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Predicting Strain in Cure Shrinkage Induced Epoxy/Metal Bilayer Beam Bending

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Cure Shrinkage Induced Bending in Epoxy/Metal Bilayer Material

- Thin layer of metal coated with thermoset epoxy

- During cure, epoxy shrinks causing bilayer to bend
  - Can cause component failure, interface cracking, etc.

SEMI-ANALYTIC MECHANICS THEORY

Timoshenko's Formulae for Eigen Strain Induced Beam Bending

- Classic 1925 paper focuses on bilayer metallic beams bending due to thermal expansion mismatch
- Same concept applicable to other eigen strains
  - Cure shrinkage
- Formula for radius of curvature (ρ)

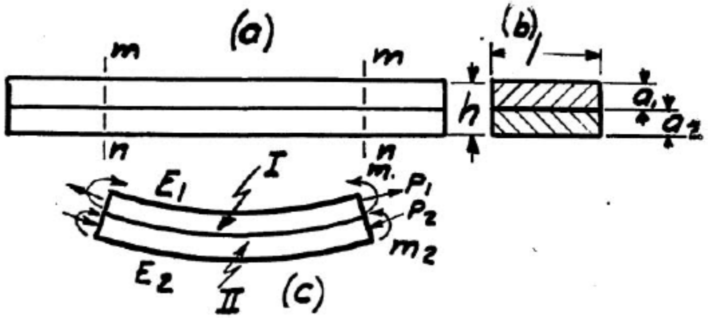


FIG. 1. *Deflection of a bi-metal strip while uniformly heated.*

$$\frac{1}{\rho} = \frac{6\varepsilon_\Delta}{h\left(3 + (1 + mn^3)\left(1 + \frac{1}{mn}\right)/(n+1)^2\right)}$$

  - Where, ε_Δ is the eigen strain mismatch, $m = \frac{E_1}{E_2}$ is the ratio of elastic moduli, $n = \frac{a_1}{a_2}$ is the ratio of thicknesses, and $h = a_1 + a_2$ is the total thickness

Predicting Strain at Bottom of Bilayer Beam

- The strain at the bottom of a beam in bending,

$$\varepsilon_{bottom} = \alpha_2 \Delta T + \frac{F_2}{E_2 a_2} + \frac{a_2}{2\rho}$$

where,

  - F_2 is the axial force placed on the second layer by the first

Simplified using, $F_2 = \frac{2(E_1 I_1 + E_2 I_2)}{h\rho}$ and the bilayer ratios, $m = \frac{E_1}{E_2}, n = \frac{a_1}{a_2}$

$$\varepsilon_{bottom} = \alpha_2 \Delta T + \frac{a_2}{\rho}\left[\frac{mn^3 + 3n + 4}{6(n+1)}\right]$$

Dependence of Bending on Geometric and Material Properties during Cure

- Timoshenko's formula depends on properties that are **not constant** during the curing process
  - $m = \hat{m}(T, x)$, **modulus of epoxy** varies with temperature and reaction extent (x)
  - $n = \hat{n}(T, x), h = \hat{h}(T, x)$, geometry can vary with temperature and reaction extent – likely negligible
  - $\varepsilon_\Delta = \hat{\varepsilon}_\Delta(T, x)$, eigen strain mismatch (**cure shrinkage**) varies with temperature and reaction extent – Important!

- Variation of moduli and cure shrinkage with temperature and reaction extent **captured by experiments**
  - SAND2013-8681

Evolution of Epoxy Shear Modulus during Cure

- Modeled as,

$$\bar{G} = \left(1 + \frac{\partial \bar{G}}{\partial T}(T - T_{ref})\right) f(x)$$

where,

- $\bar{G} = \frac{G_\infty}{G_{\infty_{ref}}}$ is the normalized equilibrium shear modulus where $G_{\infty_{ref}}$ corresponds to T_{ref}
- $f(x) = \left[\frac{x^2 - x_{gel}^2}{x_{ref}^2 - x_{gel}^2}\right]^{\frac{8}{3}}$, and x_{gel} is the reaction extent at gel, and $x_{ref} = 1$

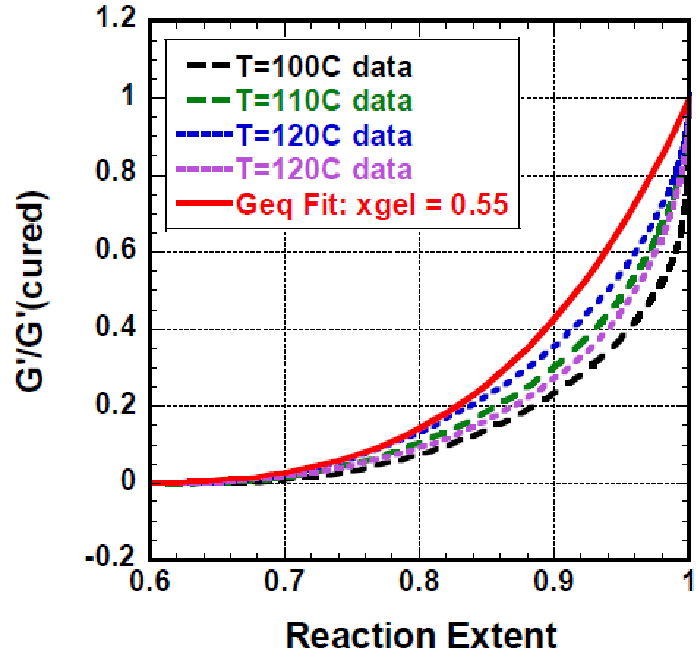


Figure 3-13. Parameterization of Equation 17 to measurements of equilibrium shear modulus as a function of reaction extent.

Source: SAND2013-8681

Modulus Ratio Function of Temperature and Reaction Extent

- The modulus ratio function $\widehat{m}$ is,

$$\widehat{m}(T, x) = \frac{E_{1_{ref}}}{E_2} \bar{G}(T, x),$$

where $E_{1_{ref}} = 2G_{\infty_{ref}}(1 + \nu_1)$

Variation of Cure Shrinkage over Reaction Extent with Temperature

- Cure shrinkage represented by volumetric strain is approximately linear after the gel point
  - $\varepsilon_{cure} = \frac{\beta_\infty}{3}\left(x - x_{gel}\right) \mid x > x_{gel}$
  - $0.060 < \beta_\infty < 0.082$
- Open questions
  - Should β_∞ be a function of temperature also?

