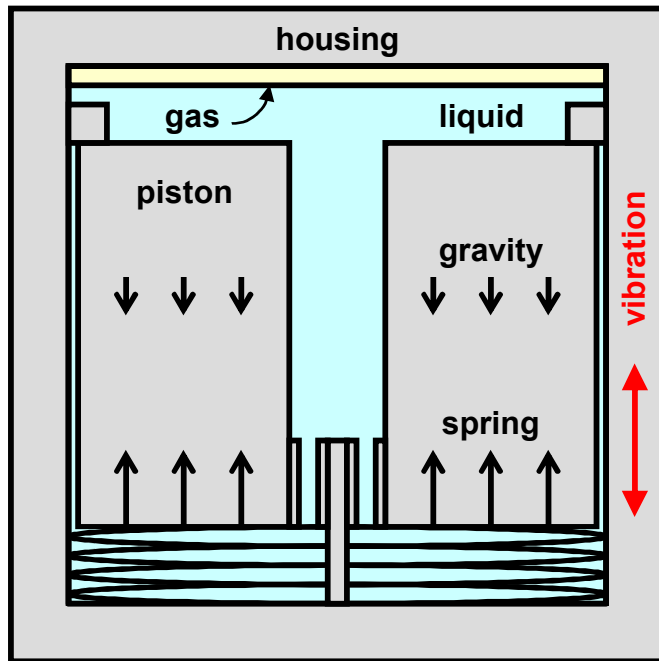
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***DFD16, American Physical Society Division of Fluid Dynamics
69th Annual Meeting; Portland, Oregon; November 20-22, 2016***

**The authors wish to thank Louis A. Romero and Gilbert L. Benavides, now
retired from Sandia National Laboratories, for many helpful interactions.**

Strange Vibration-Induced Dynamics

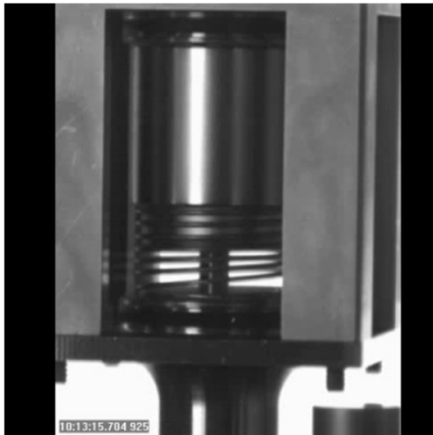


Spring-mass-damper system

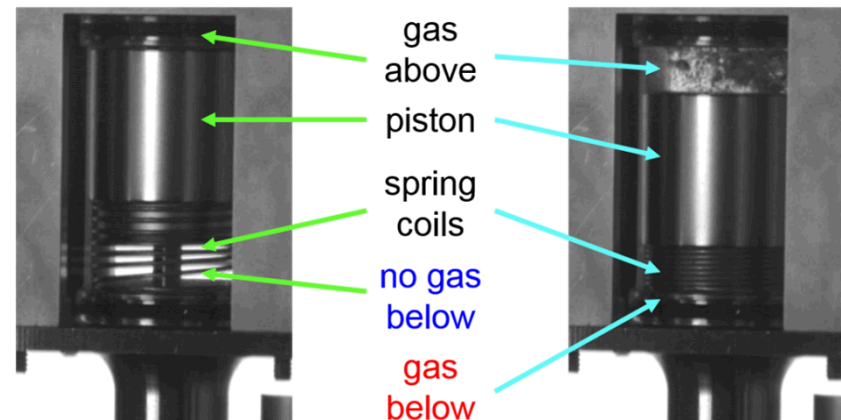
- Piston moves vertically in housing
- Spring supports it against gravity
- Viscous liquid provides damping
- Small amount of gas is present

Housing is vibrated vertically

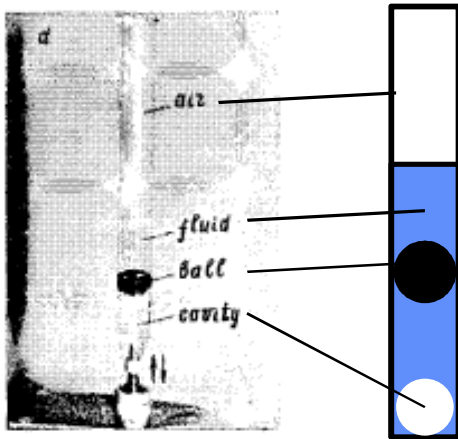
- Gas moves down below piston
- Piston moves down against spring



Spring supports piston
when vibration is **off**



Piston compresses spring
when vibration is **on**



Chelomey (1984)¹

- Heavy ball resting at bottom of liquid-filled tube
- Vibrating the tube causes the heavy ball to rise vertically in the tube
- Ball reaches a stable state with air cavity visible at the base of the tube

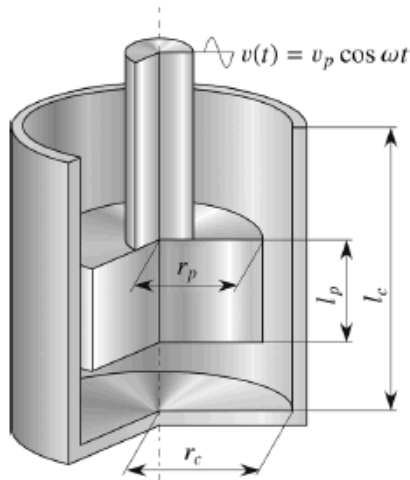


Fig. 3 Cross-sectional view of an oil damper

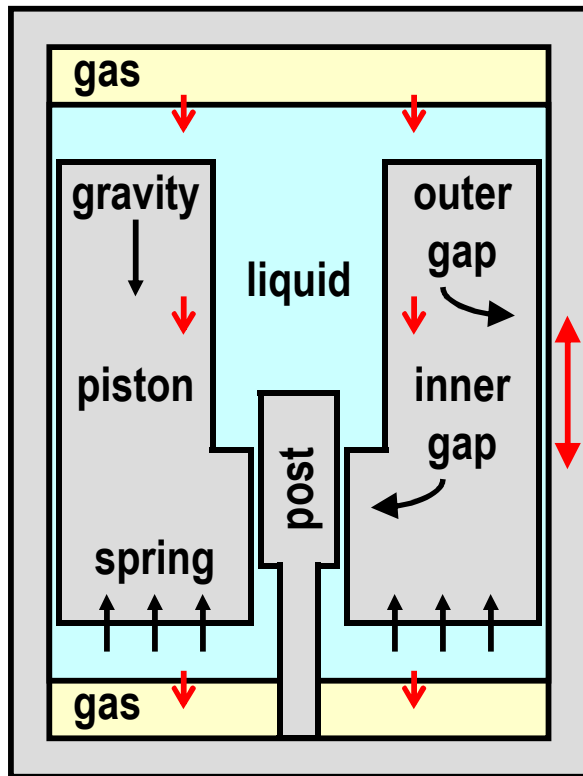
Asami et al. (2014)²

- Numerically and experimentally studied flow field around moving piston in damper
- Measured flow field above and below driven, oscillating piston
- Not multiphase, but dynamic system under vibration

1. Chelomey, V., (1985), "Paradoxes in Mechanics Caused by Vibrations," Meccanica., 20(4), pp. 314-316.

2. Asami, T., Honda, I., and Ueyama, A. (2014), "Numerical Analysis of the Internal Flow in an Annular Flow Channel Type Oil Damper," J. Fluids Engineering, 136.

