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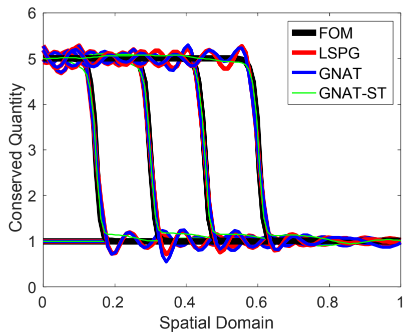

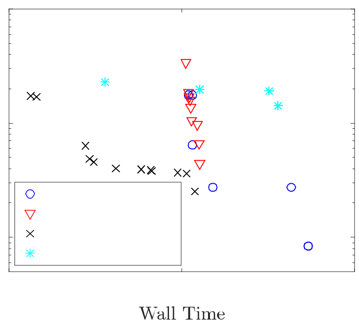

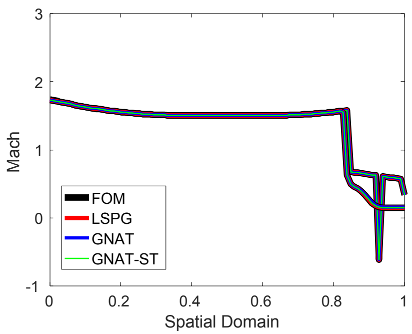

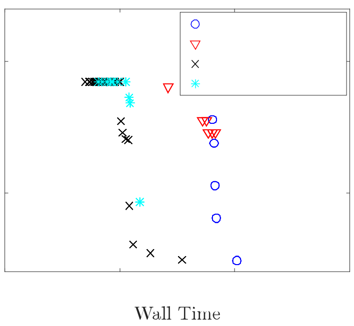

Space–time least-squares Petrov–Galerkin projection in nonlinear model reduction

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Motivation

- Typical ROMs apply spatial projection
  - \+ Reduces spatial complexity
  - \- Does not reduce temporal complexity (number of time steps remains large)
- Larger time steps with explicit time integration [Krysl et al., 2001]
  - \+ Larger stable time steps achievable with ROMs
  - \- Speedup limited by stability
  - \- Not applicable to stiff dynamics
- Space–time reduced-basis [Urban/Patera 2012], [Yano 2014], [Yano/Patera/Urban 2014]
  - \+ Dimensionality reduction in both space and time
  - \+ Error bounds grow linearly in time
  - \- *Not always practical*: requires space–time discretization in full-order model
- Forcasting with time-domain data [Carlberg/Ray/van Bloemen Waanders 2015], [Carlberg/Brencher/Haasdonk/Barth 2016]
  - \+ *Practical:* does not require space-time discretization in full-order model
  - \- *Limited reduction*: no temporal projection pursued

Proposed method

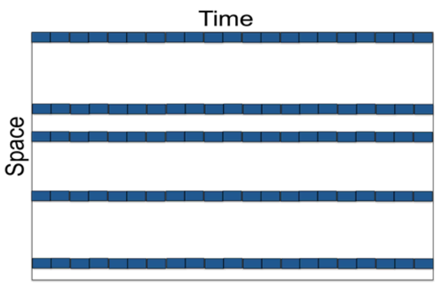
Spatial least-squares Petrov–Galerkin

$$\tilde{\boldsymbol{w}}(t^n) = \boldsymbol{w}_0 + \boldsymbol{V}\hat{\boldsymbol{w}}(t^n)$$

$$\hat{\boldsymbol{w}}^n = \arg\min_{\boldsymbol{y}} \|\boldsymbol{G}\boldsymbol{r}^n(\boldsymbol{w}_0 + \boldsymbol{V}\boldsymbol{y})\|_2^2$$

- LSPG: $\boldsymbol{G} = \boldsymbol{I}$
- GNAT: $\boldsymbol{G} = (\boldsymbol{Z}\boldsymbol{\Phi}_r)^\dagger \boldsymbol{Z}$

$\boldsymbol{Z} =$



Spatiotemporal least-squares Petrov–Galerkin

$$\tilde{\boldsymbol{w}}(t^n) = \boldsymbol{w}_0 + \sum_{i=1}^{n_s}\sum_{j=1}^{n_t^i} \boldsymbol{v}_i u_{jn}^i \tilde{w}_{ij}$$