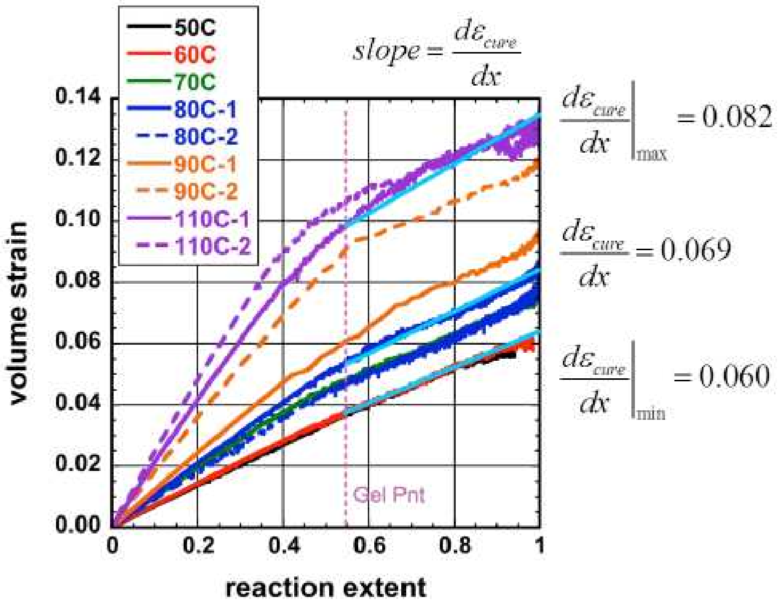


Figure 3-9. The volume strain associated with cure shrinkage versus the reaction extent. Volume shrinkage is determined based on the extrapolated polymer densities in Figure 3-7 using Equation 15. Reaction extent is determined from Equation 7 with fitting parameters described in Table 3-1 and w=0. This does not account for (1) non-isothermal conditions at early times or (2) vitrification during reaction. The 1st derivative beyond the gel point is evaluated for some datasets.

Source: SAND2013-8681

Expression of the Eigen Strain Mismatch with Cure Shrinkage and Thermomechanical Strain

- The eigen strain mismatch is the combination of the strains associated with curing and temperature

$$\varepsilon_\Delta = \varepsilon_{cure} + \alpha_2 \Delta T - \left(\alpha_1 \Delta T - \varepsilon_{ref}\right)$$

$$\hat{\varepsilon}_\Delta(T, x) = \frac{\beta_\infty}{3}\left(x - x_{gel}\right) + \alpha_2 \Delta T - \left(\alpha_1 \Delta T - \varepsilon_{ref}\right),$$

when $x \geq x_{gel}$

- The cure strain represents a volumetric shrinkage, i.e. $\boldsymbol{\varepsilon} = -\varepsilon_{cure}\mathbf{1}$
  - As it is fit as a positive magnitude, signs cancel leading to above form
- ΔT represents the temperature change from the gel point
- ε_{ref} represents the change in stress free configuration – can vary through the thickness!

Tracking the Change in Stress Free Configuration During Epoxy Cure

- The stress free configuration of the epoxy changes during the cure process due to the addition of cross links. Thus, the typical expression for thermal strain has an error.
- The offset from the original configuration is tracked through a reference strain such that,

$$\varepsilon_{ref}^{*} = \frac{1}{G_{\infty}(t_n)} \int_{t=0}^{t=t_n} G_{\infty}(t) \frac{d\varepsilon_{dev}}{dt} dt - \varepsilon_{dev}(t_n)$$

- We specifically care about the in-plane mechanical strain,

$$\varepsilon_{pp} = \frac{P}{E_1 a_1} = \frac{h}{6\rho} \left[\frac{mn^3 + 1}{mn(n+1)^2} \right]$$

$$\varepsilon_{tt} = -\frac{2\varepsilon_{pp}\lambda}{2G_{\infty}+\lambda}, \lambda \text{ is the lame constant}$$

$$\varepsilon_{pp,dev} = \varepsilon_{pp} - \frac{1}{3}\left(2\varepsilon_{pp} + \varepsilon_{tt}\right)$$

Reaction Extent dependence on Temperature

- Rate of Reaction is dependent on current reaction extent (x) and temperature

$$\frac{dx}{dt} = \hat{k}(b + x^m)(1 - x)^n$$

$$\hat{k} = \frac{k_0 \exp\left(-\frac{E_a}{RT}\right)}{(1 + wa)^\beta}$$

where,

- b, m, n, w, β, k_0 are fitted constants related to various aspects of the reaction process (See SAND2013-8681)
- E_a is the activation energy
- a represents a shift factor due to vitrification
- β is set to 0 to ignore the shift effects.

Strain in Bilayer Beam due to Cure Shrinkage Induced Bending

- Formulae for each different aspect have been discussed
  - Dependence of eigen strain mismatch on temperature and reaction extent
  - Dependence of epoxy shear modulus on reaction extent and temperature
  - Evolution of Stress Free Configuration with Modulus and Strain
  - Variation of rate of reaction (and reaction extent) on temperature
- The full solution requires:
  - Numerical integration of the rate of reaction to determine the reaction extent from the temperature data
  - Solution of evolving reference configuration and curvature
  - Calculation of strain at desired location
  - Comparison to experimental data

Rate of Reaction Equation Solution Parameters

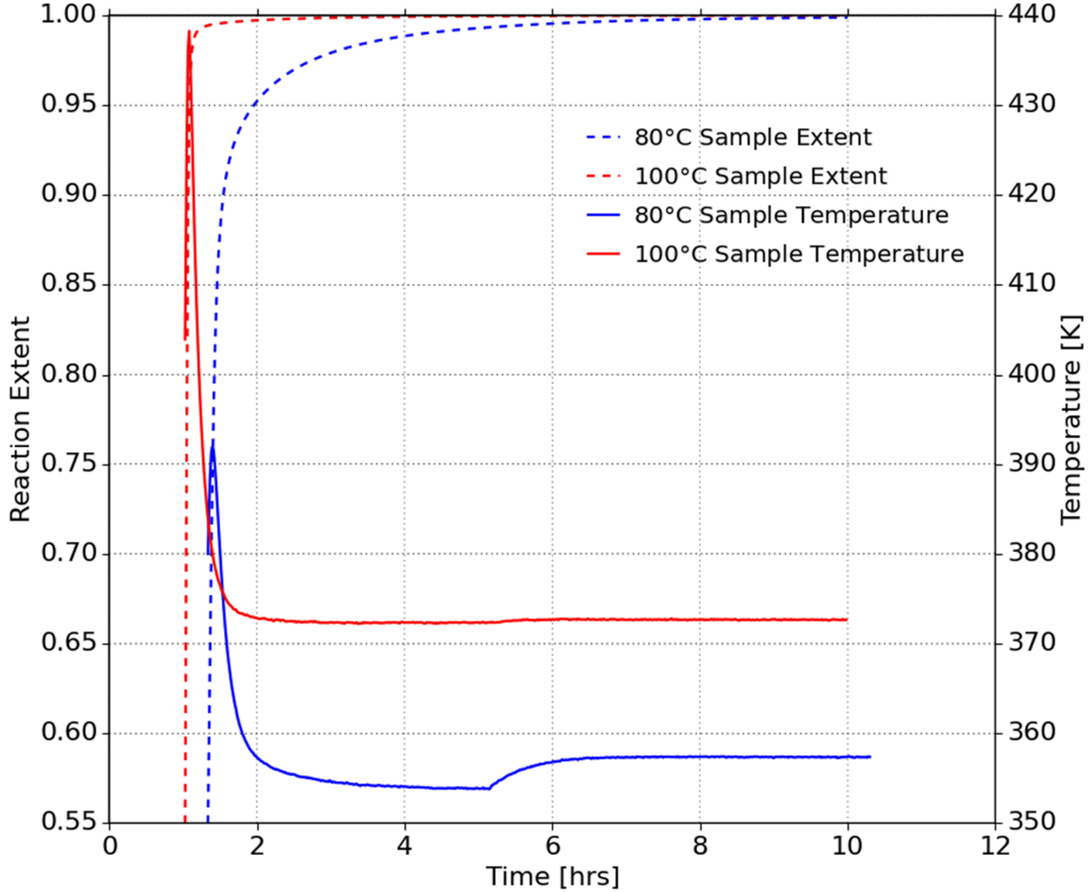
- Solution done in python
  - Scipy: odeint integrator

