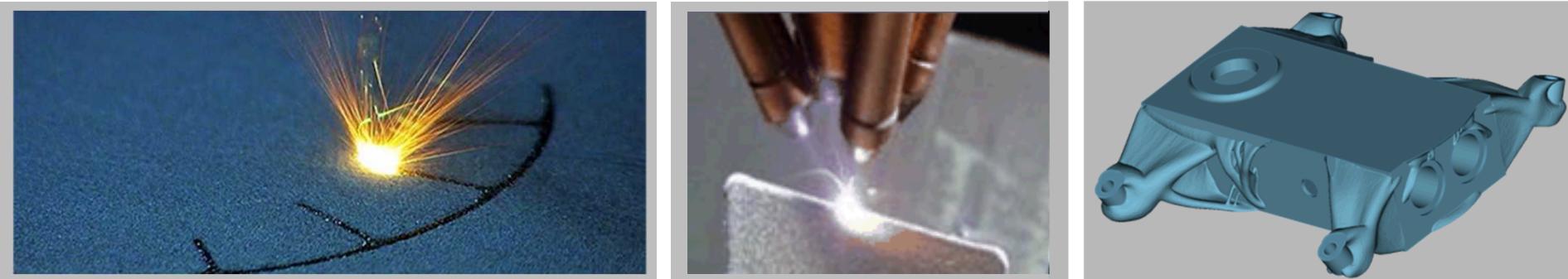


*Exceptional service in the national interest*

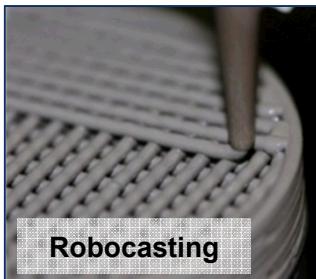
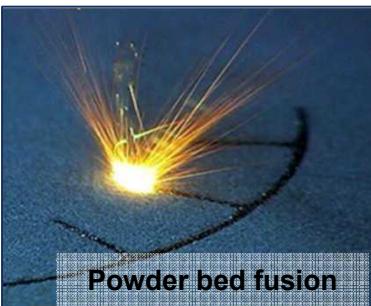


Using a common geometry description to enable  
optimization-based design

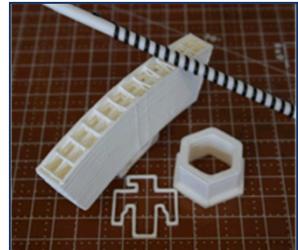
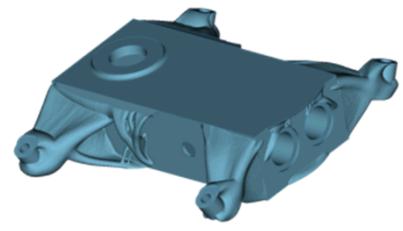
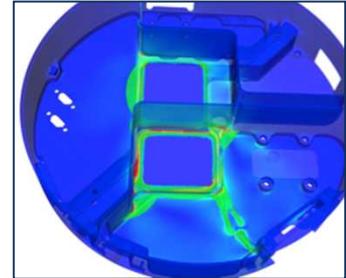
**Joshua Robbins**

# Problem Statement

## Additive Manufacturing Processes



## Design Space



Two main challenges must be overcome:

- **Qualification:** assure quality
- **Design:** effectively utilize AM

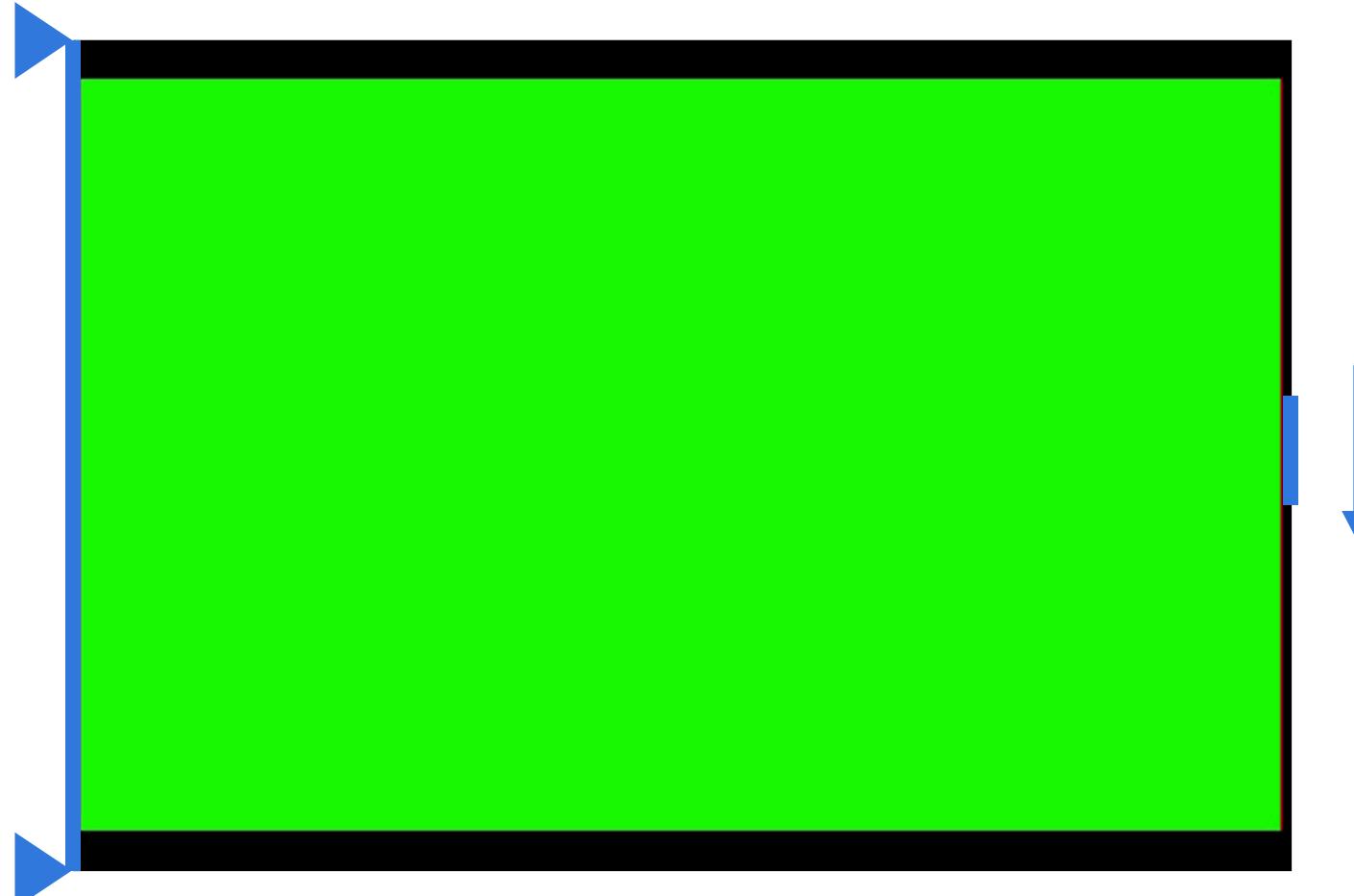
# Topology Optimization

## Forward Problem:

Design + Materials → Response

## Inverse Problem:

Design + Materials ← Response



Objective: optimal stiffness

Constraint: fixed mass

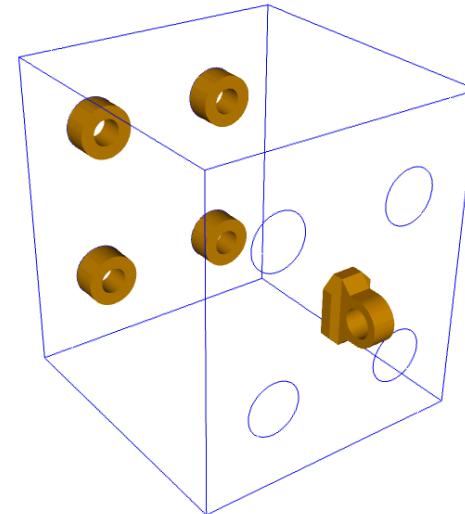
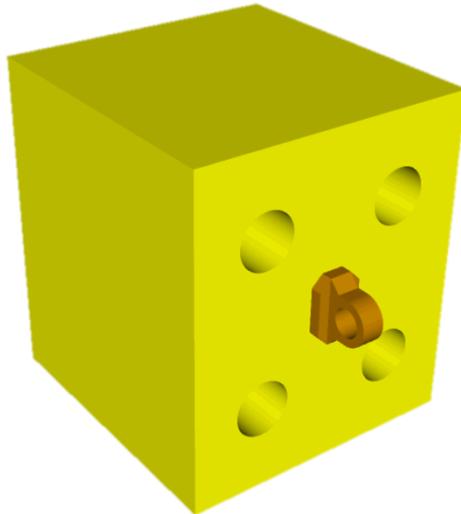
# Topology Optimization

