SAND2016-10938C

Nonlinear Finite Element Model Updating, Part II: Implementation and Simulation

Brian C. Owens
Ryan A. Schultz
Benjamin R. Pacini
Randall L. Mayes
Structural Dynamics Department
Sandia National Laboratories[1]
P.O. Box 5800 - MS0557
Albuquerque, NM, 87185
bcowens@sandia.gov
rschult@sandia.gov
brpacin@sandia.gov
rlmayes@sandia.gov

Nomenclature

M	physical mass matrix
K	physical stiffness matrix
C	physical damping matrix
f_n	natural frequency in Hertz
f	frequency in Hertz
$F(t)$	physical force vector
Φ	mode shape matrix
ω	natural frequency in radians per second
ζ	modal damping ratio
q	modal displacement degree of freedom
$\dot{q}$	modal velocity degree of freedom
$\ddot{q}$	modal acceleration degree of freedom
t	time
$f(t)$	modal force vector
$f_{nl}(q,\dot{q})$	nonlinear restoring force vector
x	physical displacement degree of freedom
$\dot{x}$	physical velocity degree of freedom
$\ddot{x}$	physical acceleration degree of freedom
σ	stress

1 Abstract

Linear structural dynamic models are often used to support system design and qualification. Overall, linear models provide an efficient means for conducting design studies and augmenting test data by recovering un-instrumented or un-measurable quantities (e.g. stress). Nevertheless, the use of linear models often adds significant conservatism in design and qualification programs by failing to capture critical mechanisms for energy dissipation. Unfortunately, the use of explicit nonlinear models

[1] Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy National Nuclear Security Administration under Contract DE-AC04-94AL85000.

can require unacceptably large efforts in model development and experimental characterization to account for common nonlinearities such as frictional interfaces, macro-slip, and other complex material behavior. The computational requirements are also greater by orders of magnitude. Conversely, modal models are much more computationally efficient and experimentally have shown the ability to capture typical structural nonlinearity. Thus, this work will seek to use modal nonlinear identification techniques to improve the predictive capability of a finite element structural dynamics model.

Part I of this paper discussed experimental aspects of this work. Part II will consider use of nonlinear modal models in finite element modeling. First, the basic theory and numerical implementation is discussed. Next, the linear structural dynamic model of a configuration of interest is presented and model updating procedures are discussed. Finally, verification exercises are presented for a high level excitation using test data and simulated predictions from a structural dynamics model augmented with models obtained in nonlinear identification efforts.

Keywords – Nonlinear System Identification, Nonlinear Simulation, Structural Dynamics, Modal Model, Finite Element Modeling

2 Introduction

Linear structural dynamics models are often used to support system design and qualification. Overall, linear models provide an efficient means for conducting design studies and augmenting test data by recovering un-instrumented or un-measurable quantities (e.g. stress). Nevertheless, the use of linear models often adds significant conservatism in design and qualification programs by failing to capture critical mechanisms for energy dissipation (e.g. joints, friction, material behavior, etc.). Unfortunately, the use of explicit nonlinear models can require unacceptably large efforts in model development and experimental characterization to account for common nonlinearities such as frictional interfaces, macro-slip, and other complex material behavior. The computational requirements are also greater by orders of magnitude.

This work uses modal nonlinear identification techniques to improve the predictive capability of structural dynamics models. Pacini and Mayes [1] have extended conventional modal testing with nonlinear system identification to develop nonlinear modal models. These experimental nonlinear models will augment linear structural dynamic finite element models allowing for the development of reduced order nonlinear structural dynamics models. A flexible, generalized framework has been created to readily accept model parameters from system identification efforts, and the predictive capability of the model is demonstrated via verification exercises. Impacts of the enhanced analysis capability are also discussed.

3 Modeling Theory and Framework

This section presents the basic theory as well as the numerical framework for developing the improved analysis capability that employs experimentally derived nonlinear structural dynamics models. The process of nonlinear identification during modal testing and derivation of nonlinear models will not be discussed here, but is presented in a companion paper by Pacini and Mayes [1].