- Rate of Reaction parameter list (SAND2013-8681)

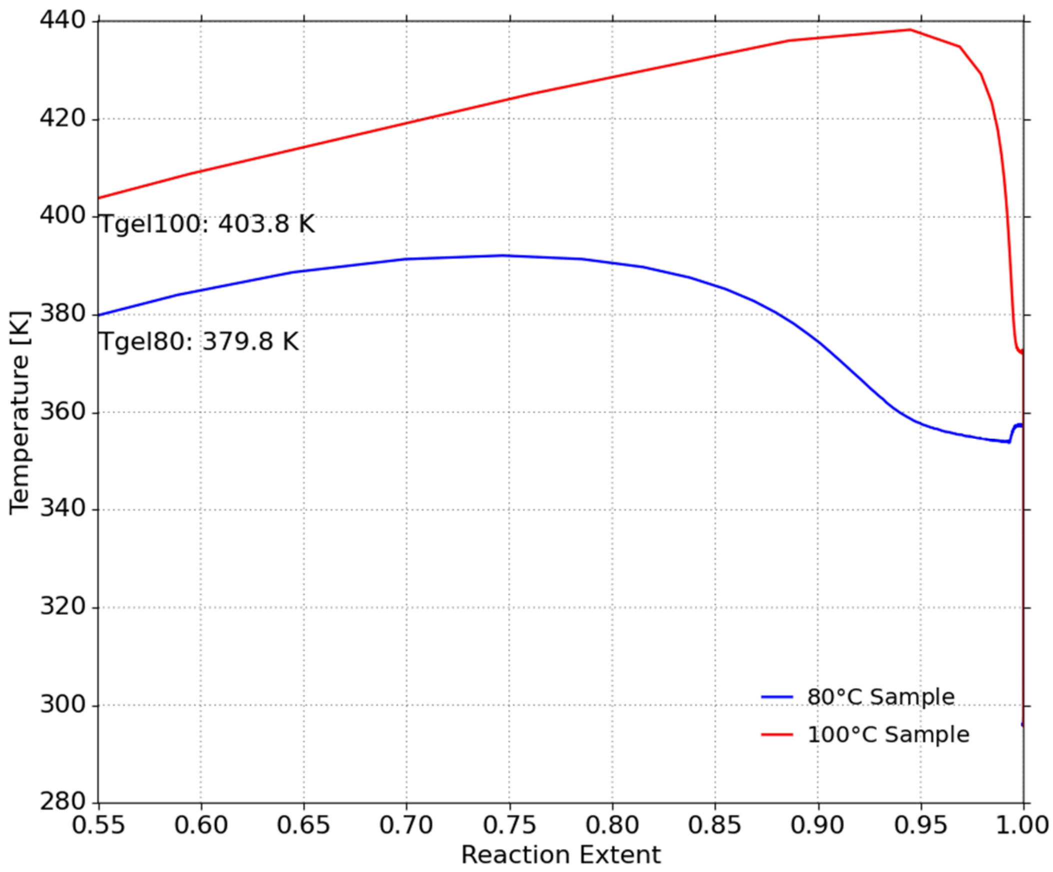
Parameter	Value
E_a	13.8 [kcal/mole]
k_0	217e+03 $[s^{-1}]$
b	0.17
m	0.33
n	1.37
w	-
a	-
β	0

WLF Shift Parameters are unused

Higher Cure Temperature Results in Higher Rate of Reaction



Temperature of Experimental Samples throughout Reaction Extent



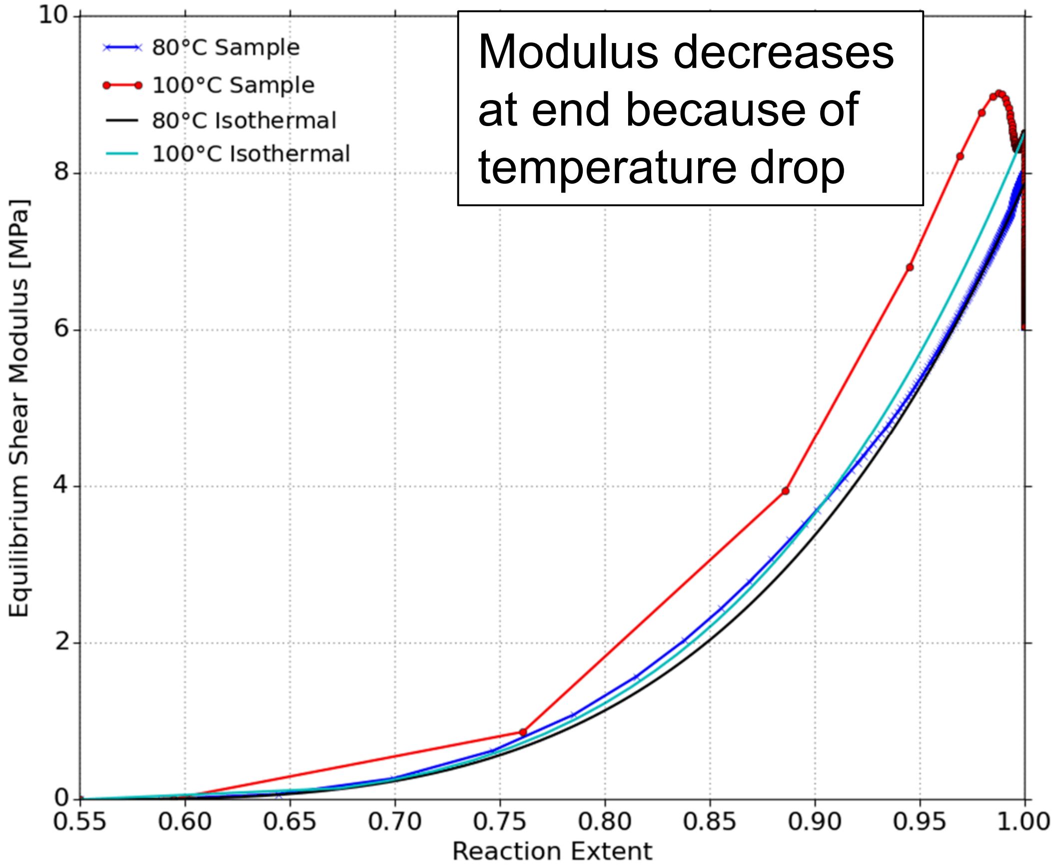
Shear Modulus Evolution Parameters

- The shear modulus is calculated using the proposed model and the calculated reaction extent for each data point

