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Accelerated Time-Domain Modeling of Electromagnetic Pulse Excitation of Finite-Length Dissipative Conductors over a Ground Plane via Function Fitting and Recursive Convolution

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Abstract

In this report we overview the fundamental concepts for a pair of techniques which together greatly hasten computational predictions of electromagnetic pulse (EMP) excitation of finite-length dissipative conductors over a ground plane. In a time-domain, transmission line (TL) model implementation, predictions are computationally bottlenecked time-wise, either for late-time predictions (about 100ns-10000ns range) or predictions concerning EMP excitation of long TLs (order of kilometers or more). This is because the method requires a temporal convolution to account for the losses in the ground. Addressing this to facilitate practical simulation of EMP excitation of TLs, we first apply a technique to extract an (approximate) complex exponential function basis-fit to the ground/Earth's impedance function, followed by incorporating this into a recursion-based convolution acceleration technique. Because the recursion-based method only requires the evaluation of the most recent voltage history data (versus the entire history in a "brute-force" convolution evaluation), we achieve necessary time speed-ups across a variety of TL/Earth geometry/material scenarios.

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FIGURES

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1. INTRODUCTION

Recently, there has been interest to provide results for the electric current induced on finite-length dissipative conductors interacting with a conducting ground, when excited by an electromagnetic pulse (EMP) [1-5]. We have developed both a frequency-domain and a time-domain method based on transmission line theory through a code we call ATLOG – Analytic Transmission Line Over Ground. In particular, the time-domain formulation described in [4-5] is essential when time-varying air conductivities are to be taken into account.

The purpose of this report is to overview recent developments in accelerating the computational modeling and prediction of EMP excitation along single-wire TLs (i.e., Earth serving as return conductor). This is offered as an alternative to a "brute-force" evaluation [4-5], which can be severely slow. Using and discretizing (in time and space) the one-dimensional TL ("Telegrapher") equations [4-6], one numerically solves the current and voltage waveform history along the TL.

The primary computational bottleneck, especially for late-time predictions of EMP waveform excitation of TLs, is performing temporal convolution to temporally update the (spatial derivative) voltage waveform at each spatial point along the line using the Earth's (temporally-integrated) impedance function and the (time derivative of the) current waveform's past history. For long lines and (especially) late-time predictions, the temporal integration interval becomes extremely long and hence the convolution becomes unwieldy and expensive to compute.

To alleviate this bottleneck, we incorporate into our time-domain ATLOG model the recursive convolution technique (detailed in [6]) that allows "re-using" past computations of the TL voltage history in subsequent voltage updates, dramatically reducing the time taken for computing mid- and late-time predictions. This technique, requiring that the ground impedance function be a complex exponential function or a linear combination thereof, is facilitated through application of the Matrix Pencil method (also known as the Generalized Pencil of Function Method [GPOF]) to systematically compute an efficient, approximate generalized Fourier (i.e., damped complex-exponential basis) decomposition of the ground impedance function [7]. The GPOF's primary advantage (compared to Prony's Fourier decomposition method used in [6]) stems from the extra Singular Value Decomposition (SVD) step used to eliminate spurious, largely redundant information from the impedance function's sampled time history that is used to extract a Fourier decomposition.

Section 2 overviews the primary "bottlenecking" equation (temporal convolution) therein, as well as the two techniques used to overcome this bottleneck (i.e., recursive convolution and GPOF). Note that some of our chosen variables may differ from those in the supporting references. Section 3 shows a few numerical examples concerning the relative accuracy of the recursive method compared to "brute-force" convolution in the case of a Bell Labs EMP excitation [8-9]. This drive waveform is being used here as an example; the theoretical model and ATLOG code are general and can be used to characterize transmission-line output for any pulse waveform.

2. BACKGROUND

We aim to model the EMP excitation problem of a finite-length wire above ground depicted in Figure 1. Our goal is to compute the temporal and spatial electric current and electric potential difference variation, excited in the wire, from such EMP coupling.