Vibration Makes Piston Move Down



Couette flow in gaps

Gas regions form pneumatic spring

- One expands, the other contracts
- Stiffness is $\sim 10^2$ of helical spring

Enables low damping Couette mode

- Piston and interfaces move together
- No liquid is forced through inner gap

Low damping gives strong resonance

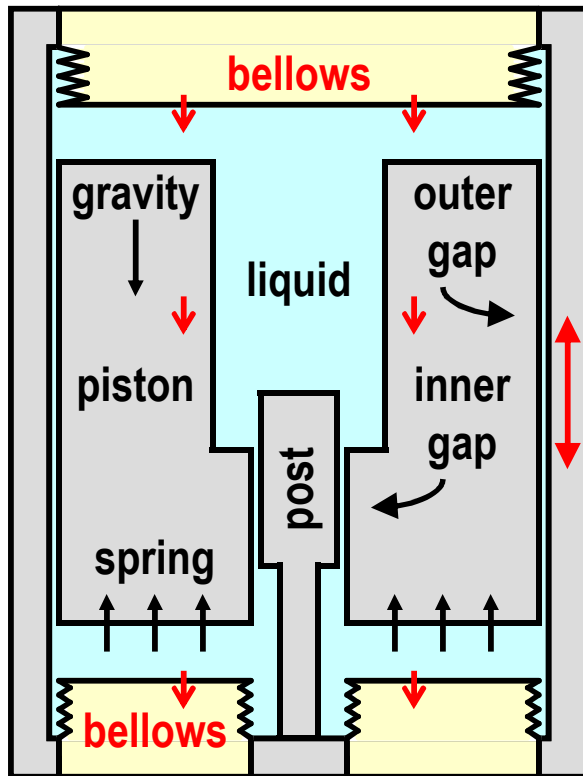
- Piston + liquid mass and gas spring

Gap nonlinearity produces net force

- Damping depends on piston position
- Moves piston down to shorten gap

$$\omega = \sqrt{\frac{K_{\text{gas}}}{M_{\text{total}}}} \quad \text{has very low damping}$$

Analyze Analogous Bellows System



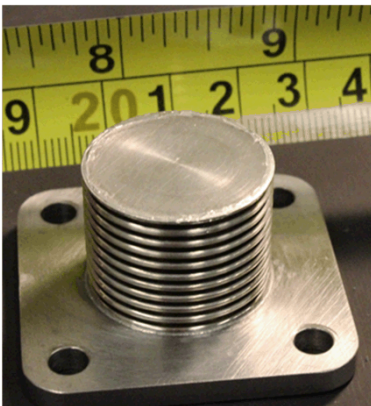
Gas regions are hard to analyze

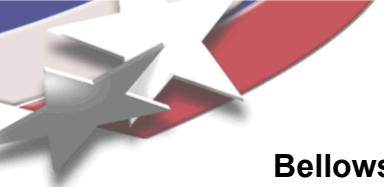
- Upper/lower split of gas is not known
- Motion is transient and complicated

So replace gas regions with bellows

- Compressibility is well characterized
- Choose pressure-volume properties to be similar to gas regions

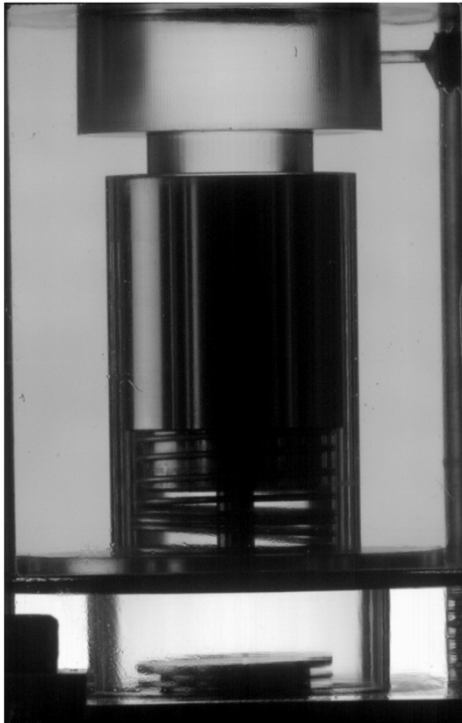
System suited to experiments, theory, and simulation





Experiments with Bellows

Bellows



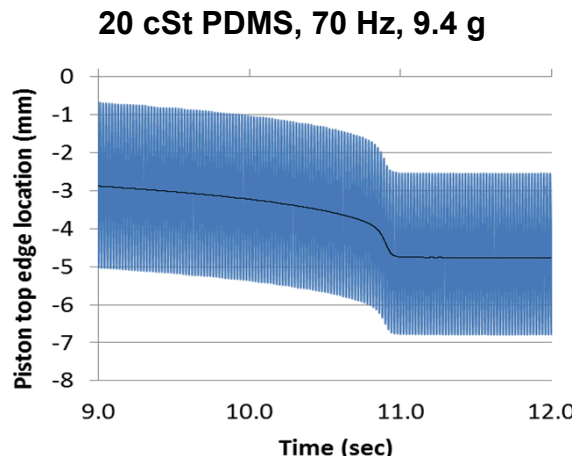
Bellows

Different bellows sizes chosen to act as bubbles of typical sizes

- Commercial Servometer bellows
 - $d = 2.5$ cm and $d = 1.9$ cm bellows span expected compressibility range
 - Control experiments without bellows
 - No bellows
 - Single bellows
- } no piston motion

Vibration conditions

- Commercial Labworks shaker/LabVIEW
- Frequency: 40-200 Hz
- Acceleration: up to 30 g (30×9.81 m/s²)
- Peak-to-peak displacement: 0-2 mm



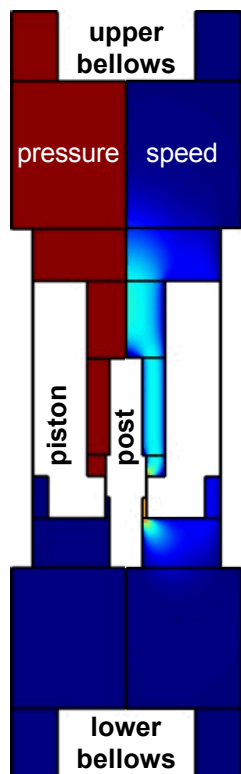


Theoretical Analysis

Theory yields 2-DOF nonlinear damped harmonic oscillator

- Newton's 2nd Law for solids, steady Stokes for liquid force
- Liquid damping & added mass depend on piston position

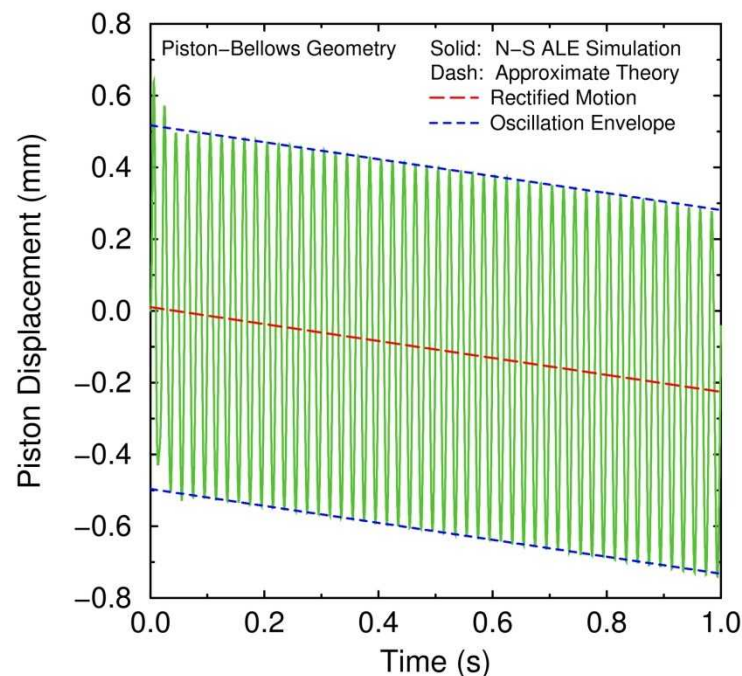
Excellent agreement with simulations for oscillation amplitude and drift