Consider the classical linear structural dynamic system shown in Eq. 1. This equation is intended to represent a dynamic system in physical coordinates. Here *M, C*, and *K* are the physical mass, damping, and stiffness matrices respectively. Physical displacement degrees of freedom (DOFs) are present in the vector x. Physical velocity and acceleration are denoted by $\dot{x}$ and $\ddot{x}$ respectively. *F(t)* is a generalized transient load vector that may act on any DOF of the system.

$$[M]\ddot{\vec{x}} + [C]\dot{\vec{x}} + [K]\vec{x} = \mathrm{F(t)} \tag{1}$$

Through assumptions of linearity and modal superposition one may express the dynamic system in modal space using the relation shown in Eq. 2 to express the system as a collection of uncoupled, single DOF systems in as shown in Eq. 3. Here, Φ is the mode shape of the system and q is the modal displacement DOF. The natural frequency and damping ratio of a mode are denoted by ω_n and ζ. The modal force is denoted by *f(t)* and is obtained through a transformation of the physical load vector.

$$\vec{x} = \Phi\vec{q} \tag{2}$$

$$\ddot{q} + 2\omega_n\zeta\dot{q} + \omega_n^2 q = f(t) \tag{3}$$

The current work augments the linear system of Eq. 3 with a nonlinear function f_{nl} as shown in Eq. 4.

$$\ddot{q} + 2\omega_n\zeta\dot{q} + \omega_n^2 q + f_{nl}(q,\dot{q}) = f(t) \tag{4}$$

This nonlinear function restoring force, f_{nl} is obtained from experimental nonlinear identification efforts. This augmented dynamic system is readily adapted for transient analysis by common explicit time integrators such as MATLAB ODE45.

Fig. 1 illustrates the framework that has been developed in this work. Finite element structural dynamic modeling efforts will be used to provide linear natural frequencies and mode shapes while experimental efforts are used to characterize linear damping and nonlinear stiffness and damping. After time integration, the system can be converted back to physical space and then traditional analysis quantities of interest such as displacement, velocity, acceleration, stress, strain, and force may be recovered at any node/element in the finite element model.

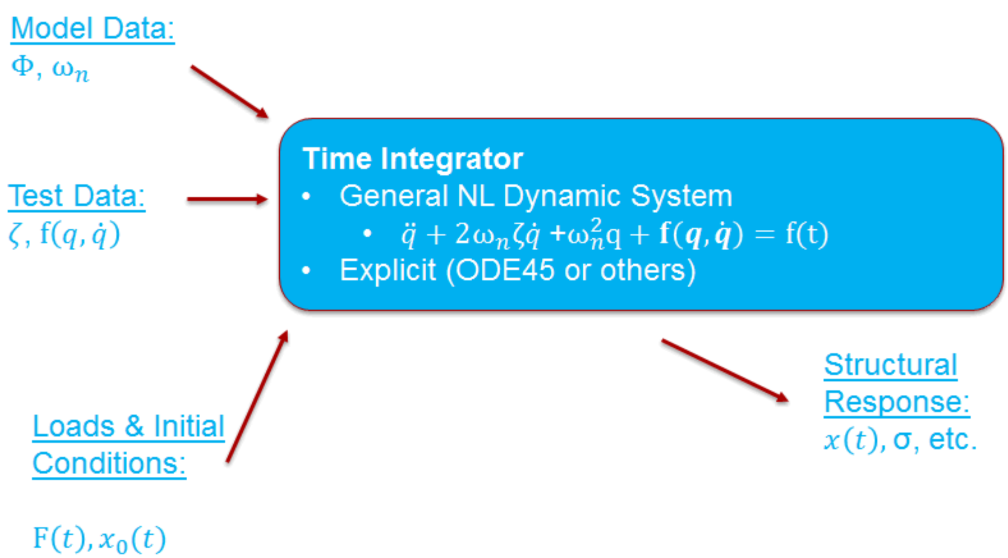


Fig. 1 Framework for nonlinear structural dynamics analysis

4 Model Description & Updating

This section describes the model, linear updating, and nonlinear dynamic model. First the configuration of interest is described and details of the corresponding finite element model are given. Next, the model is updated using modal test data. Third, the specific nonlinear modal model employed in this work is presented.

4.1 Configuration of Interest and Finite Element Model

The hardware configuration considered in this study is a jointed structure of moderate complexity. The configuration consists of a cylinder and a simple beam attached to a plate. This configuration will be referred as the Can-Plate-Beam (CPB). CPB structural components are manufactured from 6061-T6 aluminum. The cylinder and beam/plate are connected via 8 bolted joints. The jointed interface has been designed to exhibit nonlinearities. A thorough description of the hardware is given in Section 3 of Reference [1], but the experimental set-up and cross-section of the companion finite element model are shown in Fig. 2 and Fig. 3 respectively.