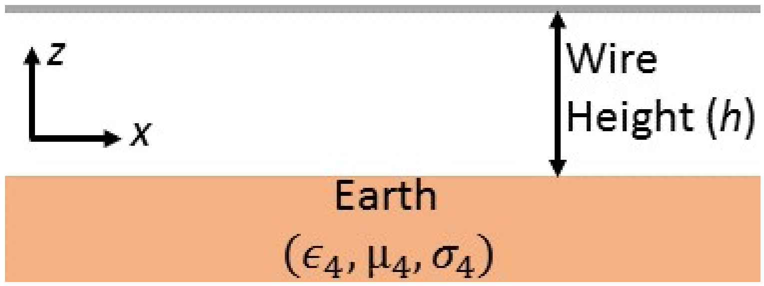

Figure 1. Problem geometry: A wire is located at height *h* from a ground plane (Earth) with electric permittivity ϵ_4, magnetic permeability μ_4, and conductivity σ_4.

Deferring the technical details of the time-domain TL formulation to [4-6], we note the primary equation whose numerical computation must be accelerated to hasten and make practical late-time predictions of EMP excitation of TLs in addition to just early-time and mid-time predictions. Namely, Eq. (22) of [6], which quantifies the contribution $CI(x,t)$ to the updated voltage, at present time *t* for some point *x* along the TL) from previous time intervals (up to time *t-Δt*) of the wire's current waveform:

$$CI(x,t) = \int_0^{t-\Delta t} \zeta(t-\tau)\frac{\partial i(x,\tau)}{\partial \tau}d\tau \quad . \tag{1}$$

In Eq. (1), $\zeta(t)$ is the (time-integrated) impedance function while $\frac{\partial i(x,\tau)}{\partial \tau}$ is the time-derivative of the current waveform causing the developed electric potential difference along the line. Note that for the (time-integrated) impedance function, we use the function given in Eq. (16) of [6], albeit replacing (based on preliminary numerical studies) the factor of 5 in the two exponential functions with 80.

2.1. Recursive Convolution Formulation for Voltage Update

Equation (1) is tractable to compute directly ("brute-force") for early-time and mid-time predictions and/or short TLs (tens or hundreds of meters), but becomes unwieldy for long-length TLs and especially late-time predictions. To accelerate this, the authors of [6] propose a recursive evaluation that, by incorporating the previously-calculated $CI(x,t)$ into the computation of its update $CI(x,t+\Delta t)$, allows for a dramatic reduction in computational burden.

Now, assume that one can synthesize the impedance function $\zeta(t)$ as a sum of R (in general complex-valued) exponentials as follows:

$$\zeta(t) = \sum_{r=1}^{R} A_r e^{\alpha_r t} \quad . \tag{2}$$

Two notes are in order. First, the exponential coefficients $\{A_r\}$ and frequencies $\{\alpha_r\}$ are, in general, complex-valued and are such that they must exhibit the standard symmetry property required for yielding a real-valued $\zeta(t)$. Second, in the computer implementation of this code, typically a small imaginary residual remains in (ideally) real-valued functions due to using finite-precision computer arithmetic. This is eliminated through taking the real part of the final computed function in question after summing.

The authors of [6] show that one can exploit the time-shift property of complex exponentials to develop a recursive convolution relation for the updated voltage history associated with a given exponential basis function $CI_r(x,t)$, as shown in Eq. (26) of [6]:

$$CI_r(x,t+\Delta t) = e^{\alpha_r \Delta t}\left[CI_r(x,t) + \int_{t-\Delta t}^{t} A_r e^{\alpha_r (t-\tau)} \frac{\partial i(x,\tau)}{\partial \tau} d\tau\right]. \tag{3}$$

Summing the $\{CI_r\}$, one for each of the R exponential basis functions, yields the updated voltage history function CI at some x. One may wonder whether this summation is efficient, given the number of exponentials that may be involved. From our experience, typically only 3-4 exponentials at most are needed.

To extract an exponential basis representation for the impedance function, we opt to use the GPOF method, overviewed in the next sub-section.

2.2. Approximate Exponential Basis Extraction

The authors of [6] use a classic complex-exponential basis extraction method known as Prony's Method. We instead opt to use a more robust extraction technique known as the GPOF, which we outline below and whose details we defer to [7]. The primary advantage of GPOF over Prony's Method is the computation of the SVD of the data matrix, which essentially allows one to set and enforce a threshold to decide and eliminate which data is largely redundant from the sampled time series of the impedance function [7].

The GPOF method does contain heuristic aspects admittedly, such as the choice of a "pencil parameter" L [7], and from our experience the method can occasionally produce spurious exponential bases that we take steps to mitigate (discussed below). However, once minor measures are taken to mitigate the shortcomings of this heuristic method, for our choice of GPOF parameters (see below) it does produce adequate early- and mid-time representations (about ≤ 100ns) of the impedance function for a broad range of environment parameter combinations (ϵ_4, σ_4, h).