**Objective:**  $\min_z F(\mathbf{u}(\hat{z}), \hat{z})$

**Measure Constraint:**  $s.t. \quad G(\hat{z}) = M$

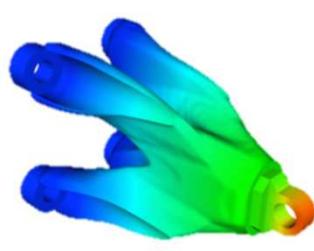
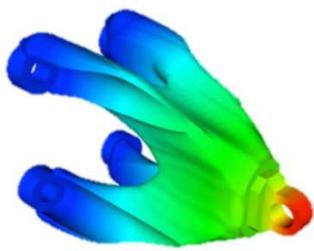
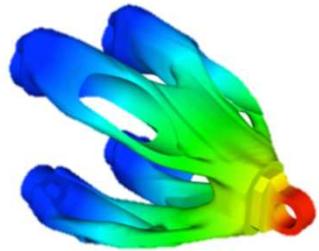
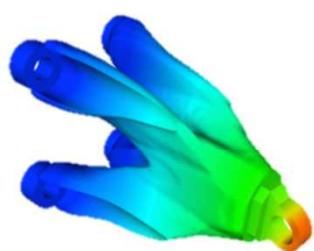
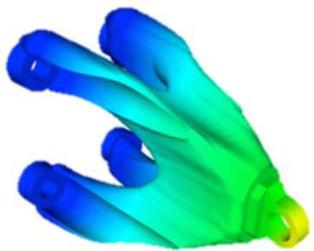
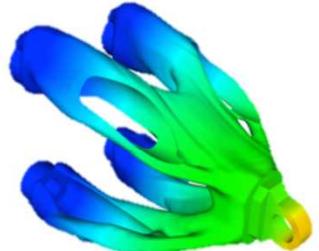
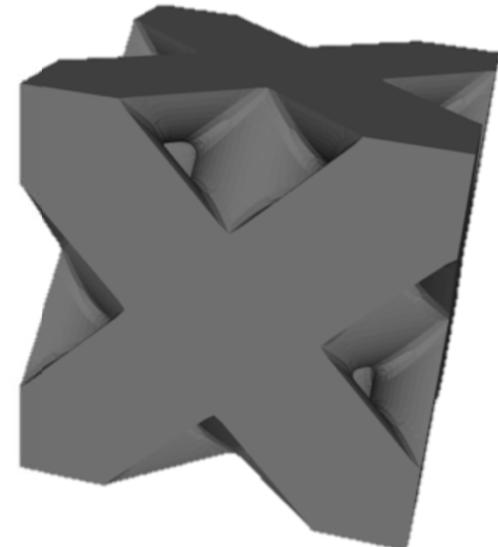
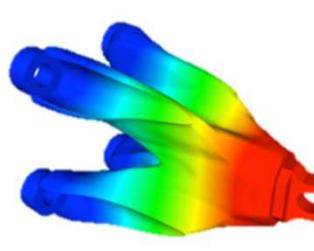
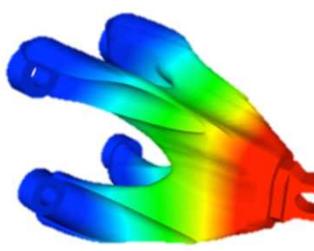
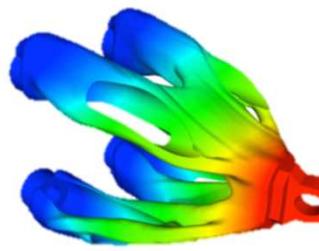
**PDE Constraint(s):**  $H_i(\mathbf{u}(\hat{z}), \hat{z}) = 0, \quad i = 1, \dots, N$

**Density Constraint:**  $\hat{z} = \{\hat{z}_1, \hat{z}_2, \dots, \hat{z}_n\}, \hat{z}_i = P_i(z_i) \in [0, 1]$

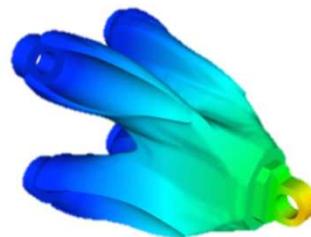
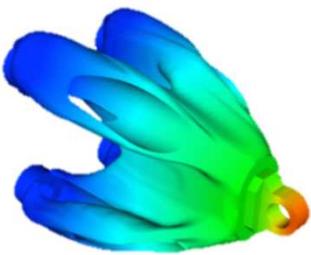
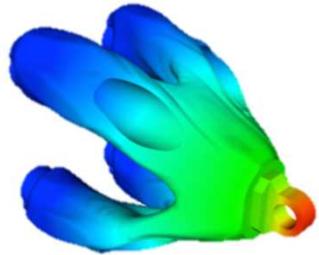
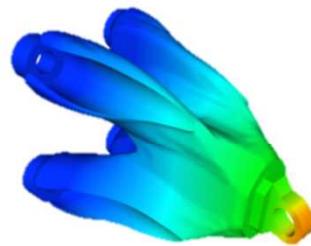
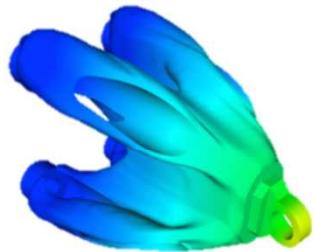
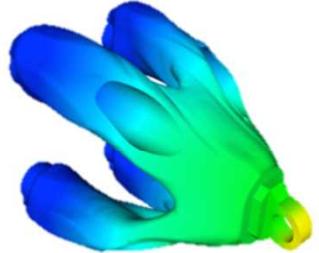
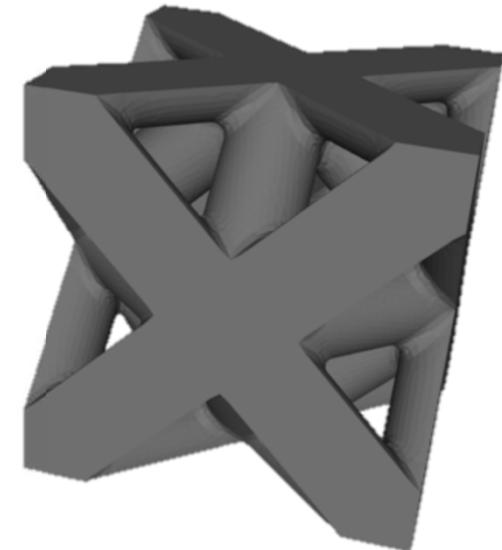
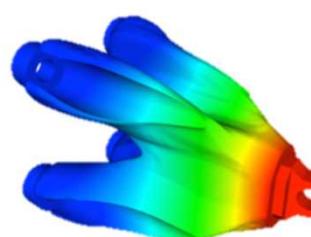
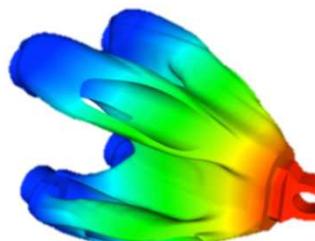
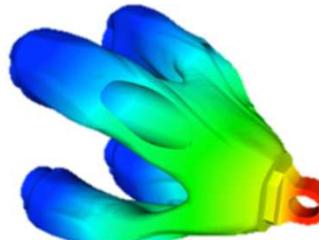


$$\min_{\rho} : \left( \frac{(1-\alpha)}{2} \int_{\partial\Omega_f} \mathbf{f}_1 \cdot \mathbf{u}_1(\rho) \, dV + \frac{(1-\alpha)}{2} \int_{\partial\Omega_f} \mathbf{f}_2 \cdot \mathbf{u}_2(\rho) \, dV + \alpha \int_{\partial\Omega_T} q \, T(\rho) \, dV \right)$$

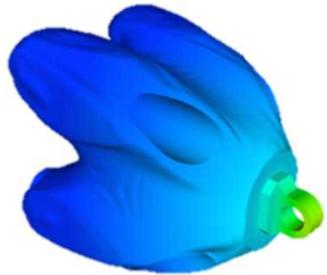
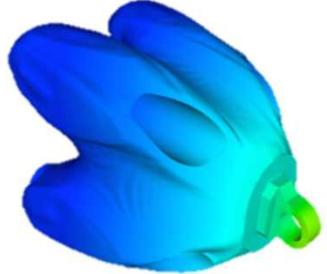
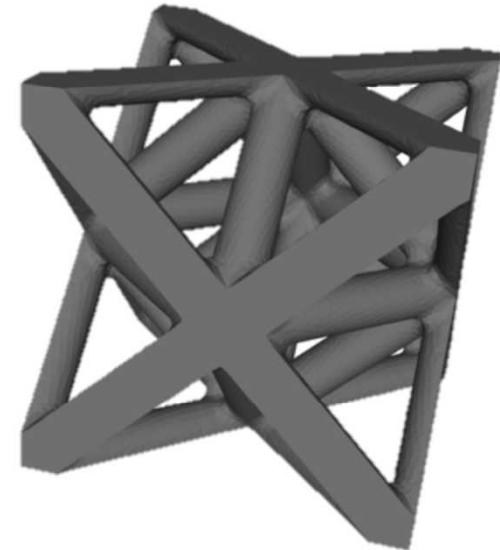
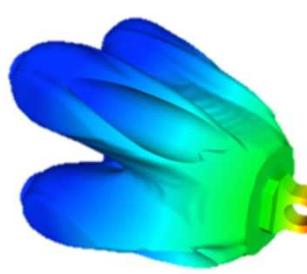
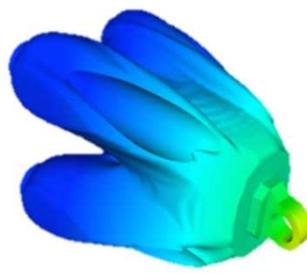
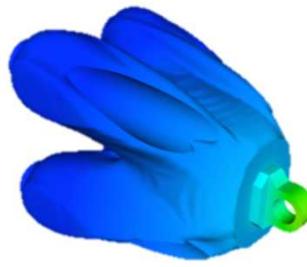
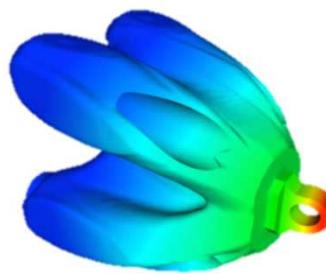
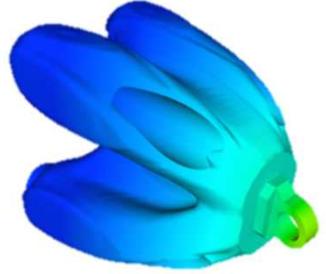
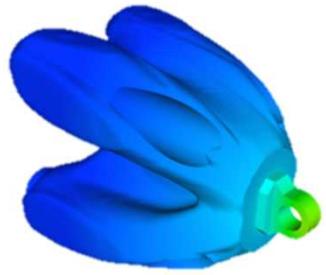
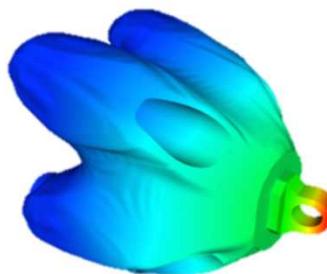
# Cellular (75% dense)

 $\alpha = 0.1$  $\alpha = 0.5$  $\alpha = 0.9$ Thermal:  $T$ Mechanical:  $u_1$ Mechanical:  $u_2$ 