The CPB finite element model is composed primarily of 20-node hexahedral elements. The jointed connections are modeled via constraints, as this provides a greater degree of flexibility in model updating procedures such as considering the contact area in tied interfaces or specification of compliance at interfaces. The finite element model also explicitly models accelerometers via concentrated masses. These masses are offset to provide a more accurate representation of mass loading as well as a more relevant comparison between measured and predicted mode shapes.

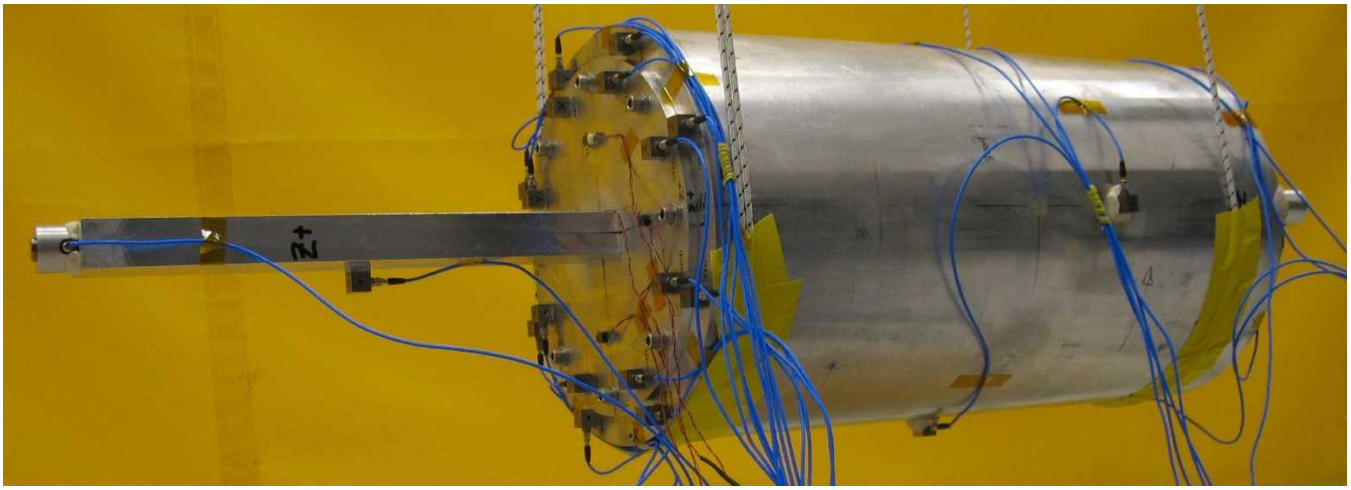


Fig. 2 Physical test hardware

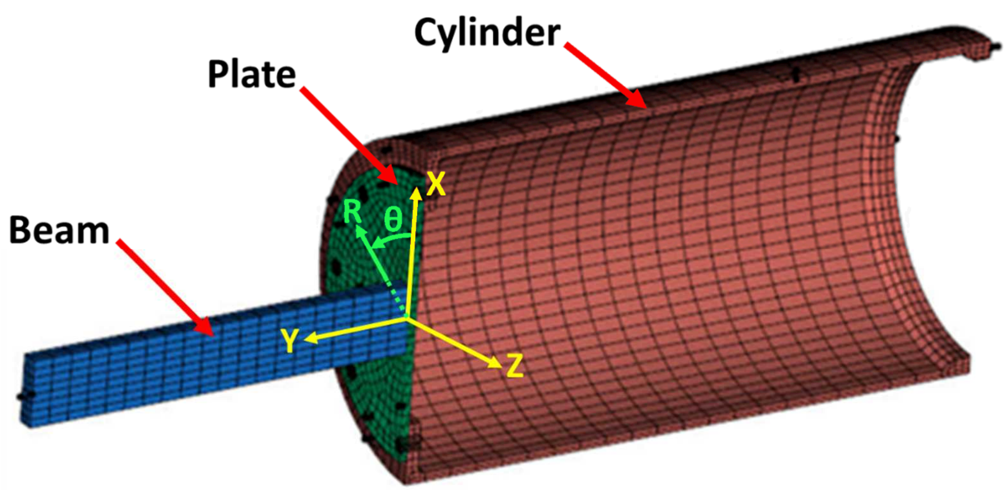


Fig. 3 CPB assembly full system solid model and coordinate systems

4.2 Linear Model Updating

First, the finite element model was updated using modal test data from low level excitation. The low-level excitation test is useful in developing an experimental linear modal model that can be used for model updating of a linear structural dynamics finite element model. Details of the linear experimental modal model are provided in Section 3.3 of Reference 1.