The basic steps of the GPOF are as follows, given N samples, pencil parameter L, uniform sampling period T, and SVD threshold P (i.e., singular values of the data matrix $[Y]$ that are below P times the value of the data matrix's largest singular value are discarded). Note that for reference, based on our preliminary numerical studies and recommendations from [7], we chose N=10, L=4, T=10ns, and P=10^{-6}.

- Take N uniformly-spaced samples of the impedance function.
- Construct a $(N-L) \times (L+1)$ matrix $[Y]$ formed from these samples (Eq. (16) of [7]).
- Take the SVD of $[Y]$ and (based on P) retain the R' dominant singular values.
- Form modified data matrices $[Y_1]$ and $[Y_2]$ (c.f. Eqs. (21)-(22) of [7]).
- Compute the pseudo-inverse of $[Y_1]$ (i.e., $[Y_1]^+$) and the R'' eigenvalues of $[Y_1]^+[Y_2]$.
- Retain the R=Min[R', R''] dominant eigenvalues λ_r (with respect to eigenvalue magnitude).
- Compute the frequency $\alpha_r = \sigma_r + i\omega_r$ (r = 1, 2, …, R) for each basis functions.
  - Damping coefficient $\sigma_r = Ln[|\lambda_r|]/T$
  - Oscillation coefficient $\omega_r = \tan^{-1}\left(Im[\lambda_r]/Re[\lambda_r]\right)/T$
- Compute the amplitudes A_r via solution of a linear system (c.f. Eq. (25) of [7]).

Since the impedance function we were interested in exhibits decay (versus time) and minimal oscillation, the GPOF algorithm often yielded purely real-valued frequencies and amplitudes (albeit marred somewhat by finite computer precision). Four practical measures, based on our numerical studies, are implemented in the computer code:

- Check that the complex frequencies and amplitudes obey the symmetry relation (to within some tolerance) required to ensure the impedance function is real-valued.
  - Moreover, to ensure the updated voltage history is real-valued, take the real part of the new history contribution before adding it to the previous history.
- Check for positive-valued σ_r coefficients, which correspond to exponentially growing basis functions (invalid on physical grounds). Discard the associated α_r and A_r.
- Set a threshold (we use $P_2 = 10^{-5}$). If the magnitude of some ω_r is less than P_2, set that $\omega_r = 0$.
- If, for some A_r, the ratio $\left|\frac{Im[A_r]}{Re[A_r]}\right| \le P_2,$ then set Im[A_r]=0.

3. NUMERICAL STUDIES

To reiterate, for all studies below we choose the following GPOF parameters: $N = 10$, $L = 4$, $T =$ 10 ns, and $P = 10^{-6}$.

3.1. Accuracy of Function-Fitting

As a preliminary check on the new algorithm's accuracy in predicting TL response to EMP excitation, in the following three pages of figures we first examine (for the same three wire/ground geometry cases explored in Section 3.2) the capability of the GPOF algorithm to accurately capture the true impedance function using an exponential basis. For all three cases, the wire is at height h = 10 m above the ground, the ground's magnetic permeability is that of vacuum ($\mu_4 = \mu_0$), and the ground's electric permittivity is $\varepsilon_4 = 10\varepsilon_0$. The three cases differ only in ground conductivity σ_4: The first, second, and third cases exhibit ground conductivity $\sigma_4 =$ 0.0015, 0.01, and 0.1 S/m respectively. Figures 2, 3, and 4 show the impedance function (reference/true function and its fit) in panels (a) and relative error between the true and fit functions in panels (b). As one can see, the early-time and mid-time exponential fit errors are quite low, while the error grows rapidly afterward. This is due to the finite number of samples taken and finite sampling period, preventing one from time-sampling and hence accurately fitting the impedance function's behavior after ~100 ns (recall N = 10 and T = 10 ns, whose product is 100 ns). That being said, since the impedance function exponentially decays in the late-time regime and is relatively small in value compared to the early-time impedance, one wouldn't expect large errors in applying the impedance function fit within the convolution integral, and this appears to be corroborated by the results in Section 3.2.