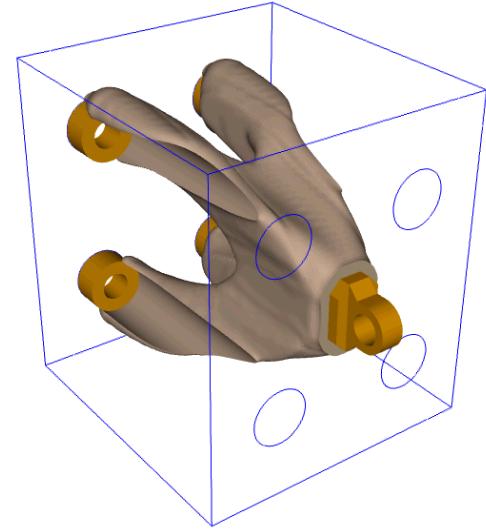
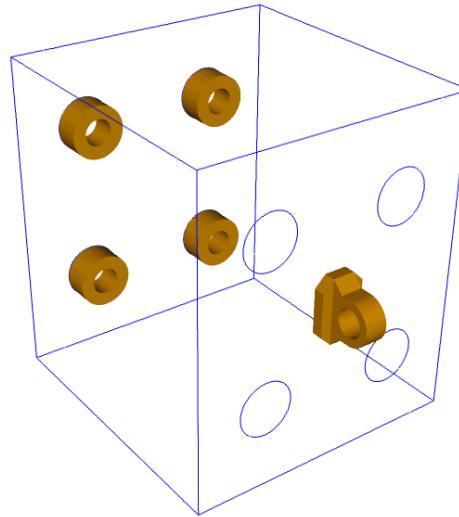
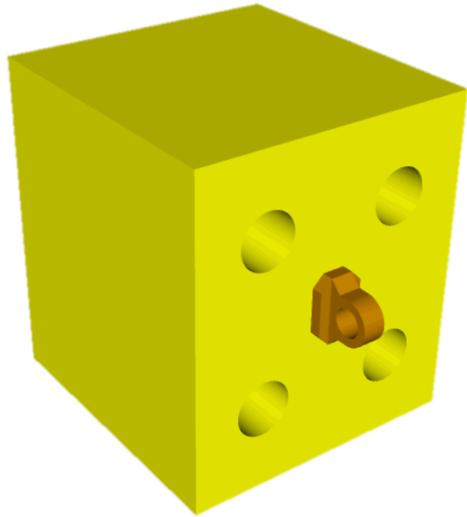
# Cellular (50% dense)

 $\alpha = 0.1$  $\alpha = 0.5$  $\alpha = 0.9$ Thermal:  $T$ Mechanical:  $u_1$ Mechanical:  $u_2$ 

# Cellular (20% dense)

 $\alpha = 0.1$  $\alpha = 0.5$  $\alpha = 0.9$ Thermal:  $T$ Mechanical:  $u_1$ Mechanical:  $u_2$ 

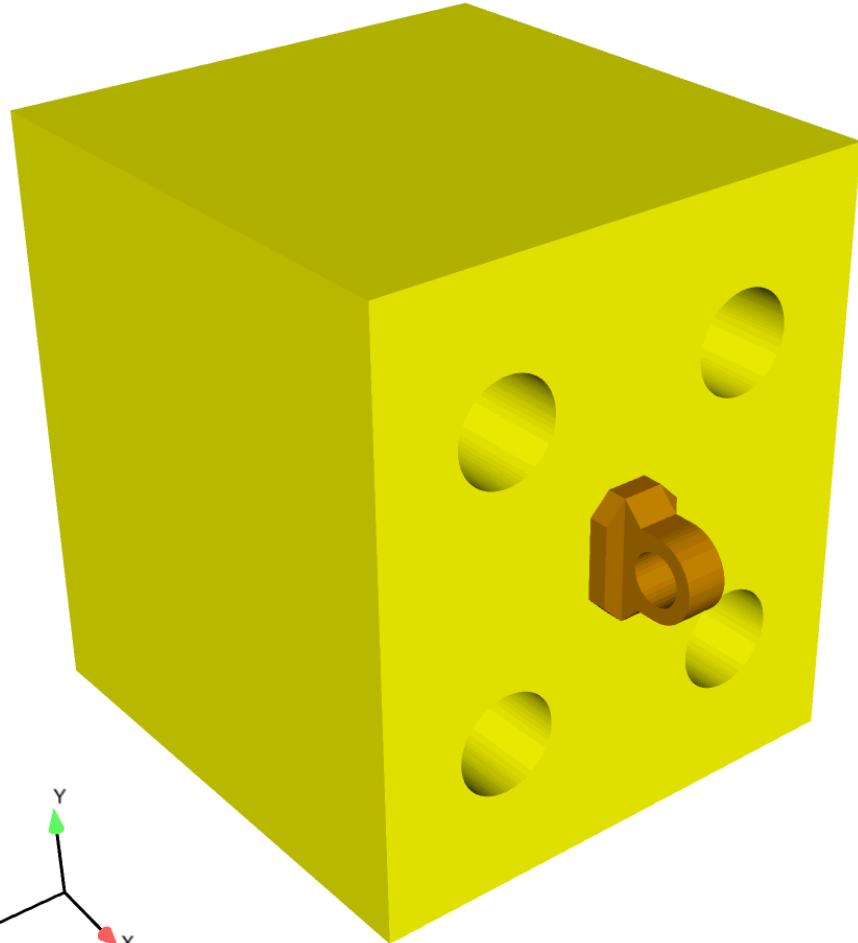
# Shape and Topology Optimization



Are the mounts in the right place, the right size, shape?

Ideally, we'd optimize the shape (number, location, etc) and the topology, concurrently.

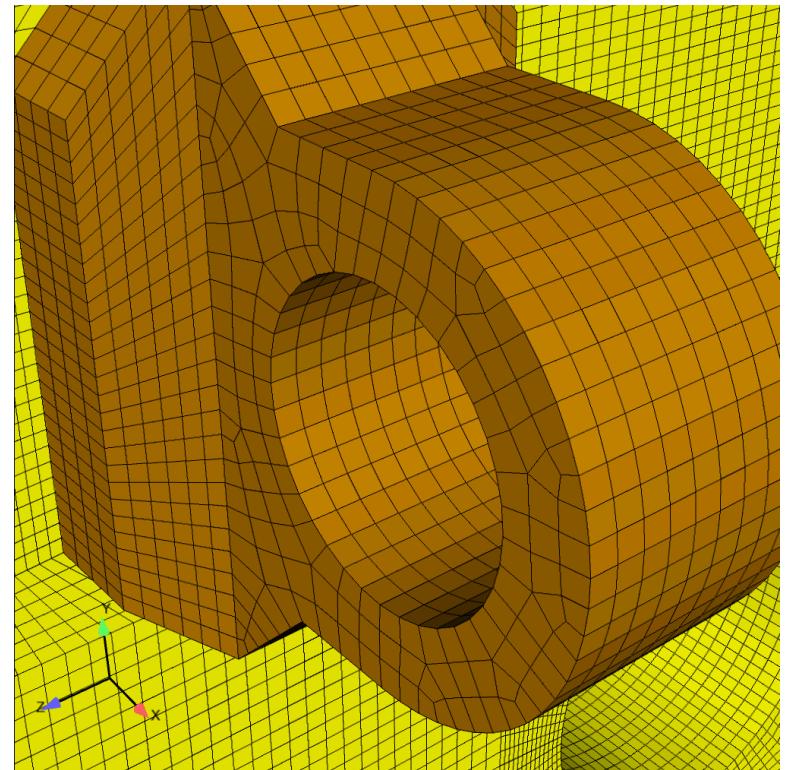
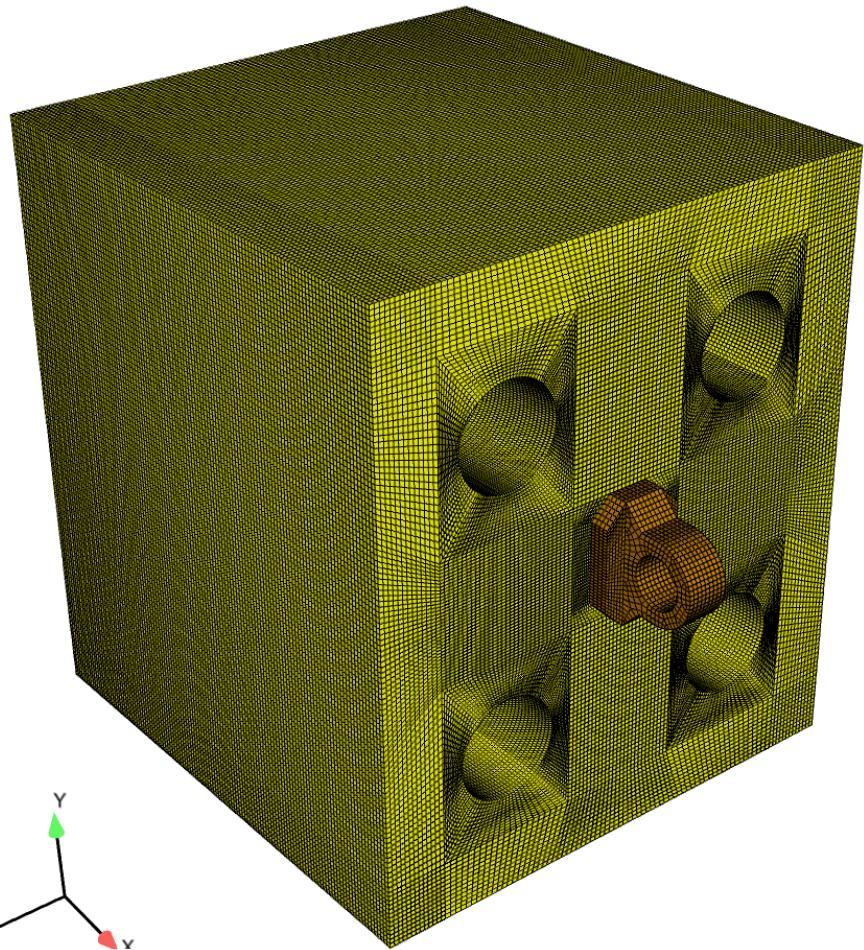
## Solid Modeling