Navier-Stokes Eqns.
Newton's 2nd Law

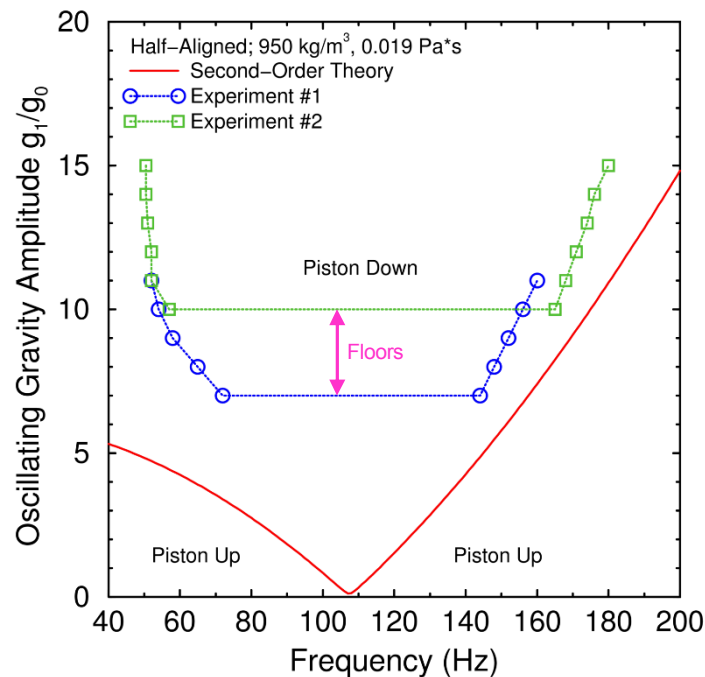
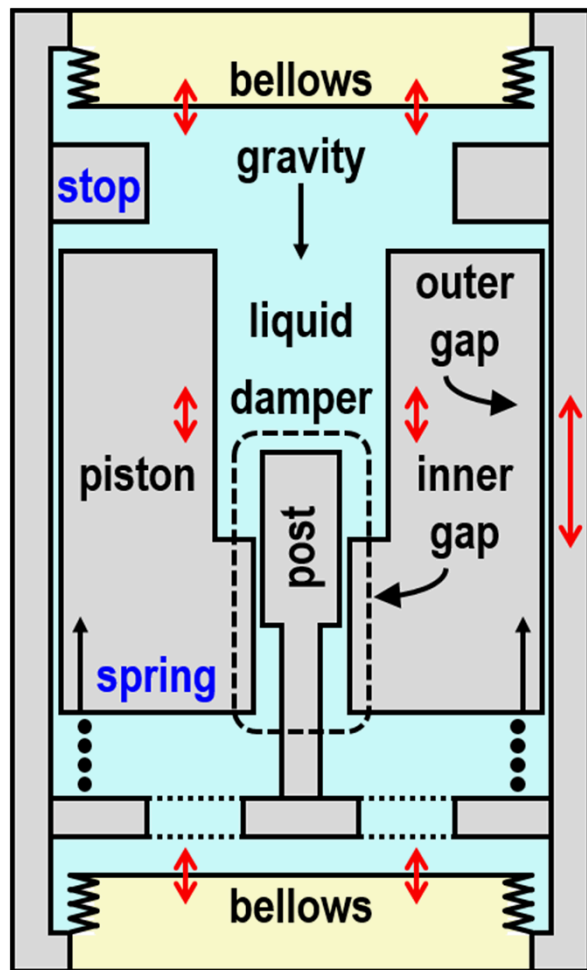
$$\lambda = \frac{A_B}{A_p + (A_G/2)}$$
$$F_{\text{rect}} = \lambda \frac{d\beta_{11}}{dz_1} \langle z_1 \dot{z}_2 \rangle$$
$$U_{\text{rect}} = \frac{F_{\text{rect}}}{\beta_{11}} \text{ (drift)}$$

liquid added masses		liquid damping coefficients		gravity-buoyancy
$(\tilde{\mathbf{M}} + \mathbf{M}) \ddot{\mathbf{Z}}$		$(\tilde{\mathbf{B}} + \mathbf{B}) \dot{\mathbf{Z}}$		$\tilde{\mathbf{K}} \mathbf{Z} = \mathbf{f}$
object masses	object damping coefficients	object spring constants		
$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}_i = 0, \quad \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma}_i = 0$	$\mathbf{Z} = \begin{pmatrix} Z_p \\ Z_B \end{pmatrix}$	$\mathbf{f} = - \begin{pmatrix} M_{PG} \\ M_{BG} \end{pmatrix} g_r \cos \omega t$		
$\mathbf{u}_i = \begin{cases} U \hat{\mathbf{e}}_z \text{ on } S_i \\ 0 \text{ on other walls} \end{cases}$	$\mathbf{B} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}$	$\tilde{\mathbf{K}} = \begin{pmatrix} K_p & 0 \\ 0 & K_B \end{pmatrix}$		
$S_i = \frac{1}{2} \left(\frac{\partial \mathbf{u}_i}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}_i^T}{\partial \mathbf{x}} \right)$	$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$	$\tilde{\mathbf{B}} = \begin{pmatrix} B_p & 0 \\ 0 & B_B \end{pmatrix}$		
	$m_{ij} = \frac{\rho}{U^2} \int_V \mathbf{u}_i \cdot \mathbf{u}_j dV$	$\tilde{\mathbf{M}} = \begin{pmatrix} M_p & 0 \\ 0 & M_B \end{pmatrix}$		