- The model uses the following parameters (SAND2013-8681)

Parameter	Value
x_{gel}	0.55
x_{ref}	1
T_{ref}	90 [deg C]
$\partial \bar{G}/\partial T$	0.0039 $[\mathrm{deg}\, K^{-1}]$

Higher Cure Temperature results in Higher Modulus during Cure

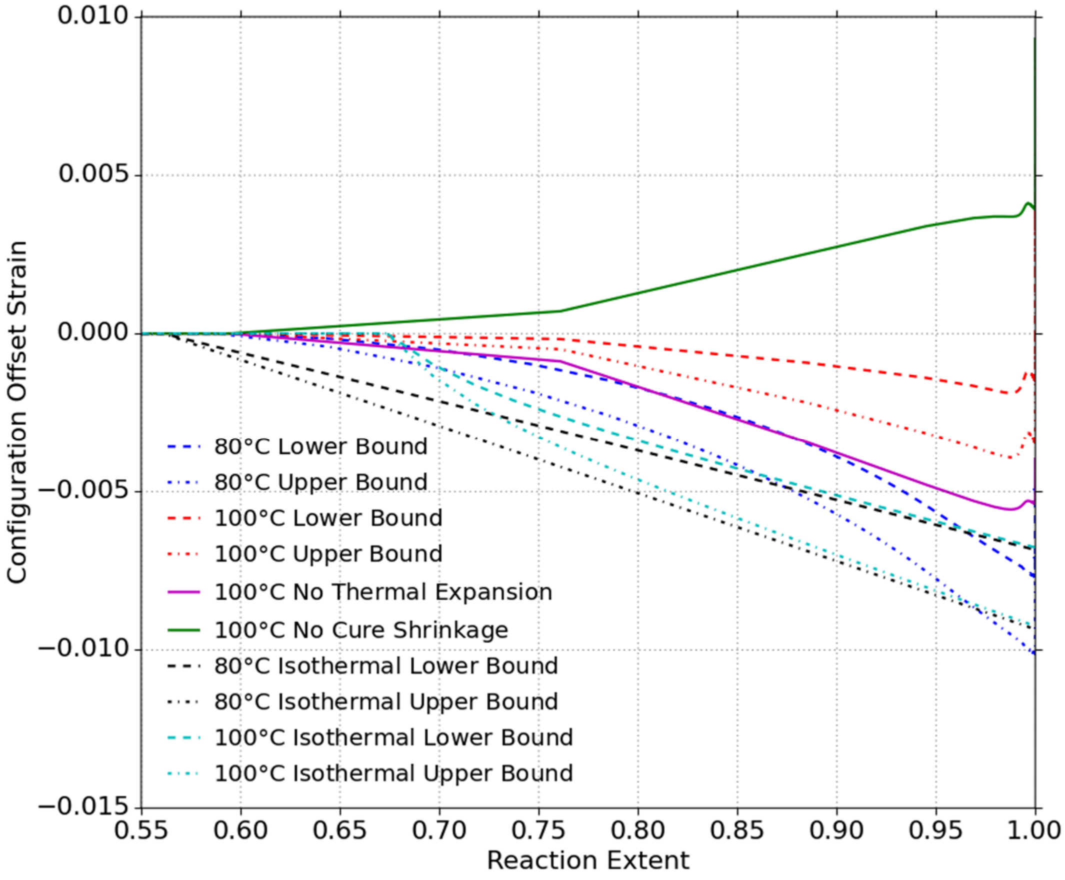


Solution to the Evolving Configuration Model

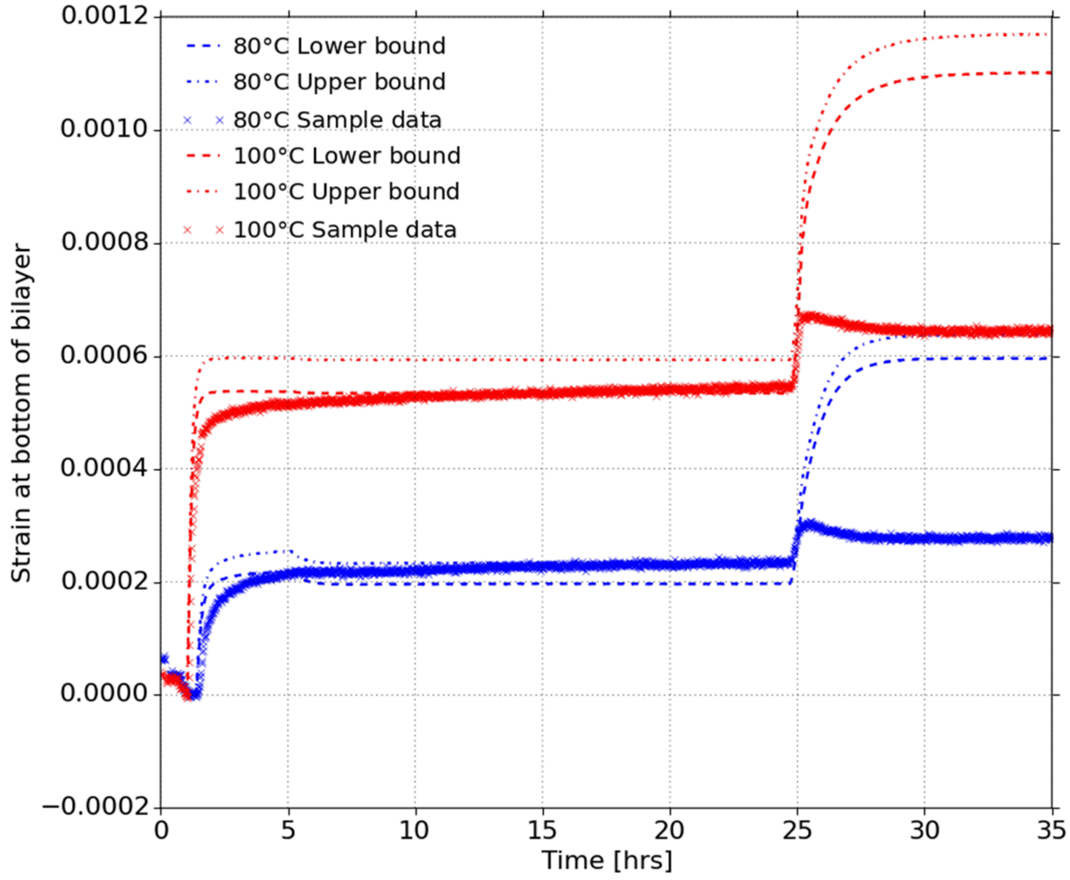
- The evolution of the stress free configuration depends on the history of the deviatoric strain
- Solve for other values
  - Curvature
  - In-plane Strain at boundary
  - Out-of-plane Strain at boundary
  - Assuming in-plane directions are equal, solve for deviatoric in-plane strain

$$\varepsilon = \begin{bmatrix} \varepsilon_{pp} & 0 & 0 \\ 0 & \varepsilon_{pp} & 0 \\ 0 & 0 & \varepsilon_{tt} \end{bmatrix}$$