Table 1 presents mode frequencies, damping, and shape descriptions for modes extracted from the low-level modal testing. These modes are considered as "truth" data for model updating. Only a high level discussion of model updating will be discussed in this paper, as a detailed description of model updating procedures is beyond the scope of this work. In general, the nominal model compared reasonably well to linear modal test data. It was notable that the frequencies of the 1st bending modes of the beam were too stiff and the frequency of the axial mode was too soft. Parametric studies were conducted to examine the effect of cylinder Young's modulus, plate to beam tied connection area, and plate to cylinder interface constraints. Two approaches were considered for modeling the bolts between the plate and cylinder:

1) the bolts were modeled using a collection of beam and rigid bar elements (see Fig. 4)

2) the interface was modeled using tied constraints at discrete patches representing the bolted interface (see Fig. 5).

The physical beam-to-plate interface has two bolted connections along with dental cement serving as an adhesive between the contact area. Therefore, the true boundary condition at this interface is somewhat ambiguous. The bolts and adhesive were not modeled explicitly, but instead the interface was modeled through tied constraints over a discrete surface area. Ultimately, the contact area was tailored as shown in Fig. 6 to provide good agreement with 1st beam bending modes from modal test data.

The modes of the system were relatively insensitive to beam parameters considered in the beam/rigid bar modeling approach employed for the plate-to-cylinder bolted connection interfaces. Therefore, the final model employed a modified contact area and tied data which resulted in good agreement with the axial mode frequency from test data. Slight modifications (approx. +5%) was also made to the cylinder Young's modulus to obtain better agreement in ovaling modes.

Table 2 shows the updated model mode predictions compared to experimental data. Overall, there is good agreement between test and simulation. The most noticeable differences are in the 1st bending mode (soft direction) of the beam, (3,0) ovaling mode, and in the 2nd bending mode of the beam. Further model updating activities may be able to provide better agreement, but this is beyond the scope of the current work which is focused on augmenting linear structural dynamics modeling with nonlinear modal data for enhanced predictions.

Table 1 – Experimental Linear Modal Parameters[1,2]

Mode	f_n (Hz)	ζ (%cr)	Reference DOF	Shape Description
7	128	0.30	31349Y	1st bend of Beam in soft direction (global X)
8	171	0.31	25449Y	1st bend of Beam in stiff direction (global Z)
9	391	0.21	34965R	(2,0) ovaling of Cylinder aligned with X-Z axes
10	395	0.03	34965R	(2,0) ovaling of Cylinder 45° from X-Z axes
11	560	0.34	53632Y	Axial mode
12	957	0.11	34965R	(3,0) ovaling of Cylinder
13	958	0.09	32916R	(3,0) ovaling of Cylinder
14	978	0.23	31349Y	2nd bend of Beam in soft direction (global X)

[1]Modes highlighted in green exhibited nonlinear behavior during test

[2]Rigid body modes (1-6) not shown.