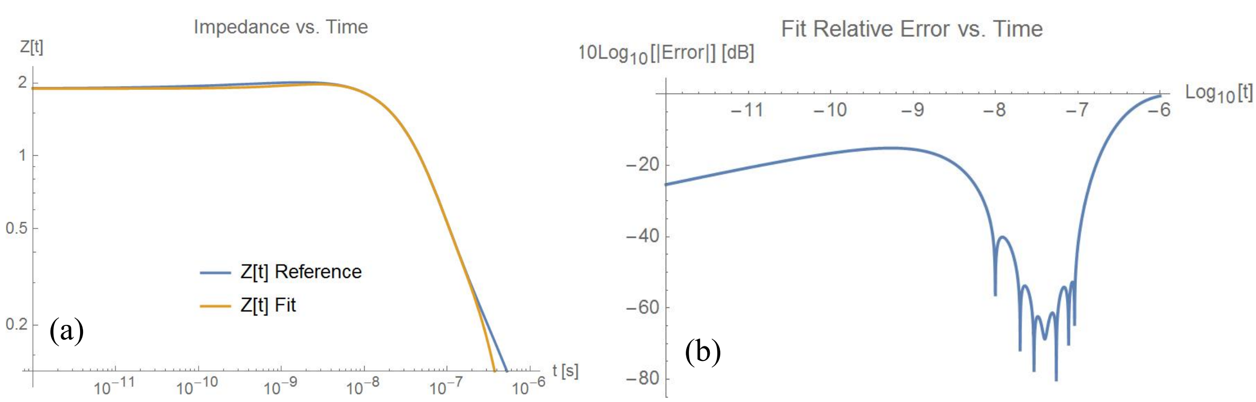

Figure 2. (a) True vs. fit impedance functions and (b) their relative difference for $\sigma_4 = 0.0015\,\mathrm{S/m}$.

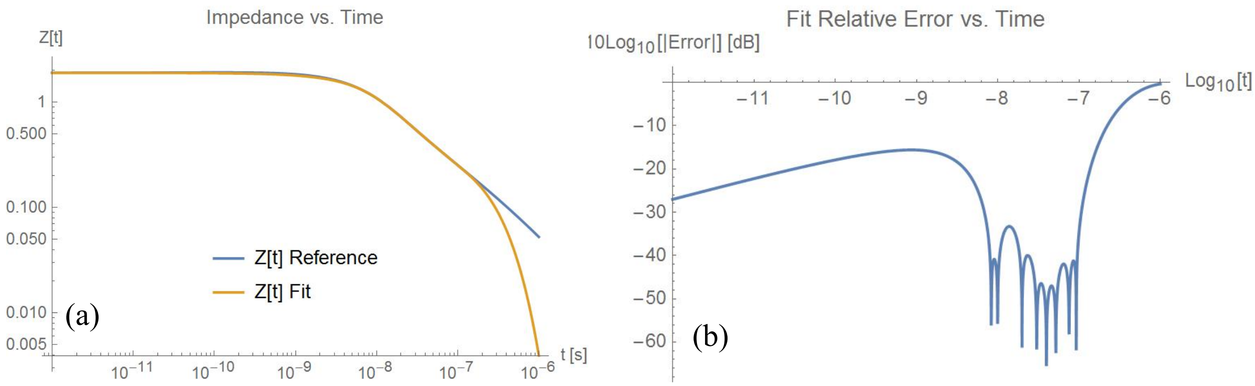

Figure 3. (a) True vs. fit impedance functions and (b) their relative difference for $\sigma_4 = 0.01\,\mathrm{S/m}$.