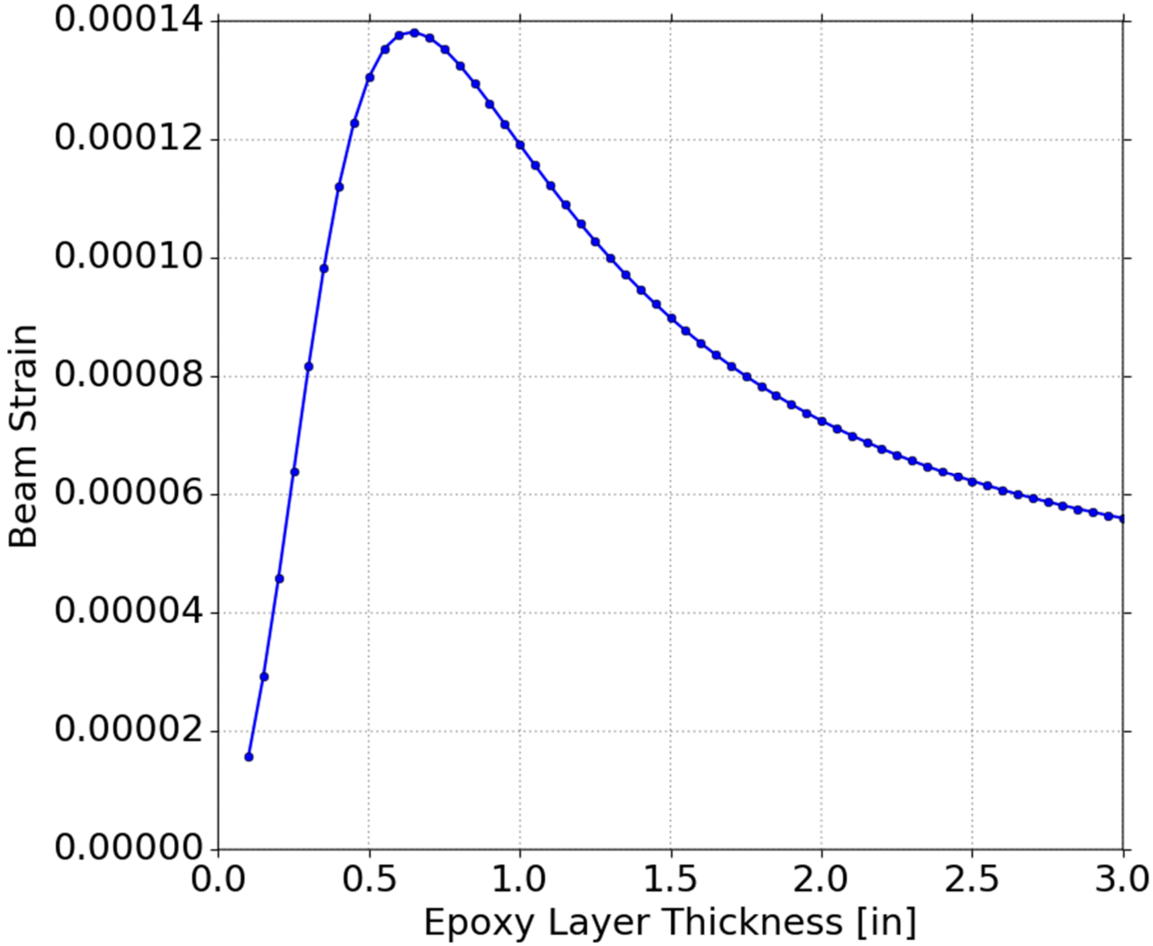
Model Suggests a Small Change in Stress Free Configuration