Theory and Simulation

Experiments Differ Significantly



Regime map: amplitude vs. frequency

- Rectified force equals spring preload

Experiments have amplitude floors

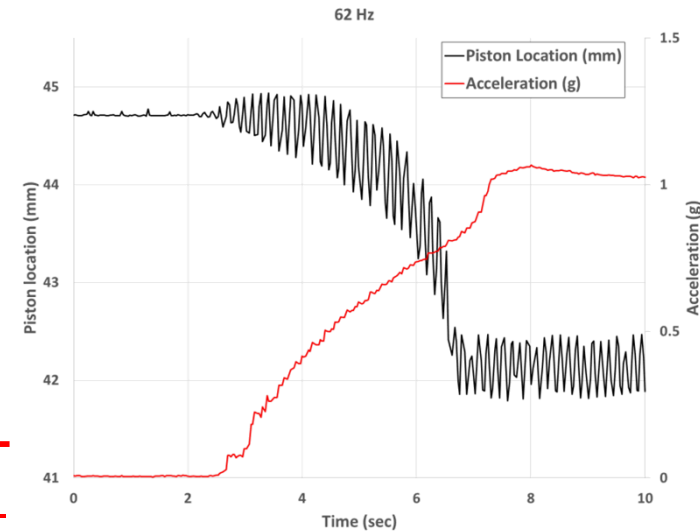
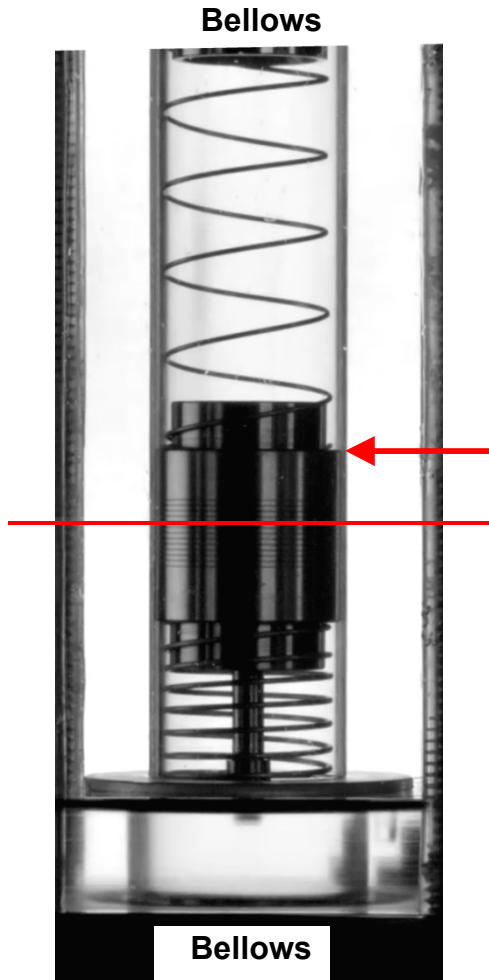
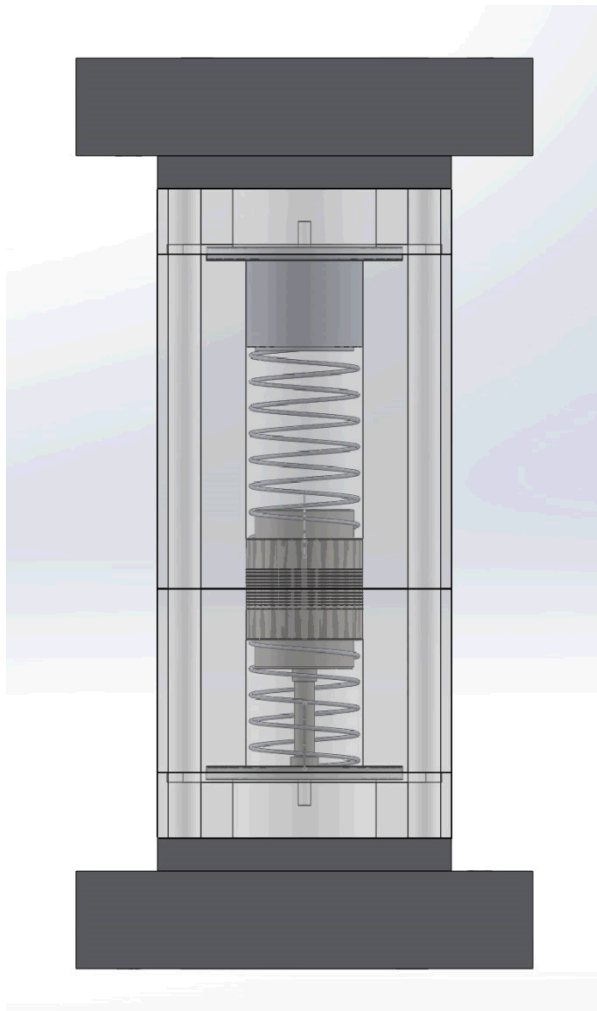
- Not reproducible from day to day

Stop may be causing the difference

Two-Spring Experiment

Replace upper piston stop with a second spring

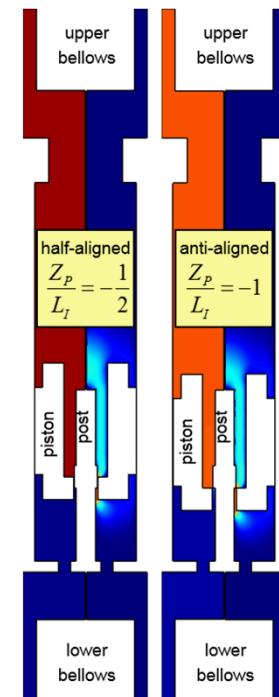
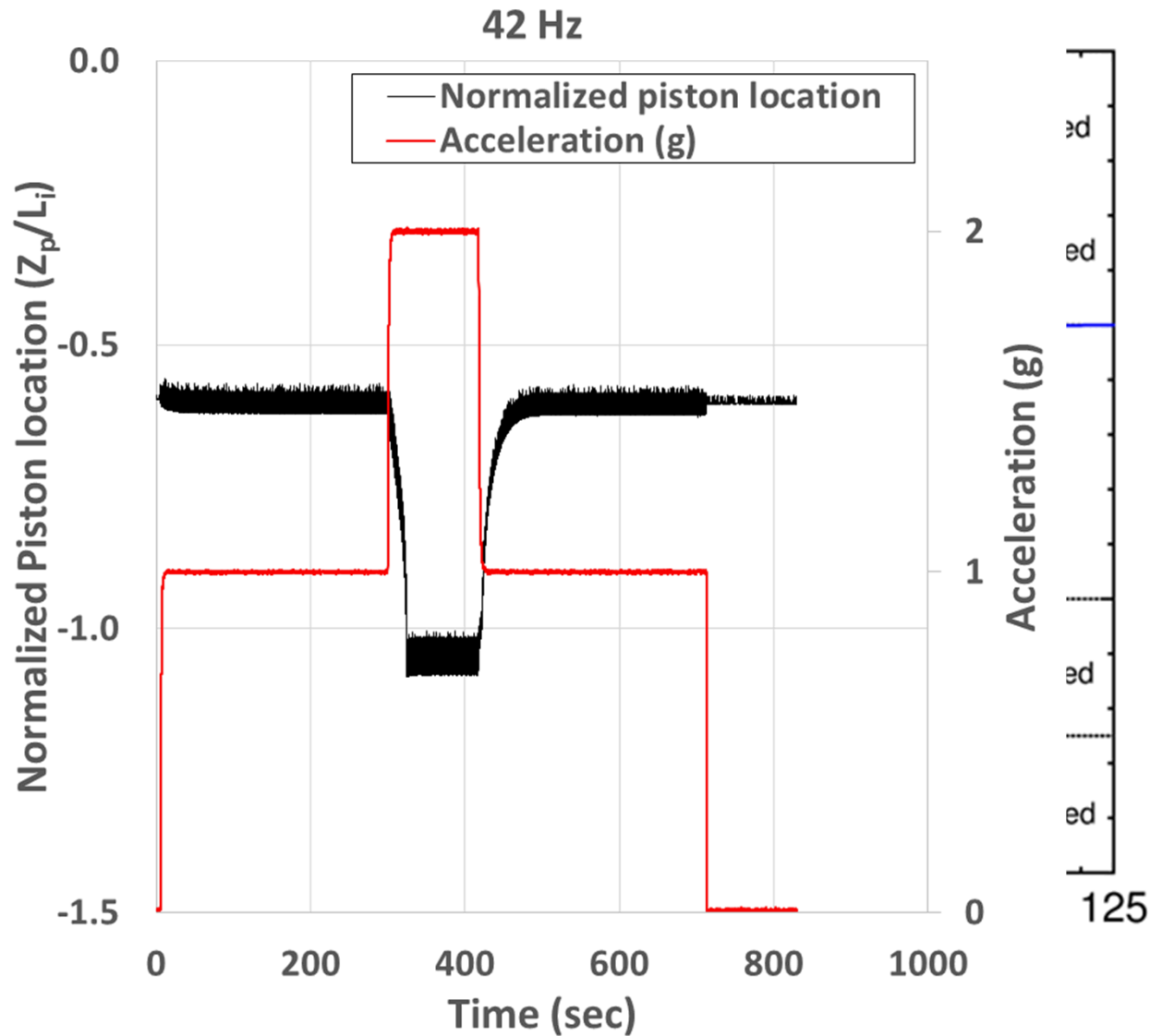
- Piston does not have to pull away from stop to start motion