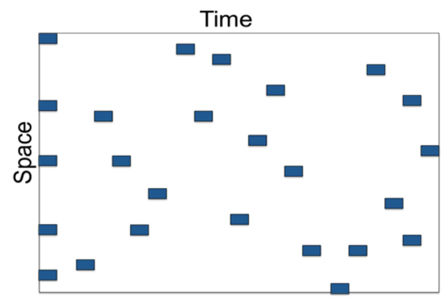
\+ Fewer degrees of freedom

$$\hat{\boldsymbol{w}}_{\mathrm{st}}^n = \arg\min_{\hat{\boldsymbol{y}}} \sum_{n=1}^{N_t} \|\bar{\boldsymbol{G}}^n \boldsymbol{r}^n(\boldsymbol{w}_0 + \sum_{i=1}^{n_s}\sum_{j=1}^{n_t^i} \boldsymbol{v}_i u_{jn}^i \hat{y}_{ij})\|_2^2$$

- LSPG: $\bar{\boldsymbol{G}}^n = \boldsymbol{I}$
- GNAT: $\bar{\boldsymbol{G}}^n =$ *n*th col block of $(\bar{\boldsymbol{Z}}\bar{\boldsymbol{\Phi}}_r)^\dagger \bar{\boldsymbol{Z}}$

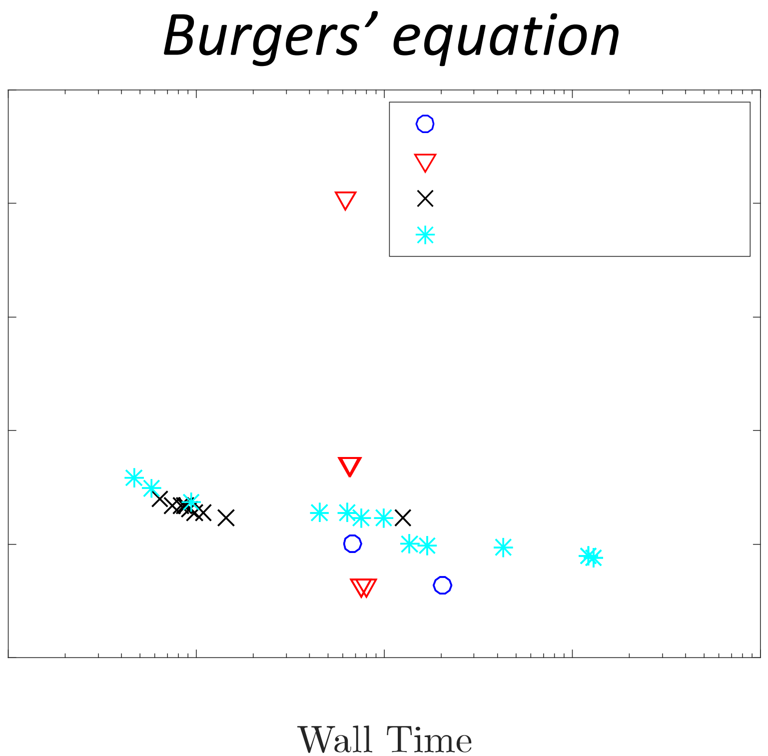
\+ Linear time-growth error bound

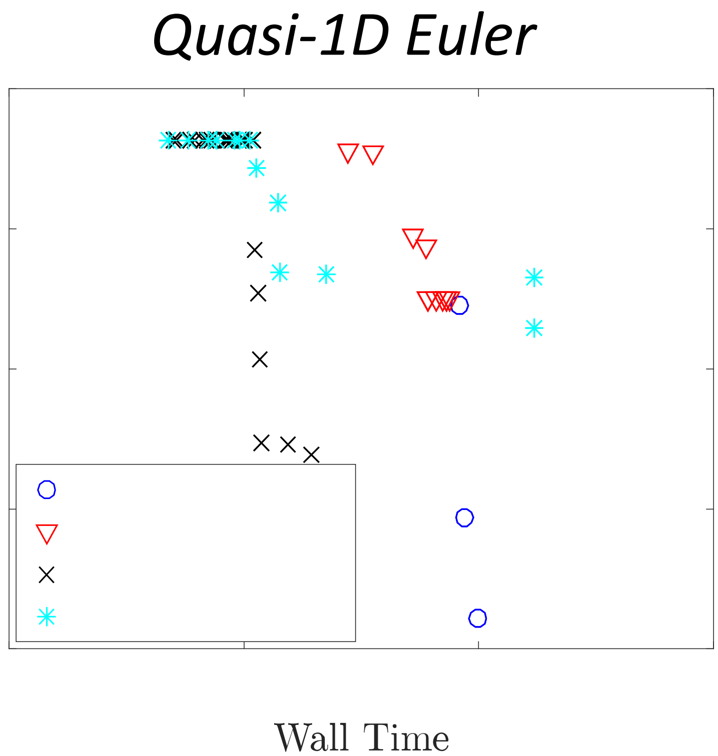
$\bar{\boldsymbol{Z}} =$



\+ Smaller complexity

Numerical results (1D Euler)





- Proposed method Pareto-optimal for small wall times
- Proposed method 100x faster than GNAT for fixed accuracy