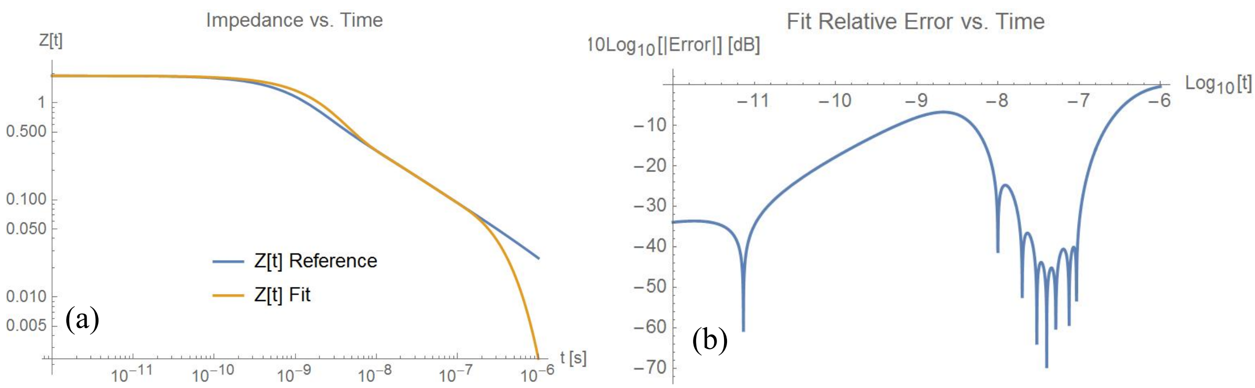

Figure 4. (a) True vs. fit impedance functions and (b) their relative difference for $\sigma_4 = 0.1\,\mathrm{S/m}$.

3.2. Current excited from a Bell Labs EMP excitation

In this section we report results from a Bell Labs EMP excitation. Following the formulation in [4-5], the parameters we take on the simulations are as follows: lossy ground with $\varepsilon_4 = 10\varepsilon_0$ and $\sigma_4 = \{0.1, 0.01, 0.0015\}\,\mathrm{S/m}$, and magnetic permeability $\mu_4 = \mu_0$, $\varepsilon_2 = \varepsilon_0$ (i.e. no dielectric coating, of radius, b surrounding the metal wire), $a = b = 1.27\,\mathrm{cm}$, and $\sigma_0 = \dfrac{1}{R\pi a^2} = 2.9281\times 10^7\,\mathrm{S/m}$ using $R = 6.74\times 10^{-5}\,\Omega/\mathrm{m}$. We consider a 100 m long wire above ground with height $h = 10\,\mathrm{m}$. The finite line is left open-circuited at both ends.

The induced current in the middle of the 100 m line computed using the frequency-domain and the time-domain (both “brute-force” and with the recursive method) implementations of ATLOG is reported in Figure 5 to Figure 7 for various ground conductivity conditions as described in the figure caption. While the “brute-force” time domain simulations complete in about 5 hours, the GPOF simulations complete in less than a minute. Great agreement is observed among the different methods, especially for the high conductivity case of $\sigma_4 = 0.1\,\mathrm{S/m}$. For smaller ground conductivities, we observe a time delay at late times when using the GPOF algorithm, likely due to a non-optimal fit for the ground impedance. Nonetheless, the agreement is remarkable. Note that $R = \dfrac{1}{\sigma_0 \pi a^2}$ is taken as a constant in the time domain solutions whereas the resistance per unit length R (and the internal inductance) of the wire vary in the frequency domain solution.

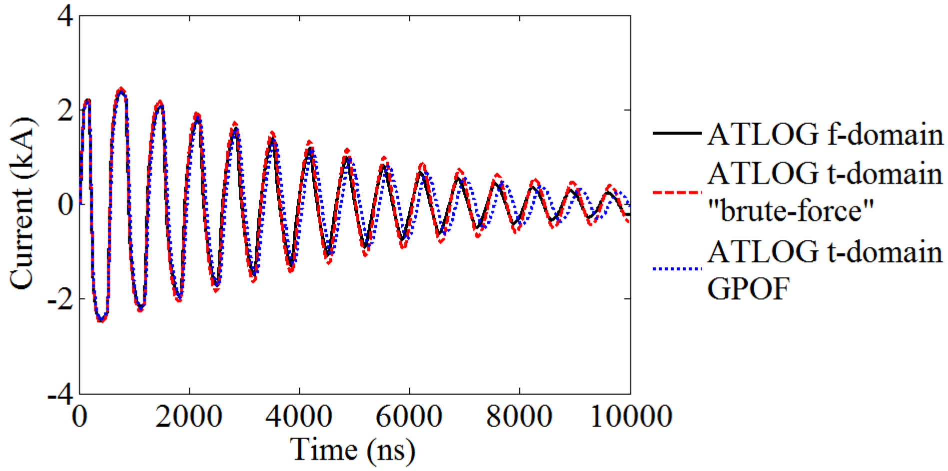

Figure 5. Current versus time for the Bell Labs excitation for a 100 m long line with lossy ground with $\sigma_4 = 0.0015\,\mathrm{S/m}$. Results are based on the time-domain ATLOG model (both "brute-force" and GPOF) and the frequency-domain ATLOG model. The current is evaluated at the center of the wire.