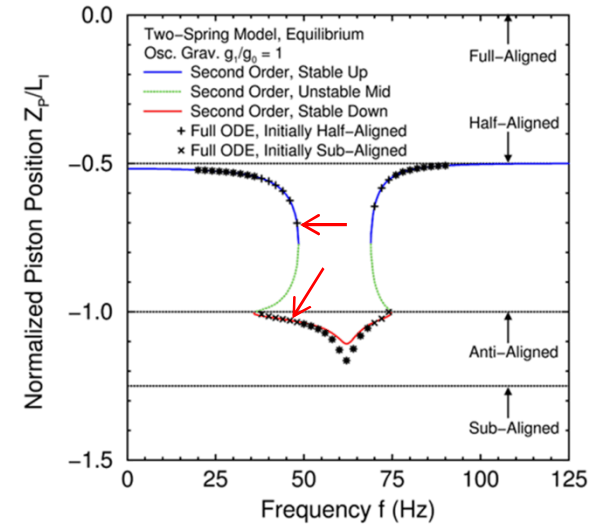
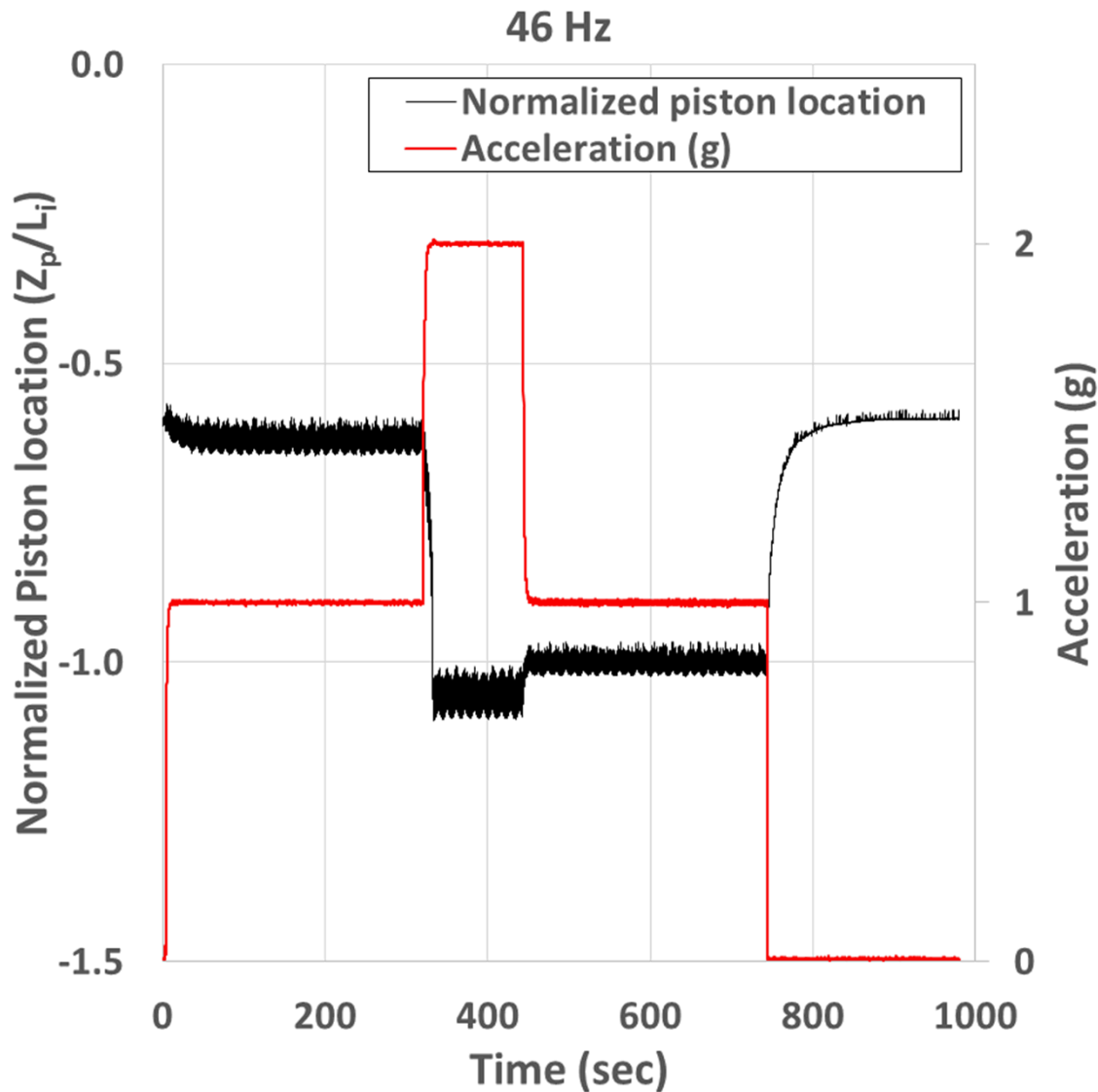
- Piston motion requires lower acceleration
 - (e.g., 1 g vs. 20 g)
- Motion much more repeatable