**Surface-based solid model**

# Design Revisited: Volume based geometry

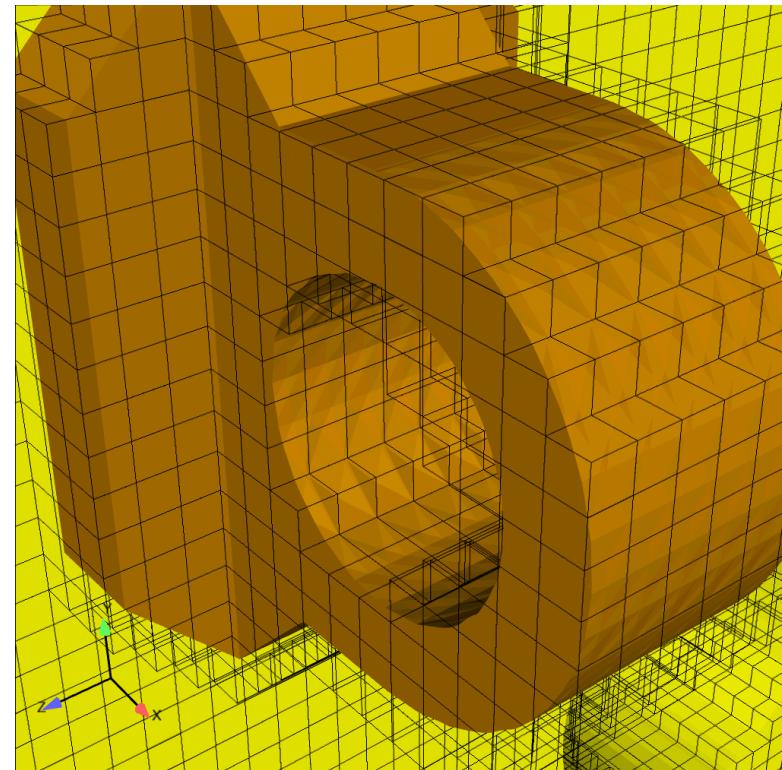
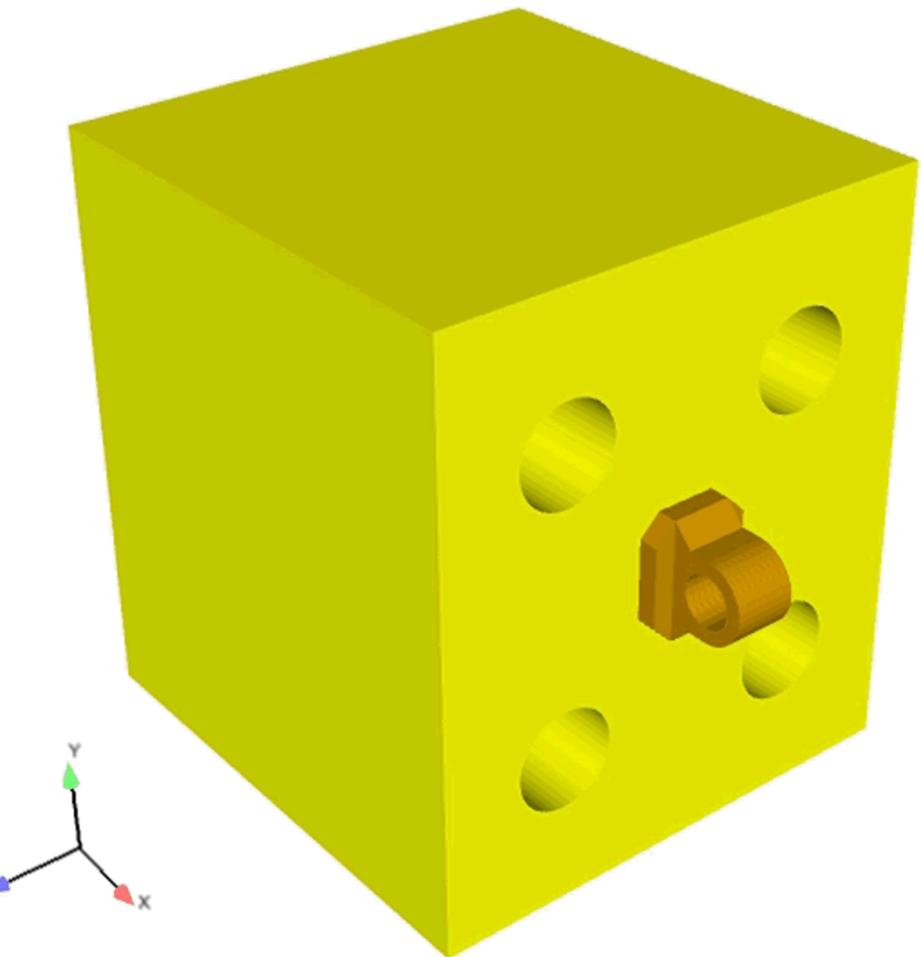
## Simulation



Discrete conformal volume-based solid model

# Design Revisited: Volume based geometry

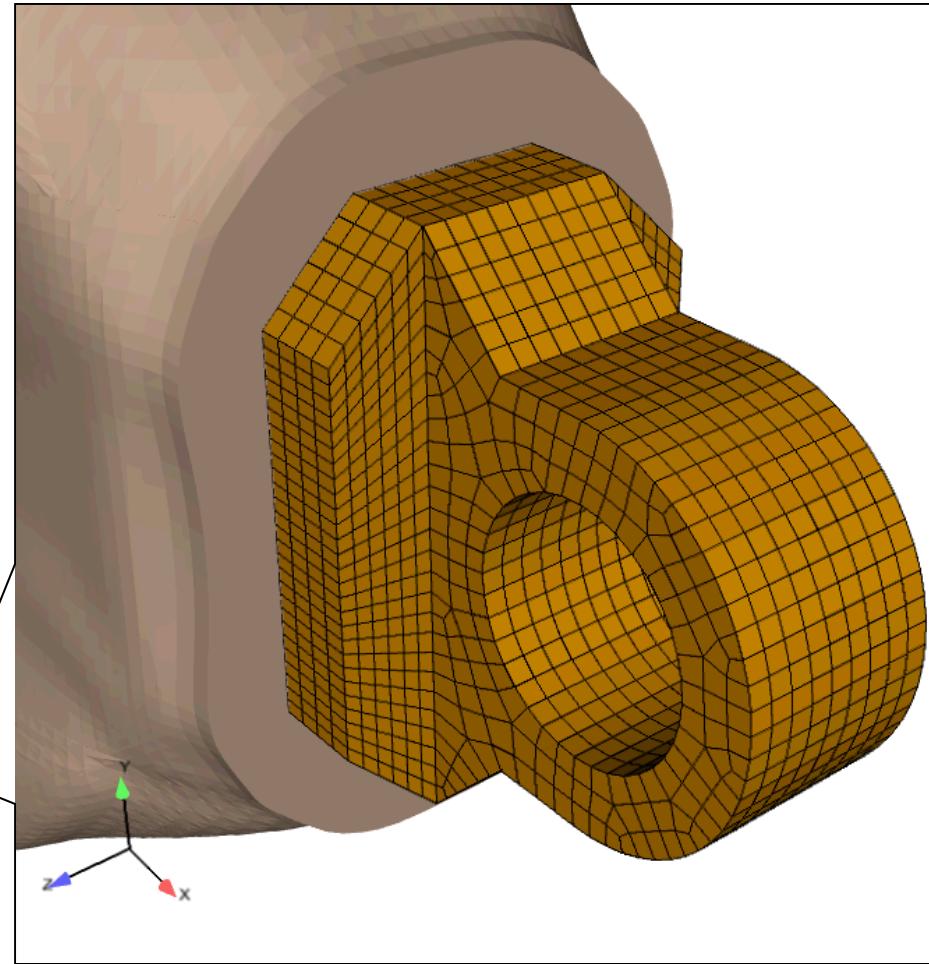
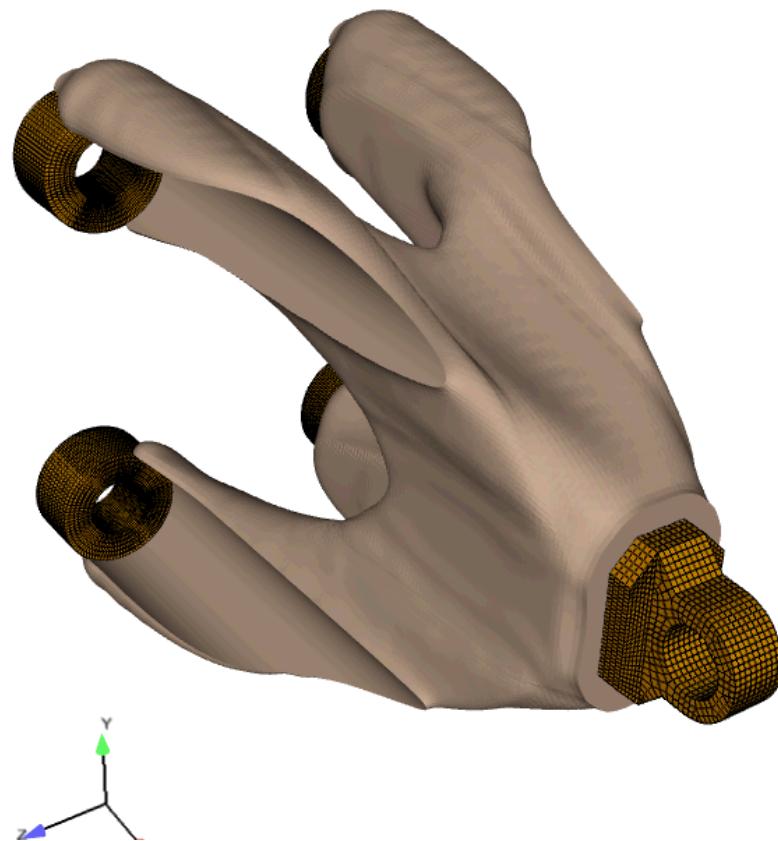
## Simulation