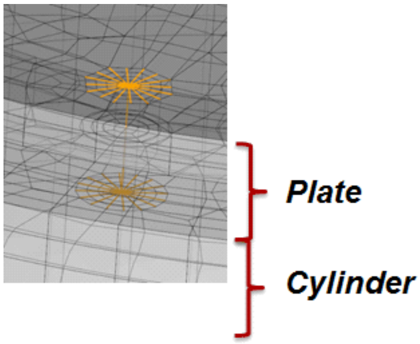


Fig. 4 Beam/rigid bar bolt interface modeling approach

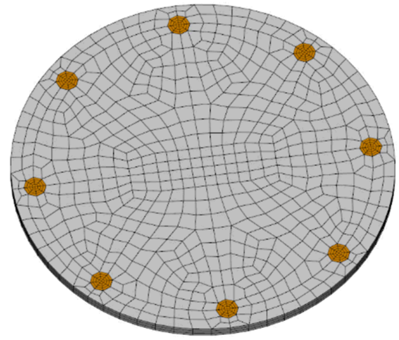
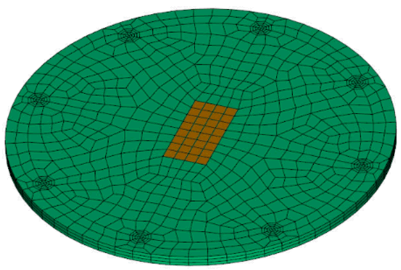
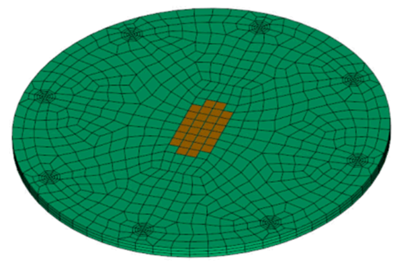


Fig. 5 Element "patches" used for tied constraint bolt interface modeling approach



a.) Initial tied constraint contact area



b.) Updated tied constraint contact area

Fig. 6 Contact area used for tied constraint of beam to plate interface

Table 2 – Comparison of Experimental and Simulation Modes

Mode #	Test Frequency (Hz)	Simulation Frequency (Hz)	% Difference	Mode Shape Description
7	128	120	-6.3	1st bend of Beam in soft direction (global X)
8	171	173	1.2	1st bend of Beam in stiff direction (global Z)
9	391	392	0.3	(2,0) ovaling of Cylinder aligned with X-Z axes
10	395	393	-0.5	(2,0) ovaling of Cylinder 45° from X-Z axes
11	560	549	-2.0	Axial mode
12	957	943	-1.5	(3,0) ovaling of Cylinder
13	958	913	-4.7	(3,0) ovaling of Cylinder
14	978	1011	3.4	2nd bend of Beam in soft direction (global X)

4.3 Nonlinear Modeling

Further modal testing was conducted at higher force input levels to drive the configuration into the nonlinear response regime. The nonlinear identification techniques discussed in Reference 1 were used to develop a nonlinear pseudo-modal model using fits for cubic stiffness and damping in the nonlinear restoring force term. The form of this modal model is shown in Eq. 5 and the model parameters are shown in Table 3. The f_{nl} term defined in Eq. 5 is employed in Eq. 4 and serves as input to the numerical framework depicted in Fig. 1. As noted in Table 1, modes 7, 8, 11, and 14 behave nonlinearly, while the other modes are predominantly linear. Therefore, nonlinear models were only developed for these modes.

$$f_{nl}(q(t),\dot{q}(t)) = c_1|\dot{q}(t)|\dot{q}(t) + c_2\dot{q}^3(t) + k_1|q(t)|q(t) + k_2q^3(t) \quad (5)$$

Table 3 - Damping and Stiffness Coefficients [1]

Mode	c_1	c_2	k_1	k_2
7	2.6	-1.95	-9.15E+07	3.77E+10
8	3.17	-3.28	-2.86E+08	1.92E+11
11	318	-834	-2.05E+10	1.54E+14
14	-188	811	-6.16E+09	8.41E+13

3 Verification Exercise

After linear and nonlinear characterization of the CPB structure, a verification exercise was performed. The CPB was excited at a location on the flange of the cylinder in the axial (Y) direction via a 0.3 second chirp (i.e. a very fast sine sweep) from 50 to 1400 Hz. Data was recorded for 4 seconds. This DOF was chosen since it excited three of the four nonlinear modes. The amplitude of the sweep was varied in order to maximize the response of each nonlinear mode without exceeding the maximum voltage limit of any accelerometer. The input location and an output location on the beam tip are depicted in Fig. 7. The input chirp signal is depicted in Fig. 8.