Model Closely Predicts Strain in Experiments

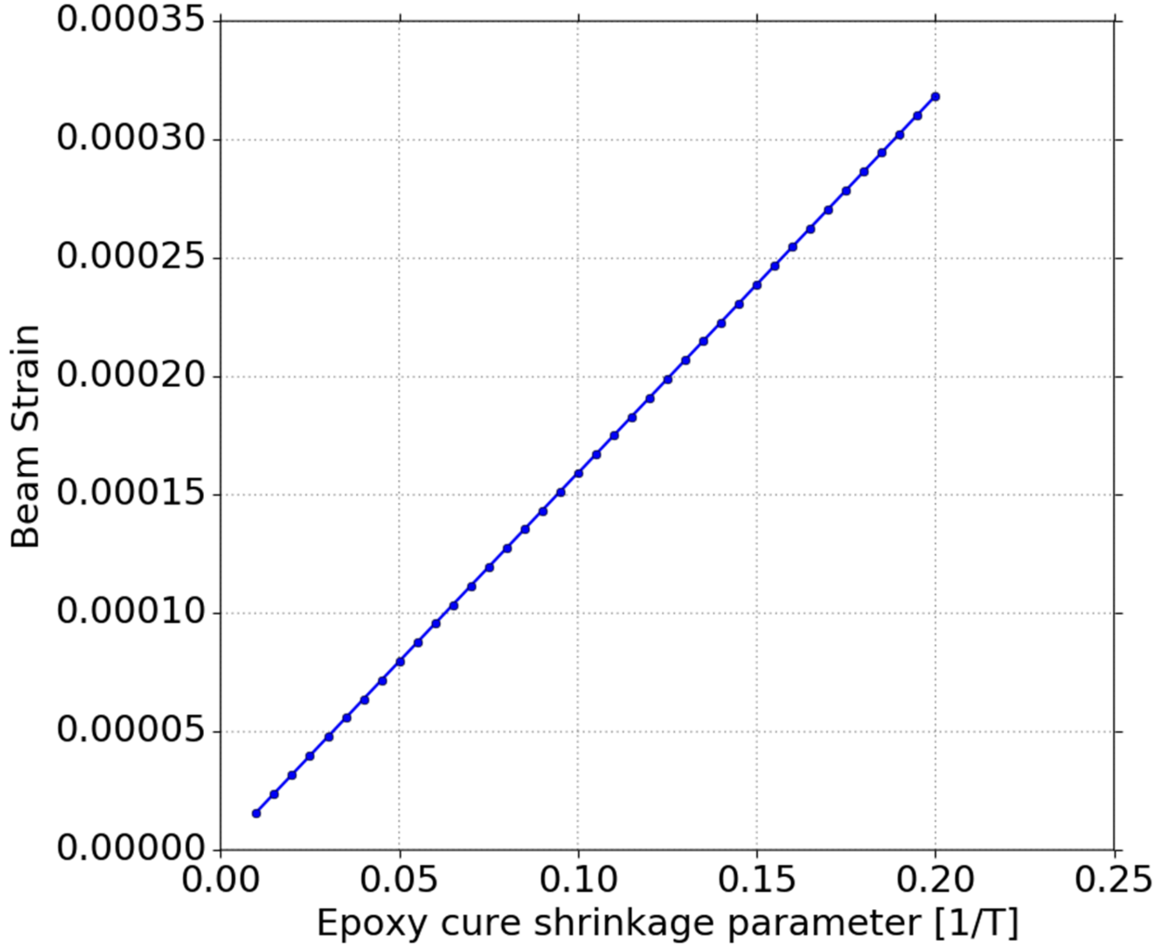


Sensitivity of Strain Output to Epoxy Thickness

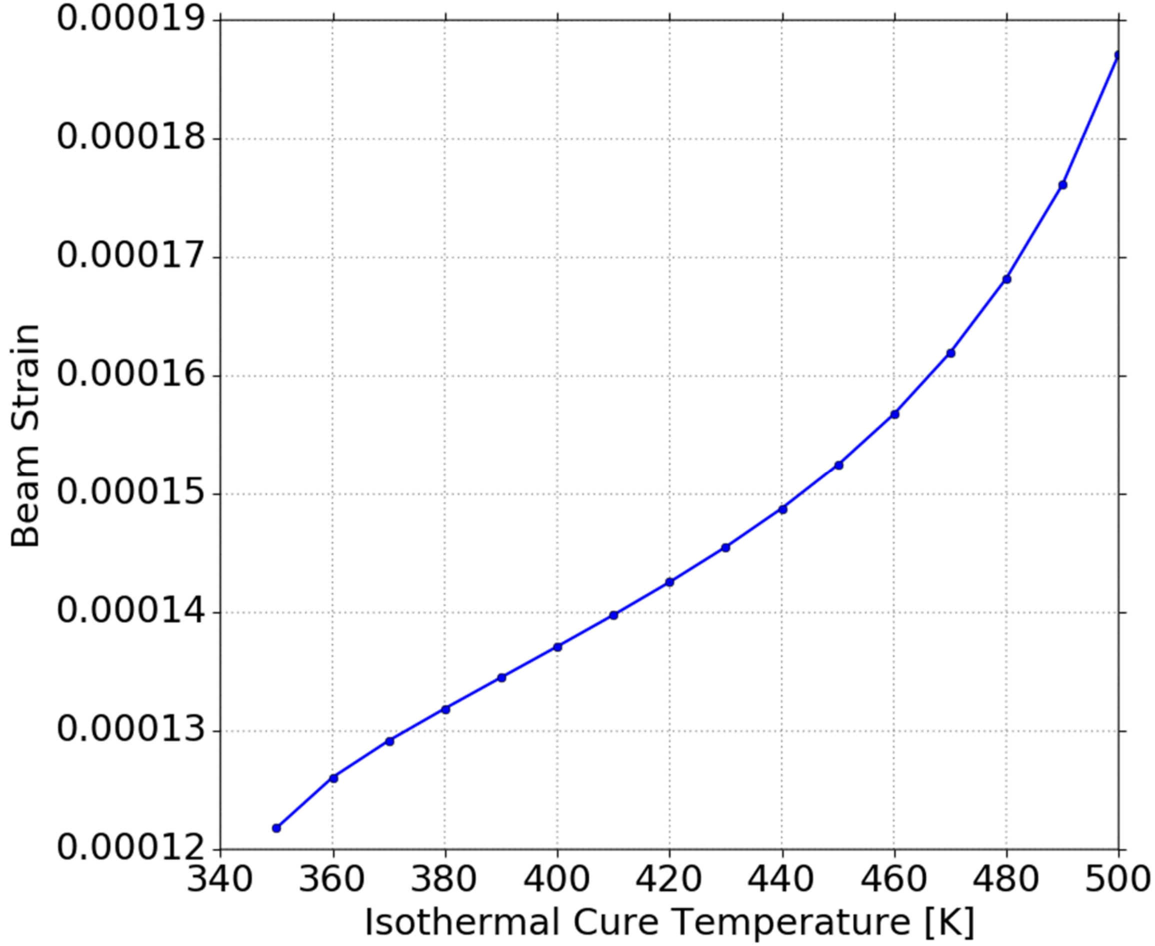


Sensitivity of Strain Output on Cure Shrinkage Rate

Sensitivity on cure shrinkage rate scales proportionally with beam strain with change in thickness



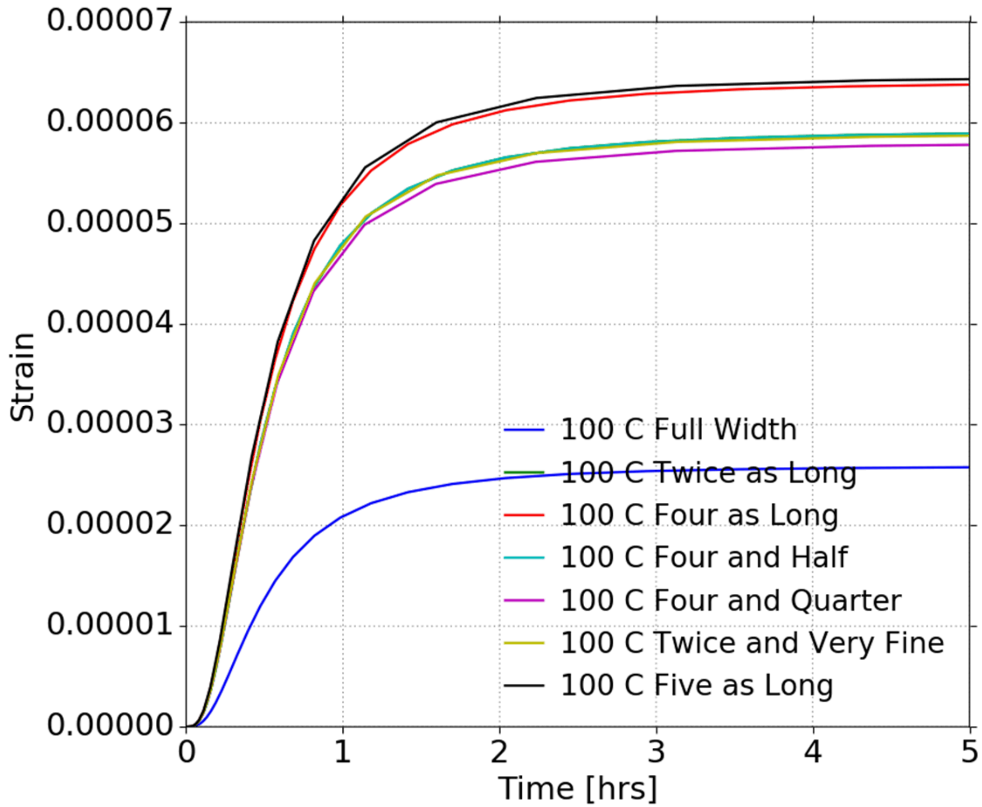
Sensitivity of Beam Strain on Cure Temperature