Discrete nonconformal volume-based solid model

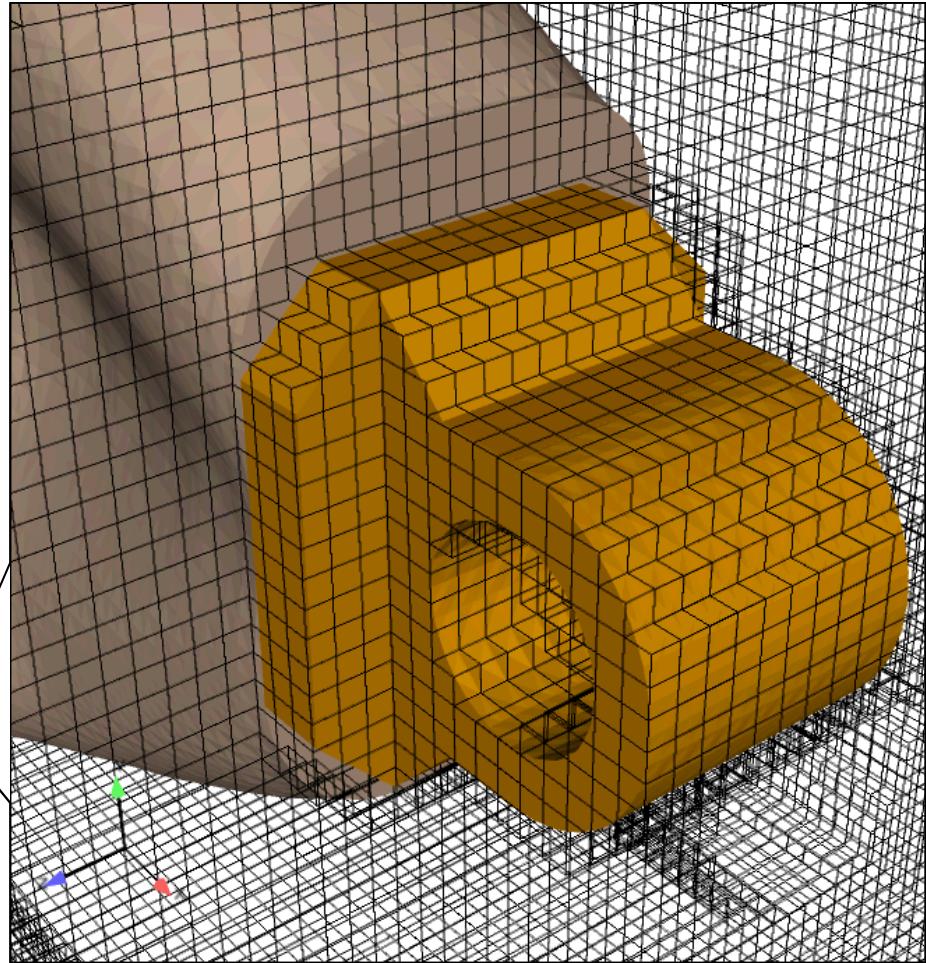
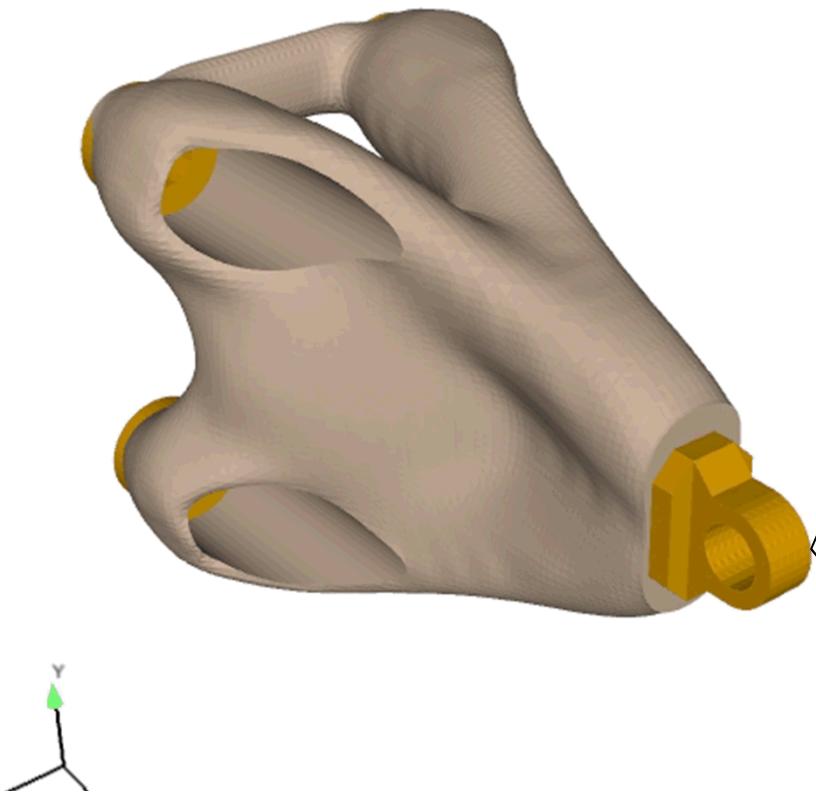
# Design Revisited: Volume based geometry

## Optimization-based design



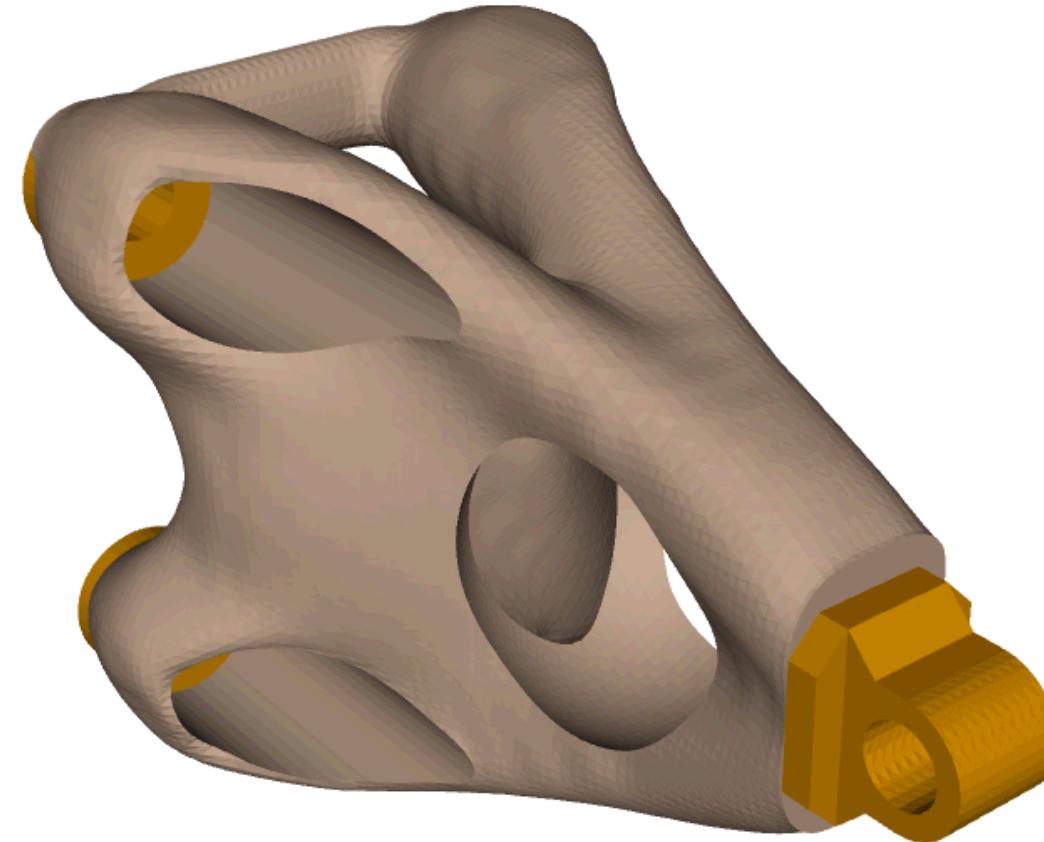
Discrete nonconformal volume-based solid model

## Optimization-based design



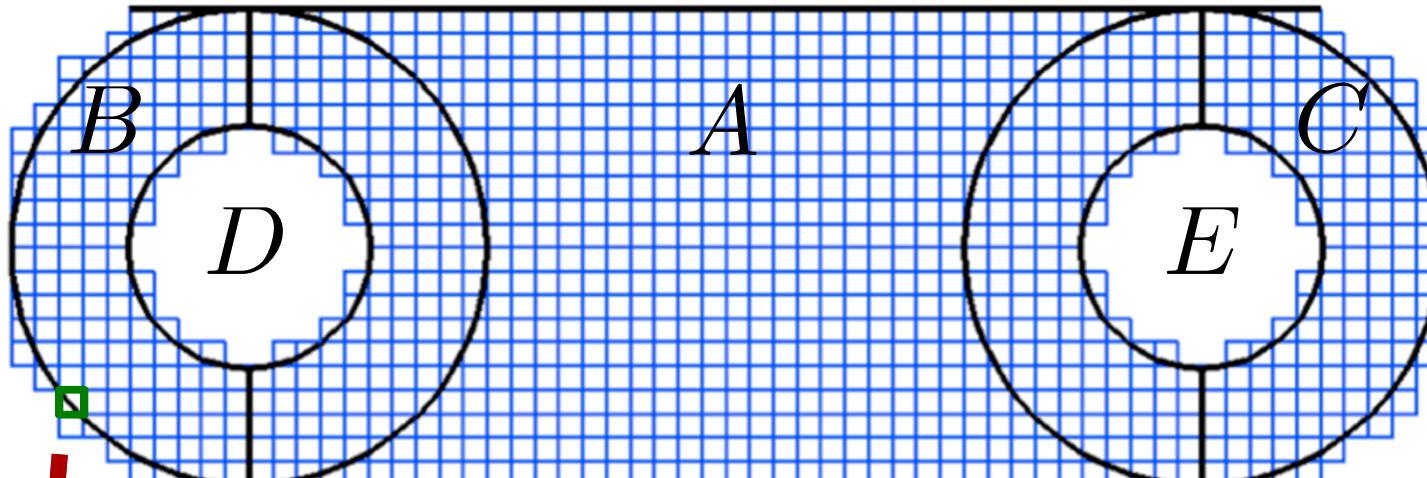
Discrete nonconformal volume-based solid model

## Interaction / Modification

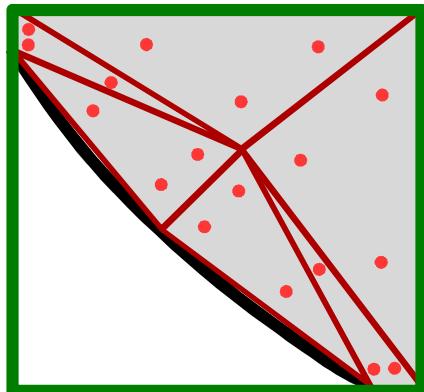


**Discrete nonconformal volume-based solid model**

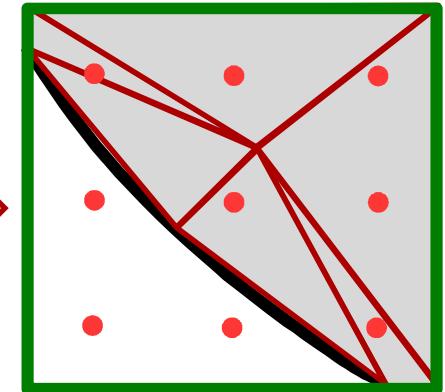
# Shape and topology optimization



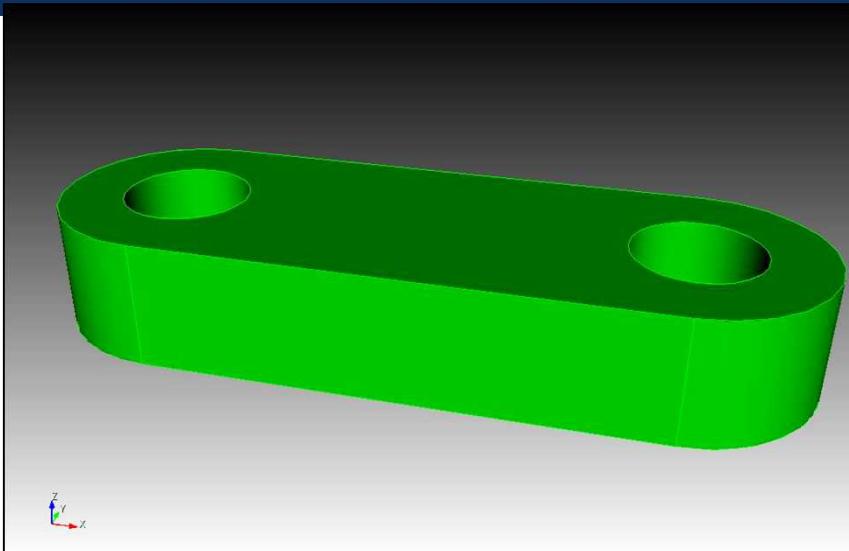
$$(A \vee B \vee C) \wedge \neg(D \vee E)$$