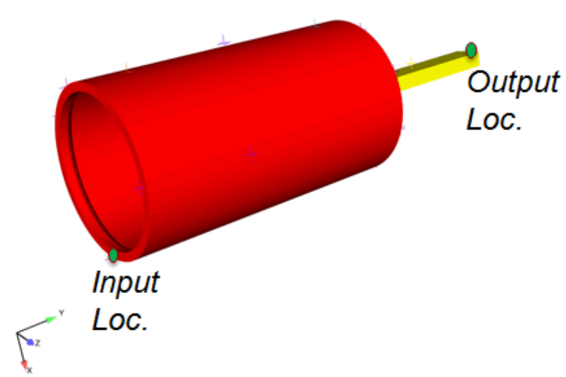


Fig. 7 Input and output locations considered in verification study.

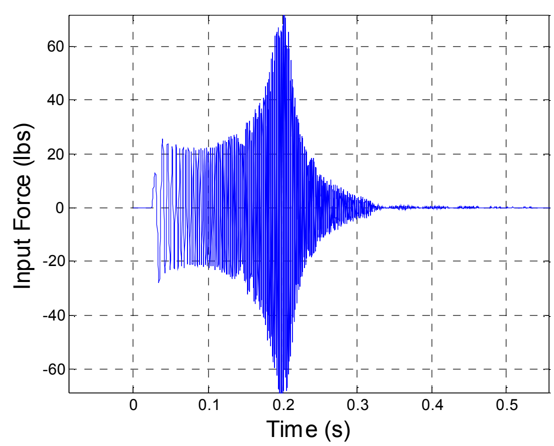


Fig. 8 Chirp excitation

The nonlinear framework described in Section 3 was used to perform a virtual experiment using the updated linear model, nonlinear modal model, and chirp excitation. This simulation employed an explicit time integration routine (ODE45) from 0 to 1 seconds of simulation time. The time step of this simulation was adaptively chosen by the ODE45 solver. Acceleration time history predictions were recovered at the drive point and beam tip location for comparison to experimental data. Note that only the first 1 second of the experimental data was considered in the comparison to model predictions for a more meaningful comparison. Time histories were transformed to the frequency domain via the Fast Fourier Transform (FFT). Use of the FFT allows for a succinct description of the frequency content and levels in a time series. Furthermore, environmental specifications and transfer functions are typically developed in the frequency domain.

Fig. 9 shows acceleration FFTs of the experimental data, linear model, and nonlinear model. Overall, reasonable agreement to experimental data is observed. This level of agreement is more than sufficient for design studies and environmental specifications informed by model data. Differences in model data vs. experimental data can be attributed to error in the linear model predictions as well as differences in experimental and numerical mode shapes. It is likely that further model development could result in improved agreement, but the current level of agreement is useful for assessing the nonlinear modeling approach under development.

The axial mode is the dominant nonlinear mode of this system, and Fig. 10 shows a “zoomed in” FFT of this mode. It is notable that the nonlinear model simulation both captures the amplitude reduction and “softening” of the frequency. In fact, the linear model has levels approximately 2x higher than the nonlinear model. Thus, using a linear model could drive overly conservative designs (e.g. over prediction of stress response) or environmental specifications that are used for testing and qualification of a design. Furthermore, the good agreement in modal frequency for the linear axial mode and experimental data is simply due to differences in the linear axial mode observed in modal testing and that in the updated finite element model. Inspection of Table 2 shows the axial mode prediction is approximately 2% “soft”/low. Therefore, the experimental and nonlinear model “soften” at this mode due to nonlinear effects. In other words, a more accurate representation of the linear axial mode frequency would have resulted in better agreement between the nonlinear model predictions and experimental data for the axial mode FFT. The beam tip response was also examined with similar trends in verification. Therefore, these results are omitted from this paper for brevity.

Note that the secondary peak in the FFT around 550 Hz is a consequence of using the FFT on a nonlinear signal. At early times, the high level input excites nonlinearities in the axial mode, but at later times the response decays and becomes primarily linear. Since this mode has a softening behavior, this linear modal frequency appears as a “secondary peak”/irregularity at a higher frequency on the FFT.