FEA VALIDATION

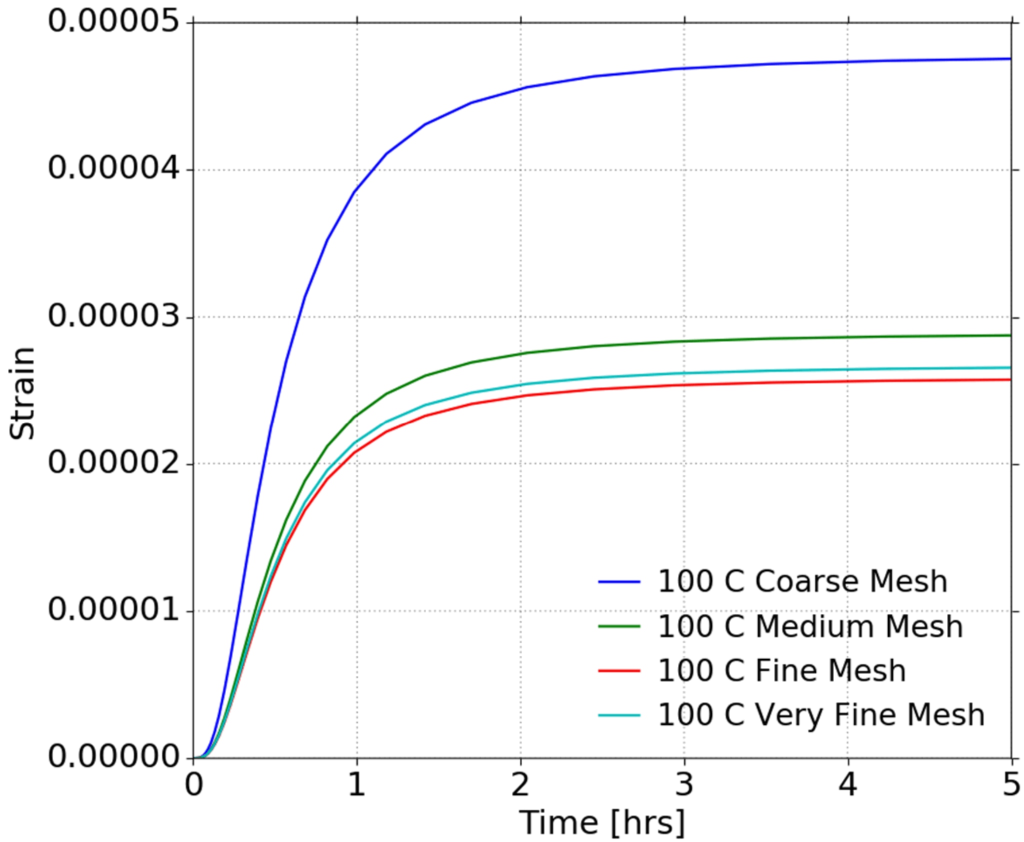
Finite Element Model Setup

- Geometries of various lengths – 1”, 2”, 4”
- Quarter symmetry exercised
- Final Geometry: 1” x 4” with quarter symmetry (Mesh: 0.5” x 2”)



FEA Mesh at 3 Densities

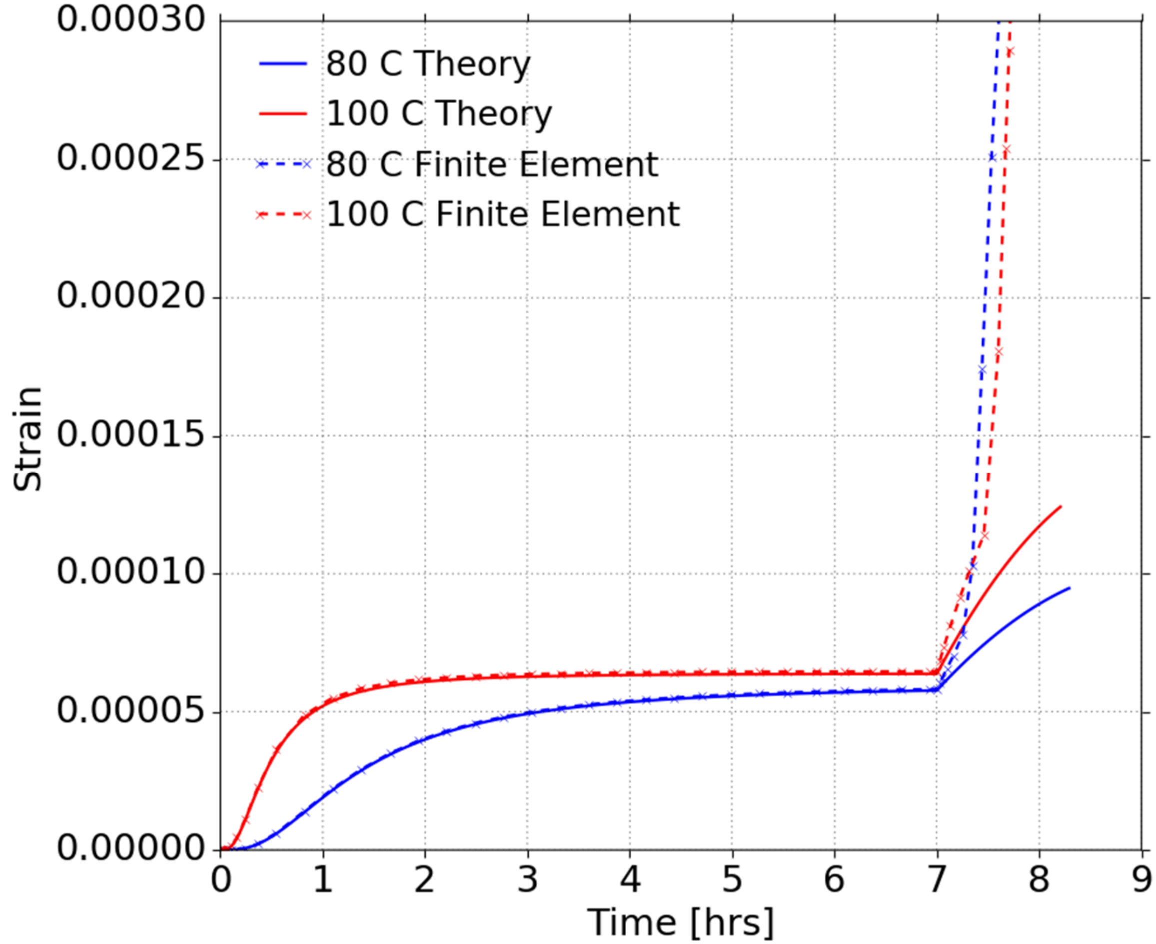
- Coarse, Medium, Fine meshes to prove mesh convergence
  - ~5K elements
  - ~38K elements
  - ~300K elements