62 Hz, 1.0 g, 20 cSt PDMS, 1" bellows

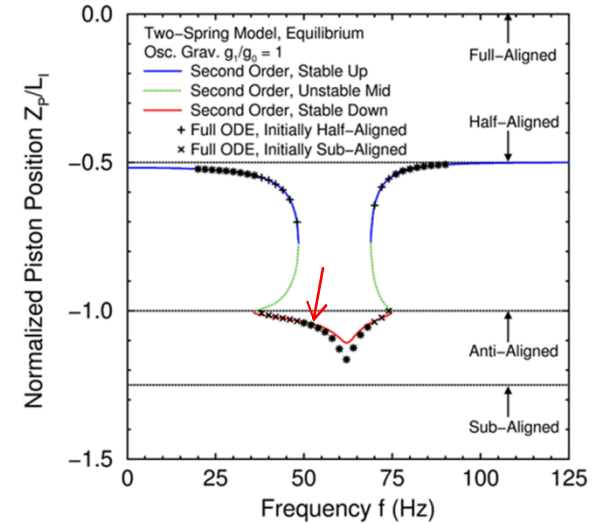
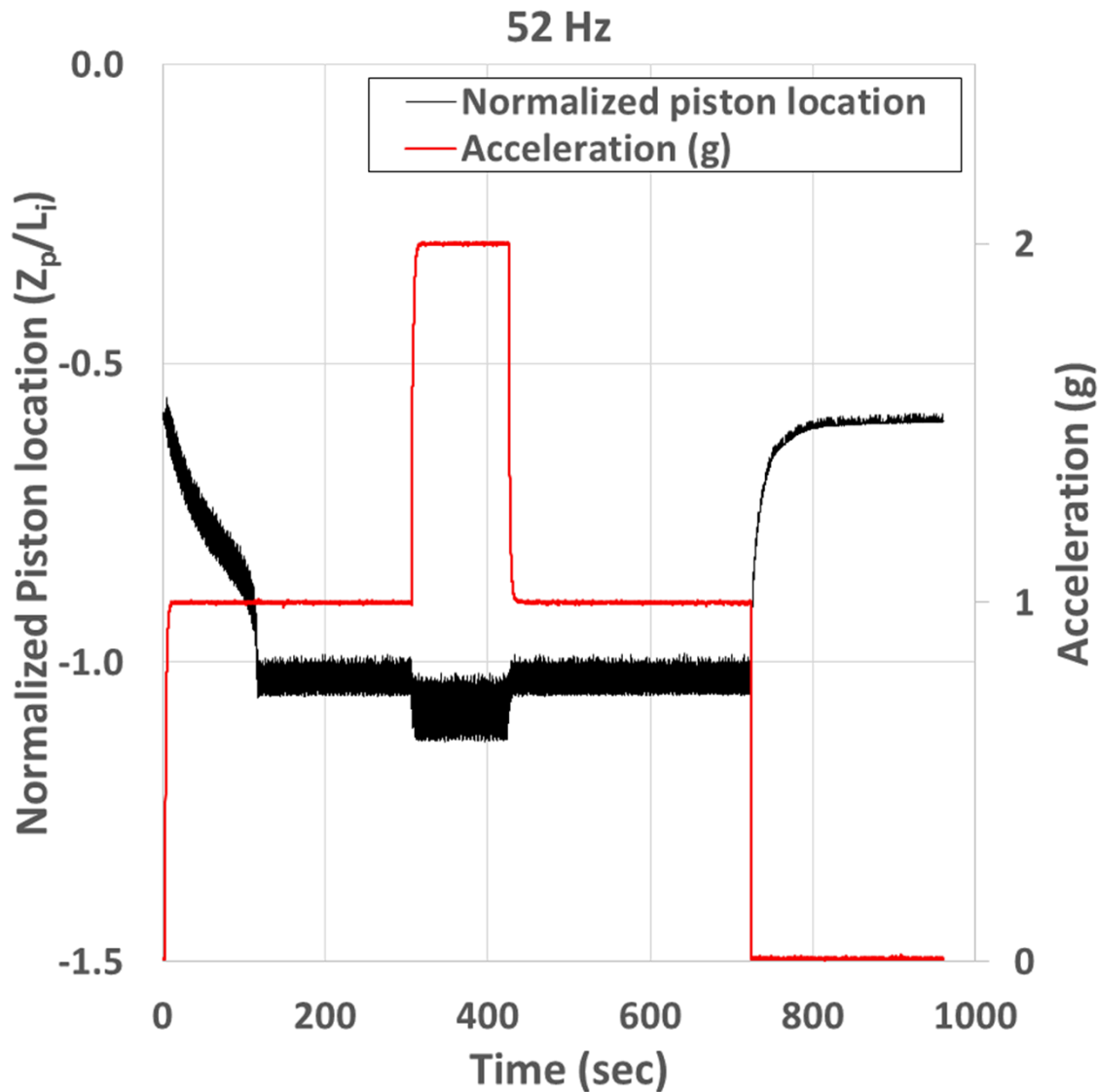
Two-Spring Experiment – Validation Data



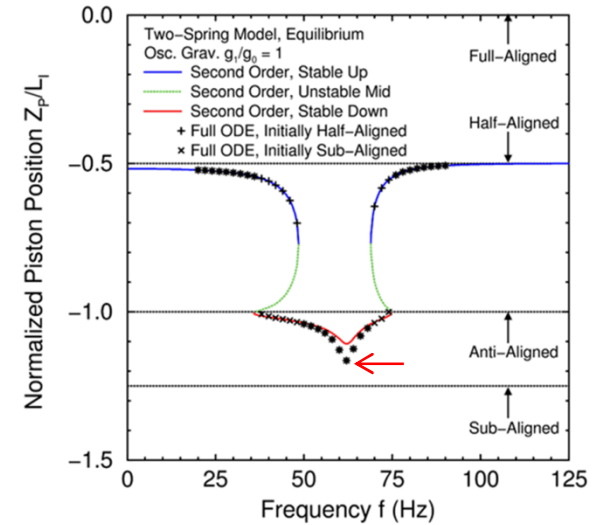
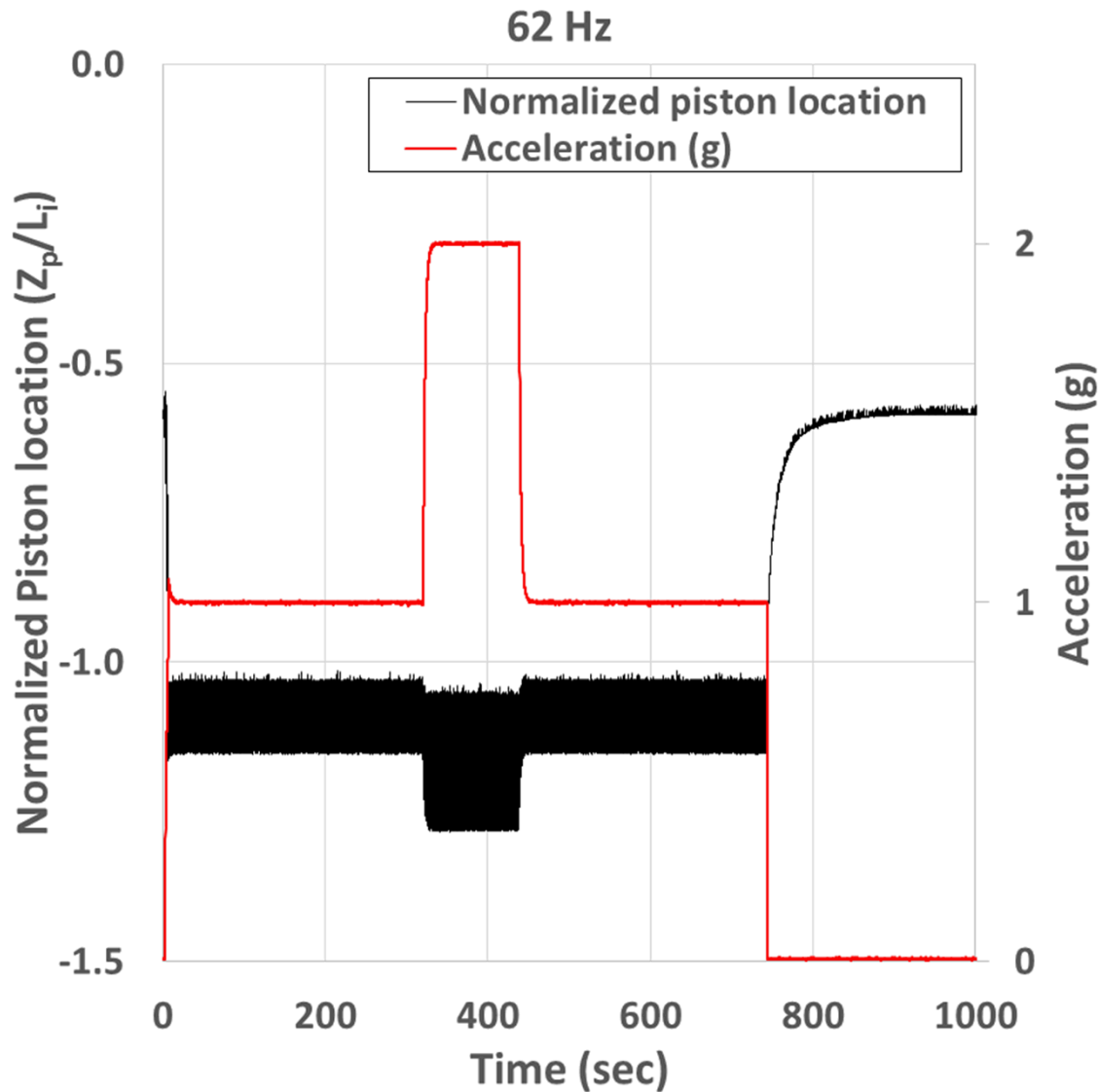
Two-Spring Experiment – Validation Data