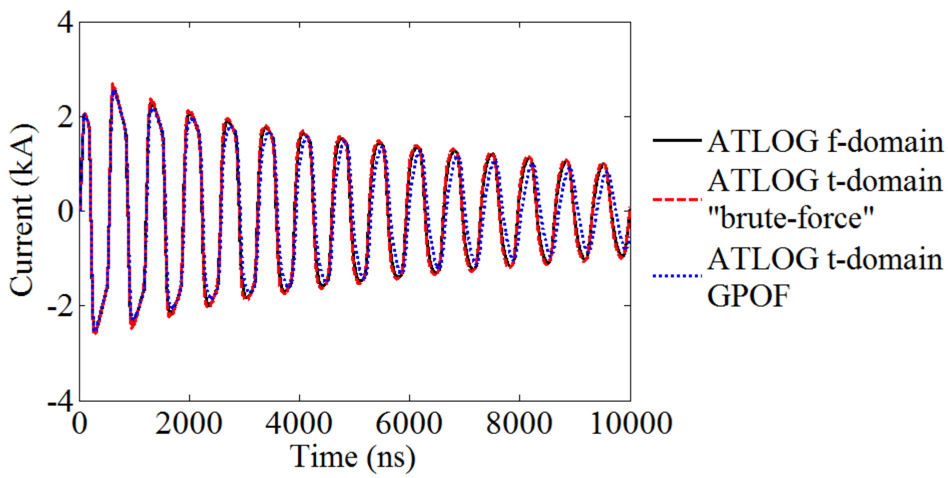

Figure 6. Current versus time for the Bell Labs excitation for a 100 m long line with lossy ground with $\sigma_4 = 0.01\,\mathrm{S/m}$. Results are based on the time-domain ATLOG model (both "brute-force" and GPOF) and the frequency-domain ATLOG model. The current is evaluated at the center of the wire.

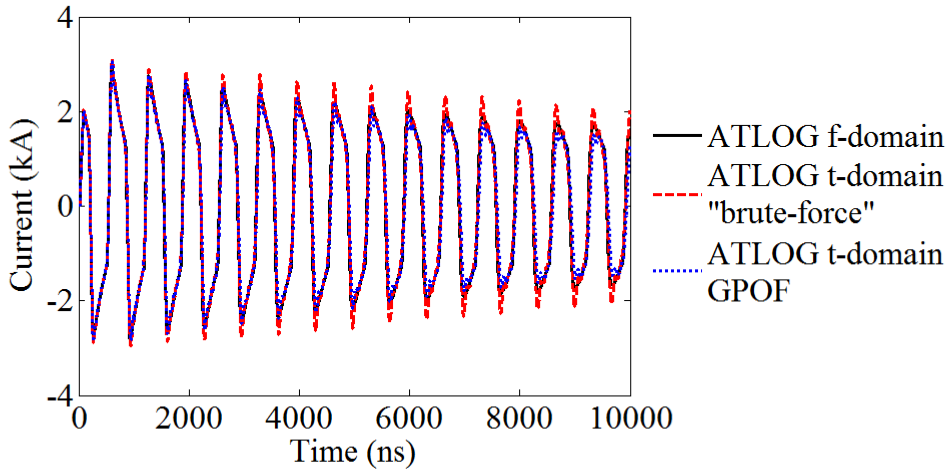

Figure 7. Current versus time for the Bell Labs excitation for a 100 m long line with lossy ground with $\sigma_4 = 0.1\,\mathrm{S/m}$. Results are based on the time-domain ATLOG model (both "brute-force" and GPOF) and the frequency-domain ATLOG model. The current is evaluated at the center of the wire.

4. CONCLUSIONS

In this report we overviewed recent developments in accelerating the production of computational predictions concerning EMP excitation of TLs of both short and long lengths. Namely, this report contains an overview of the fundamentals concerning the incorporated techniques. Results indicate significant computation speed-ups when compared to a "brute-force" implementation, allowing the practical simulation of much longer TLs than previously possible for a given computing resource limitation.

Note that in general other geometrical and material aspects, such as wire length, wire dielectric coating thickness and material parameters (if any coating present), and radius of the wire's metal core may factor into the overall simulation involving the TLs and Earth taken together, and are mentioned in the numerical examples when relevant. Our main focus herein, however, is discussion of accelerating the temporal convolution-based voltage update associated with such simulations, and the geometry/material aspects relevant to that voltage update.

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