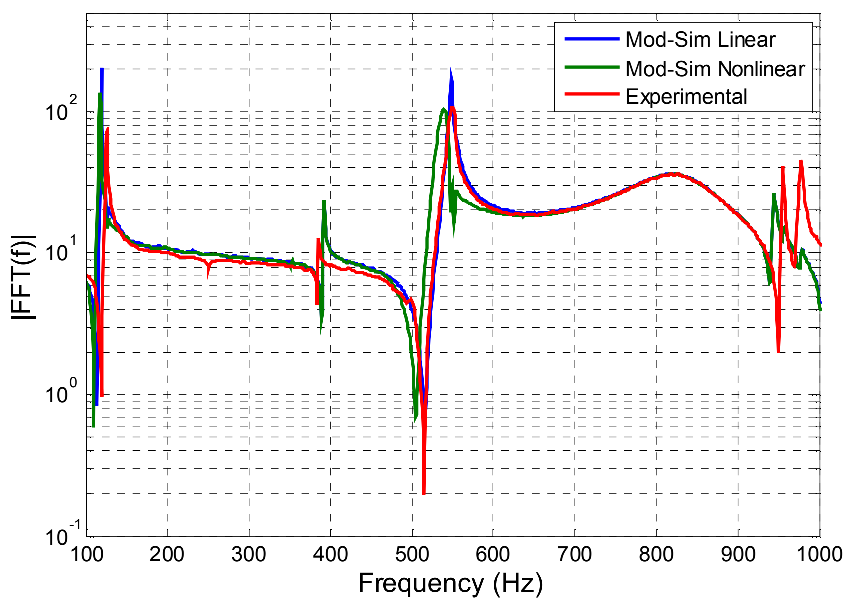


Fig. 9 Comparison of FFT of drive-point acceleration

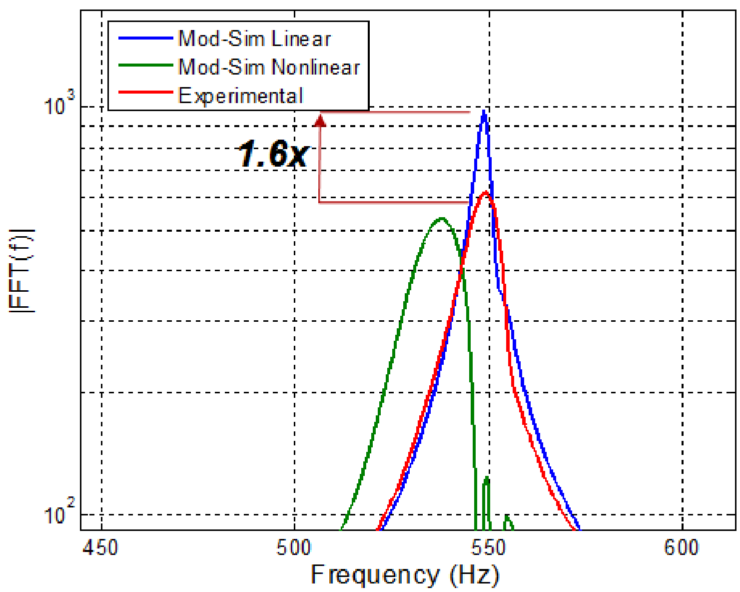


Fig. 10 Comparison of FFT of drive-point acceleration for axial mode

4 Conclusions

This work has successfully demonstrated the ability for efficient nonlinear dynamic modeling through means of nonlinear identification methods in modal testing. Part I of this work showed the capability of using a pseudo-modal model to capture nonlinearities of a real structure using cubic polynomials for the stiffness and damping forces. Part II of this work developed a numerical framework that would accept a linear finite element structural dynamic model and augment the dynamic system with nonlinear data from test efforts. A hardware configuration of moderate complexity with nonlinear interfaces was considered and finite element model was created and updated using modal test data. The baseline dynamic system was augmented with nonlinear pseudo-modal models from experimental nonlinear identification efforts. This capability was demonstrated against an experimental verification case. The results showed good agreement in the dynamic behavior of the primary nonlinear mode of the system. Nonlinear model predictions were able to replicate frequency "softening" as well as reduction in amplitude levels associated with nonlinear effects. This improved modelling approach could significantly benefit design studies or environmental specifications informed by model data in that a more accurate representation of system dynamics would reduce conservatism in modeling efforts.

Notice: This manuscript has been authored by Sandia Corporation under Contract No. DE-AC04-94AL85000 with the U.S. Department of Energy. The United States Government retains and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.

References

1. Pacini, B.R., Mayes, R.L., Owens, B.C., and Schultz, R.A. "Nonlinear Finite Element Model Updating, Part I: Experimental Techniques and Nonlinear Modal Model Parameter Extraction", presented at the 35th International Modal Analysis Conference, January 2017.