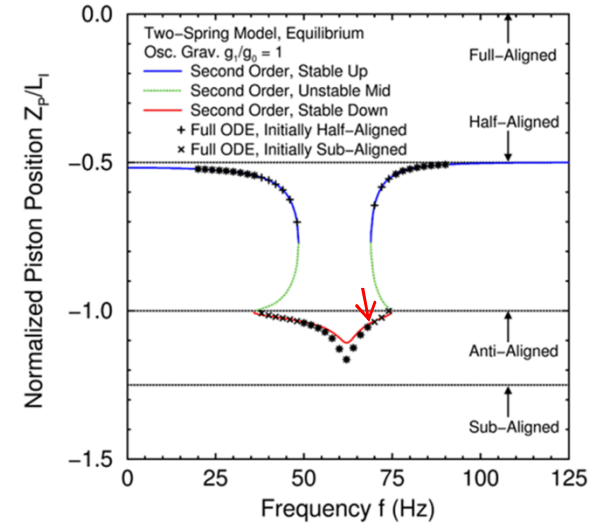
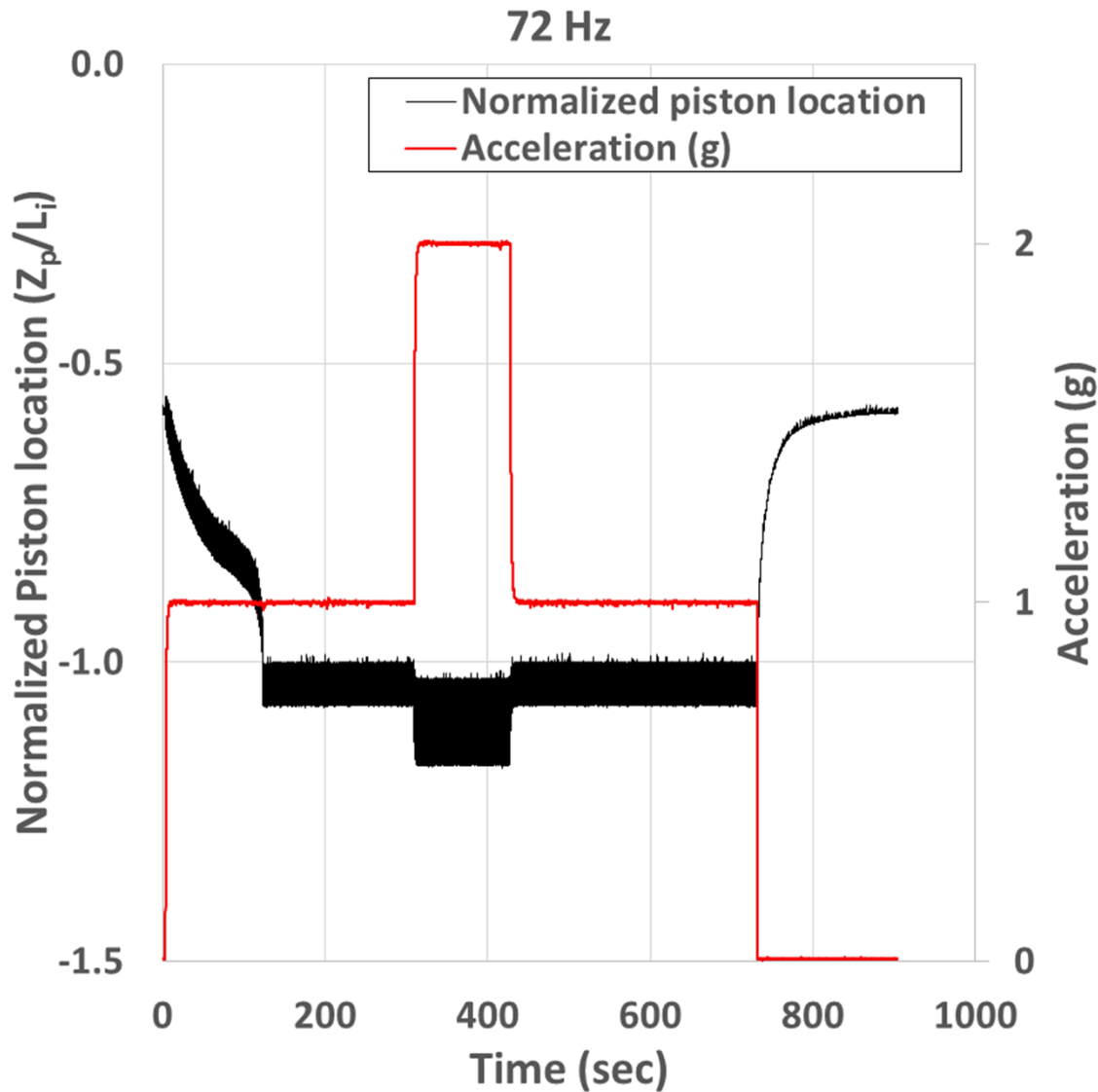
Two-Spring Experiment – Validation Data