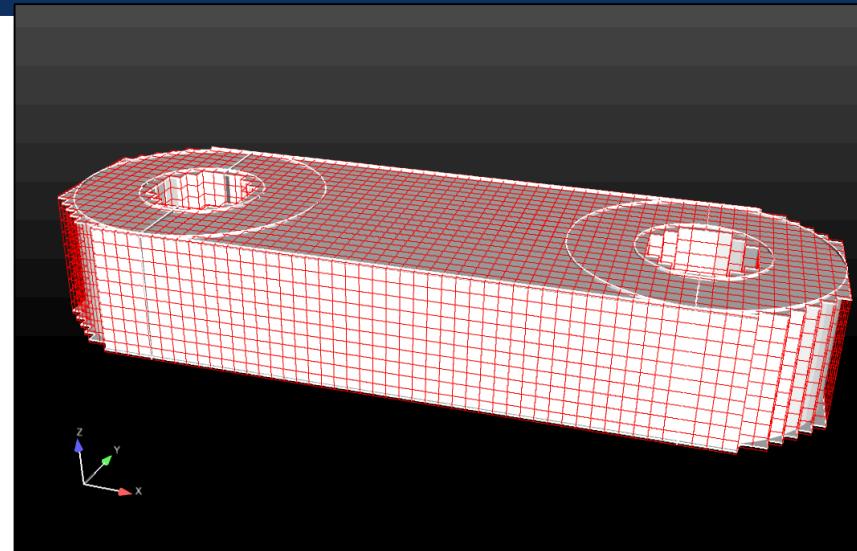
$$\int_{\Omega} g_j(\mathbf{x}) d\Omega = \sum_i \omega_i g_j(\xi_i) \det J_{\mathbf{x}\xi}(\xi_i)$$



# Common Geometry Representation



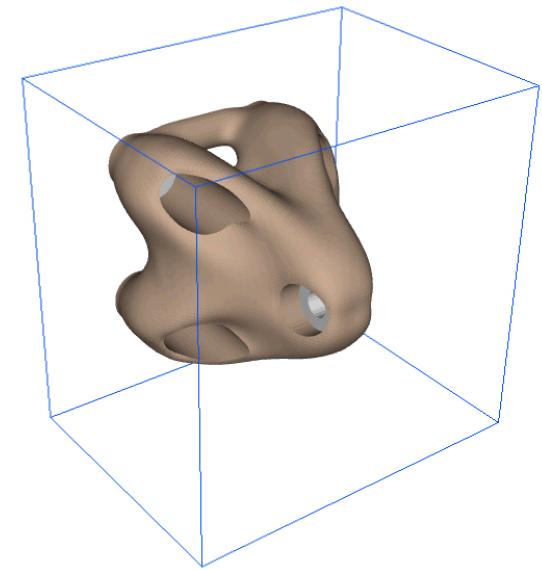
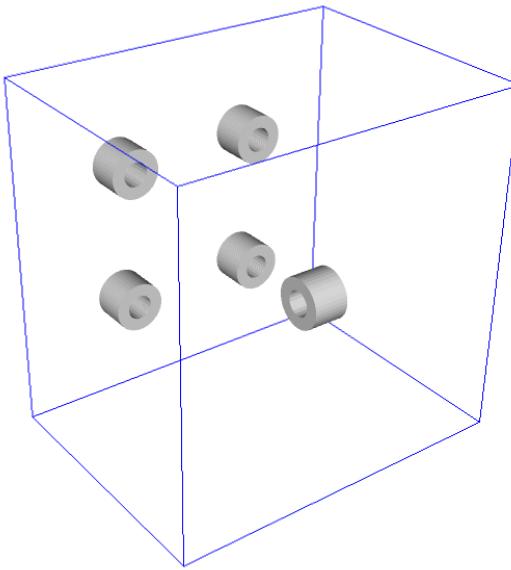
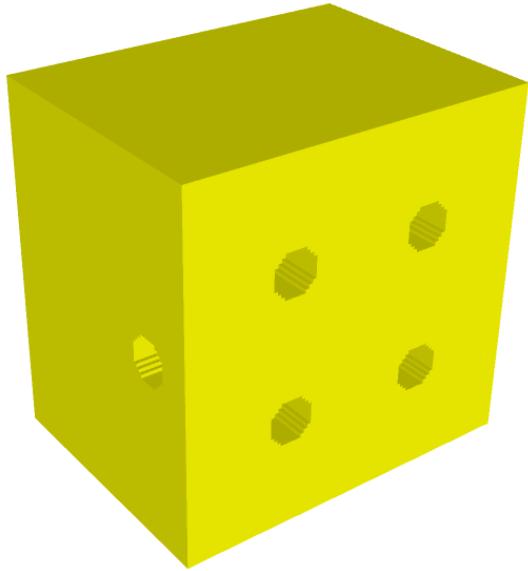
```
create brick x 2 y 1 z 0.5
create cylinder height 0.5 radius 0.5
move volume 2 x 1
unite volume 1 2
create cylinder height 0.5 radius 0.5
move volume 3 x -1
unite volume 1 3
create cylinder height 0.5 radius 0.25
move volume 4 x 1
subtract volume 4 from volume 1
create cylinder height 0.5 radius 0.25
move volume 5 x -1
subtract volume 5 from volume 1
```



```
body = GMeshTools.Body()
brick = GMeshTools.Brick(2.0, 1.0, 0.5)
body.Add(brick)
cylinder = GMeshTools.Cylinder(0.5,0.5,[1,0,0])
body.Add(cylinder)
cylinder = GMeshTools.Cylinder(0.5,0.5,[-1,0,0])
body.Add(cylinder)
cylinder = GMeshTools.Cylinder(0.5,0.25,[1,0,0])
body.Subtract(cylinder)
cylinder = GMeshTools.Cylinder(0.5,0.25,[-1,0,0])
body.Subtract(cylinder)
gmesh.Imprint(body)
```

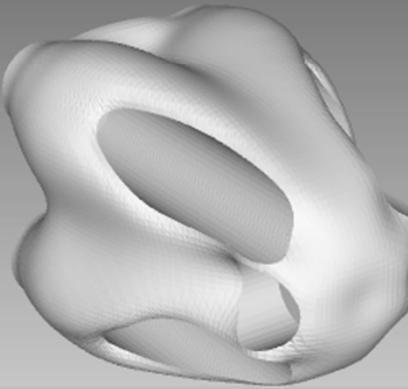
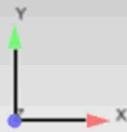
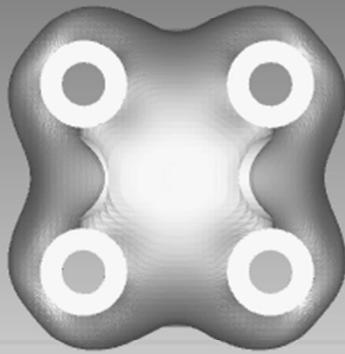
Goal is to implement geometry creation utilities in a stand-alone library (Cogent) that is used by **the design tool, the mod/sim/STMO code, and the post-processor**.

# Topology Optimization with Shape Parameter Study



$$\min_{\rho} : \left( \frac{(1 - \alpha)}{2} \int_{\partial\Omega_f} \mathbf{f}_1 \cdot \mathbf{u}_1(\rho) \, dV + \frac{(1 - \alpha)}{2} \int_{\partial\Omega_f} \mathbf{f}_2 \cdot \mathbf{u}_2(\rho) \, dV + \alpha \int_{\partial\Omega_T} q \, T(\rho) \, dV \right)$$

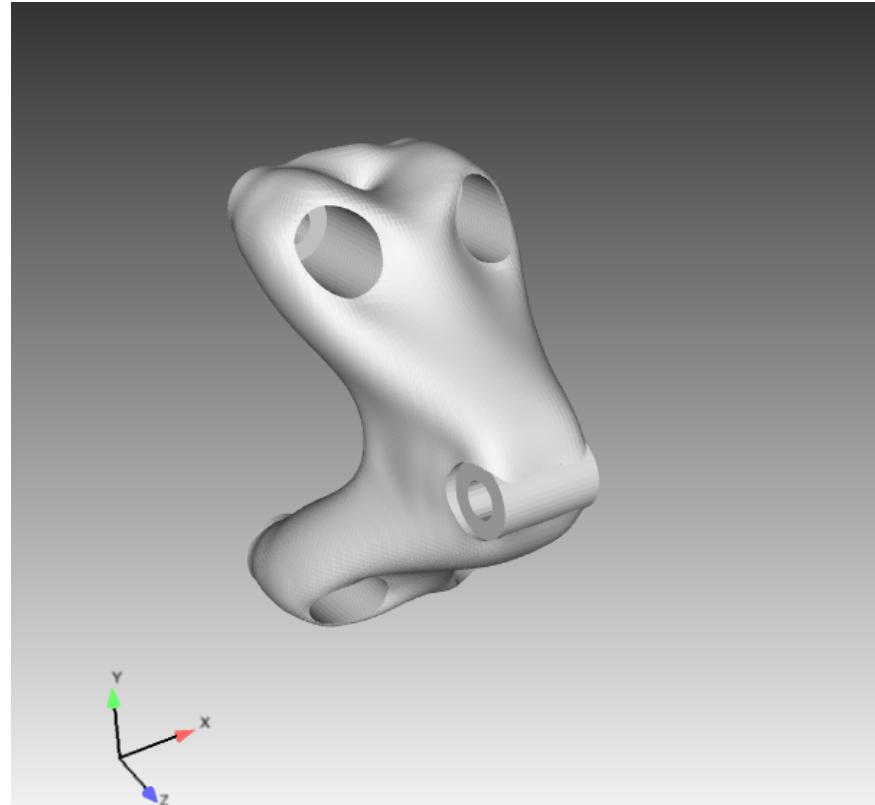
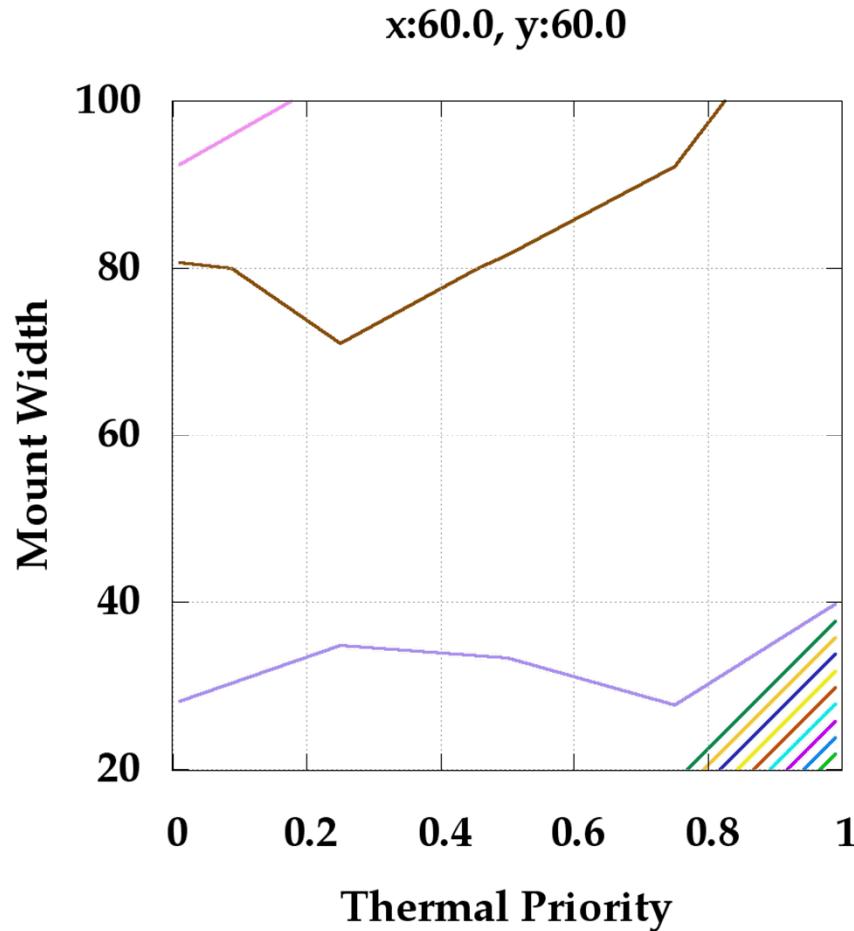
# Topology Optimization with Shape Parameter Study