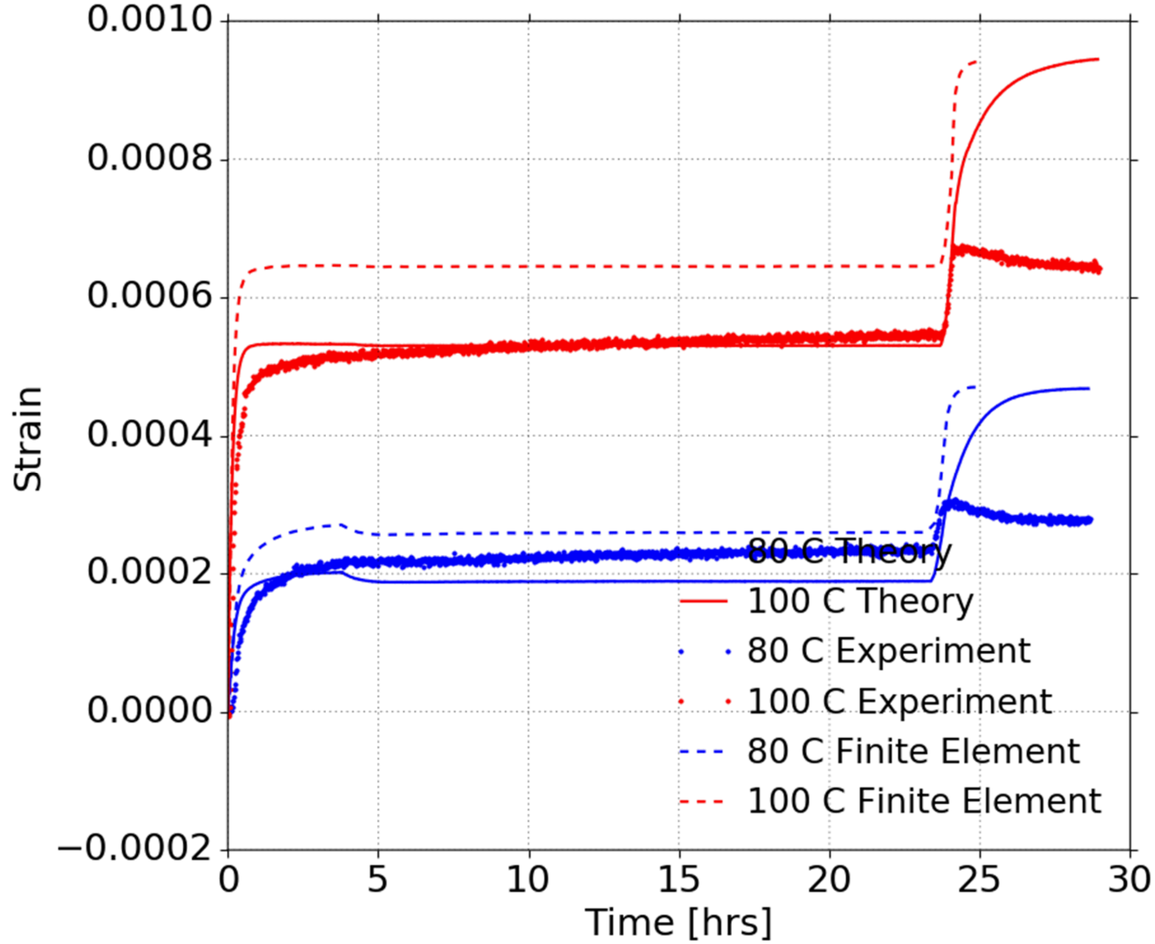
Material Models

- Linear Elastic Aluminum
  - Small strains, good assumption
- Universal Curing Model

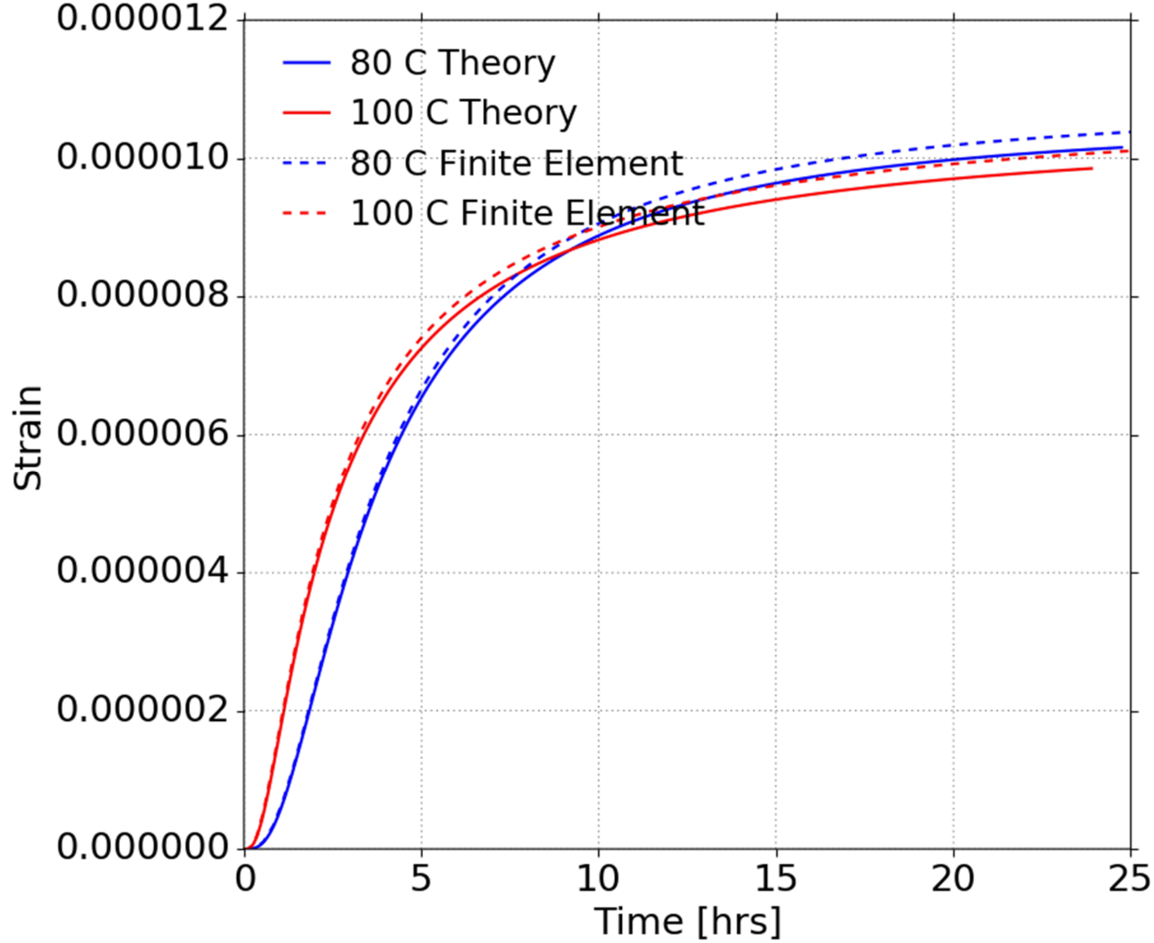
Comparison of FEA and Theory for Isothermal Case



FEA, Theory, and Experiment Comparison



Preliminary DEA Results



Conclusions

- For 828/T403:
  - Analytical model captures major strain behaviors from the bilayer beam experiment.
  - FEA and Analytical models show a discrepancy for changing temperatures – possibly points to a difference in treatment of thermal strain
- For 828/DEA:
  - FEA and Analytical models agree fairly well for isothermal case
  - Experimental data is inadequate for comparison, still additional understanding needed of the 828/DEA Epoxy