Two-Spring Experiment – Validation Data

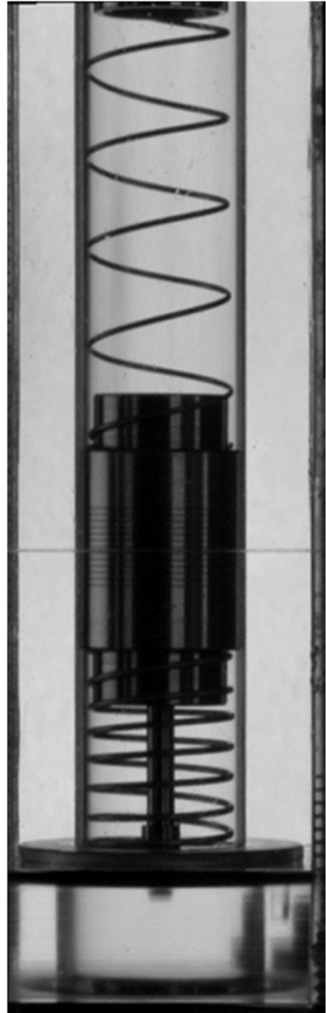


Two-Spring Experiment – Validation Data

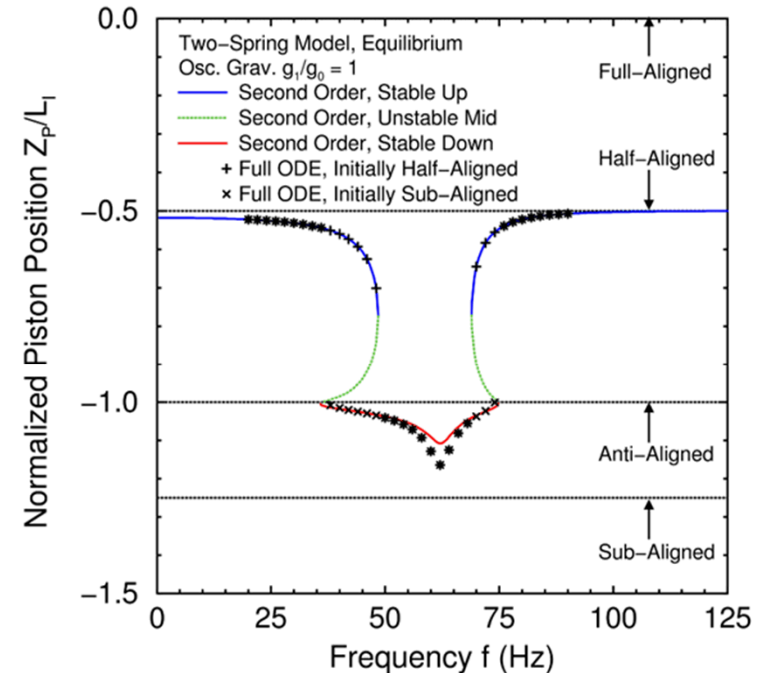
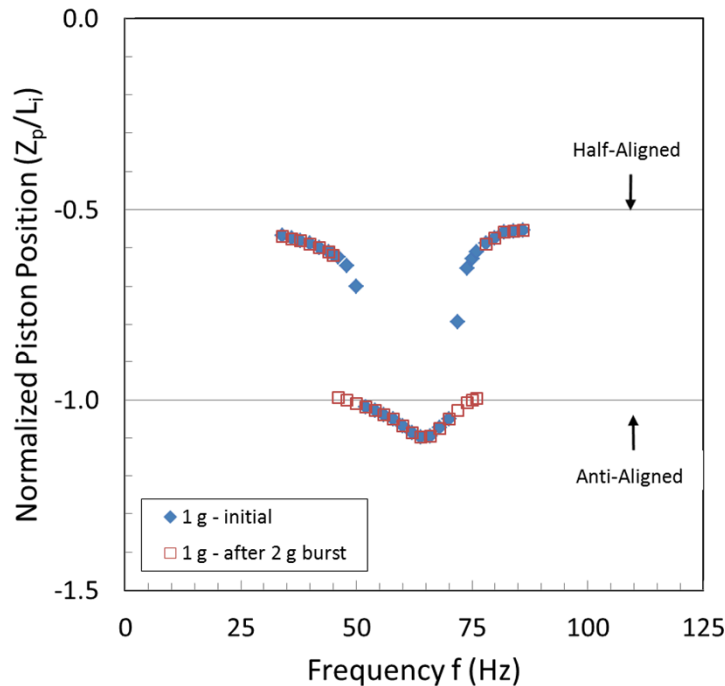


Two-Spring Experiment – Validation Data

Bellows



Bellows



Experiments and Models in good agreement

- Still need to run with real spring constants, bellows characteristics for direct comparison
- Much better experimental repeatability and agreement with theory without the stops

Conclusions and Future Work

gas above + vibration = gas below



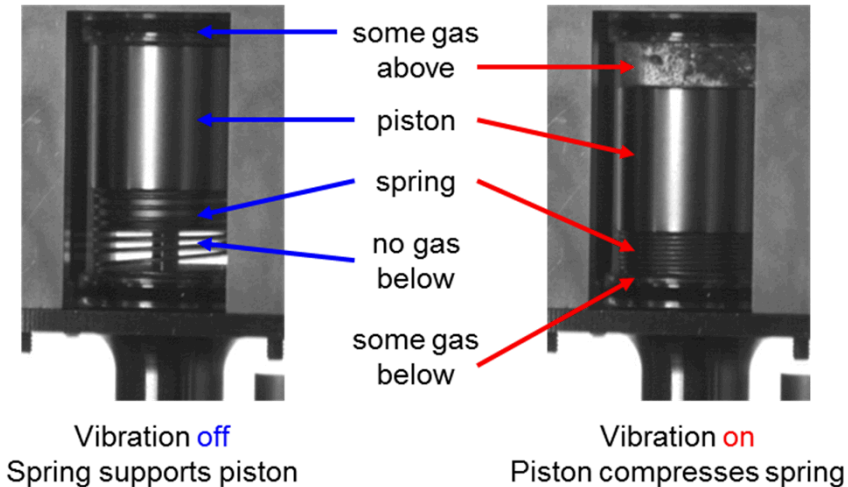
gas above + gas below = gas spring



gas spring + piston mass = resonance



resonance + gap nonlinearity = net motion



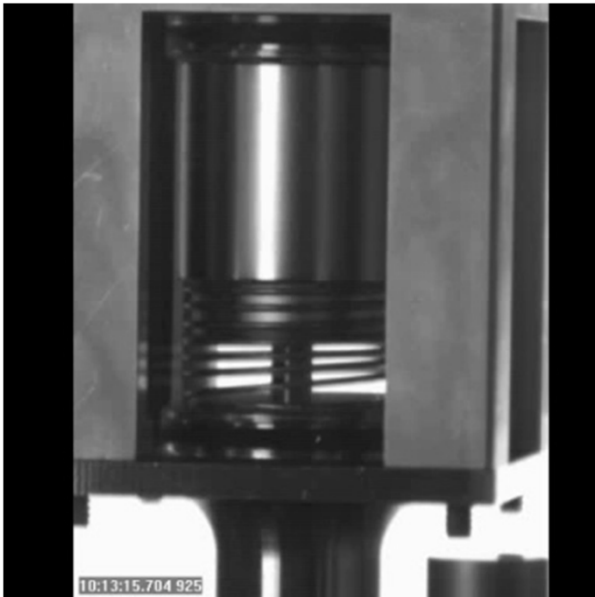
Gas changes how an immersed body responds to vibration

- Clear physical picture of route to net motion (rectification)
- Couette mode dominates Poiseuille mode in gap-controlled case of interest
- Good agreement between theory & simulation (bellows)

Much work remains to be done for complete understanding

- Compare theory, simulation, and experiments in detail
- Investigate effects of friction and contact forces (at stops in experiment)
- Study how gas divides between upper & lower regions

“Gas-Enabled Resonance and Rectified Motion of a Piston in a Vibrated Housing Filled with a Viscous Liquid,” (2016), Romero, L. A., Torczynski, J. R., Clausen, J. R., O’Hern, T. J., Benavides, G. L., *ASME Journal of Fluids Engineering*, 138(6), 554-573.



Questions?

The nonlinear dynamics of a piston supported by a spring in a vibrated liquid-filled housing is investigated experimentally. The housing containing the piston and the liquid is subjected to vibrations along its axis. A post fixed to the housing penetrates a hole through the piston and produces a flow resistance that depends on piston position. Flexible bellows attached to the housing ends enable the piston, liquid, and bellows to execute a collective motion that forces little liquid through the flow resistance. The low damping of this motion leads to a resonance, at which the flow-resistance nonlinearity produces a net force on the piston that can cause it to compress its spring. Experiments are performed to investigate the nonlinear dynamics of this system, and these results are compared to theoretical and numerical results.