$$\min_{\rho} : \left( \frac{(1-\alpha)}{2} \int_{\partial\Omega_f} \mathbf{f}_1 \cdot \mathbf{u}_1(\rho) \, dV + \frac{(1-\alpha)}{2} \int_{\partial\Omega_f} \mathbf{f}_2 \cdot \mathbf{u}_2(\rho) \, dV + \alpha \int_{\partial\Omega_T} q \, T(\rho) \, dV \right)$$

# Topology Optimization with Shape Parameter Study

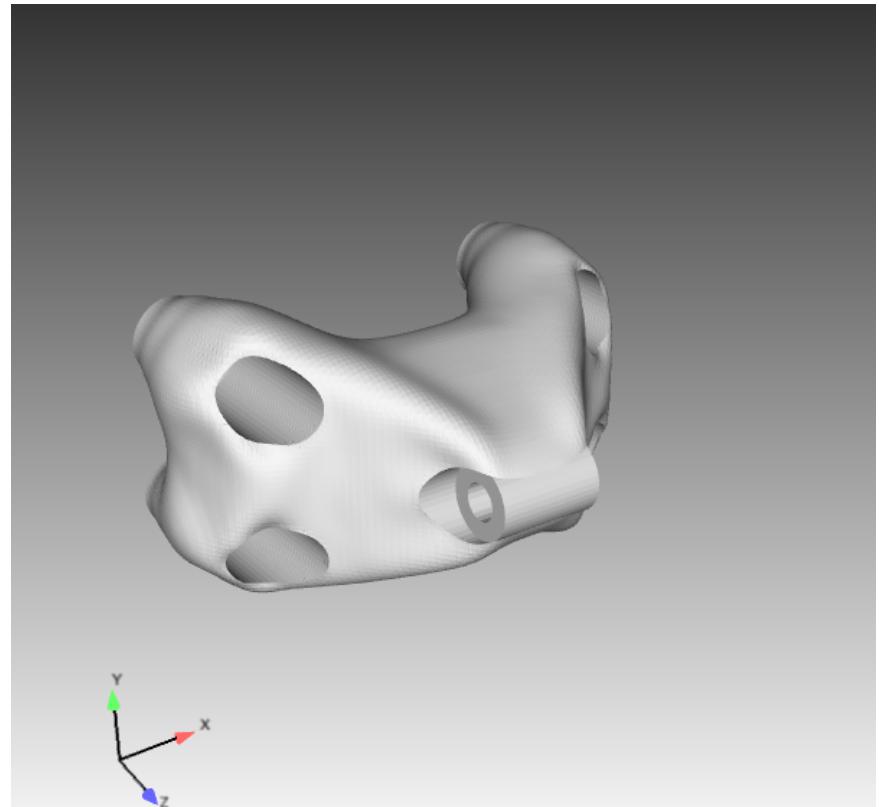
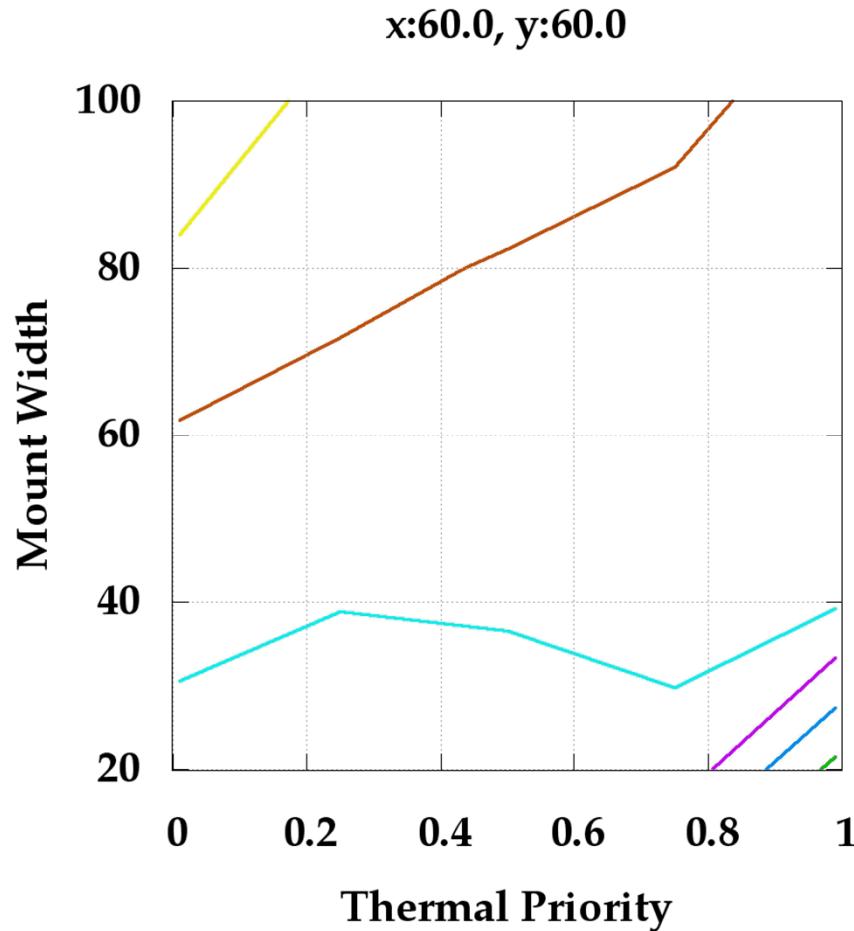
## Vertical Displacement:



X:60.0, Y:140, Mount width: 40.0

# Topology Optimization with Shape Parameter Study

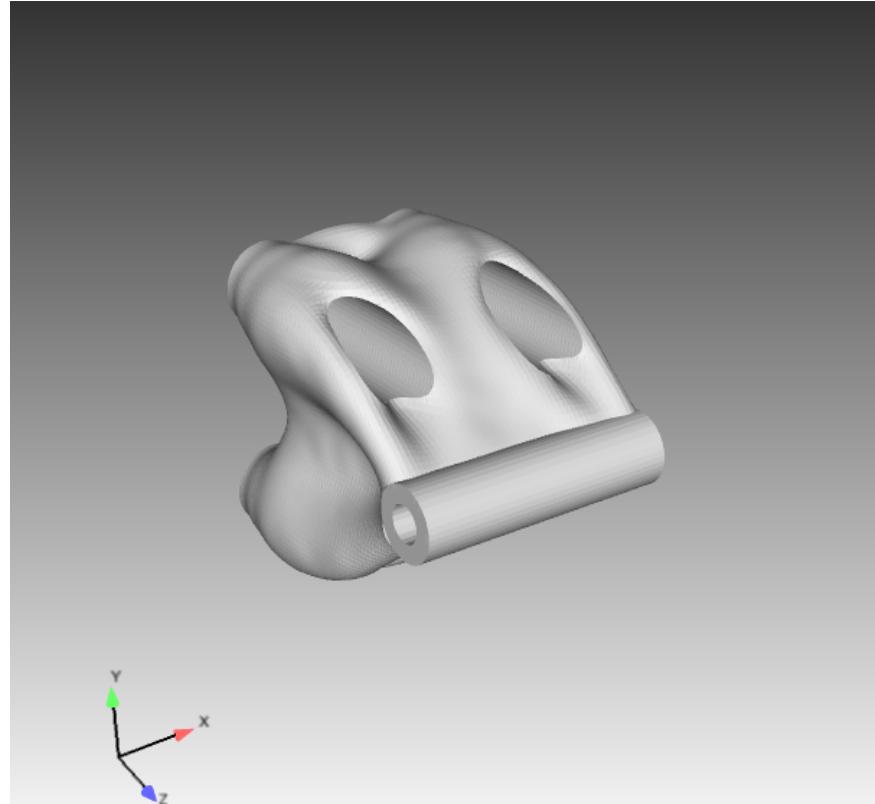
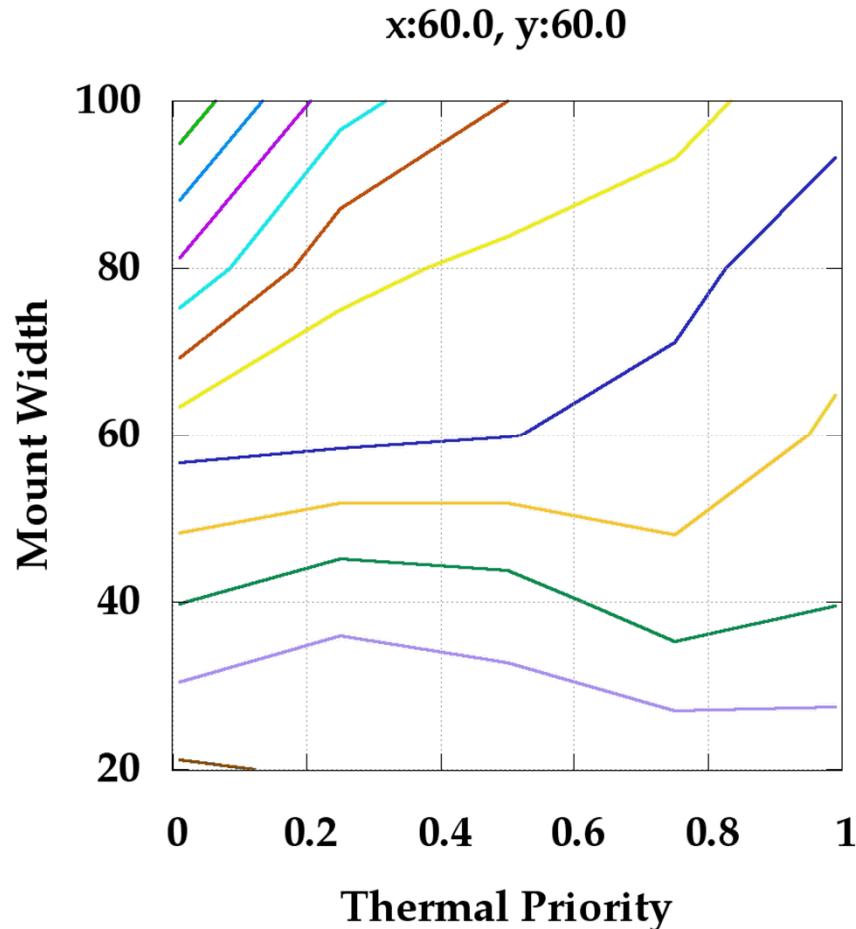
## Lateral Displacement:



X:140.0, Y:60, Mount width: 40.0

# Topology Optimization with Shape Parameter Study

## Temperature Difference:



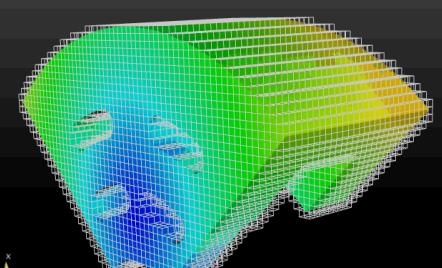
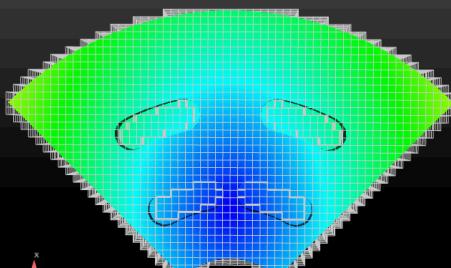
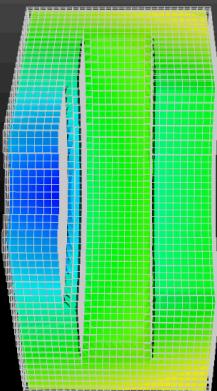
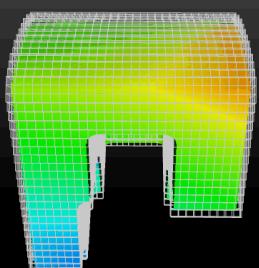
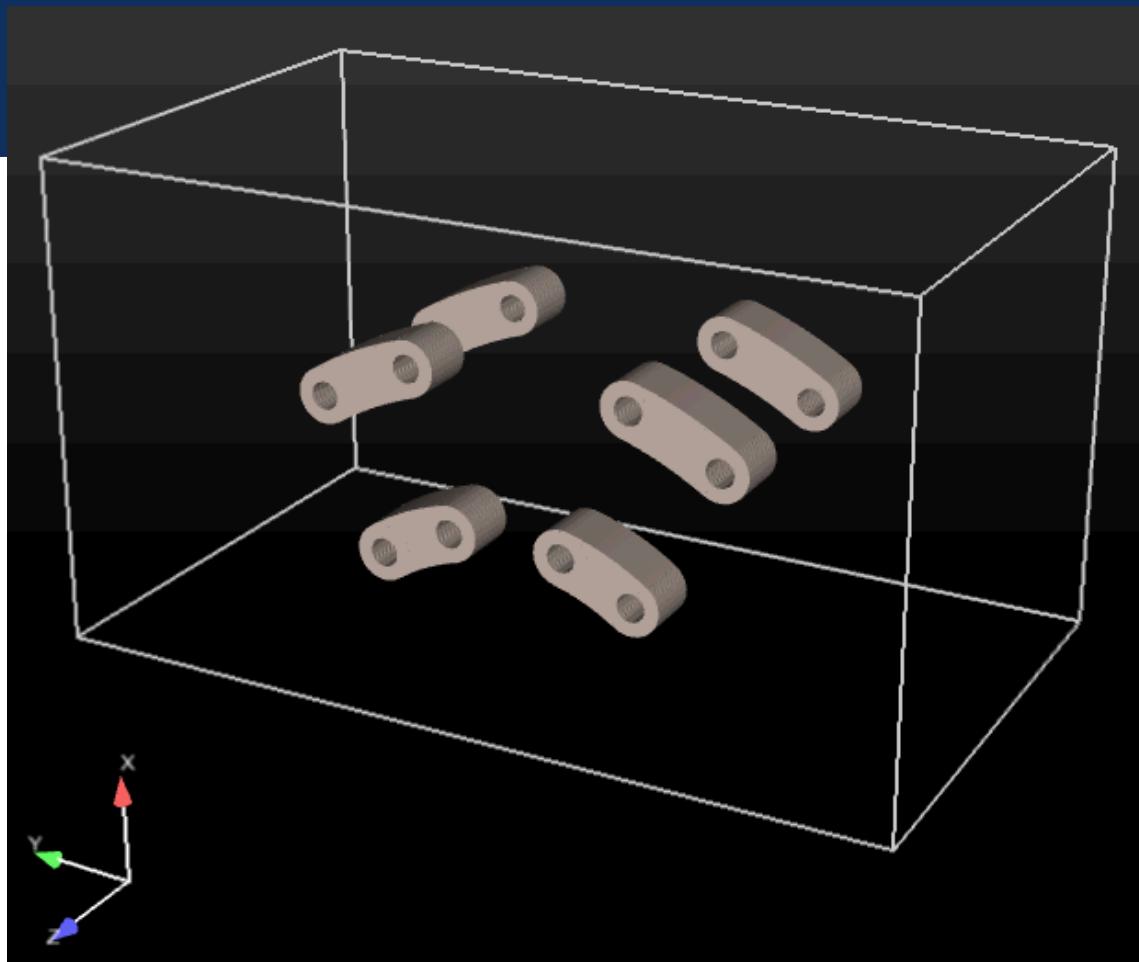
X:60.0, Y:80, Mount width: 100.0

# Case Study

**Objective:** conduction and stiffness

**Constraint:** volume

**Shape parameters:** ~20

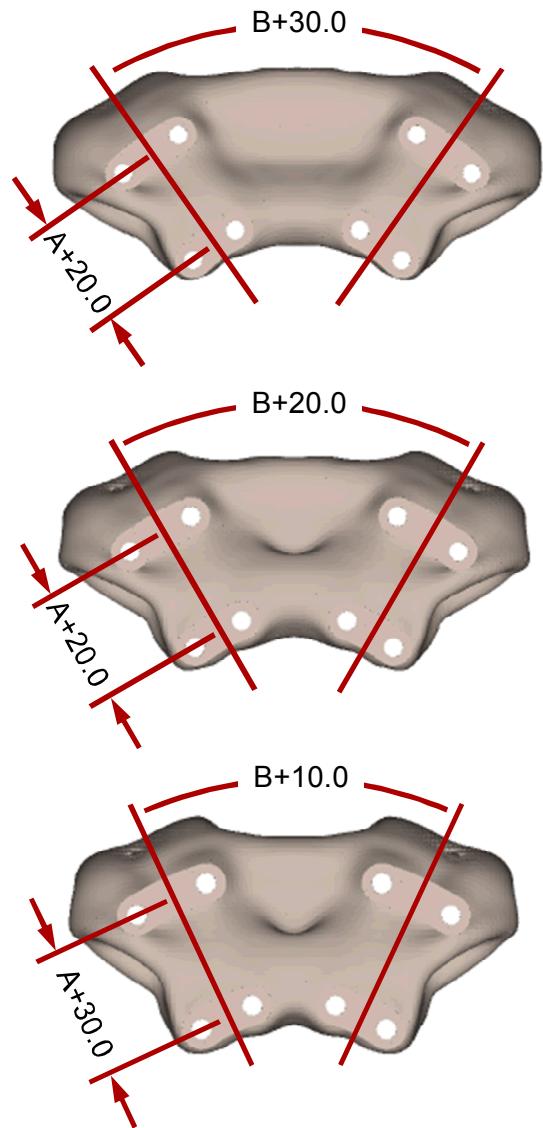
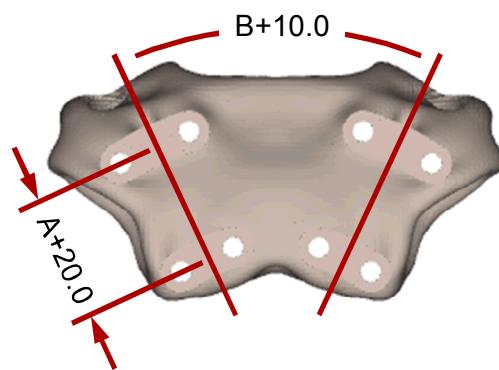
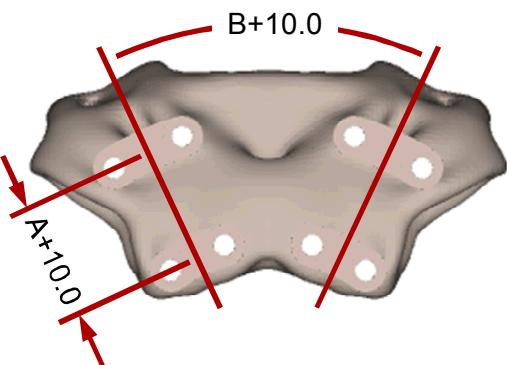


# Topology Optimization Case Study

- Parameter space is too large for brute force approach
- Gradient based approach is under development:

$$\frac{dF}{dw_j} = \frac{dF}{d\hat{z}_k} \frac{d\hat{z}_k}{dw_j}$$

- Sensitivities computed via adjoint method
- Derivatives wrt shape parameters via AD
- Gradient based approach enables nested or concurrent shape and topology optimization



# Concluding Remarks

## **Fully non-conformal geometry:**

- greatly simplifies concurrent treatment of shape and topology
- eliminates (separate) meshing step
- eliminates “un-meshing” step
- simplifies modification of computed designs
- is proving robust even for complicated models
- creates interesting possibilities for computational efficiency, geometric multi-level, automatic distributed meshing, and ?
- seamlessly integrates modeling and simulation

## **Ongoing and future work:**

- Robust preconditioners
- Gradient based shape optimization
- General-use constraints
- Tools for pre and post

# Thank you